Image Segmentation

- Image analysis:
  - Extracting information from an image.
- First step:
  - To segment the image
  - i.e. to subdivide an image into its constituent regions or objects.

Image Segmentation

- Segmentation is based on two basic properties of gray-level values:
  - Discontinuity, i.e. to partition the image based on abrupt changes in intensity (gray levels), e.g. edges
  - Similarity, i.e. to partition the image into similar (according to predefined criteria) regions, e.g. thresholding, region growing, region splitting and merging

Detection of Discontinuities

- 3 basic types of gray-level discontinuities:

  Points

  Lines

  Edges

- Common method of detection: run a mask through the image.

Image Segmentation

FIGURE 10.1 A general 3×3 mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Point Detection

- T: nonnegative threshold: $|R| \geq T$

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i$$

−1	−1	−1
−1	8	−1
−1	−1	−1

Point Detection

- A point has been detected at the location on which the mask is centered if: $|R|>T$

- The gray level of an isolated point will be quite different from the gray levels of its neighbors

  - measure the weighted differences between the center point and its neighbors

Line Detection

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

−1	−1	2
−1	2	−1
2	−1	−1

+ 45°

−1	2	−1
−1	2	−1
−1	2	−1

Vertical

2	−1	−1
−1	2	−1
−1	−1	2

−45°

R_1 R_2 R_3 R_4

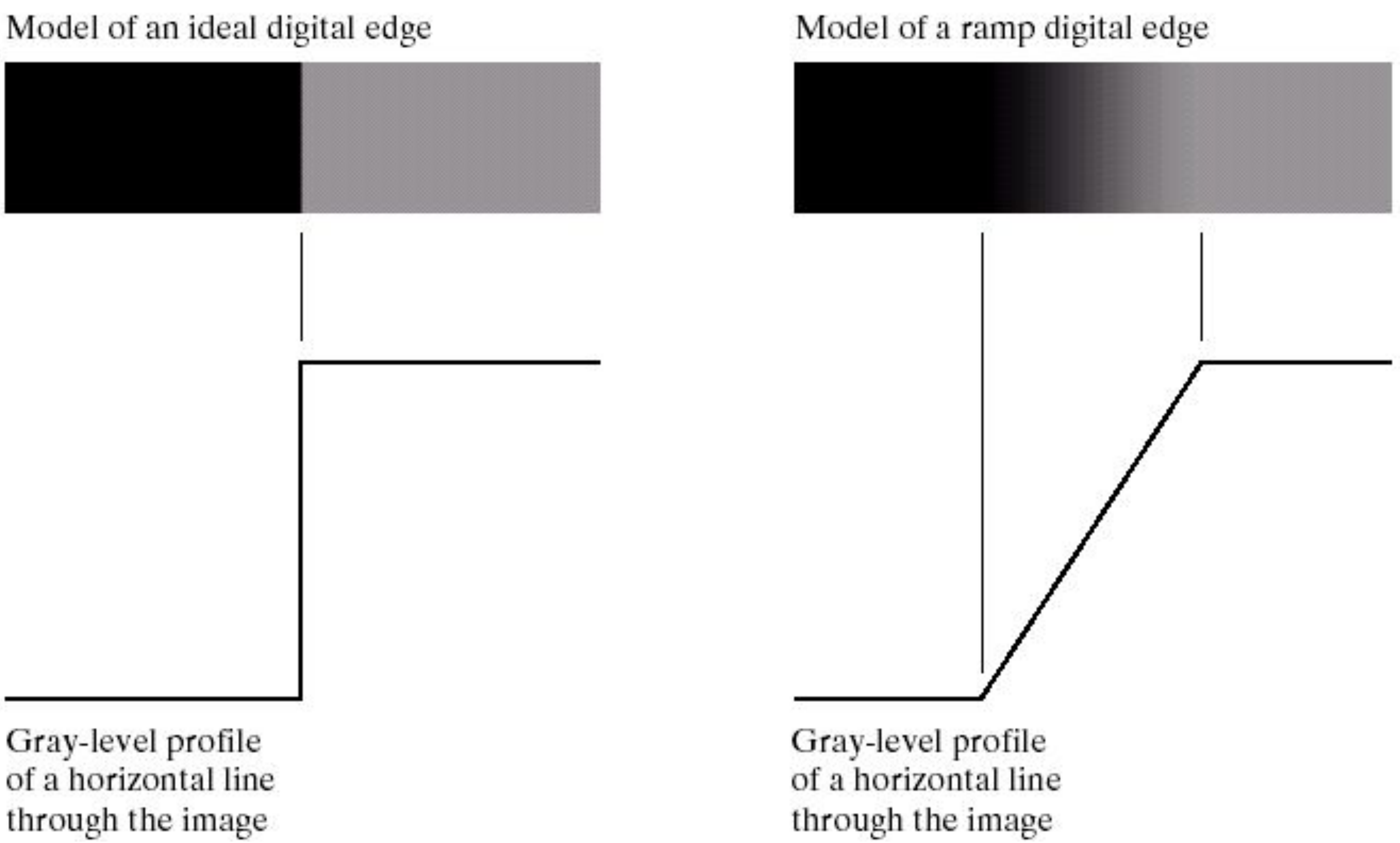
- If at a certain point $|R_i|>|R_j|$, this point is more likely associated with a line in the direction of mask i.

Edge Detection

- Edge (a set of connected pixels):
  - the boundary between two regions with relatively distinct gray-level properties.
  - Note: edge vs. boundary

- Assumption:
  - the regions are sufficiently homogeneous, so that the transition between two regions can be determined on the basis of gray-level discontinuities alone.

Image Segmentation



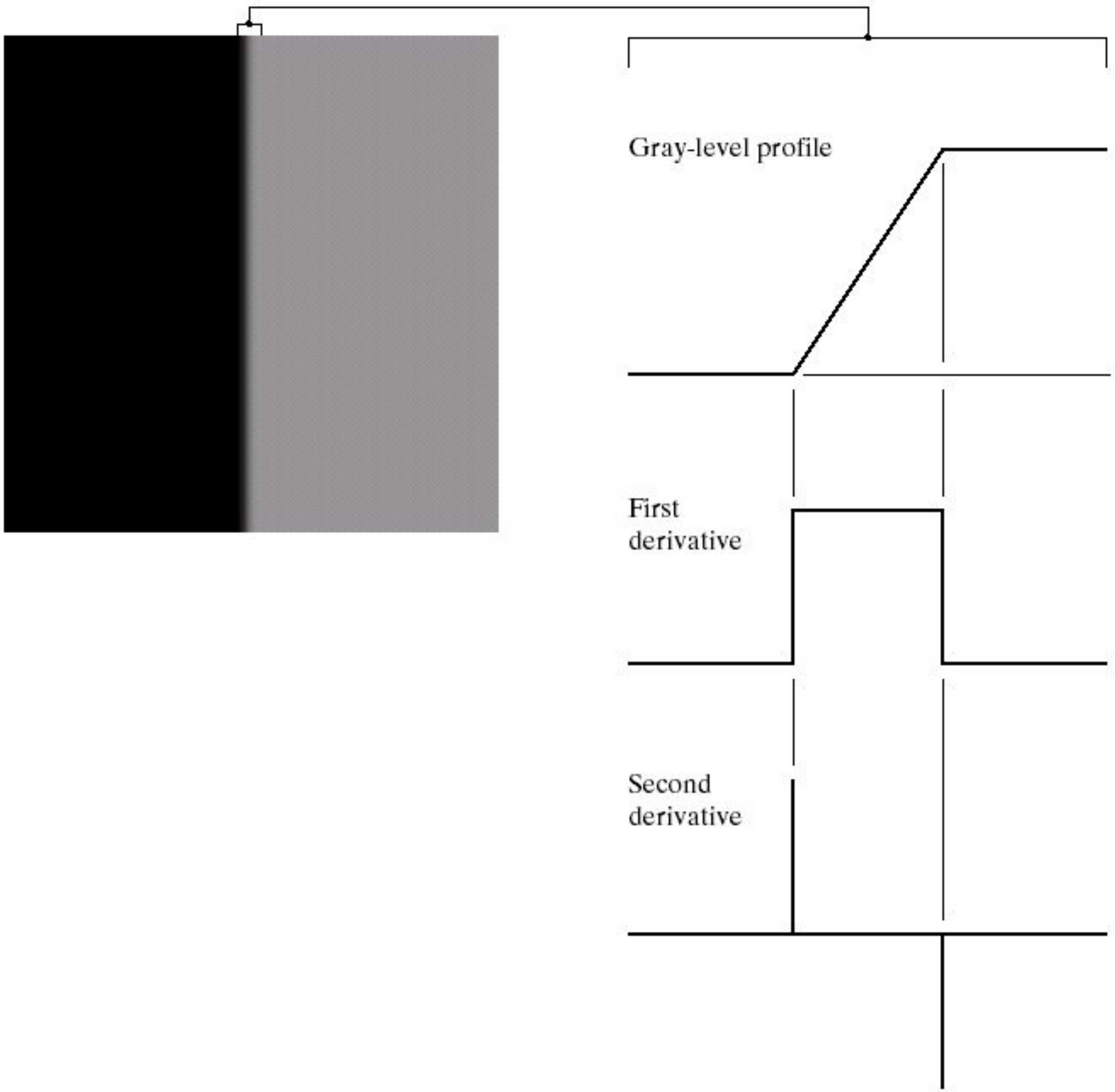
a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

Image Segmentation

a b

FIGURE 10.6
(a) Two regions separated by a vertical edge.
(b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.



Edge Detection

- Basic Idea:
  - A profile is defined perpendicularly to the edge direction and the results are interpreted.
  - The magnitude of the first derivative is used to detect an edge (if a point is on a ramp)
  - The sign of the second derivative can determine whether an edge pixel is on the dark or light side of an edge.
- Remarks on second derivative:
  - It produces two responses for every edge
  - The line that can be formed joining its positive and negative values crosses zero at the mid point of the edge (zero-crossing)

Edge Detection

- Computation of a local derivative operator
  - A profile is defined perpendicularly to the edge direction and the results are interpreted.
  - The first derivative is obtained by using the magnitude of the gradient at that point.
  - The second derivative is obtained by using the Laplacian.

Gradient Operators

$$\nabla F = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

The gradient vector points in the direction of maximum rate of change of f at (x,y).

Gradient Operators

Gradient: $\nabla f = mag(\nabla F) = [G_x^2 + G_y^2]^{1/2}$

(maximum rate of increase of f(x,y) per unit distance)

$$\nabla f \approx | G_x | + | G_y |$$

Direction angle of ∇f at (x,y): $a(x, y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$

Image Segmentation

a	
b	c
d	e
f	g

FIGURE 10.8 A 3 × 3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

Image Segmentation

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a	b
c	d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

Image Segmentation

a b
c d

FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

Image Segmentation

a b
c d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5×5 averaging filter.

Image Segmentation

a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

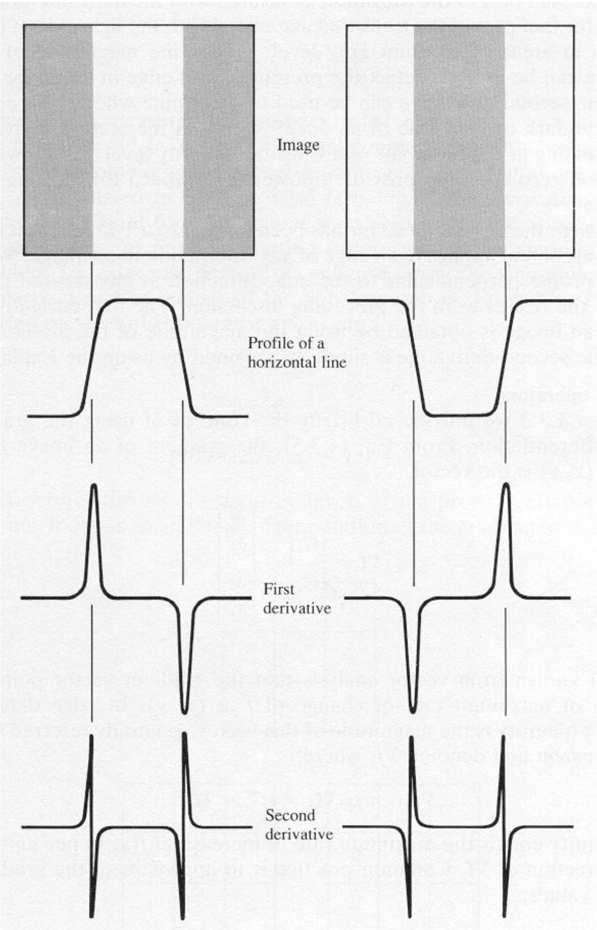
Gradient Operators

- Computation of the gradient of an image:
  - Soebel operators provide both a differencing & a smoothing effect:

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

Summary: Gradient Operators



Smooth edges due to blurring (result of sampling)

Positive: leading
Negative: trailing
Zero: in constant gray levels

Positive: from dark side
Negative: from light side
Zero: in constant gray levels

Summary

- The magnitude of the first derivative detects the presence of an edge and the sign of the second detects whether the edge pixel lies on the dark or light side of an edge.
- The second derivative has a zero-crossing at the mid-point of a transition.

Laplacian

- (of a 2-D function f(x,y)): $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

- A 3 x 3 discrete mask based on the above is:

$$\nabla^2 f = 4z_5 - (z_2 + z_4 + z_6 + z_8)$$

Laplacian

- The idea:
  - Coefficient of center pixel should be positive
  - Coefficients of outer pixels should be negative
  - Sum of coefficients should be zero (the Laplacian is a derivative)

0	–1	0
–1	4	–1
0	–1	0

Image Segmentation

FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Laplacian

- The Laplacian is seldom used in practice, because:
  - It is unacceptably sensitive to noise (as second-order derivative)
  - It produces double edges
  - It is unable to detect edge direction

Laplacian

- An important use of the Laplacian:
  - To find the location of edges using its zero-crossings property.
- Plus, the Laplacian plays only the role of detector of whether a pixel is on the dark or light side of an edge.

Laplacian

- Convolve an image with the Laplacian of a 2D Gaussian function of the form:

$$h(x,y) = -\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

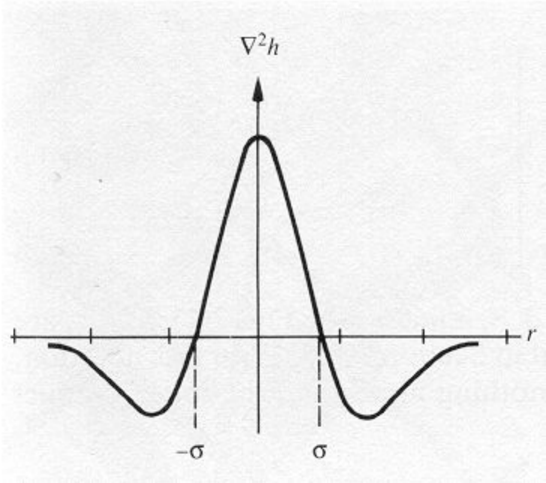
where σ is the standard deviation.

Laplacian

- The Laplacian of the above Gaussian is:

$$\nabla^2 h = -\left(\frac{r^2 - \sigma^2}{\sigma^4}\right)\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

where $r^2 = x^2 + y^2$.



σ determines the degree of blurring that occurs.

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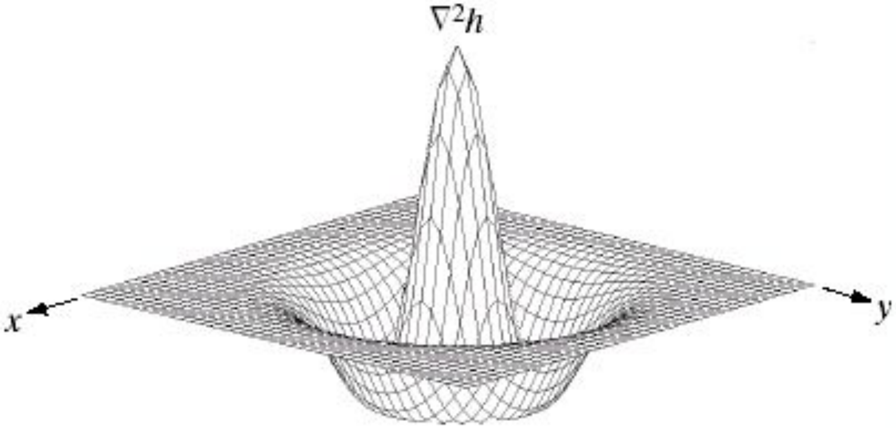
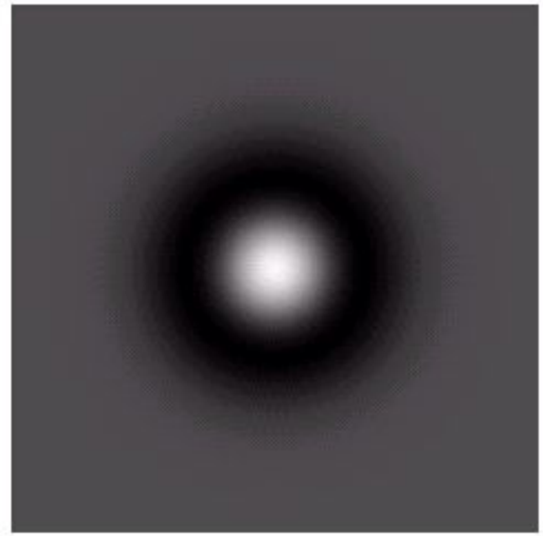
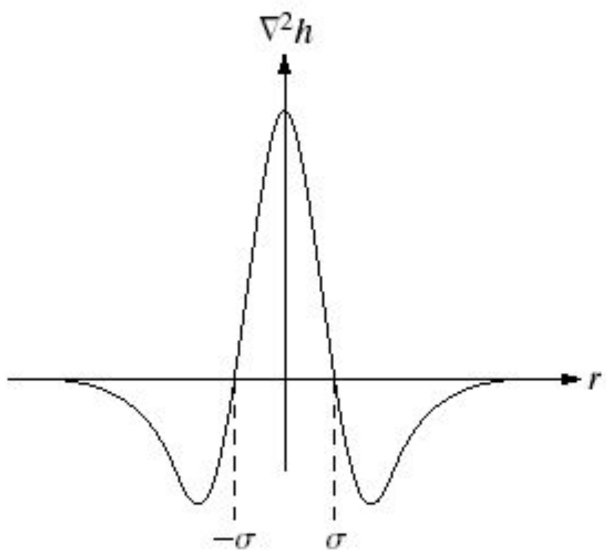




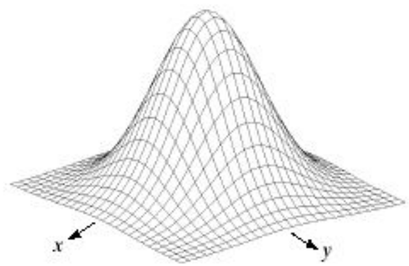


0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

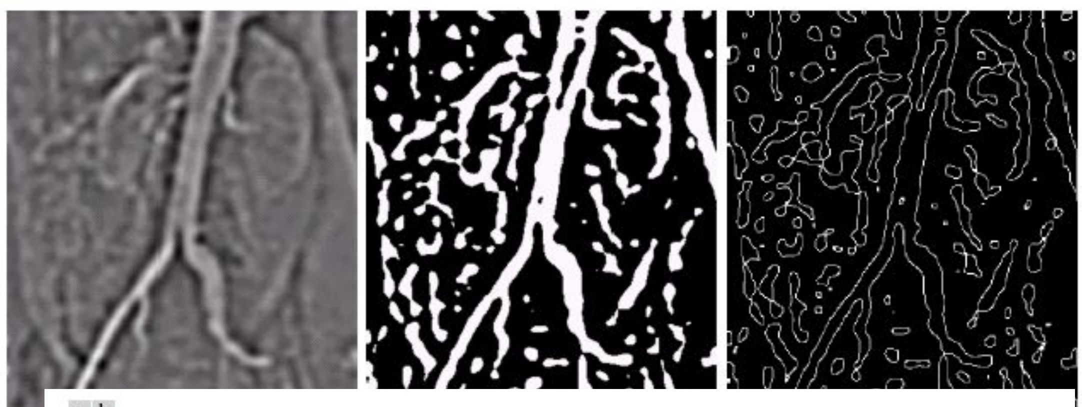
a	b
c	d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

Image Segmentation



−1	−1	−1
−1	8	−1
−1	−1	−1



a b
c d
e f g

FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)