Subject :

Question No.	1	2	3	4	5	6	7	8	9	10	11	12	Total
Marks Obtained													

8- Queens Problem

1 → no 2 queens in a row

2 → no 2 queens in a column

3 → no 2 queens in left diagonal

4 → no 2 queens is right diagonal

→ Matrix is filled column by column. Hence we need to satisfy remaining 3 conditions.

Apply Branch & Bound approach.

→ create 3 arrays to check conditions ① , ③ & ② ④.

→ Develope a numbering system to specify which queen is placed.

<u>Preprocess</u>

Create 2 NXN matrices, one for ① top-left to bottom right

② Top right to bottom-left

Fill them in such manner that 2 queens sharing same TL-BR diagonal will have same value in matrix 1 and 2 queens sharing same top-right - bottom-left diagonal will have same value in matrix 2.

matrix 1

$$\begin{array}{c|cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 2 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 3 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 \\ 4 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 \\ 5 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 \\ 6 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 \\ 7 & 14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 \end{array}$$

mat1[row][col] = row - col + (N-1)

matrix 2

$$\begin{array}{c|cccccccc} 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array}$$

mat2[row][col] = row + col.

To place queen check,

1. whether row 'j' is used or not
2. whether i+j is used or not
3. whether i-j+7 is used or not.

→ if anwer is True, try another location for queen i on row j. ~~and di~~

→ mark row and diagonals

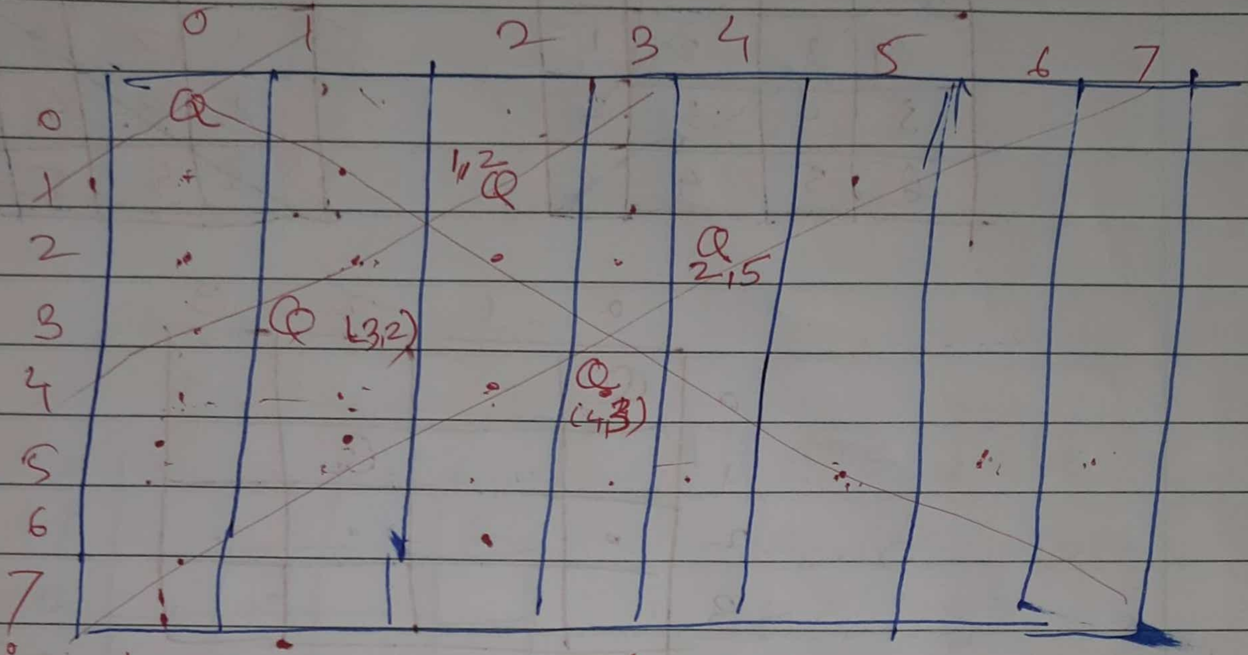
→ recur for queen i+1

row 0

(size n) row_bool_value [F F F f F F F F]

size 2(n+1) mat1_ dia_bool [ ]

size 2*(n-1) mat2_ dia _bool [ ]



i = co~~lum~~n = 0 ; ~~row = 0~~ j col = 0

i = 0

→ ~~row~~ col _bool_val [T F F F F F F F]

mat1_dia_bool [i] [col] = T

0 0

mat 2_dia_ bool [0] [col] = T

[0,0]

chk row = 1 (i) & col [0] ie (1,0) position.

row1 col_bool_val [T F T F F F F F]

row2 [T F T F T F F F]

row3 → [T T F F T F F F]

row4 → T T T T T F F F

0 1 2 3 4

bool array [F F F F F]

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

	0	1	2	3
0	Q			
1			Q	
2				
3				

row [0] - (0,0)

col_bool_array [T F F F]

row [1] [1,0] × same column

[1,1] × - mat 1

[1,2]

col_bool array [T F T F]

0 1 2 3

~~ro~~ row [2] , [2,0] - same colu.

[2,1] × mat2

[2,2] × same colu

[2,3] × from mat 1

Backtrack → col_bool_array [T F F F]

Q			

Changes in mat 1 & mat 2 ~~as~~ well.

row (1,3)

mat1

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

mat2

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

check row(1,3)

	0	1	2	3
0	Q			
1				Q
2				
3				

all are false
Hence place
at (1,3)

col-bool-array = [T F F T]

row (2,0) – sam col.
row (2,1) – possible to place
as false in all 3, bool-array, mat1, mat2

⇩

Q			
			Q
	Q		

mat1

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

mat2

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

↳ row (3,0) – X same col
(3,1) X same col
(3,2) X not possible from mat1
(3,3) X same col.

Hence backtrack
from row 3 – all posi X not possible

from row 2, (2,2) – not allowed from mat1
(2,3) X same col.

no further for place ⟶ frow row1 → backtrack to row 0 row 0 ~~to~~ earlier Queen at (0,0)

need to explore further.

reset array, mat1, mat2.

col_bool_array [F F F F]

mat 1

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

mat 2

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

Soln

	0	1	2	3
0		Q		
1				
2				
3				

row (0,1) - possible with all false at array, mat1, mat2

Top left

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

BR

TR

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

BL

Boll arra

[F T F F]

no

row (1,0) → X by mat2

row (1,1) → X by array

row (1,2) — X by mat1

row (1,3) — possible

place Queen at (1,3)

mat1

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

mat2

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

array

[F T F ~~F~~ F]

0 1 2 3

Soln

	Q		
			Q

row (2,0)

→ OK by col_array,
mat 1, mat 2

place Queen
at (2,0).

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3	2	1	6
4	3	2	1
5	4	3	2
6	5	4	3

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

[T T ~~F~~ T]

Soln

		Q	
			Q
Q			

row(3,0) – X by same col array.
row(3,1) – X by mat 1
row(3,2) – OK by col array
– OK by mat 1
– OK by mat 2

3	2	1	0
4	3	2	1
5	4	3	2
6	5	4	3

0	1	2	3
1	2	3	4
2	3	4	5
3	4	5	6

[? T T T]

Solution

●	Q		
			Q
Q			
		Q	