

**Questions should be —**

**COMPUTER LASER PRINTED OR TYPED OR WRITTEN IN LEGIBLE HANDWRITING IN BLACK INK.  
SIGNS, SKETCHES OR FIGURES IF ANY BE DRAWN IN NEAT BLACK INK,  
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Questions should be written one below the other and in every front page only.

Duration ..... Hours.

Total Marks assigned to the paper .....

Marks assigned to each question should be stated against each question.

**Instructions to the candidates, if any :—**

**N.B. :**

Johnson's 1

Q. No.

Marks

Johnson's Algo. -  $O(V^2 \log V + VE)$

All pair  $\therefore$  asymptotically better than Warshall

— uses Dijkstra's & Bellman Ford.

→ technique of reweighting.

→ if all positive edges in Graph,  
compute by Dijkstra's by computing  
for each vertex as source vertex.

→ if negative edges, compute new set  
of nonnegative edges, that can  
allow to use Dijkstra's Shortest Path  
method.

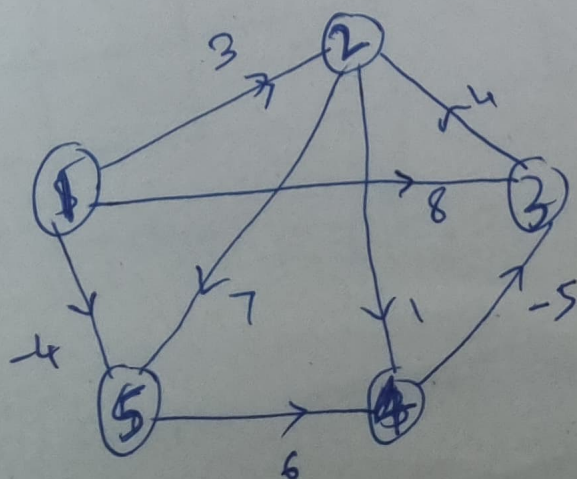
The new edges must have 2 properties. Johnson's - 2

① all vertices  $u, v \in V$ ; shortest path from  $u$  to  $v$  using weight function  $w$  is also a shortest path from  $u$  to  $v$  using weight function  $\hat{w}$ .

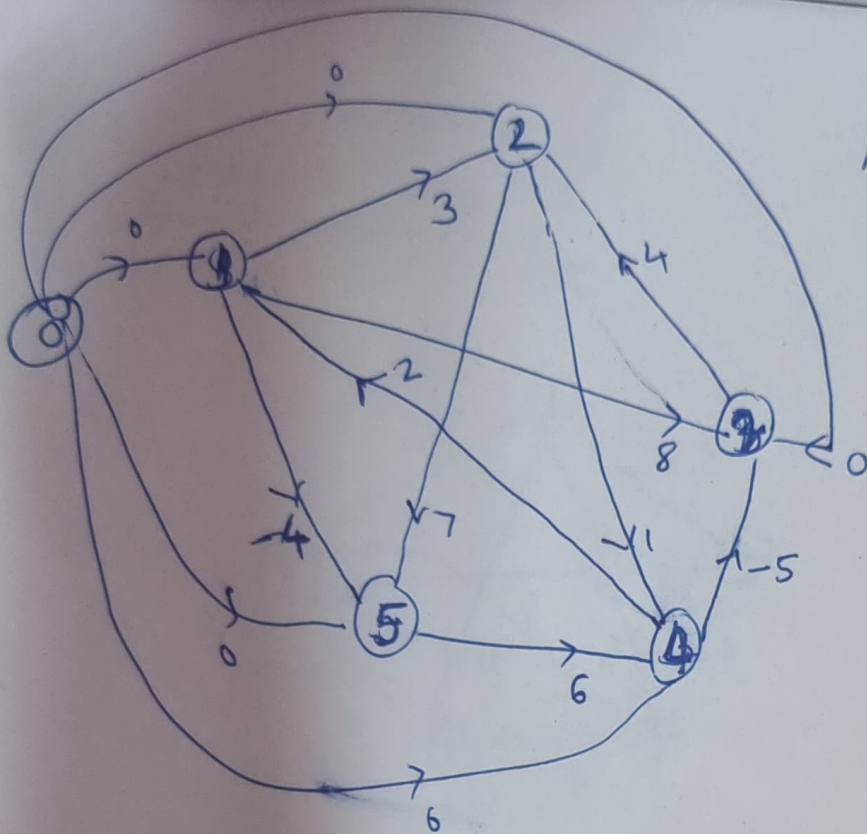
② For all edges  $(u, v)$ , the new weight  $\hat{w}(u, v)$  is nonnegative.

Produce nonnegative edges by reweighting.

- make a new graph  $G'(V', E')$
- Use Bellman Ford algo.
- Add a new source  $s$  to  $G'$ .







Johnson's - 3  
Add new source 0

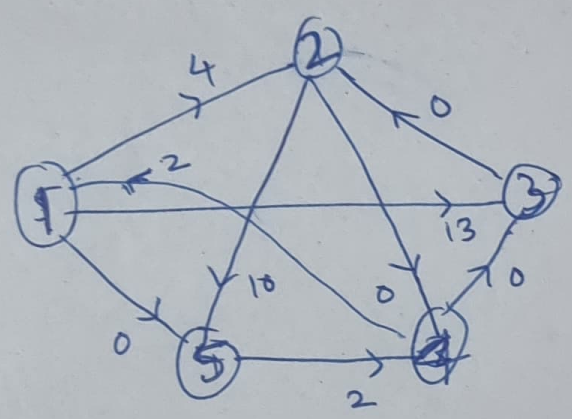
Source	1	2	3	4	5
1 edge in bet	0	0	0	0	0
2 edge in bet	0	0	-5	0	-4
3 edge in bet	0	-1	-5	0	-4
4 edge in bet	0	-1	-5	0	-4
5	"	"	"	"	"

~~from~~ H function  $[ ] = [0, -1, -5, 0, -4]$

→ remove source & rewrite edges as,

$$w'(u, v) = w(u, v) + h(u) - h(v)$$

Rewrite edges



$$w'(3,2) = \cancel{w(3,2)} = 4 + (-5) - (-1) = 0$$

$$w'(1,2) = 3 + (0) - (-1) = 4$$

$$w'(1,3) = 8 + 0 - (-5) = 13$$

Apply Dijkstra's Shortest Path Algo.

Consider Source 1

	Source	Selected	1	2	3	4/5
Initial	1	1	0	4	13	$\infty$ 0
1		5	-	4	13	<del>2</del> 0
	1,5	4	-	4	2	2 0
Source 1	1,5,4	3	-	2	2	2 0

Similarly do for remaining vertices.

Source 2	V	1	3	4	5
		2	6	0	2

Name of the Faculty  
To, The HOD

Computers

Page No. ....

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Johnson/-5

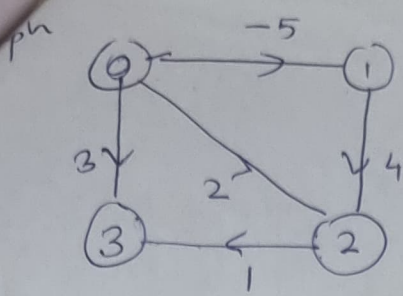
Q. No. Marks

Source 3	1	2	4	5
	2	0	0	2

Source 4	1	2	3	5
	1	0	0	2

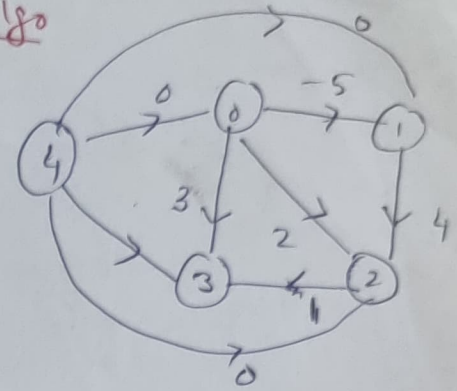
Source 5	1	2	3	4
	4	2	2	2





Johnson's algo

add source



① Bellman Ford

Source		4	0	1	2	3
K		4	0	1	2	3
0		0	∞	∞	∞	∞
1		0	0	0	0	0
2			0	-5	-1	

from 4 to 0, 1, 2, 3 are

(0, -5, -1, 0)

$$\begin{aligned} dist^2(0) &= \min(dist^1(0), \\ &\quad \min(dist^1(i) + cost(i,0)) \\ &= 0, ( \end{aligned}$$

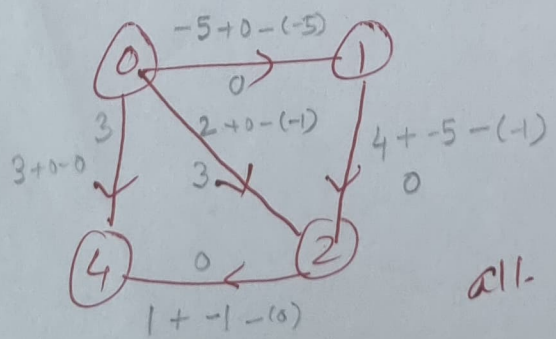
$$\begin{aligned} dist^2(1) &= \min(dist^1(1), \\ &\quad \min(dist^1(i) + cost(i,1)) \\ &= (-5, 0-5, \end{aligned}$$

$$\begin{aligned} dist^2(2) &= \min(dist^1(2), \\ &\quad \min(dist^1(i) + cost(i,2)) \\ &= 0, 0+2, \end{aligned}$$

②  $h[] = \{0, -5, -1, 0\}$  calculation

③ remove source, rewrite edges.

$$by \ w(u,v) = w(u,v) + h[u] - h[v]$$



all costs +ve

④ run Dijkstra's for every vertex as source

Time complexity  $O(V^2 \log V + VE)$