Chapter 2
Simulation Examples

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Discrete-Event System Simulation

Purpose

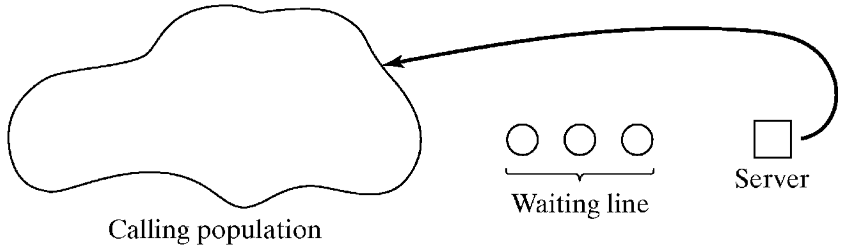
- To present several examples of simulations that can be performed by devising a simulation table either manually or with a spreadsheet.
- To provide insight into the methodology of discrete-system simulation and the descriptive statistics used for predicting system performance.

Outline

- The simulations are carried out by following steps:
  - Determine the input characteristics.
  - Construct a simulation table.
  - For each repetition *i*, generate a value for each input, evaluate the function, and calculate the value of the response y_i.
- Simulation examples are in queueing, inventory, reliability and network analysis.

Simulation of Queueing Systems

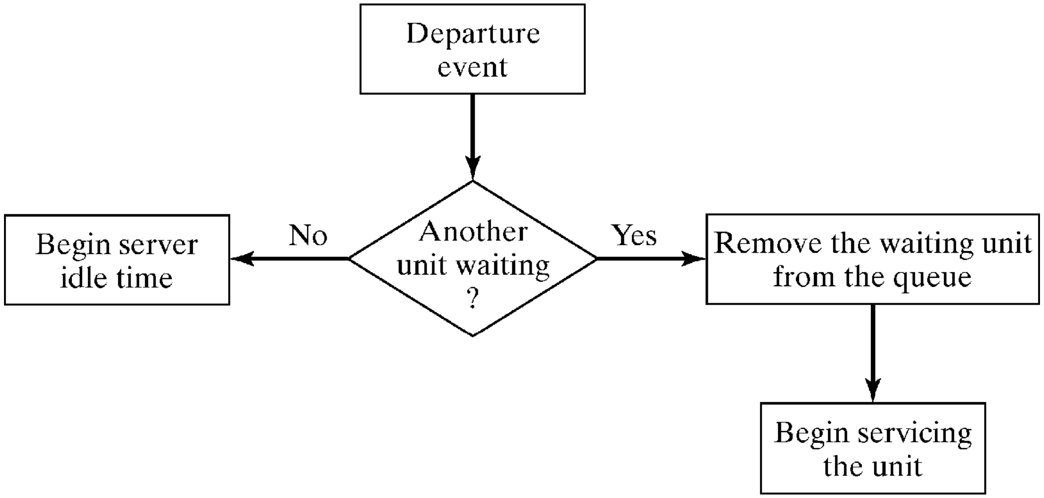
- A queueing system is described by its calling population, nature of arrivals, service mechanism, system capacity and the queueing discipline (details in Chapter 6.)
  - A simple single-channel queuing system:



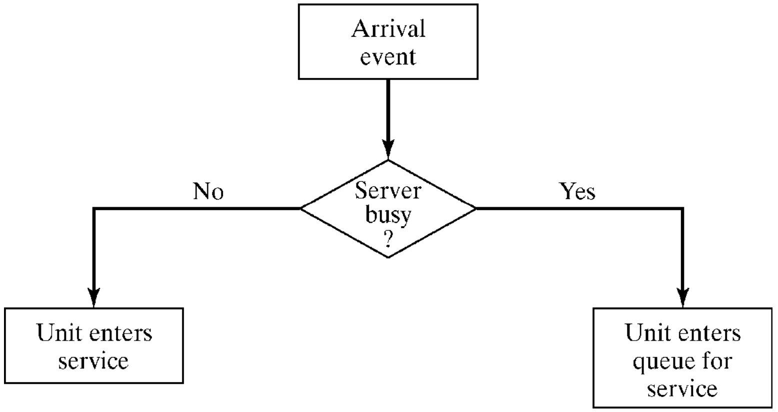
- In a single-channel queue:
  - The calling population is infinite.
  - Arrivals for service occur one at a time in a random fashion, once they join the waiting line, they are eventually served.
- Arrivals and services are defined by the distribution of the time between arrivals and service times.
- Key concepts:
  - The system state is the number of units in the system and the status of the server (busy or idle).
  - An event is a set of circumstances that causes an instantaneous change in the system state, e.g., arrival and departure events.
  - The simulation clock is used to track simulated time.

Simulation of Queueing Systems

- If a unit has just completed service, the simulation proceeds in the manner shown below:



- The flow diagram for the arrival event:



Simulation of Queueing Systems

- Potential unit actions upon arrival:

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

- Server out comes after the completion of service:

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Simulation of Queueing Systems

- Event list: to help determine what happens next.
  - Tracks the future times at which different types of events occur. (this chapter simplifies the simulation by tracking each unit explicitly.)
  - Events usually occur at random times.
- The randomness needed to imitate real life is made possible through the use of random numbers, they can be generated using:
  - Random digits tables: form random numbers by selecting the proper number of digits and placing a decimal point to the left of the value selected, e.g., Table A.1 in book.
  - Simulation packages and spreadsheets.
  - Details in chapter 7.

Simulation of Queueing Systems

- Single-channel queue illustration:
  - Assume that the times between arrivals were generated by rolling a die 5 times and recording the up face. Input generated:

Customer	Interarrival Time	Arrival Time on Clock
1	-	0
2	2	2
3	4	6
4	1	7
5	2	9
6	6	15

    - The 1st customer is assumed to arrive at clock time 0. 2nd customer arrives two time units later (at clock time 2), and so on.
  - Assume the only possible service times are 1,2,3 and 4 time units and they are are equally likely to occur. Input generated:

Customer	Service Time	Customer	Service Time
1	2	4	2
2	1	5	1
3	3	6	4

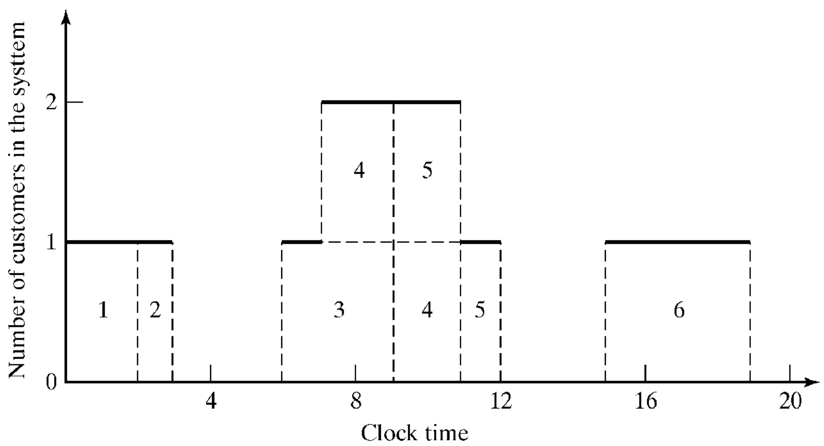
Simulation of Queueing Systems

- Resulting simulation table emphasizing clock times:

Customer Number	Arrival Time (clock)	Time Service Begins (Clock)	Service Time (Duration)	Time Service Ends (clock)
1	0	0	2	2
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

- Another presentation method, by chronological ordering of events:

Event Type	Customer Number	Clock Time
Arrival	1	0
Departure	1	2
Arrival	2	2
Departure	2	3
Arrival	3	4
Arrival	4	7
Departure	2	8
Arrival	5	8
Departure	4	11
Departure	5	12
Arrival	6	15
Departure	6	19



Simulation of Queueing Systems

- Grocery store example: with only one checkout counter.
  - Customers arrive at random times from 1 to 8 minutes apart, with equal probability of occurrence:

Time between Arrivals (minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.125	0.125	001-125
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.000	876-000

  - The service times vary from 1 to 6 minutes, with probabilities:

Time between Arrivals (minutes)	Probability	Cumulative Probability	Random Digit Assignment
1	0.10	0.100	001-100
2	0.20	0.300	101-300
3	0.30	0.600	301-600
4	0.25	0.850	601-850
5	0.10	0.950	851-950
6	0.05	1.000	951-000

Grocery Store Example

- To analyze the system by simulating arrival and service of *100* customers.
  - Chosen for illustration purpose, in actuality, *100* customers is too small a sample size to draw any reliable conclusions.
  - Initial conditions are overlooked to keep calculations simple.
- A set of uniformly distributed random numbers is needed to generate the arrivals at the checkout counter:
  - Should be uniformly distributed between *0* and *1*.
  - Successive random numbers are independent.
- With tabular simulations, random digits can be converted to random numbers.
  - List *99* random numbers to generate the times between arrivals.
  - Good practice to start at a random position in the random digit table and proceed in a systematic direction (never re-use the same stream of digits in a given problem.

Grocery Store Example

[Simulation of Queueing Systems]

- Generated time-between-arrivals:

Customer	Random Digits	Interarrival Times (minutes)
1	-	-
2	64	1
3	112	1
4	678	6
5	289	3
6	871	7
…	…	…
100	538	4

- Using the same methodology, service times are generated:

Customer	Random Digits	Service Times (minutes)
1	842	4
2	181	2
3	873	5
4	815	4
5	006	1
6	916	5
É	É	É
100	266	2

Grocery Store Example

[Simulation of Queueing Systems]

- For manual simulation, Simulation tables are designed for the problem at hand, with columns added to answer questions posed:

2nd customer was in the system for 5 minutes.

Customer	Interarrival Time (min)	Arrival Time (clock)	Service Time (min)	Time Service Begins (clock)	Waiting Time in Queue (min)	Time Service Ends (clock)	Time customer spends in system (min)	Idle time of server (min)
1		0	4	0	0	4	4	
2	1	1	2	4	3	6	5	0
3	1	2	5	5	4	11	9	0
4	6	8	4	11	3	15	7	0
5	3	11	1	15	4	16	5	0
6	7	18	5	18	0	23	5	2
…	…	…	…	…	…	…	…	…
100	5	415	2	416	1	418	3	
Totals	415		317		174		491	0

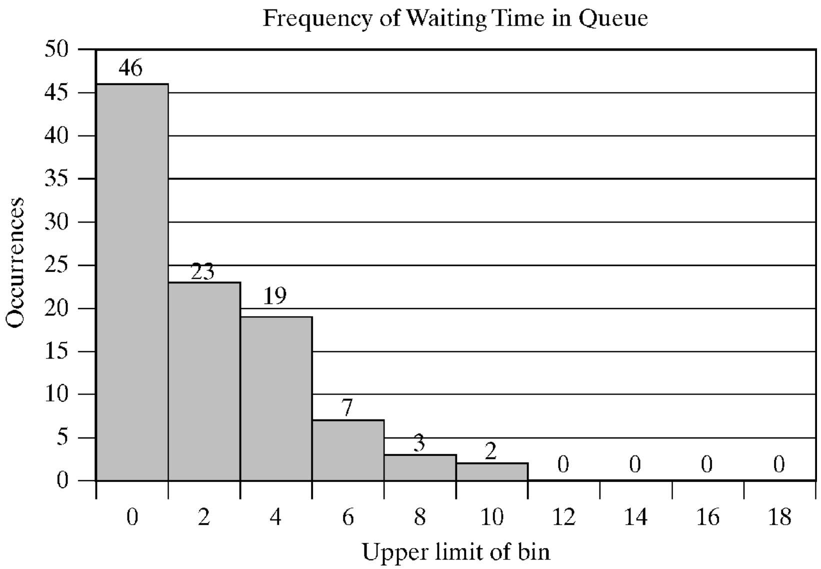
Service could not begin until time 4 (server was busy until that time)

Service ends at time 16, but the 6th customer did not arrival until time 18. Hence, server was idle for 2 minutes

Grocery Store Example

[Simulation of Queueing Systems]

- Tentative inferences:
  - About half of the customers have to wait, however, the average waiting time is not excessive.
  - The server does not have an undue amount of idle time.

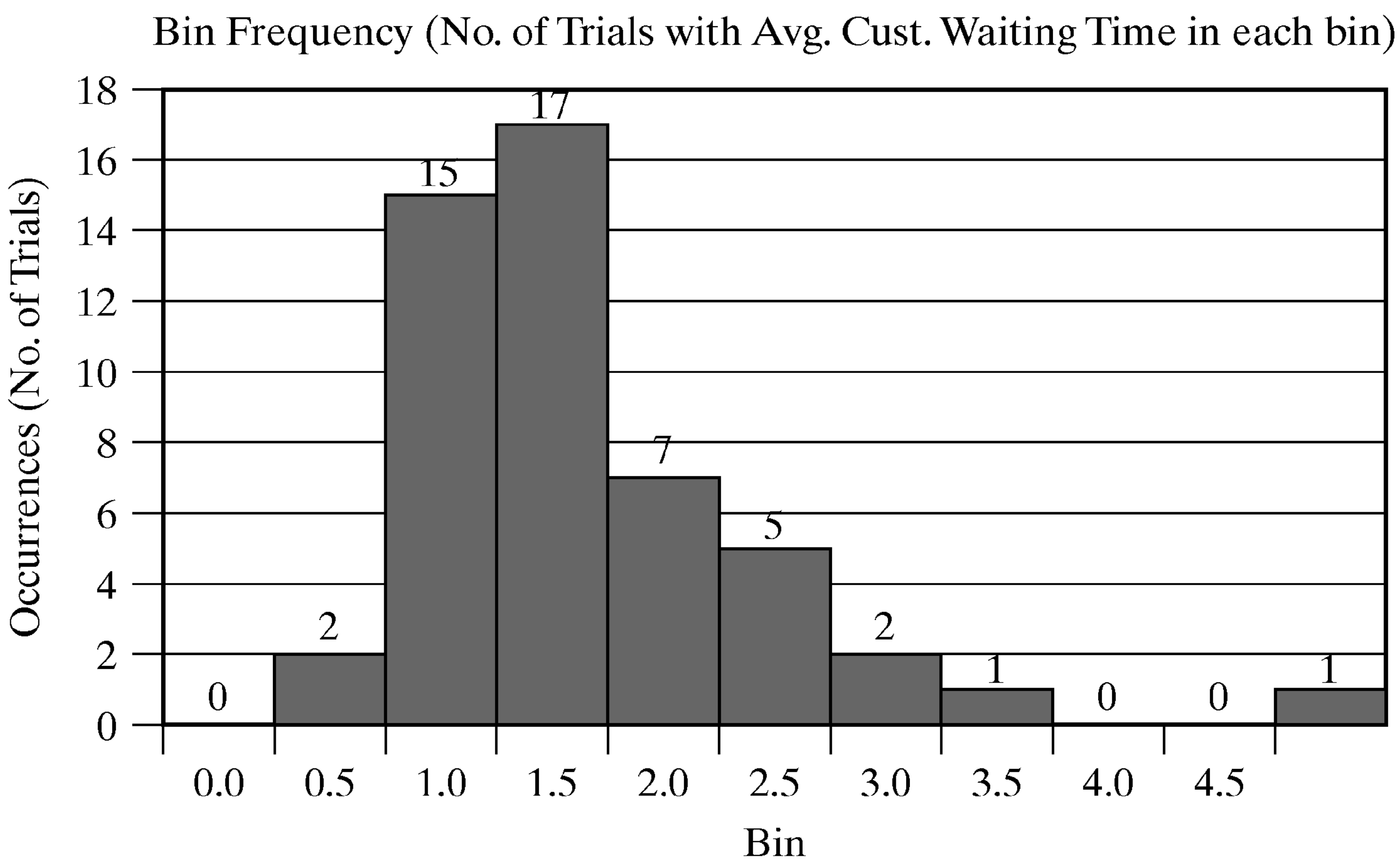


- Longer simulation would increase the accuracy of findings.
- Note: The entire table can be generated using the Excel spreadsheet for Example 2.1 at www.bcnn.net

Grocery Store Example

[Simulation of Queueing Systems]

- A histogram of the 50 average waiting times for the 50 trials:



Grocery Store Example

[Simulation of Queueing Systems]

- Key findings from the simulation table:

$$\begin{array}{c}\text{Average waiting}\\ \text{time (min)}\end{array} = \frac{\text{total time wait in queue (min)}}{\text{total number of customers}} = \frac{174}{100} = 1.74\,\text{min}$$

$$\text{Probability(wait)} = \frac{\text{numbers of customers who wait}}{\text{total number of customers}} = \frac{46}{100} = 0.46$$

$$\begin{array}{c}\text{Probability of}\\ \text{idle server}\end{array} = \frac{\text{total idle time of server (min)}}{\text{total run time of simulation (min)}} = \frac{101}{418} = 0.24$$

$$\text{Hence: Probability of busy server} = 1 - 0.24 = 0.76$$

$$\begin{array}{c}\text{Average service}\\ \text{time (min)}\end{array} = \frac{\text{total service time (min)}}{\text{total number of customers}} = \frac{317}{100} = 3.17\,\text{min}$$

$$\begin{aligned}&\text{Check: Expected service time}\\ &= 1(0.1) + 2(0.2) + 3(0.3 = 4(0.25) + 5(0.1) + 6(0.05)\\ &= 3.2\,\text{min}\end{aligned}$$

$$\begin{array}{c}\text{Average interarrival}\\ \text{Times (min)}\end{array} = \frac{\text{sum of all interarrival times (min)}}{\text{number of arrivals} - 1} = \frac{415}{99} = 4.19\,\text{min}$$

$$\text{Check: Expected interarrival time} = (1+8)/2 = 3.2\,\text{min}$$

Able-Baker Call Center Example

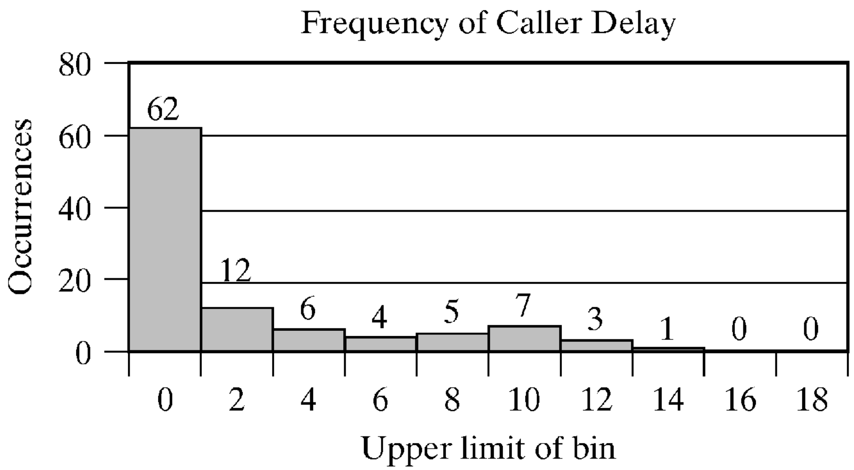
[Simulation of Queueing Systems]

- A computer technical support center with two personnel taking calls and provide service.
  - Two support staff: Able and Baker (multiple support channel).
  - A simplifying rule: Able gets the call if both staff are idle.
  - Goal: to find how well the current arrangement works.
  - Random variable:
    - Arrival time between calls
    - Service times (different distributions for Able and Baker).
  - A simulation of the first 100 callers are made
    - More callers would yield more reliable results, 100 is chosen for purposes of illustration.

Able-Baker Call Center Example

[Simulation of Queueing Systems]

- The steps of simulation are implemented in a spreadsheet available on the website (www.bcnn.net).
  - In the first spreadsheet, we found the result from the trial:

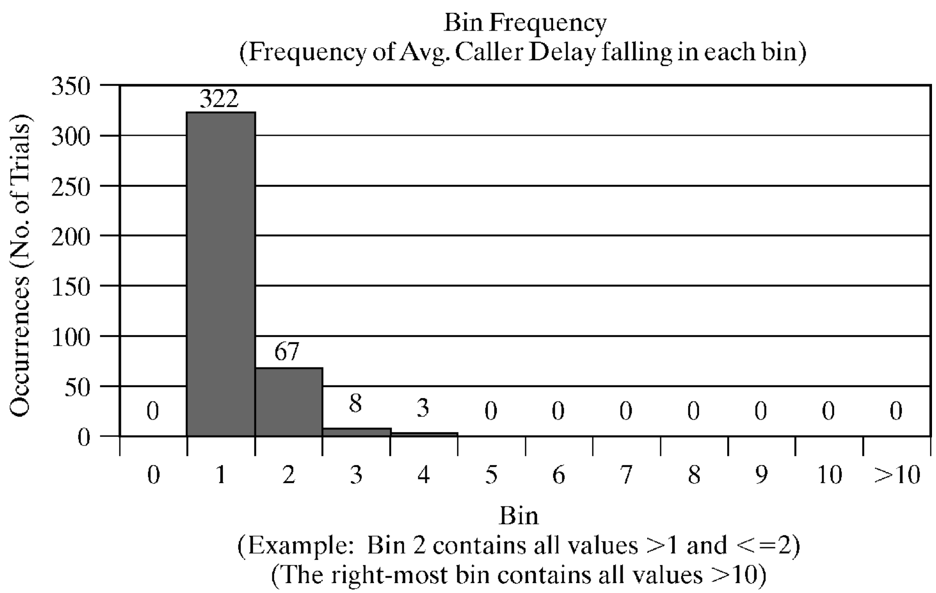


- 62% of the callers had no delay
- 12% had a delay of one or two minutes.

Able-Baker Call Center Example

[Simulation of Queueing Systems]

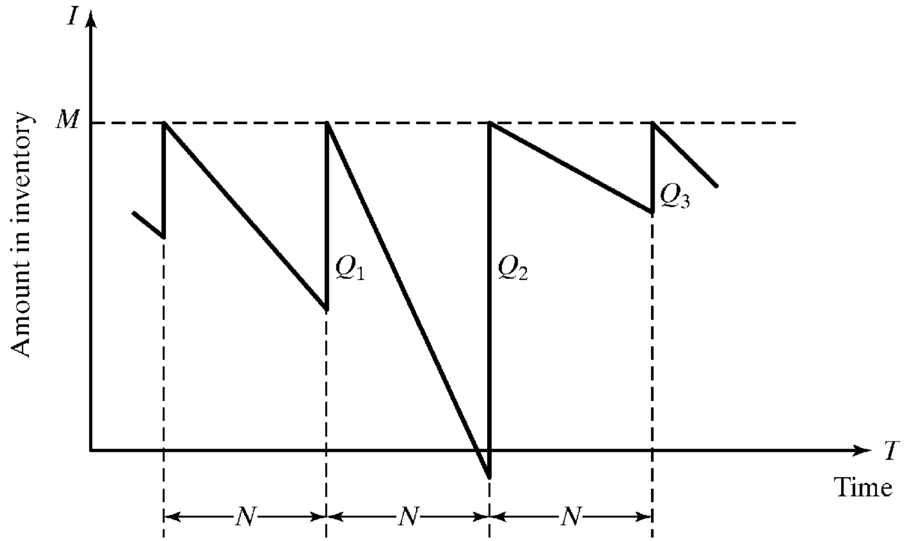
- In the second spreadsheet, we run an experiment with 400 trials (each consisting of the simulation of 100 callers) and found the following:



  - 19% of the average delays are longer than two minutes.
  - Only 2.75% are longer than 3 minutes.

Simulation of Inventory Systems

- A simple inventory system, an *(M, N)* inventory system:



  - Periodic review of length, *N*, at which time the inventory level is checked.
  - An order is made to bring the inventory up to the level *M*.
  - At the end of the i^{th} review period, an order quantity, Q_i, is placed.
  - Demand is shown to be uniform over time. However, in general, demands are not usually known with certainty.

Simulation of Inventory Systems

- A simple inventory system (cont.):
  - Total cost (or profit) of an inventory system is the performance measure.
    - Carrying stock in inventory has associated cost.
    - Purchase/replenishment has order cost.
    - Not fulfilling order has shortage cost.

Simulation of Inventory Systems

- The News Dealer's Example: A classical inventory problem concerns the purchase and sale of newspapers.
  - News stand buys papers for 33 cents each and sells them for 50 cents each.
  - Newspaper not sold at the end of the day are sold as scrap for 5 cents each.
  - Newspaper can be purchased in bundles of 10 (can only buy 10, 20,… 50, 60…)
  - Random Variables:
    - Types of newsdays.
    - Demand.

Profits = (revenue from sales) – (cost of newspaper)
– (lost profit form excess demand)
+ (salvage from sale of scrap papers)

News Dealer's Example

[Simulation of Inventory Systems]

- Three types of newsdays: "good"; "fair"; "poor"; with probabilities of 0.35, 0.45 and 0.20, respectively.

Type of Newsday	Probability	Cumulative Probability	Random digit Assignment
Good	0.35	0.35	01-35
Fair	0.45	0.80	36-80
Poor	0.20	1.00	81-00

- Demand and the random digit assignment is as follow:

	Cumulative Distribution			Random Digit Assignment		
Demand	**Good**	**Fair**	**Poor**	**Good**	**Fair**	**Poor**
40	0.03	0.10	0.44	01-03	01-10	01-44
50	0.08	0.28	0.66	04-08	11-28	45-66
60	0.23	0.68	0.82	09-23	29-68	67-82
70	0.43	0.88	0.94	24-43	69-88	83-94
80	0.78	0.96	1.00	44-78	89-96	95-00
90	0.93	1.00	1.00	79-93	97-00	
100	1.00	1.00	1.00	94-00		

News Dealer's Example

- Simulate the demands for papers over 20-day time period to determine the total profit under a certain policy, e.g. purchase 70 newspaper
- The policy is changed to other values and the simulation is repeated until the best value is found.

News Dealer's Example

[Simulation of Inventory Systems]

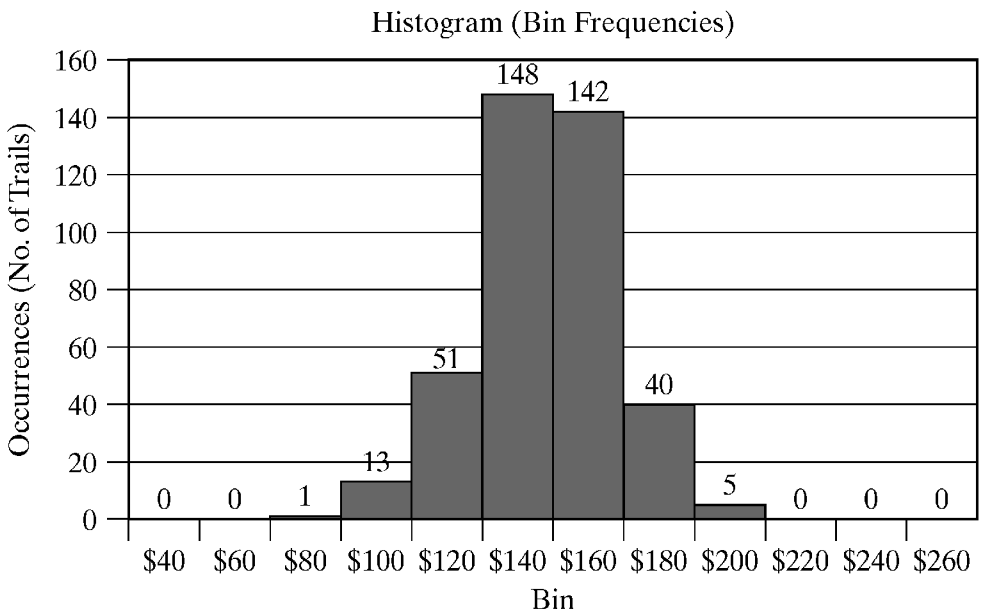
- From the manual solution
  - The simulation table for the decision to purchase 70 newspapers is:

Day	Random Digits for Type of Newsday	Type of Newsday	Random Digits for Demand	Demand	Revenue from Sales	Lost Profit from Excess demand	Salvage from Sale of Scrap	Daily Profit
1	58	Fair	93	80	$35.00	$1.70	-	$10.20
2	17	Good	63	80	$35.00	$1.70	-	$10.20
3	21	Good	31	70	$35.00	-	-	$11.90
4	45	Fair	19	50	$25.00	-	$1.00	$2.90
5	43	Fair	91	80	$35.00	$1.70		$10.20
…	…	…	…	…	…	…	…	…
19	18	Good	44	80	$35.00	$1.70	-	$10.20
20	98	Poor	13	40	$20.00	-	$1.50	-$1.60
			Total		$600.00	$17.00	$10.00	$131.90

News Dealer's Example

[Simulation of Inventory Systems]

- From Excel: running the simulation for 400 trials (each for 20 days)

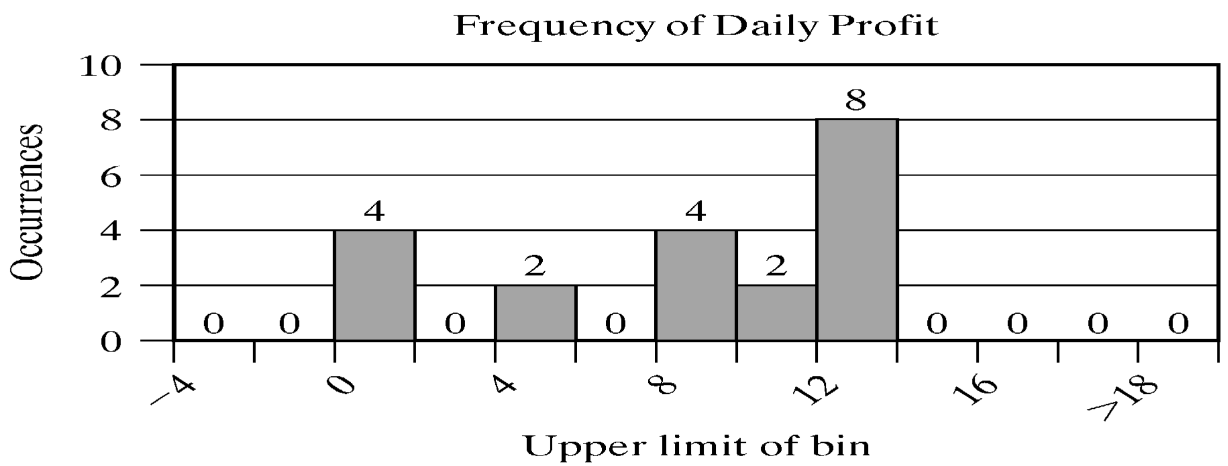


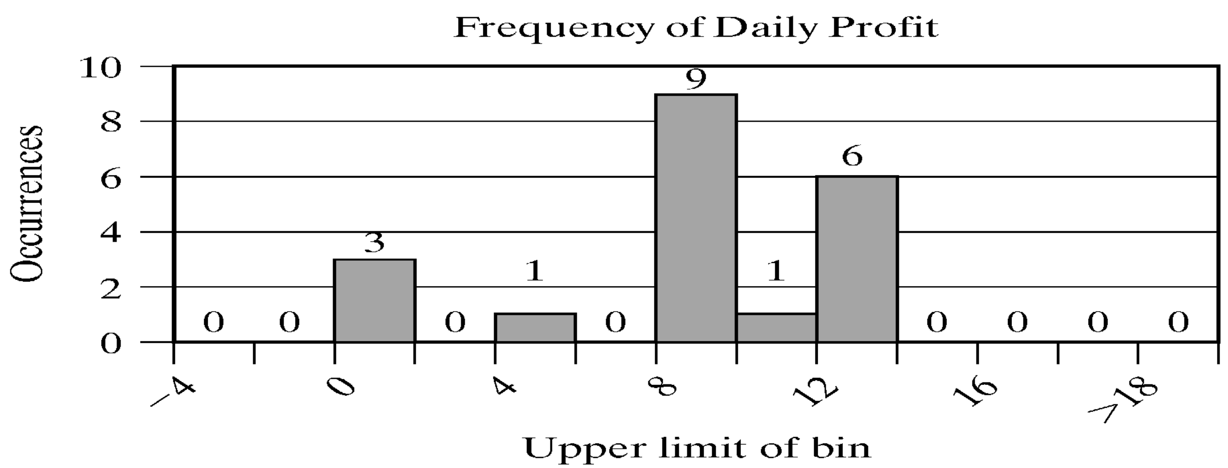
  - Average total profit = $137.61.
  - Only 45 of the 400 results in a total profit of more than $160.

News Dealer's Example

[Simulation of Inventory Systems]

- First two histograms of daily profit





News Dealer's Example

- The manual solution had a profit of $131.00, not far from the average over 400 days, $137.61.
- But the result for a one-day simulation could have been the minimum value or the maximum value.
- Hence, it is useful to conduct many trials.
- On the "One Trial" sheet in Excel spreadsheet of Example 2.3.
  - Observe the results by clicking the button 'Generate New Trail.'
  - Notice that the results vary quite a bit in the profit frequency graph and in the total profit.

Order-Up-To Level Inventory Example

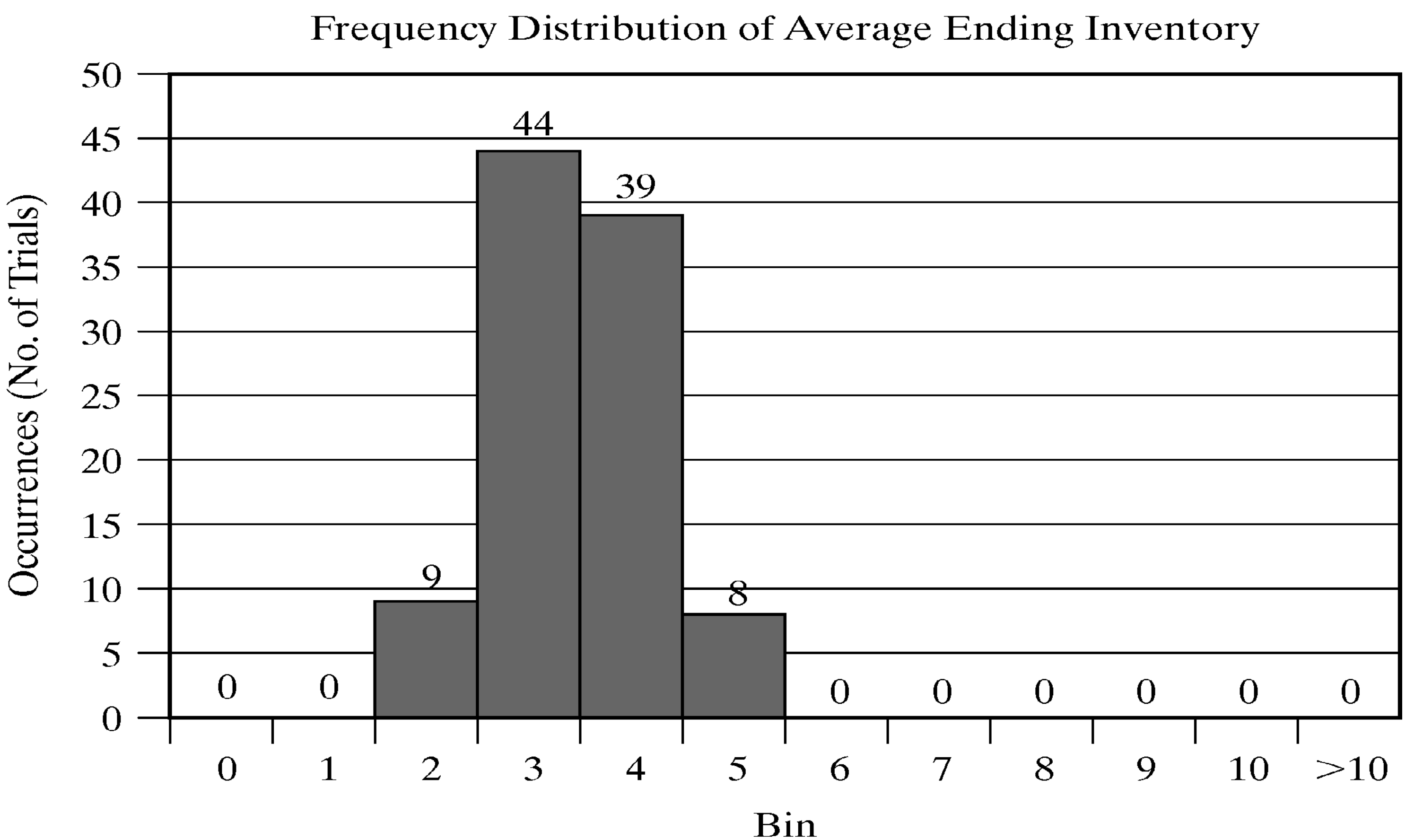
- A company sells refrigerators with an inventory system that:
  - Review the inventory situation after a fixed number of days (say *N*) and order up to a level (say *M*).

    Order quantity = (Order-up-to level) - (Ending inventory) + (Shortage quantity)
  - Random variables:
    - Number of refrigerators ordered each day.
    - Lead time: the number of days after the order is placed with the supplier before its arrival.
  - See Excel solution for Example 2.4 for details.

Order-Up-To Level Inventory Example

[Simulation of Inventory Systems]

- Average ending inventory for 100 trials(each 25 days)



Other Examples of Simulation

- A Reliability Problem:
  - A machine with different failure types of which repairman is called to install or repair the part.
  - Downtime for the mill : $10 per minute.
  - On-site cost of the repairperson : $30 per hour.
  - It takes 20 minutes to change one bearing , 30 minutes to change two bearings , and 40 minutes to change three bearings.
  - The delay time of the repairperson's arriving :

Delay Time (Minutes)	Probability	Cumulative Probability	Random Digit Assignment
5	0.6	0.6	1—6
10	0.3	0.9	7—9
15	0.1	1	0

A Reliability Problem

- Bearing-Life Distribution:

Bearing Life(Hours)	Probability	Cumulative Probability	Random Digit Assignment
1000	0.1	0.1	01 —10
1100	0.13	0.23	11 —23
1200	0.25	0.48	24 —48
1300	0.13	0.61	49 —61
1400	0.09	0.7	62 —70
1500	0.12	0.82	71 —82
1600	0.02	0.84	83 —84
1700	0.06	0.9	85 —90
1800	0.05	0.95	91 —95

- Evaluate the proposal of replacing all three bearings whenever a bearing fails:
  - Measure of performance : total cost per 10000 bearing-hours

A Reliability Problem

- In a simulation of 15 bearing changes under the current method of operation,the total delay was (110 + 110 + 105) minutes and the total life of the bearings was (22,300 + 18,700 + 18,600) hours.
  - Cost of bearing: 45 bearings * $32/bearing = $1,440
  - Cost of delay time: (110 + 110 + 105)minutes * $10/minute = $3,250
  - Cost of downtime during repair: 45 bearings * 20 minutes/bearing * $10/ minute = $9,000
  - Cost of repairpersons: 45 bearings * 20 minutes.bearing * $30/60 minutes = $450
  - Total cost: $1,440 + $3,250 + $9,000 + $450 = $14,140
  - Total life of bearings: 22,300 + 18,700 + 18,600 = 59,600 hours.
  - Total cost per 10,000 bearing-hours($14,140/5.96) = $2,372.

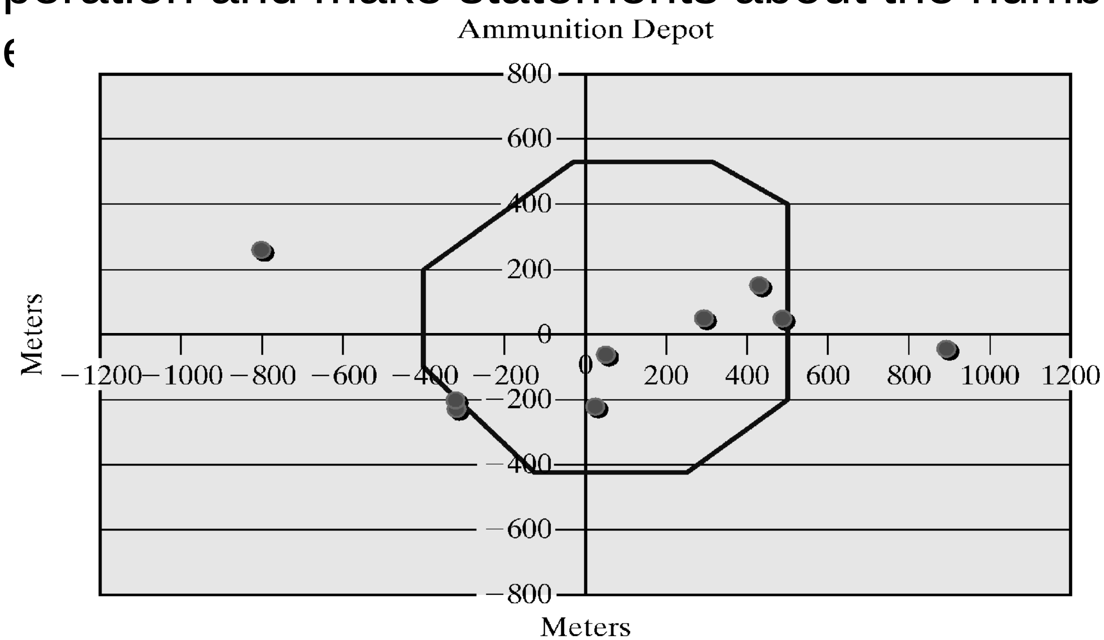
A Reliability Problem

- In a simulation of proposed method:
  - Cost of bearings: 45 bearings * $32/bearing = $1,440
  - Cost of delay time: 110 minutes * $10/minute = $1,100
  - Cost of downtime during repairs: 15 sets * 40 minutes/set * $10/minute = $6,000
  - Cost of repairpersons: 15 sets * 40 minutes/set * $30/60 minutes = $3000
  - Total cost: $1,440 + $1,100 + $6,000 + $300 = $8,840
  - Total life of the bearings : (17,000 * 3) = 51,000 hours.
  - The total cost per 10,000 bearing-hours :

    ($8,840 /5.1) = $1,733
  - The new policy generates a saving of $634 per 10,000 hours.

Random Normal Numbers

- A bomber wants to destroy an ammunition depot.
- If bomb falls on the target, a hit is scored (otherwise miss)
- The bomber flies in horizontal direction and carries 10 bombs.(the aiming point: (0,0)).
- The pint of impact is normally distributed around the aiming point(σ_x = 400 and σ_y = 200).
- Simulate the operation and make statements about the number of bombs on targe



Random Normal Numbers

- If X ~ N(μ ,σ^2) and Z ~ N(0,1) :

$$Z = \frac{X-\mu}{\sigma}$$

- If (X,Y) be the simulated coordinates of the bomb , after it has fallen,we have:
  - X = Zσ_x → X = 400Z_i
  - Y = Zσ_y → Y = 200Z_j
- I and J indicate that the values of Z should be different.
- The values of Z are random normal numbers and can be generated from uniformly distributed random numbers.

Random Normal Numbers

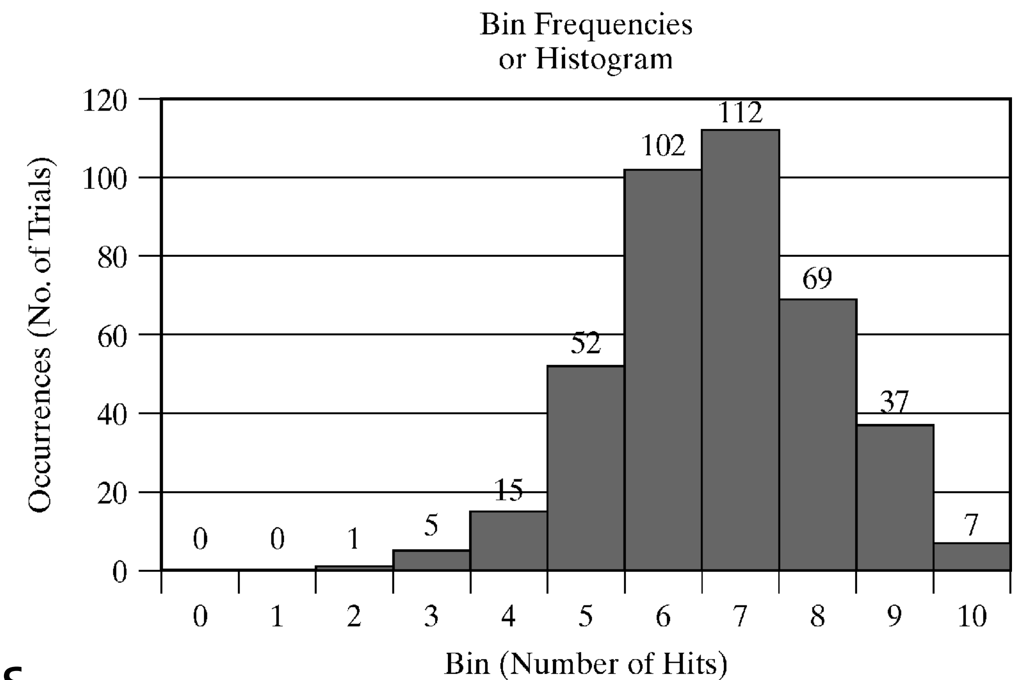
- Results of a simulated run:

Bomb	RNNx	X Coordinate (400 RNNx)	RNNy	Y Coordinate (200 RNNy)	Result
1	2.2296	891.8	-0.1932	-38.6	Miss
.	.	.	.	.	.
.	.	.	.	.	.

- RNNx : Random Normal Number to compute the x coordinate(corresponds to Z_i).

Random Normal Numbers

- Using the spreadsheet,an experiment was run with 400 trials(each trial being 10 bombs):



- The average:

  6.72 hits
- 44% of the

bombing runs:

there are 6 or fewer hits
- In 71% of cases: there were 6 , 7 or 8 hits.

Other Examples of Simulation

- Lead-time demand:
  - Lead time is the random variable: the time from placement of an order until the order is received.
  - Other possible random variable: demand.
  - Possible decision variables: how much and how often to order.
  - The daily demand is given by the following distribution:

Daily Demand (Rolls)	**3**	**4**	**5**	**6**
Probability	0.20	0.35	0.30	0.15

  - Lead time is a random variable given by the following distribution:

Lead Time (Days)	**1**	**2**	**3**
Probability	0.36	0.42	0.22

Other Examples of Simulation

- Lead-time demand:
  - Random digit assignment for Demand

Daily Demand	Probability	comulative probabilty	Random Digit Assignment
3.00	0.20	0.20	01-20
4.00	0.35	0.55	21-55
5.00	0.30	0.85	56-85
6.00	0.15	1.00	86-00

  - Random digit assignment for Lead Time

Daily Demand	Probability	comulative probabilty	Random Digit Assignment
1	0.36	0.36	01-36
2	0.42	0.78	37-78
3	0.22	1.00	79-00

Other Examples of Simulation

- Simulation Table for Lead Time Demand:

cycle	Ramdom Digits for Lead Time	Lead Time (Days)	Ramdom Digits for Demand	Demand	Lead-Time Demand
1	57	2	11	3	
			64	5	8
2	33	1	37	4	4
3	46	2	13	3	
			80	5	8
4	91	3	27	4	
			66	5	
			47	4	13
.	.		.	.	.
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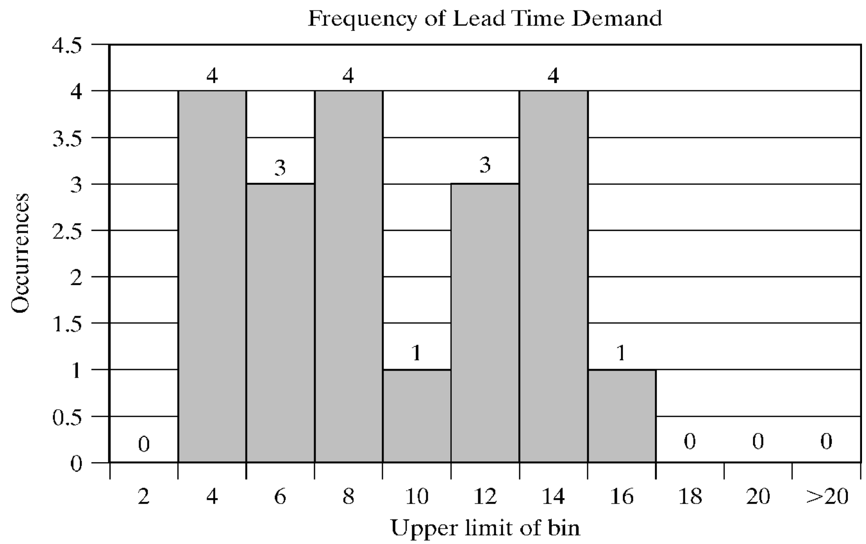
Other Examples of Simulation

- The resulting distribution of lead time demand on a 20-cycle trial may be like in following histogram:



- The project that requires the completion of a number of activities can be represent by a network of activities.

Other Examples of Simulation

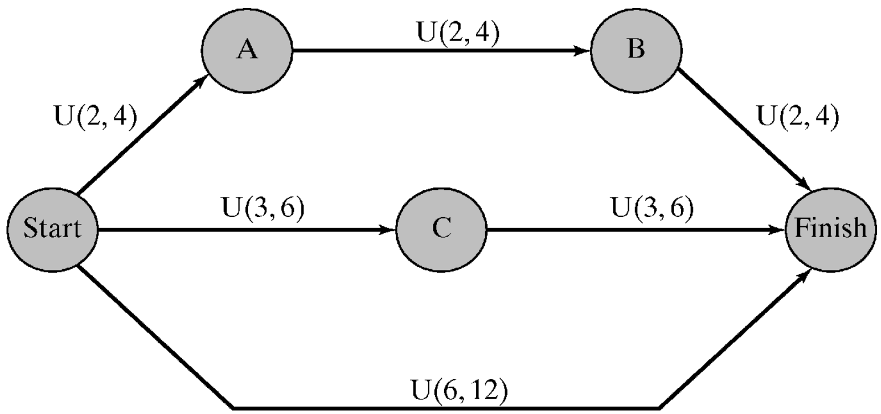
- Project simulation:
  - A project can be represented as a network of activities: some activities must be carried out sequentially, others can be done in parallel.
  - Possible random variables: times to complete the activities.
  - Possible decision variables: sequencing of activities, number of workers to hire.
  - Cook bully, egg, and toast example:

Top path:

Start	$\longrightarrow$	A	Crack egg
A	$\longrightarrow$	B	Scramble egg
B	$\longrightarrow$	Finish	Cook egg
Start	$\longrightarrow$	C	Make toast
C	$\longrightarrow$	Finish	Butter Toast
Start	$\longrightarrow$	Finish	Fry the bully

Other Examples of Simulation

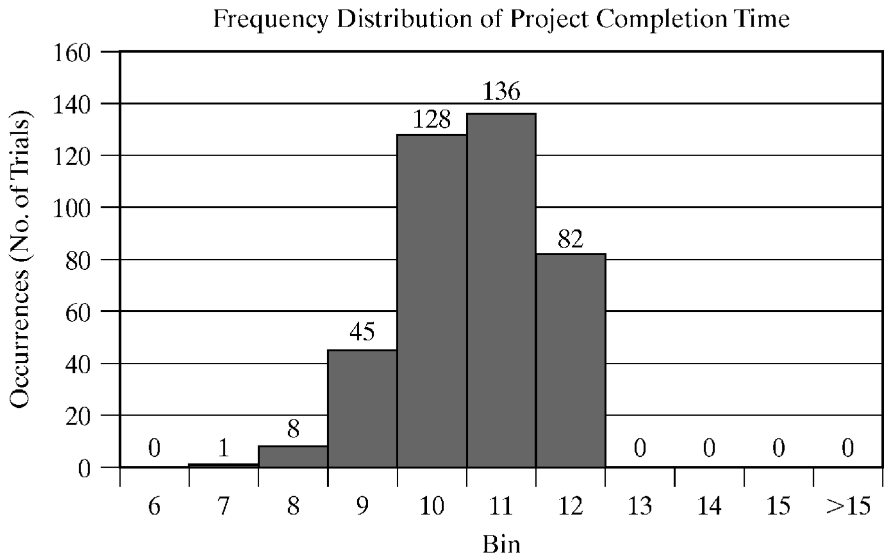
- Project simulation:
- Time of each activity can be represent by a uniform distribution between a lower and upper limit as shown in the following graph:



- For a uniform distribution a simulated activity time is :
  - Simulated activity = Lower limit + (Upper limit – Lower limit)*Random number
  - For example the activity start $\longrightarrow$ A if Random number = 0.794 is 2 + (4 – 2)*0.7943 = 3.59

Other Examples of Simulation

- Project simulation:
- The resulting frequency the project completion time with 400 trial and using default seed is shown in following diagram:



  - 13.5% of the time (54 of 400) breakfast will e ready in 9min or less
  - 20.5% of the time (82 of 400) breakfast will take from 11 to 12 min

Summary

- Introduced simulation concepts by means of examples, illustrated general areas of application, and motivated the remaining chapters.
- Ad-hoc simulation tables were used:
  - Events in tables were generated by using uniformly distributed random numbers, and resulting responses were analyzed.
  - Ac-hoc simulation table may fail due to system complexities. More systematic methodology, e.g., event scheduling approach, is described in Chapter 3.
- Key takeaways:
  - A simulation is a statistical experiment and results have variation.
  - As the number of replications increases, there is an increased opportunity for greater variation.