TUTORIAL 0 THE NUMBER SYSTEMS

Overview

- **We will review the following concept in this tutorial:**
  - The conversion of numbers between base 2 (binary), base 10 (decimal) and base 16 (hexadecimal).
- **Work with number conversion examples**

The Decimal (Base 10) representation

- We are using the decimal (base 10) number system to represent numbers in our daily lives.

- For example when we say a year has $365_{(10)}$ days, this decimal number $365_{(10)}$ really means the following:

$$365_{(10)} = 300 + 60 + 5$$
$$365_{(10)} = 3 \times 100 + 6 \times 10 + 5 \times 1$$
$$365_{(10)} = 3 \times 10^2 + 6 \times 10^1 + 5 \times 10^0$$

- When we represent a fractional number in the decimal (base 10) format like in the following, it really means:

$$365.25_{(10)} = 3 \times 10^2 + 6 \times 10^1 + 5 \times 10^0 + 2 \times 10^{-1} + 5 \times 10^{-2}$$

The General Base *r* Representation

- In general we can represent a number using any *base* (or radix) *r:*

$$\underbrace{(a_n \ldots\ a_5 a_4 a_3 a_2 a_1 a_0}_{\text{Integer part}}.\underbrace{a_{-1} a_{-2} a_{-3} a_{-4} \ldots a_{-m}}_{\text{Fractional part}})_r \qquad a_i < r$$

- For example: the following fractional number represented in base *r* is really:

$$(a_n \ldots\ a_5 a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3} a_{-4} \ldots a_{-m})_r$$

$$= a_n \times r^n + a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \cdots + a_0 \times r^0 + a_{-1} \times r^{-1} + \ldots + a_{-m} \times r^{-m}$$

A Binary (Base 2) Representation Example

- From the equation on the last page, we can convert a binary fractional number $(111.01)_2$ into the decimal representation easily:

$$(111.01)_2 = (?)_{10}$$

$$\begin{aligned}(111.01)_2 &= 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^{-1} + 1 \times 2^{-2} \\ &= (1 + 2 + 4 + 0.25)_{10}\end{aligned}$$

A Hexadecimal (Base 16) Representation Example

- **From the same equation two slides before, we can convert a hexadecimal fractional number $(\boldsymbol{A05.3F})_{16}$ into the decimal representation easily:**

$$(A05.3F)_{16} = (?)_{10}$$

$$(\mathrm{A05.3F})_{16} = 10 \times 16^2 + 0 \times 16^1 + 5 \times 16^0 + 3 \times 16^{-1} + 15 \times 16^{-2}$$

$$= 2565.24609375_{10}$$

Therefore $A05.3F_{16}$ equals to 2565.24609375_{10}

Converting a Decimal Integer to the Binary Representation

- To convert a decimal integer to the binary representation, we have to divide the decimal integer repeatedly by the base 2, until we have obtained the quotient of zero

$(26)_{10} = (?)_2$

Division	Quotation	Generated Remainder
26 / 2	13	0 Least Significant Bit
13 / 2	6	1
6 / 2	3	0
3 / 2	1	1
1 / 2	0	1 Most Significant Bit

So the corresponding binary number is $11010_{(2)}$

Converting a Decimal Integer to the Hexadecimal Representation

- **Similarly, to convert a decimal integer to a base *r* number, we will be dividing the decimal integer repeatedly by *r*, until the quotient of 0 has been obtained:**

$(426)_{10} = (?)_{16}$

Division	Quotation	Generated Remainder	
426 / 16	26	10	Least Significant Bit
26 / 16	1	10	
1 / 16	0	1	Most Significant Bit

So the corresponding binary number is $1AA_{(16)}$

Quick Conversion Table for Different Bases

Decimal $r = 10$	Binary $r = 2$	Ternary $r = 3$	Quaternary $r = 4$	Octal $r = 8$	Hexadecimal $r = 16$
0	0	0	0	0	0
1	1	1	1	1	1
2	10	2	2	2	2
3	11	10	3	3	3
4	100	11	10	4	4
5	101	12	11	5	5
6	110	20	12	6	6
7	111	21	13	7	7
8	1000	22	20	10	8
9	1001	100	21	11	9
10	1010	101	22	12	A
11	1011	102	23	13	B
12	1100	110	30	14	C
13	1101	111	31	15	D
14	1110	112	32	16	E
15	1111	120	33	17	F
16	10000	121	100	20	10
17	10001	122	101	21	11
18	10010	200	102	22	12
19	10011	201	103	23	13
20	10100	202	110	24	14

Binary (Base 2) to Hexadecimal (Base 16) Conversion

- **Starting from the Least Significant Bit (LSB) of the binary integer, we group four digits at a time and replace them with the hexadecimal equivalent of those groups and we can get the final hexadecimal number. We may need to pad extra zeros at the left of the binary integer to make up for enough digits:**

$(\mathbf{101}\ \mathbf{1110}\ )_2 = (?\ )_{16}$

<table>
<tr><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td></tr>
<tr><td colspan="4">5</td><td colspan="4">E</td></tr>
</table>

Additional 0 padded to the left

Therefore $(\mathbf{101}\ \mathbf{0110}\ )_2 = (\mathbf{5E})_{16}$

Hexadecimal (Base 16) to Binary (Base 2) Conversion

- **If a hexadecimal number is given and you are asked to convert it into its binary equivalent, then each hexadecimal digit is converted into a 4-bit-equivalent binary number and by combining all those digits we get the final binary equivalent:**

$(\mathbf{5E})_{16} = (?)_2$

5				E			
0	1	0	1	1	1	1	0

Therefore $(\mathbf{5E})_{16} = (\mathbf{0101\ 1110})_2$