

Practice problems: Chapter 2

1. Consider two events A and B .
 - (a). Show that $P(AB^c) = P(A) - P(AB)$.
 - (b). Show that the probability that exactly one of the events A or B occurs equals $P(A) + P(B) - 2P(AB)$.
2. (Compute the probabilities of 5-card poker hands). A poker hand consists of 5 unordered cards from a standard deck of 52. Calculate the probability of each of the standard kinds of poker hands.
 - (a). **Royal Flush.** This hand consists of values 10,J,Q,K,A, all of the same suit.
 - (b). **Straight Flush.** A straight flush consists of five cards with values in a row, all of the same suit.
 - (c). **Straight.** A straight consists of five values in a row, not all of the same suit.
 - (d). **Flush.** A flush consists of five cards, all of the same suit (Excludes royal and straight flushes).
 - (e). **Full House.** 3 cards of one denominator and 2 cards of another. For example, 3 aces and 2 kings is a full house.
 - (f). **Four of a Kind.** This hand consists of 4 cards of one value, and 1 more card of different value.
 - (g). **Three of a Kind.** This hand consists of three cards of one value, and two more cards, each of different values.
 - (h). **Two Pairs.** This hand consists of two pairs of different values, and a fifth card of another different value.
 - (i). **One Pair.** This hand consists of a pair of one value, and three additional cards, each of different value.
 - (j). What is the probability that the poker hand is nothing as above?
3. Suppose that A and B are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$. What is the probability that
 - (a). either A or B occurs?
 - (b). A occurs but B does not?
 - (c). both A and B occur?
4. A pair of fair dice is rolled. What is the probability that the second die lands on a higher value than does the first?

Solutions:

1. Just show your steps.
2. (a). Royal Flush: $\frac{4}{\binom{52}{5}} = 0.000001539$; (b). Straight Flush: $\frac{40}{\binom{52}{5}} = 0.00001539$; (c). Straight: $\frac{10(4^5-4)}{\binom{52}{5}} = 0.003925$; (d). Flush: $\frac{4\binom{13}{5}-40}{\binom{52}{5}} = 0.001965$; (e). Full House: $\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}} = 0.001441$; (f). Four of a Kind: $\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}} = 0.000240$; (g). Three of a Kind: $\frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}} = 0.021129$; (h). Two Pairs: $\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}} = 0.047539$; (i). One Pair: $\frac{\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3}{\binom{52}{5}} = 0.422569$; (j). Nothing: 50.1177%.
3. (a). 0.8; (b). 0.3; (c). 0
4. 5/12