Tutorial Notes 1.1 of MATH2421

A brief summary of course material

1. (The Basic Principle of Counting)

Suppose that two experiments are to be performed:

If experiment 1 can result in $\mathbf{m}$ possible outcomes, and for each outcome of experiment 1 experiment 2 can result in $\mathbf{n}$ possible outcomes; then together there are $\mathbf{mn}$ possible outcomes of the two experiments.

2. (General Principle in Permutations)

Suppose there are $\mathbf{n}$ (distinct) objects, then the total number of different arrangements is

$$n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 = n!$$

with the convention that $0! = 1$.

3. (General Principle in Permutations)

For $\mathbf{n}$ objects of which $\mathbf{n_1}$ are alike, $\mathbf{n_2}$ are alike, . . . , $\mathbf{n_r}$ are alike, there are

$$\frac{n!}{n_1!n_2!\cdots n_r!}$$

different permutations of the $\mathbf{n}$ objects.

Tutorial Notes 1.2 of MATH2421

A brief summary of course material

1. (Combinations)

Number of ways of choosing r items from n items:

$$\frac{n!}{r!(n-r)!} = \binom{n}{r}$$

2. (Some Combinatorial Identities)

For $r = 0, 1, \ldots, n$, we have:

$$\binom{n}{r} = \binom{n}{n-r} \qquad \text{and} \qquad \binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

3. (The Binomial Theorem)

Let n be a nonnegative integer, then

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

4. (Multinomial Coefficients)

A set of n distinct items is to be divided into r distinct groups of respective sizes $n_1, n_2, \ldots, n_r$ where $\sum_{i=1}^{r} n_i = n$. There are

$$\frac{n!}{n_1!n_2!\cdots n_r!} = \binom{n}{n_1, n_2, \ldots, n_r}$$

possible divisions.

5. (The Multinomial Theorem)

$$(x_1 + x_2 + \ldots + x_r)^n = \sum_{(n_1,\ldots,n_r): n_1+\cdots+n_r=n} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} \cdots x_r^{n_r}$$

Example

1. (Permutation)

Five people, designated as A, B, C, D, E, are arranged in linear order.

(1) How many ways to arrange these five people? (120)

(2) How many ways to arrange these five people, if they are arranged in a circle? (24)

2. (Permutation)

How many different ways can 3 red, 4 yellow and 2 blue bulbs be arranged in a string of Christmas tree lights with 9 sockets? (1260)

3. (Combinations)

A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if

(a) there are no restrictions?

(b) one particular person must be chosen on the committee?

(c) there are to be 3 men and 2 women?

(d) there is to be a majority of women?

4. (Permutations and Combinations)

If 4 Maths books are selected from 6 different Maths books and 3 English books are chosen from 5 different English books, how many ways can the seven books be arranged on a shelf (linear order):

(a) If there are no restrictions?

(b) If 4 Maths books remain together?

(c) If Maths and English books alternate?

(d) If a Math book is at the beginning of the shelf?

5. (The Binomial/Multinomial Theorem)

There are some simple applications of Binomial therorem.

(a) Suppose we expand $(x+y)^4$, what is the coefficient of x^3y?

(b) Suppose we expand $(x+2y)^4$, what is the coefficient of x^3y?

(c) Suppose we expand $(2x-3y)^4$, what is the coefficient of x^3y?

(d) Suppose we expand $(2x+3y+4z)^4$, what is the coefficient of x^2yz?

Tutorial Notes 2.1 of MATH2421

A brief summary of course material

1. Sample Space S; Outcomes; Event A
2. Intersection of Events; Disjoint and Mutually Exclusive of events
3. Union of Events; Exhaustive Events; Partition
4. Associative Law; Distributive Law; De Morgan's Law
5. For any event A: $P(\emptyset) = 0 \leq P(A) \leq 1 = P(S)$
6. For any event A, B and C: $P(A \cup B) = P(A) + P(B) - P(AB)$

Example

1. (Properties of Probability)

Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing
(1) a ring or a necklace?
(2) a ring and a necklace?

2. (Sample Spaces Having Equally Likely Outcomes)

Five people, designated as A, B, C, D, E, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that
(1) there is exactly one person between A and B?
(2) there are exactly two people between A and B?
(3) there are three people between A and B?

Extra exercise

(Accounting and Permutation)

How many different number-plates for cars can be made if each number-plate contains four of the digits 0 to 9 followed by a letter A to Z, assuming that

(a) no repetition of digits is allowed? (131,040)

(b) repetition of digits is allowed? (260,000)

(Permutation)

In how many ways can the six letters of the word "mammal" be arranged in a row? (60)

(Combinations)

In a poker hand consisting of 5 cards (chosen from 52 cards, without replacement). How many poker hands are there if

(a) there are no restrictions?

(b) 4 Kings?

(c) 2 Clubs and 3 Hearts?

(Understanding Sample Space)

List the elements of each of the following sample spaces:

(a) A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

(b) A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

(1) Give the sample space of this experiment.

(2) Let A be the event that the patient is in serious condition. Specify the outcomes in A.

(3) Let B be the event that the patient is uninsured. Specify the outcomes in B.

(4) Give all the outcomes in the event $B^c \cup A$.