Tutorial Notes 2 of MATH2421

A brief summary of course material

1. Sample Space S; Outcomes; Event A
2. Intersection of Events; Disjoint and Mutually Exclusive of events
3. Union of Events; Exhaustive Events
4. Associative Law; Distributive Law; De Morgan's Law
5. For any event A: $P(\emptyset) = 0 \leq P(A) \leq 1 = P(S)$
6. For any event A, B and C:

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

7. Let A and B be two events in a sample space S with $P(B) > 0$. The Conditional Probability of A given B is defined as

$$P(A|B) = \frac{P(AB)}{P(B)}$$

8. Multiplication rule:
   (i) For any two events A and B with $P(A) > 0$, $P(BA) = P(B|A)P(A)$;
   (ii) For any three events A, B and C with $P(BC) > 0$ and $P(AB) > 0$, we have

$$P(ABC) = P(A|BC)P(B|C)P(C)$$

$$P(ABC) = P(A)P(B|A)P(C|AB)$$

   (iii) More generally, for k events $A_1, A_2 \cdots A_k$, if $P(A_1 A_2 \cdots A_{k-1}) > 0$, we have

$$P(A_1 A_2 \cdots A_k) = P(A_1)P(A_2|A_1)P(A_3|A_1 A_2) \cdots P(A_k|A_1 \cdots A_{k-1})$$

Example

1. (Properties of Probability)

Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing
(1) a ring or a necklace?
(2) a ring and a necklace?

2. (Sample Spaces Having Equally Likely Outcomes)

Five people, designated as A, B, C, D, E, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that
(1) there is exactly one person between A and B?
(2) there are exactly two people between A and B?
(3) there are three people between A and B?

3. (Conditional Probability)

A robot manufacturer produces robots which contain hundreds of circuits. From years of production, it's known that 2 out of 1000 robots are defective. For a certain test method, if a robot is not defective, the test is negative 97% of the time, while if a robot is defective, the test is positive 99.5% of the time.
(a) What is the probability that a randomly selected robot will get a negative result?
(b) If a randomly selected robot gets a positive result, what is the probability that it is defective?

4. (General Multiplication Rule)

An ordinary deck of 52 playing cards is randomly divided into 4 $distinct$ piles of 13 cards each. Compute the probability that each pile has exactly 1 ace.