



COMP 2211 Exploring Artificial Intelligence  
A Proof of Bayes' Theorem  
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# Bayes' Theorem

- As mentioned in class, Bayes' theorem is defined as:

$$\begin{aligned} P(B|E) &= \frac{P(B)P(E|B)}{P(E)} \\ &= \frac{P(B)P(E|B)}{P(E|B)P(B) + P(E|NOT\ B)P(NOT\ B)} \end{aligned}$$

- In other words, Bayes' theorem gives us the conditional probability of B given that E has occurred as long as we know  $P(E|B)$ ,  $P(E|NOT\ B)$ , and  $P(B) = 1 - P(NOT\ B)$ .

# A Proof of Bayes' Theorem

- By the definition of conditional probability:

$$P(E|B) = \frac{P(E \text{ AND } B)}{P(B)} \quad (1)$$

$$P(E \text{ AND } B) = P(E|B)P(B) \quad (2)$$

- By the law of total probability:

$$\begin{aligned} P(E) &= P((E \text{ AND } B) \text{ OR } (E \text{ AND } (\text{NOT } B))) \\ &= P((E \text{ AND } B) + (E \text{ AND } (\text{NOT } B))) \end{aligned} \quad (3)$$

- By substituting Equation (2) to Equation (3):

$$P(E) = P(E|B)P(B) + P(E|\text{NOT } B)P(\text{NOT } B) \quad (4)$$

where on Equation (4), that fact that  $E \text{ AND } B$  and  $E \text{ AND } (\text{NOT } B)$  are mutually exclusive events was used.

## A Proof of Bayes' Theorem

- Since  $E \text{ AND } B = B \text{ AND } E$ , by Equation (1) and (2), we have:

$$\begin{aligned} P(B|E) &= \frac{P(B \text{ AND } E)}{P(E)} \\ &= \frac{P(E|B)P(B)}{P(E)} \end{aligned} \tag{5}$$

- By merging Equation (4) to (5), we get the following:

$$P(B|E) = \frac{P(E|B)P(B)}{P(E|B)P(B) + P(E|\text{NOT } B)P(\text{NOT } B)} \tag{6}$$

Equation (6) is what we wanted to prove. :)