Practice problems: Chapter 3

1. Let us take 10 marbles from a bag of N marbles, of which $r > 10$ are red, without replacement. Let X_i be an indicator equal to 1 if and only if the ith marble is red.

   (a). With what probability is the first marble red? The second? Third? The tenth?

   (b). Given that the first marble is red, with what probability is the second marble red? The third? The tenth?

   (c). Given that the first and fifth marbles are red, with what probability is the second marble red? The tenth?

   (d). What is the probability that the first two marbles are red? The second and third? The ninth and tenth?

   (e). Given that the first marble is red and the second is not red, what is the probability that the seventh marble is red and the ninth marble is not red?

2. Your neighbor has 2 children. You learn that he has a son, Joe. What is the probability that Joes sibling is a brother?

3. Two cards from an ordinary deck of 52 cards are missing. What is the probability that a random card drawn from this deck is a spade?

4. Urn 1 contains 5 white balls and 7 black balls. Urn 2 contains 3 whites and 12 black. A fair coin is flipped; if it is Heads, a ball is drawn from Urn 1, and if it is Tails, a ball is drawn from Urn 2. Suppose that this experiment is done and you learn that a white ball was selected. What is the probability that this ball was in fact taken from Urn 2? (i.e., that the coin flip was Tails)

5. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find (a). $P(B)$; (b). $P(A|B)$; and (c). $P(B^c|A)$.

Solutions:

1. (a). $\frac{r}{N}$; (b). $\frac{r-1}{N-1}$; (c). $\frac{r-2}{N-2}$; (d). $\frac{r(r-1)}{N(N-1)}$; (e). $\frac{r-1}{N-2}\frac{N-r-1}{N-3}$
2. $\frac{1/4}{3/4} = 1/3$
3. 1/4 (You may consider the events A_i's where A_i is event that i spades are missing from the deck, for $i = 0, 1, 2$)
4. 12/37
5. (a). $P(B) = 2/5$; (b). $P(A|B) = 1/4$; (c). $P(B^c|A) = 3/5$