Hyper-Reduction Approaches for Contact Modeling with Small Tangential Displacements: Applications for a Bolted Joint

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1 Introduction and Motivation

Model order reduction of the non-linear dynamical models of structures with frictional contacts is of high importance in making simulations of such systems tractable. Even for relatively simple bolted structures, the representational requirements as well as the amount of non-linear evaluations necessary can make the system very large and very expensive for simulations. Projection-based techniques for model order reduction (see, for instance [7]), which are based on finding an appropriate reduced representation of the solution in a lower-rank subspace (and solving the problem there), are a very popular approach in the literature. One of the main drawbacks with such approaches is that it is generally not trivial to establish a reduced representation of non-linearities on the chosen subspace itself. In other words, evaluation of non-linearities in such models is conducted by first transforming the unknowns into the full-order model, evaluating the non-linearities, and then transforming them back into the reduced domain. This procedure, while beneficial for a lot of cases, becomes computationally cumbersome when it comes to very large models wherein the evaluation of the non-linear function becomes an important computational bottleneck. In order to alleviate this issue, there have been several hyper-reduction approaches (see, for instance [8]), which seek to develop reduced order modeling strategies with the non-linearities completely represented in the reduced domain itself. One promising approach for this is to develop a data-based representation of the non-linearity on the subspace (see [5] for an application) based on several non-linear function evaluations on the full-order model. Such an approach, however, suffers from input-level dependence, i.e., the trained model performs only as good as the training data-set.

The current paper, following previous efforts [4], explores model reduction through the development of reduced representations of the interface while retaining the physical meaning of the degrees-of-freedom of the reduced model. This allows for relatively easy definitions of the non-linearities consistently in the reduced domain, thereby avoiding the need to transform back to the full problem. Two approaches are presented with their merits and shortcomings discussed: (1) an improved whole-joint formulation (similar to the ones used in [6]) applied to regions on the interface selected based on a binned field objective; and (2) an interface remeshing approach based on efficient representation of a continuous field objective.

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2 Description of Approaches

As already mentioned, the two approaches are closely tied together in the sense that both are aimed at developing a representation of the interface that best represents a particular field quantity over the interface (contact pressure, dissipation fluxes, etc.). Therefore, a scalar-valued field objective, denoted $\mathcal{P}(\underset{\sim}{x})$ in the following, has to first be identified so that the reduced model can be developed.

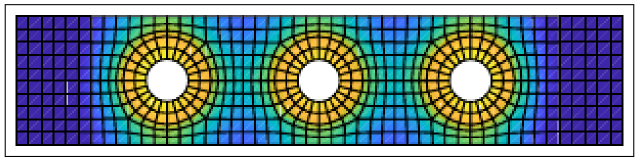
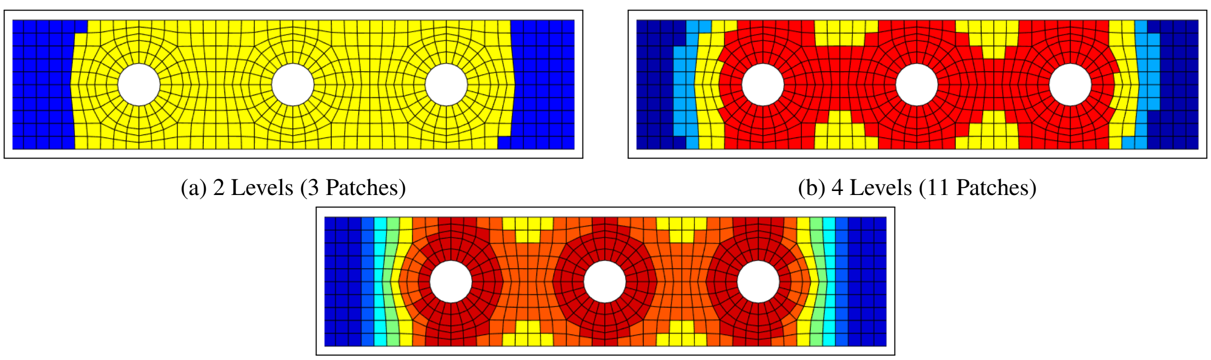


Figure 1: Reference mesh. The static contact pressure mapped to [0, 1], used as the field objective in the following, is denoted using colours.

All of the results shown in the following are based on the interfacial mesh shown in fig. 1, which corresponds to the bolted joint interface for a finite-element model of the Brake-Reuß Beam (BRB), a three-bolted lap-joint structure [3]. The field objective that will be used here will be the normal contact pressure from static bolt-prestress simulations (with each bolt applying a prestress of 11.58 kN) used as the field objective (re-ranged to [0,1] and denoted using color here).

2.1 Whole Jointed Approach Based on Binned Field Objective



(a) 2 Levels (3 Patches)

(b) 4 Levels (11 Patches)

(c) 7 Levels (21 Patches)

Figure 2: Binned objective patches for the Brake-Reuß Beam (BRB) interface. The colors indicate different patches.

For the first approach, the field $\mathcal{P}(\underset{\sim}{x})$ is first divided into a set of discrete levels $\mathcal{P}_{min} = \mathcal{P}_0 < \mathcal{P}_1 < \cdots < \mathcal{P}_{N_{lev}} = \mathcal{P}_{max}$. Elements corresponding to each bin i are selected as the elements that have $\mathcal{P}(\underset{\sim}{x}) \in [\mathcal{P}_{i-1}, \mathcal{P}_i)$. Following this, disconnected trees of the ensuing graph are identified and separated into separate "patches". Repeating this procedure for all the chosen levels yields a set of patches (defined as element-sets).

An improved stiffness-preserving whole-joint formulation[1] (see [2] for formulation) is then employed to represent each of these patches by a single six-Degree-of-Freedom (six-DoF) virtual node (three displacements and three rotations). Finally, conducting a CMS (Component-Mode Synthesis) procedure on this gives the effective reduced order model. Since this is a model that is based on a set of physical DoF's, contact models may be used on these nodes directly (making the model hyper-reduced).

Figure 2 indicates the patches identified for three different binning levels for the interface of .

[1]Similar, but not identical to the RBE3 elements in ANSYS terminology

2.2 Interface Remeshing Approach

The second approach comes from the idea that not all nodes in an interface may be necessary to accurately represent a field variable. Therefore, information about the local gradients of the field quantity are employed to guide the design of a new mesh on top of the initial mesh in the interface. Consider a "full mesh", denoted by $\mathcal{T}$, and a "reduced mesh", denoted by $\mathcal{T}_r$. Denoting the nodal DoF vectors in each as $\underset{\sim}{u}$ and $\underset{\sim}{u_r}$ respectively, the vector $\underset{\sim}{u}$ may be approximated by interpolating $\underset{\sim}{u_r}$ on the mesh $\mathcal{T}_r$ using its shape functions as follows:

$$\underset{\sim}{u} \approxeq \mathbf{Q}_r \underset{\sim}{u_r}. \tag{1}$$

Here, the matrix $\mathbf{Q}_r$ denotes the interpolation matrix developed using the corresponding shape functions of $\mathcal{T}_r$. Note that the same relationship (in eq. (1)) may be used to approximate nodal tractions on the original mesh but not forces.

For a dynamical system of the form

$$\mathbf{M}\underset{\sim}{\ddot{u}} + \mathbf{K}\underset{\sim}{u} + \underset{\sim}{f_{nl}}(\underset{\sim}{u}) = \underset{\sim}{f_{ext}}(t), \tag{2}$$

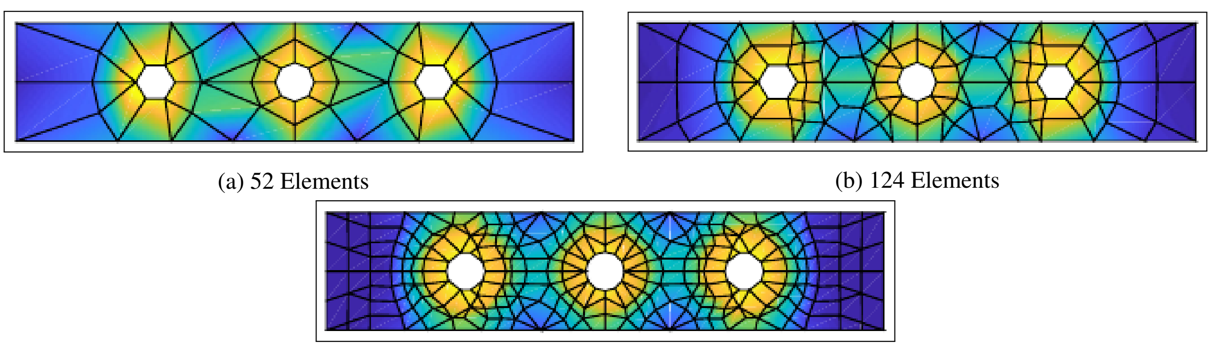
a Galerkin-projection-based Reduced-Order-Model (ROM) may be developed if eq. (1) is interpreted as a projection onto a reduced basis. Equation (2), under this projection, becomes

$$\left[\mathbf{Q}_r{}^T\mathbf{M}\mathbf{Q}_r\right]\underset{\sim}{\ddot{u}_r} + \left[\mathbf{Q}_r{}^T\mathbf{K}\mathbf{Q}_r\right]\underset{\sim}{u_r} + \underbrace{\mathbf{Q}_r{}^T\underset{\sim}{f_{nl}}(\mathbf{Q}_r\underset{\sim}{u_r})}_{\underset{\sim}{f_{nl}^r}} = \mathbf{Q}_r{}^T\underset{\sim}{f_{ext}}(t). \tag{3}$$

Note here that the non-linear force f_{nl} still has to be evaluated as many times as in the original model, i.e., the model is not hyper-reduced yet. This is achieved by evaluating the non-linear tractions in $\mathcal{T}_r$ (denoted as $t_r(\underset{\sim}{u_r})$), interpolating it onto $\mathcal{T}$ (becoming $\mathbf{Q}_r t_r(\underset{\sim}{u_r})$), integrating this in on $\mathcal{T}$ (computed with some quadrature integration matrix $\mathbf{P}$ and denoted as $\mathbf{P}\mathbf{Q}_r t_r(\underset{\sim}{u_r})$), and finally applying the Galerkin projection to obtain the forcing for the ROM. This procedure may be summarized as,

$$\underset{\sim}{f_{nl}^r} = \mathbf{Q}_r{}^T\underset{\sim}{f_{nl}}(\mathbf{Q}_r\underset{\sim}{u_r}) = \underbrace{\mathbf{Q}_r{}^T\mathbf{P}\mathbf{Q}_r}_{\mathbf{T}_m^f} t_r(\underset{\sim}{u_r}). \tag{4}$$

Here, the non-linearities need to be evaluated only in the reduced mesh $\mathcal{T}_r$, and projected onto the ROM using a single matrix $\mathbf{T}_m^r$. Since integrals of tractions on $\mathcal{T}_r$ directly do not have any meaning for the ROM, the reduced mesh may not be interpreted as a regular finite element mesh that discretizes the interface in a reduced fashion. It merely represents a reduced representation of the original mesh with consistent (but approximate) mappings.



(a) 52 Elements

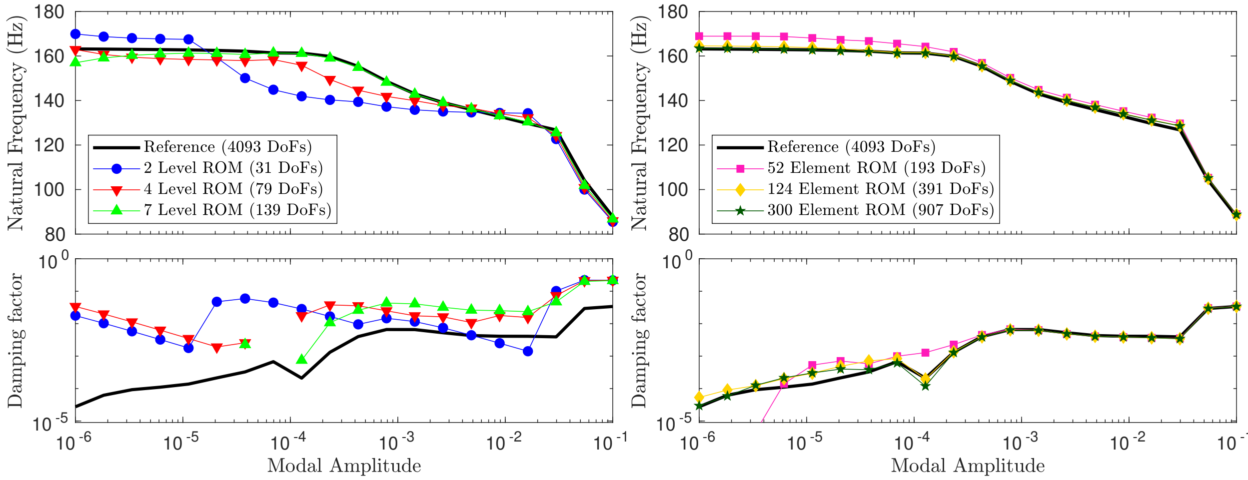
(b) 124 Elements

(c) 300 Elements

Figure 3: Examples of different reduced meshes using approach 2. The colors indicate the field-objective function, with blue indicating 0 and yellow indicating 1.

Figure 3 presents sample reduced meshes developed for the BRB interface using the contact pressure as the objective field (see fig. 1 for the field on full mesh). The pressure values are re-scaled to range from 0 to 1 for convenience. For the meshes shown in the figure, nodes were biased to lie in regions with large gradients or changes in the objective field.

3 Results



(a) (b)

Figure 4: Results for the (a) Whole Jointed Approach and the (b) Remeshing approach. Reference results are calculated using the full mesh (fig. 1); whole joint ROMs are developed based on the patches shown in fig. 2; and remeshing ROMs are developed based on the meshes shown in fig. 3.

Figure 4 shows the results for amplitude-dependent non-linear modal simulations using the technique developed in [1] for the first mode of vibration. Unilateral linear springs and planar elastic dry friction elements (2D Jenkins models) are used in the traction-sense as the normal and tangential contact constitutive relationships for the reference model and the remeshing ROMs. The same contact laws are employed for the whole joint ROMs but since these require phenomenological models (force-displacement models), the traction-stiffnesses used in the reference are scaled by the areas of each patch to obtain consistent model parameters for the ROMs.

In terms of performance, one can see that both demonstrate good convergence in capturing the reference trends in the stiffness characteristics (amplitude-dependent natural frequency). However, the remeshing approach seems to represent the dissipative characteristics (amplitude-dependent damping factor) much better than the whole joint approach. This may be an observation that could be related to the formulations of the ROMs, or just numerical relics of the estimated dissipation metric. Further investigations are currently being conducted to draw more conclusions.

4 Discussiond and Conclusions

The paper presents the development and relative comparison of hyper-reduction approaches for structures with small displacement contacting interfaces. The ROMs developed are consistently hyper-reduced, i.e., simulations of the full non-linear model are not necessary for the hyper-reduction. An additional advantage of this is that a developed ROM can potentially be used to conduct cheap simulations over any amplitude range of interest accurately.

In addition to extracting conclusive inferences from observations such as in section 3, the current investigation also explores the use of different choices of objective functions such as modal strains from linear modal analyses, dissipation fields from non-linear modal analyses, etc., as well as weighted combinations of these.

Another aspect that is taken up in the current work is the fact that it may not always be possible to come up with asymptotic accuracy analyses for the approaches considered here. For the first approach, having a very large number of binning levels leads to a system where some patches will be constituted with just a single element. These present numerical difficulties since in this case all the nodes will be shared by the patch with adjacent patches. For the second approach, increasing the number of required elements starts failing after a point since the elements one can come up with, while following the specified objective, start losing shape-regularity and yield bad quality elements. In most cases, however, this has been encountered when the sizes of some elements in the reduced mesh start becoming significantly smaller than those of the reference mesh.

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