# Hyper-Reduction Approaches for Contact Modeling with Small Tangential Displacements: Applications for a Bolted Joint

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October 16, 2019

**Keywords:** Interface reduction, hyper-reduction, jointed structures

#### 1 Introduction and Motivation

Model order reduction of the non-linear dynamical models structures with frictional contacts is of high importance in making computational modeling of such systems tractable. Even for relatively simple bolted structures, the representational requirements as well as the amount of non-linear evaluations necessary can make the system very large and very expensive for simulations. Projection-based techniques for model order reduction (see, for instance [6]), which are based on finding an appropriate reduced representation of the solution in a lower-rank subspace (and solving the problem there), are fairly standard now and a very popular approach for this in the literature. One of the main drawbacks with such approaches is that it is generally not trivial to establish a reduced representation of non-linearities on the chosen subspace itself. In other words, evaluation of non-linearities in such models is conducted by first transforming the unknowns into the full-order model, evaluating the non-linearities, and then transforming them back into the reduced domain. This procedure, while beneficial for a lot of cases, becomes computationally cumbersome when it comes to very large models wherein the evaluation of the non-linear function becomes an important computational bottleneck. In order to alleviate this issue, there have been several hyper-reduction approaches (see, for instance ?? ]), which seek to develop reduced order modeling strategies with the non-linearities completely represented in the reduced domain itself. One promising approach for this is to develop a data-based representation of the non-linearity on the subspace (see [4] for an application) based on several non-linear function evaluations on the full-order model. Such an approach, however, suffers from input-level dependence, i.e., the trained model performs only as good as the training data-set.

The current paper, following previous efforts [3], explores model reduction through the development of reduced representations of the interface while retaining the physical meaning of the degrees-of-freedom of the reduced model. This allows for relatively easy definitions of the non-linearities consistently in the reduced domain, thereby avoiding the need to transform back to the full problem. Two approaches are presented with their merits and shortcomings discussed: (1) an improved whole-joint formulation (similar to the ones used in [5]) applied to regions on the interface selected based on a thresholded field objective; and (2) an interface remeshing approach based on efficient representation of a continuous field objective.

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# 2 Description of Approaches

As already mentioned, the two approaches are closely tied together in the sense that both are aimed at developing a representation of the interface that best represents a particular field quantity over the interface (contact pressure, dissipation fluxes, etc.). Therefore, a scalar-valued field objective, say  $\mathcal{P}(\underline{x})$ , is first identified before the reduced model is developed.

#### 2.1 Whole Jointed Approach Based on Thresholded Field Objective

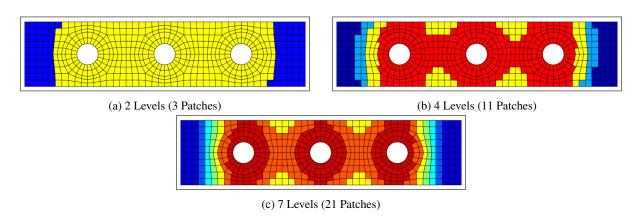


Figure 1: Thresholded objective patches for the Brake-Reuß Beam (BRB) interface. The objective used was the normal contact pressure on the elements; the colors indicate different patches.

For the first approach, the field  $\mathcal{P}(\underline{x})$  is first thresholded into a set of discrete levels  $\mathcal{P}_i$ ,  $i=1,\ldots,N_{lev}$ . Elements corresponding to each threshold level i are selected as the elements that have  $\mathcal{P}(\underline{x}) \in [\mathcal{P}_{i-1}, \mathcal{P}_i)$ . Following this, disconnected trees of the ensuing graph are identified and separated into separate "patches". Repeating this procedure for all the chosen levels yields a set of patches (defined as sets of elements).

An improved stiffness-preserving whole-joint formulation<sup>1</sup> (see [1] for formulation) is then employed to represent each of these patches by a single six-Degree-of-Freedom (six-DoF) virtual node (three displacements and three rotations). Finally, conducting a CMS (Component-Mode Synthesis) procedure on this gives the effective reduced order model. Since this is a model that is based on a set of physical DoF's, contact models may be used on these nodes directly (making the model hyper-reduced).

Figure 1 indicates the patches identified for three different thresholding levels for the interface of Brake-Reuß Beam (BRB), a three-bolted lap-joint structure [2]. The field objective used here is the normal contact pressure developed upon application of a static bolt-prestress loading. This was obtained here by conducting a full order simulation with a non-linear (penalty-spring with elastic dry friction) contact model in the interface.

### 2.2 Interface Remeshing Approach

The second approach comes from the idea that not all nodes in an interface may be necessary to accurately represent a field variable. Therefore, information about the local gradients of the field quantity are employed to guide the design of a new mesh on top of the initial mesh in the interface. Clearly, a model expressed in terms of the nodes of the reduced mesh is of smaller size than the original model. However, the reduced mesh may not be treated as an usual finite element mesh since integrals over the elements of this mesh do not have any direct meaning. Therefor, for calculating the non-linear forces, tractions are evaluated on the quadrature locations here, interpolated and integrated on the original mesh, and then projected back onto the reduced mesh for the simulations. The procedure can be achieved using a single matrix multiplication on the tractions evaluated on the reduced mesh.

Figure 2 presents sample reduced meshes developed for the BRB interface using the contact pressure as the objective field (see fig. 2a for the field on full mesh). The pressure values are re-scaled to range from 0 to 1 for convenience.

<sup>&</sup>lt;sup>1</sup>Similar, but not identical to the RBE3 elements in ANSYS terminology

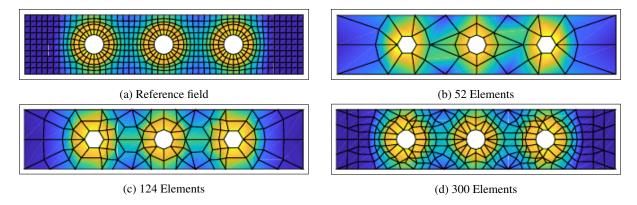


Figure 2: Examples of different reduced meshes using approach 2. The field objective used here, is the contact normal pressure re-scaled to range from 0 to 1. The colours indicate the objective function with blue indicating 0 and yellow indication 1.

For the meshes shown in the figure, nodes were biased to lie in regions with large gradients or changes in the objective field.

## 3 Discussions

From previous investigations [3] it was concluded that using just static prestress as the objective function leads to reduced order models that do not perform very well. Therefore the current investigation explores the use of different choices of objective functions such as modal strains from linear modal analyses, dissipation fields from non-linear modal analyses, etc. and weighted combinations of these.



Another aspect that is taken up in the current work is the fact that it may not always be possible to come up with asymptotic accuracy analyses for the approaches considered here. For the first approach, having a large number of thresholding levels leads to a system where some patches will be constituted with just a single element. These present numerical difficulties since in this case all the nodes will be shared by the patch with adjacent patches. For the second approach, increasing the number of required elements starts failing after a point since the elements one can come up with, while following the specified objective, start losing shape-regularity.

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