NNM Results for 2-DOF Nonlinear System

2023/02/06

Contents

The general form of the nonlinear system considered is,

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\\ddot{x}_2\end{Bmatrix}+\beta\begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}\begin{Bmatrix}\dot{x}_1\\\dot{x}_2\end{Bmatrix}+\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\\ddot{x}_2\end{Bmatrix}+\begin{Bmatrix}\alpha x_1^3\\\gamma\dot{x}_2^3+f_{fr}(\dot{x}_2)\end{Bmatrix}=\begin{Bmatrix}F\cos\Omega t\\0\end{Bmatrix}, \quad (1)$$

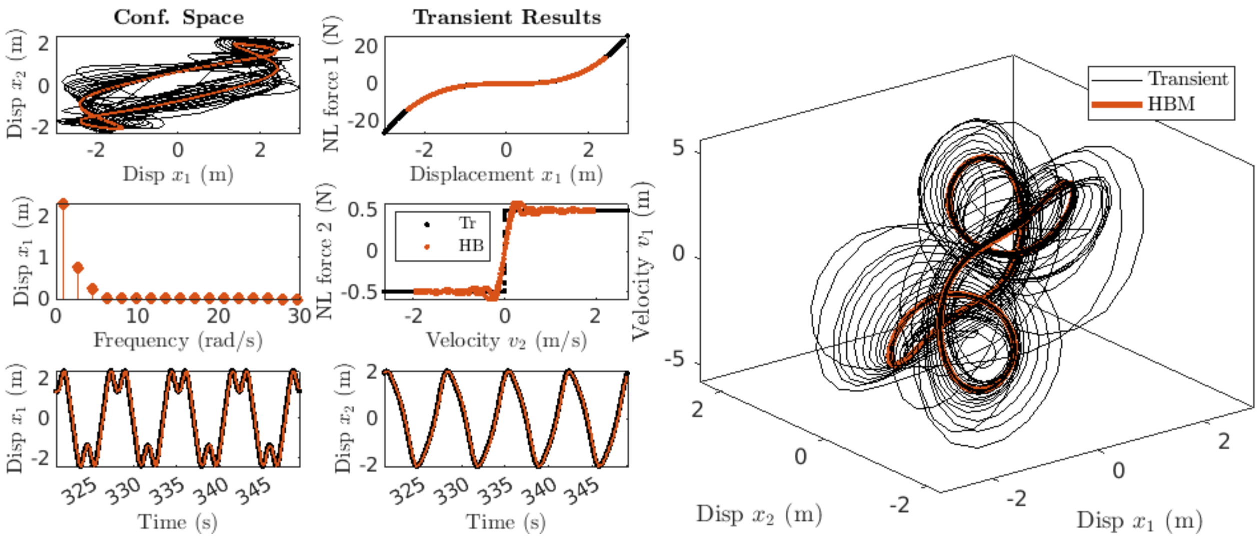
$$\text{with, } f_{fr}(\dot{x}_2)\begin{cases}\in(-\mu N,\mu N) & \dot{x}_2=0\\ \mu N=sgn(\dot{x}_2) & otherwise\end{cases}. \quad (2)$$

The friction model $f_{fr}(\cdot)$ is a **set-valued Coulomb law** that is implemented using a **Dynamic Lagrangian Framework**.

For all the results that follow, the damping parameter $\beta = 0.1Nsm^{-1}$ and $\gamma = 0$.

1 Comparisons with Transient Simulations

Note: Transients calculated using an average acceleration implicit Newmark-β implementation.



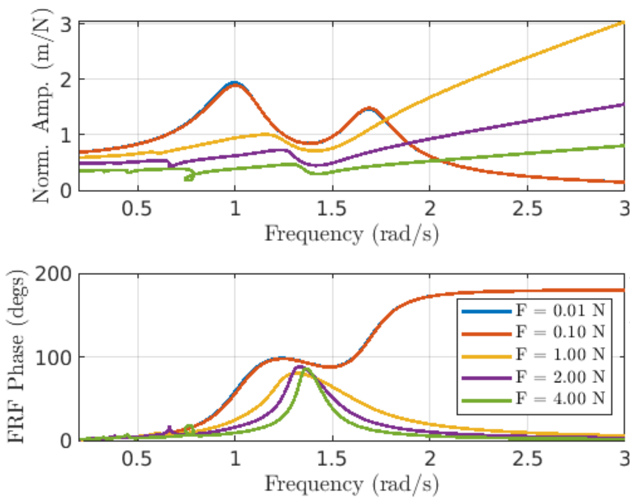
(a) (b)

Figure 1: Comparisons between transient simulations and Harmonic Balance for Forced Response

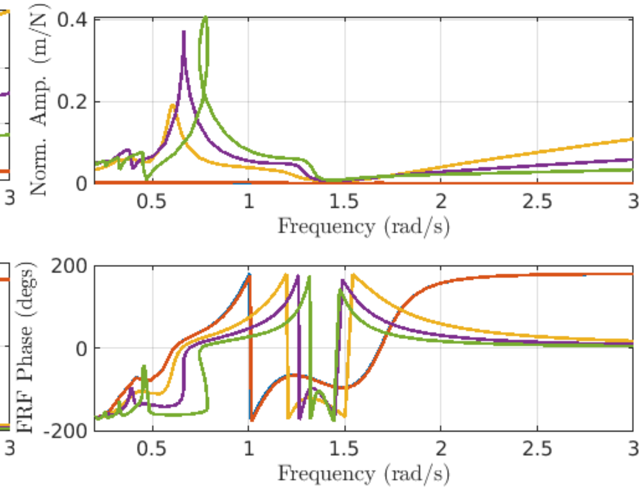
Table 1: Parameters Used

Parameter	Value
β	$0.1\,Nsm^{-1}$
α	$1\,Nm^{-1}$
γ	$0\,Ns^3m^{-3}$
μN	$0.5\,N$
Ω	$0.9\,rad/s$
F	$10\,N$

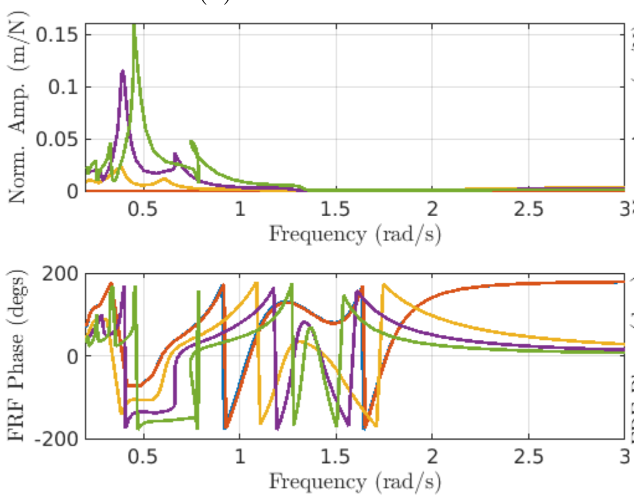
2 Forced Response Results



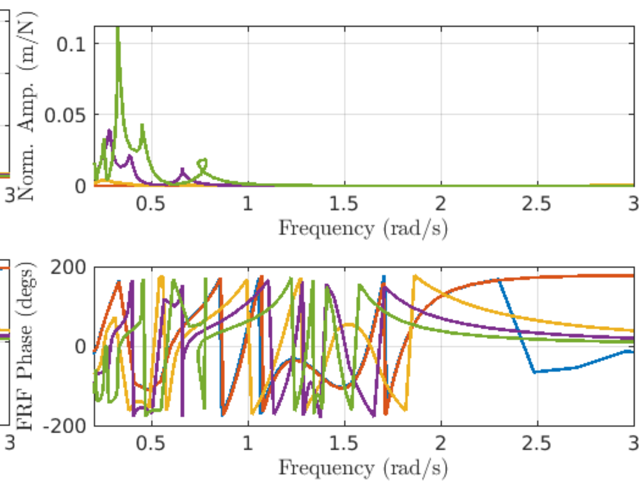
(a) First Harmonic



(b) Third Harmonic



(c) Fifth Harmonic



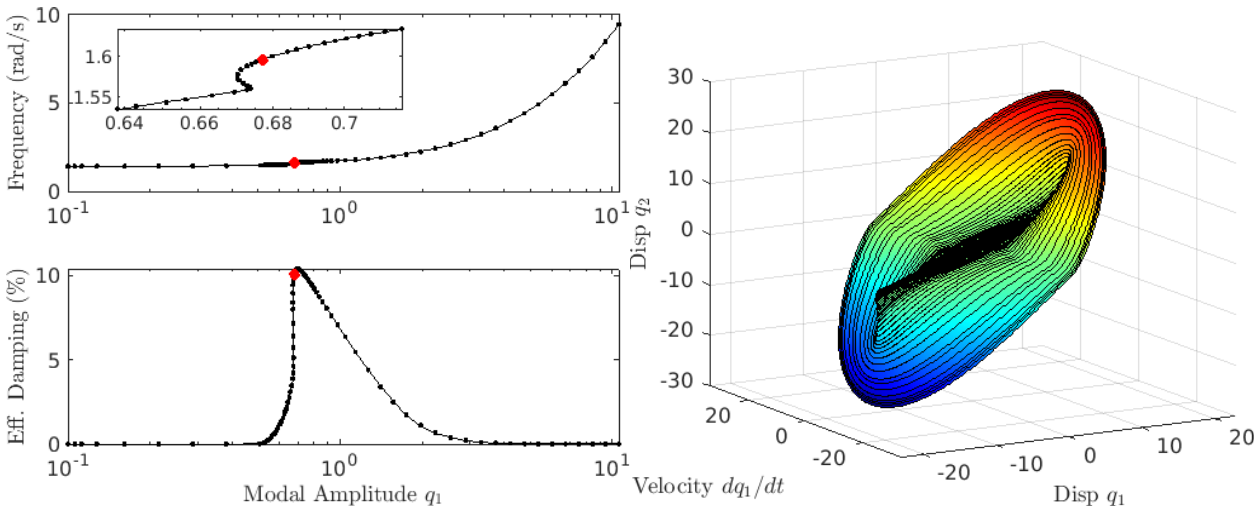
(d) Seventh Harmonic

Figure 2: Different harmonic components of system response (plotted, x_1) to steady state forcing (a total of 33 harmonics were considered for the balance)

Table 2: Parameters Used

Parameter	Value
β	$0.1\,Nsm^{-1}$
α	$1\,Nm^{-1}$
γ	$0\,Ns^3m^{-3}$
μN	$0.5\,N$
Ω	$[0.1,\,3.0]\,rad/s$
F	$[0.01,\,4.0]\,N$

3 EPMC Results



(a) Backbone (b) "Invariant" Manifold

Figure 3: Backbone and "invariant" manifold computed using EPMC (33 harmoincs considered for the balance). Manifold plotted until the red colored point in the backbone.

Table 3: Parameters Used

Parameter	**Value**
β	$0.1\,Nsm^{-1}$
α	$1\,Nm^{-1}$
γ	$0\,Ns^3m^{-3}$
μN	$0.5\,N$
Ω	N/A
F	N/A

4 Influence of Harmonic Truncation

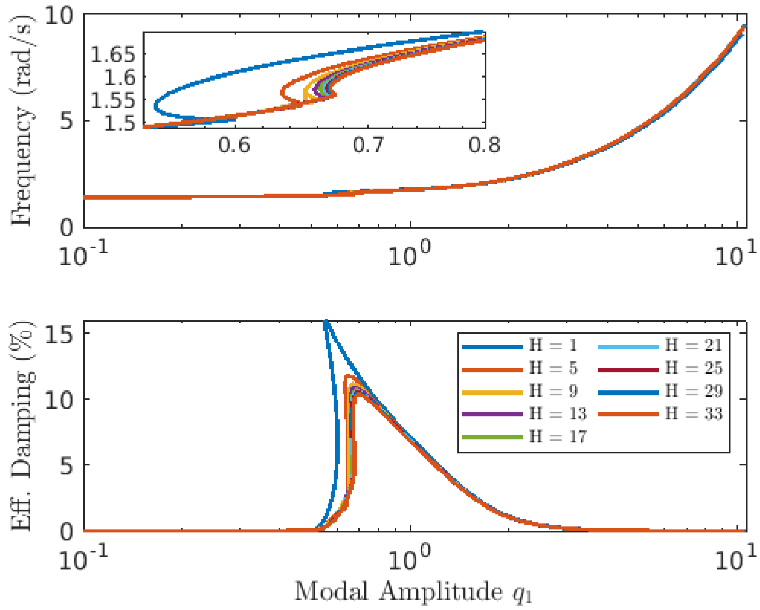


Figure 4: Influence of Harmonic Truncation on the Computed Backbones

Table 4: Parameters Used

Parameter	**Value**
β	$0.1\,Nsm^{-1}$
α	$1\,Nm^{-1}$
γ	$0\,Ns^3m^{-3}$
μN	$0.5\,N$
Ω	N/A
F	N/A

5 Influence of Parameters

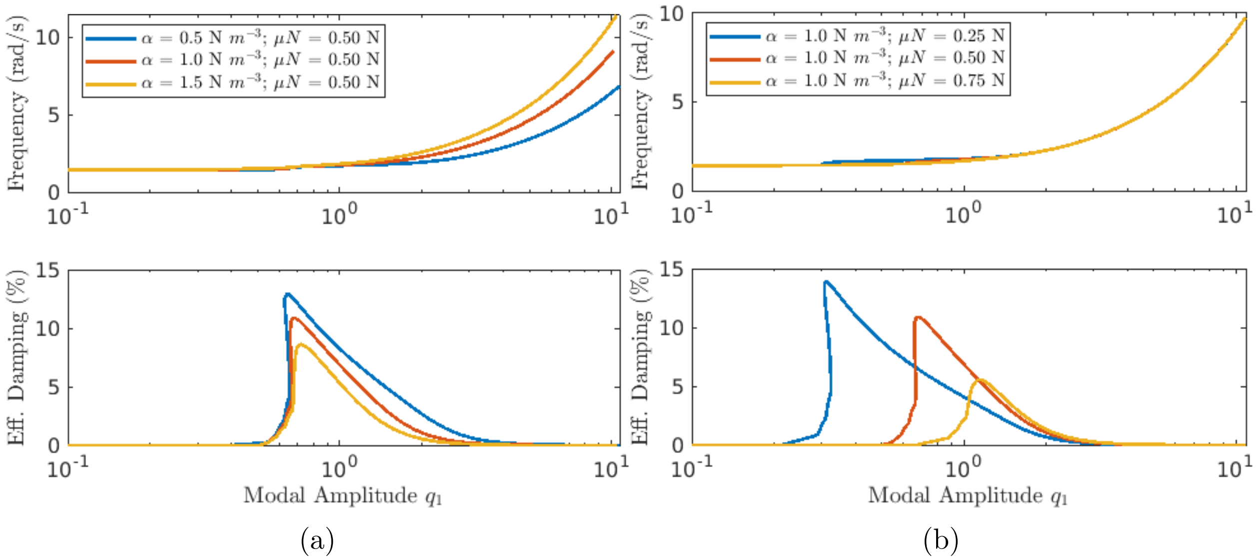


Figure 5: Influence of (a) α and (b) μN on the backbones when the other is kept constant (13 harmonics used for balance)

Table 5: Parameters Used

Parameter	**Value**
β	$0.1\,Nsm^{-1}$
α	$[0.5,\ 1.5]\,Nm^{-1}$
γ	$0\,Ns^3m^{-3}$
μN	$[0.25\,0.75]\,N$
Ω	N/A
F	N/A

6 Friction-Only and Cubic Spring-only Backbones

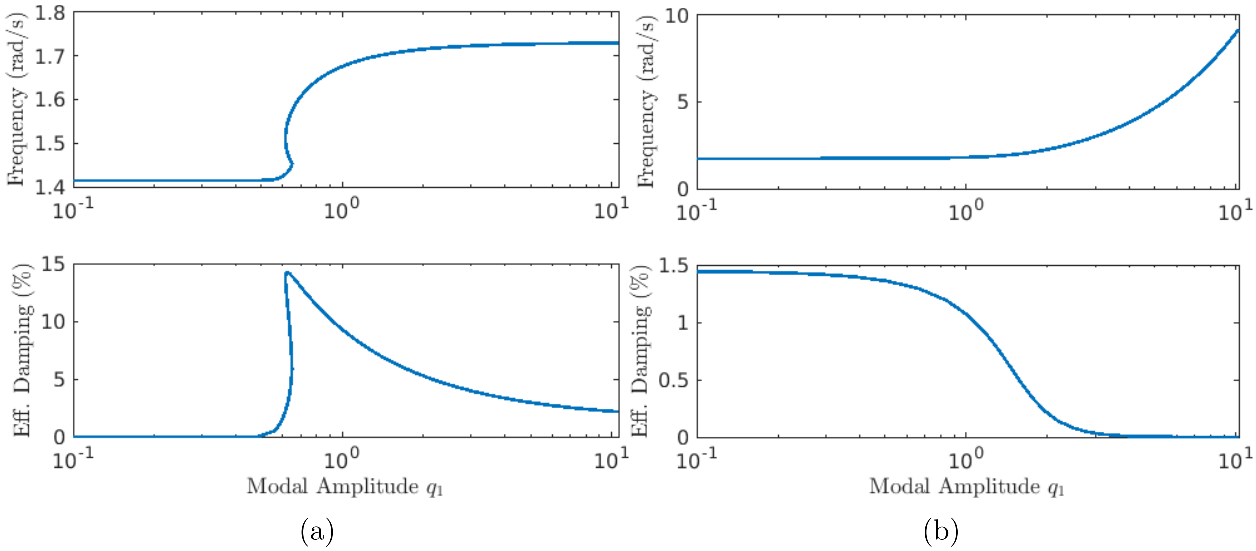


Figure 6: Backbones for (a) friction-only ($\alpha = \gamma = 0$) and (b) cubic spring only ($\mu N = \gamma = 0$) cases (33 harmonics used for balance)

The remaining parameters are identical to before.