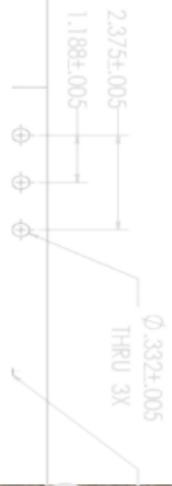


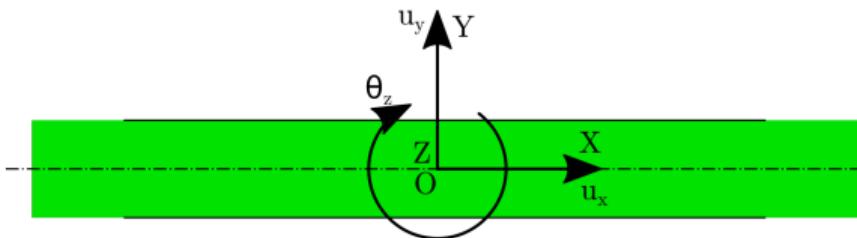
Wave-Based Modeling for Jointed Structures

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Dispersion of the Timoshenko Beam I



- ▶ The coordinate systems that will be used here are given in the figure above. The beam is taken to be in the XY plane and the degrees of freedom for each point on the beam are taken to be the displacements u_x , u_y , and rotation θ_z .
- ▶ The equations of motion used for the Timoshenko beam are:

$$\begin{bmatrix} \rho A & 0 & 0 \\ 0 & \rho A & 0 \\ 0 & 0 & \rho I_z \end{bmatrix} \frac{\partial^2}{\partial t^2} \begin{Bmatrix} u_x \\ u_y \\ \theta_z \end{Bmatrix} - \begin{bmatrix} EA & 0 & 0 \\ 0 & GA & 0 \\ 0 & 0 & EI_z \end{bmatrix} \frac{\partial^2}{\partial x^2} \begin{Bmatrix} u_x \\ u_y \\ \theta_z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & GA \\ 0 & -GA & 0 \end{bmatrix} \frac{\partial}{\partial x} \begin{Bmatrix} u_x \\ u_y \\ \theta_z \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & GA \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ \theta_z \end{Bmatrix} = \begin{Bmatrix} f_x \\ f_y \\ m_z \end{Bmatrix}.$$

Dispersion of the Timoshenko Beam II

- ▶ Dispersion relationships will be discussed using the wave representation $\bar{u} = \bar{U}e^{i(\kappa x - \omega t)}$.
- ▶ The dispersion relationship for the axial vibration (longitudinal waves) is

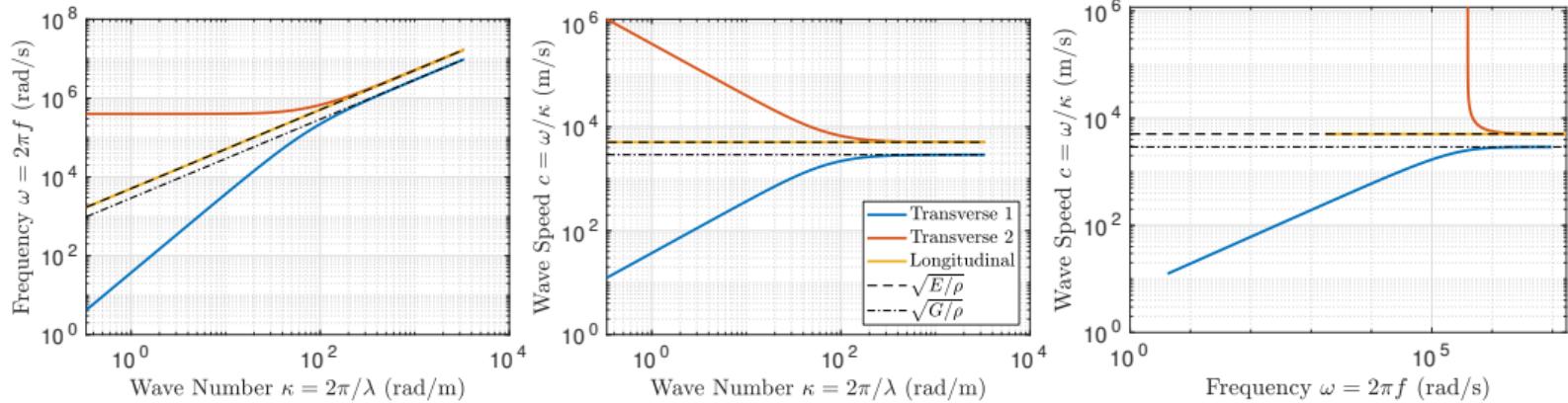
$$\omega = \sqrt{\frac{E}{\rho}} \kappa$$

- ▶ For bending vibration (transverse waves), the dispersion relationship is

$$(\rho^2 A I_z) \omega^4 - [\rho G A^2 + \kappa^2 \rho A I_z (G + E)] \omega^2 + (E G A I_z) \kappa^4 = 0$$

Dispersion of the Timoshenko Beam III

- The dispersion relationship is presented below graphically:



- Asymptotes corresponding to the case $\omega \rightarrow \infty$ are depicted using dashed lines in the figure