Balancing zero harmonics in the Wave-Based Formalism

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I will take the example of the Euler Bernoulli Beam whose dispersion relationship is given by

$$k = \left(\frac{\rho A_r}{E_y I_y} w^2 \right)^{1/4}. \tag{1}$$

The solution is expanded through the wave-based formalism as,

$$u(x,t) = (a_1^+ e^{kx} + a_1^- e^{-kx} + a_2^+ e^{ikx} + a_2^- e^{-ikx}) e^{-i\omega t} + c.c., \tag{2}$$

where $a_1^\pm$ and $a_2^\pm$ are the evanescent and traveling wave components respectively.

Considering a joint between two points, say A and B, with relative displacement denoted by $\delta u(x,t)|_{x=x_J}$, the joint force balance equation can be written as,

$$V(x,t)|_{x=x_J} = -E_y I_y \frac{\partial^3 u}{\partial x^3} = f_{joint}(\delta u), \tag{3}$$

where $f_{joint}(\cdot)$ is the joint constitutive model, $E_y I_y$ is the flexural rigidity, and $V(x,t)|_{x=x_J}$ is the shear force at the joint.

Substituting the wave-based solution ansatz from eq. 2 into eq. 3 yields,

$$-E_y I_y k(\omega)^3 \begin{bmatrix} 1 & -1 & -i & i & 0 & 0 & 0 & 0 \\ 1 & -1 & -i & i & -1 & 1 & i & -i \end{bmatrix} \begin{bmatrix} a_1^+ \\ a_1^- \\ a_2^+ \\ a_2^- \\ b_1^+ \\ b_1^- \\ b_2^+ \\ b_2^- \end{bmatrix} = \begin{bmatrix} f_{joint}(\delta u) \\ 0 \end{bmatrix}. \tag{4}$$

In the above vector equation, the first row equation represents the force balance from eq. 3 and the second row equation represents the fact that the interface force is identical across the joints.

Question

When ω is set to zero, $k(\omega) = 0$ (see eq. 1). As a consequence, eq. 4 represents a null expression (matrix in lhs multiplied by zero). In other words, this **requires** the static component of the joint force to be zero. Does this make sense or is something wrong? **What if I have a joint that provides a static force? How can I represent this in this formalism?** This also leads to the more general question of **can the wave based approach accommodate constant terms in the PDE?**