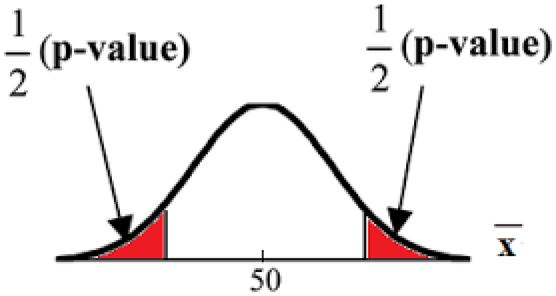
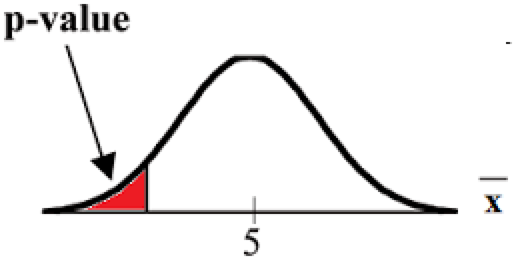
STATISTICAL INFERENCE

HYPOTHESIS TESTING

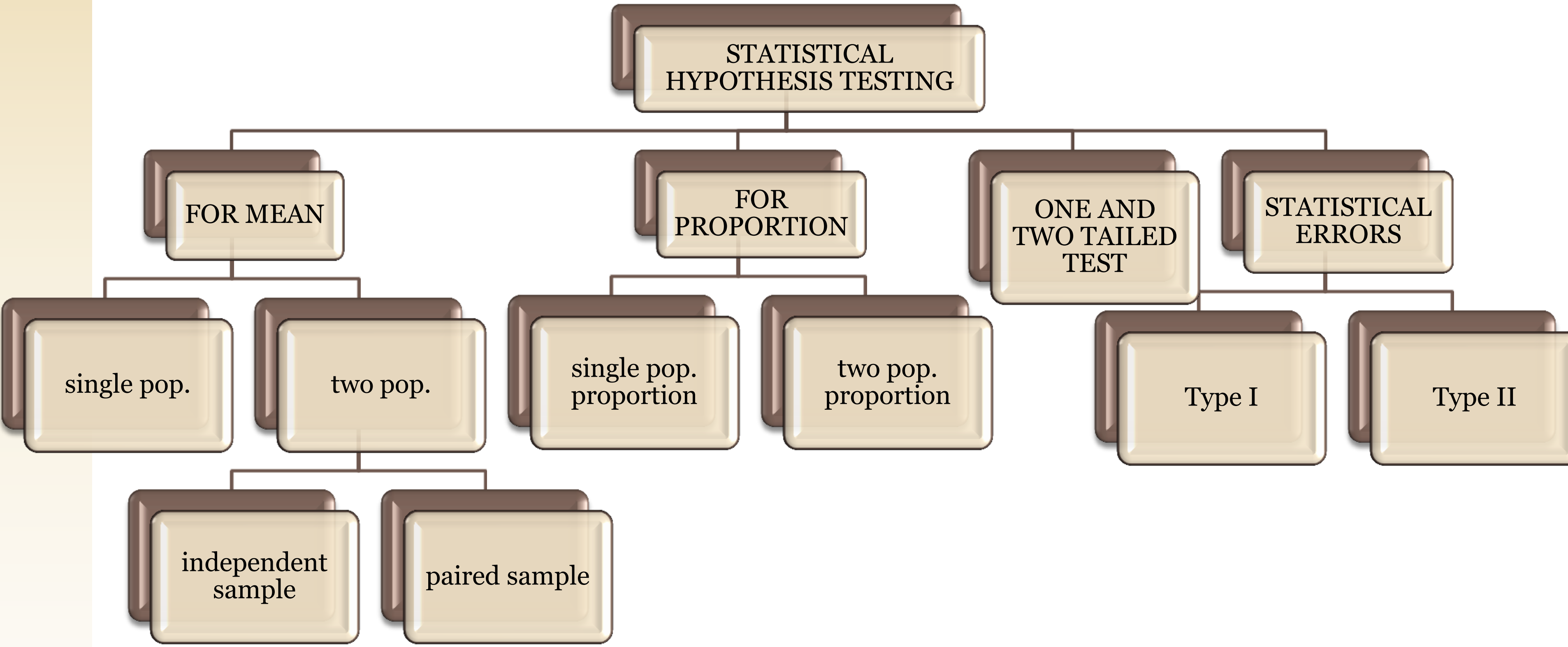
STATISTICAL INFERENCE

HYPOTHESIS TESTING:
SINGLE POPULATION PARAMETER
TWO POPULATION PARAMETER





WHAT WE WILL DISCUSS IN HYPOTHESIS TESTING?



RECAP: INFERENTIAL STATISTICS

ESTIMATION

- "guessing" the value of the parameter
- provide the measure of the quality (reliability) of the guess

HYPOTHESIS TESTING

- making a "yes-no" decision regarding the parameter
- understand the chances of making incorrect decision

WHY YOU NEED TO KNOW ABOUT HYPOTHESIS TESTING?

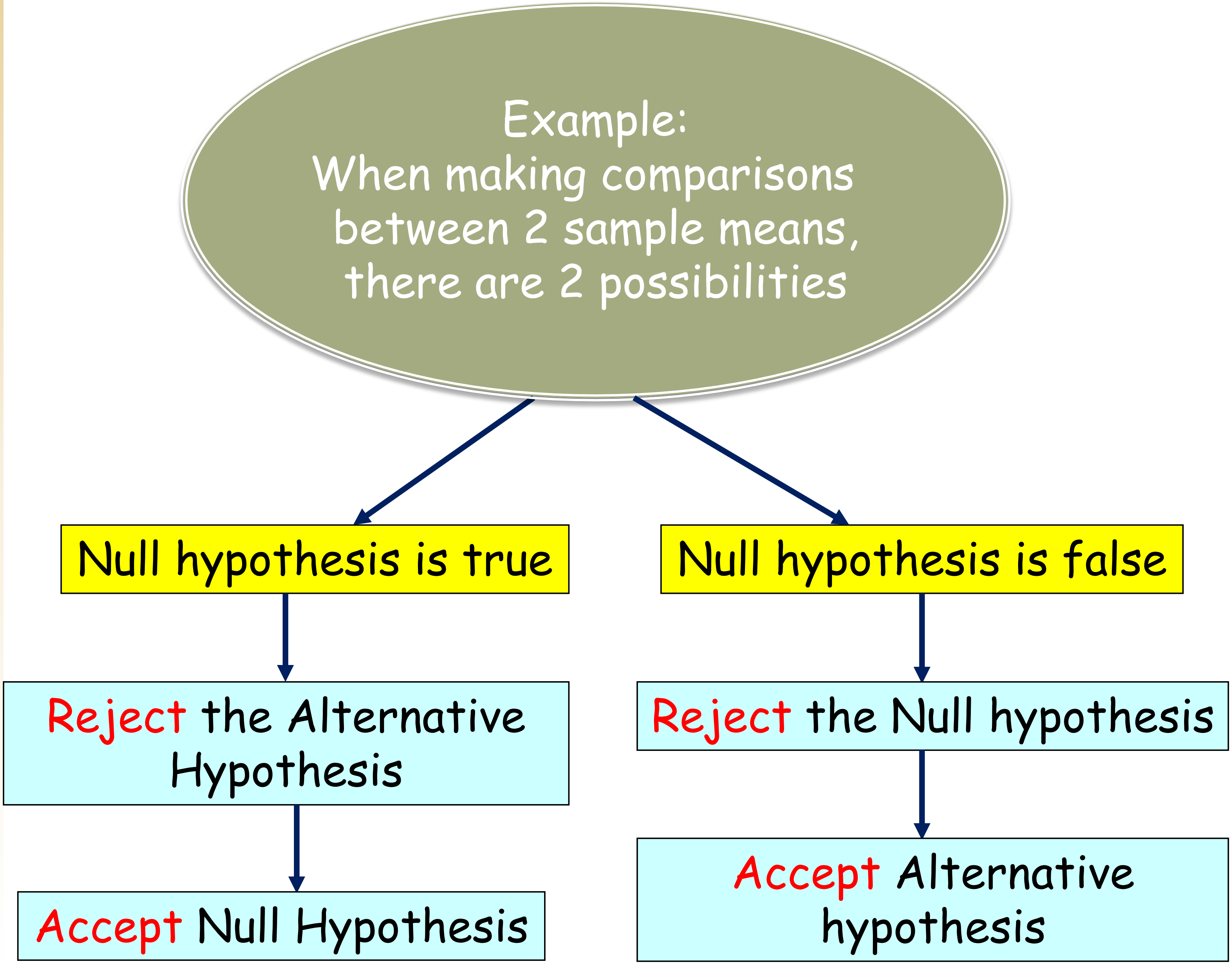
- Hypothesis testing is performed regularly in many industries.
  - Example: Companies in the pharmaceutical industry must perform many hypothesis tests on new drug products before they are deemed to be safe and effective by the federal Food and Drug Administration (FDA).

- Describe a relationship among variables
  - Can state that as one variable increases, the other will decrease; as one variables increases, the other will increase, and so on.

FORMULATING THE HYPOTHESIS

Two hypotheses are formulated:

- Null Hypothesis (H_o):
  - The statement about the population parameter that will be assumed to be true during the conduct of the hypothesis test.
  - The null hypothesis will be rejected only if the sample data provide substantial contradictory evidence.
  - contains an equality sign, such as "=", "≤" or "≥"
- Alternative Hypothesis (H_A):
  - The alternative hypothesis will be selected only if there is strong enough sample evidence to support it.
  - The alternative hypothesis is true if the null hypothesis is rejected.


Example:
When making comparisons between 2 sample means, there are 2 possibilities
Null hypothesis is true
Reject the Alternative Hypothesis
Accept Null Hypothesis
Null hypothesis is false
Reject the Null hypothesis
Accept Alternative hypothesis

TYPES OF HYPOTHESIS

Type of Hypothesis Test	Null Hypothesis	Alternative Hypothesis
Comparison of samples	The samples are the same; the samples come from the same population; the characteristics of the samples differ only because of sampling randomness	The samples are fundamentally different
Correlation	There is no significant correlation between the variables	The correlation is statistically significant
Normality	The data are normally distributed	The data are not normally distributed

FORMULATING THE HYPOTHESIS

Example:

In today's economy, university students often work many hours to help pay for the high costs of a college education. Suppose a university in the Midwest is considering changing its class schedule to accommodate students working long hours. The registrar has stated a change is needed because the mean number of hours worked by undergraduate students at the university is more than 20 per week.

Steps to solution:

1. **Determine the population parameter of interest.**

Population parameter is mean, μ number of hours worked by undergraduate students

2. **Identify the hypothesis of interest**

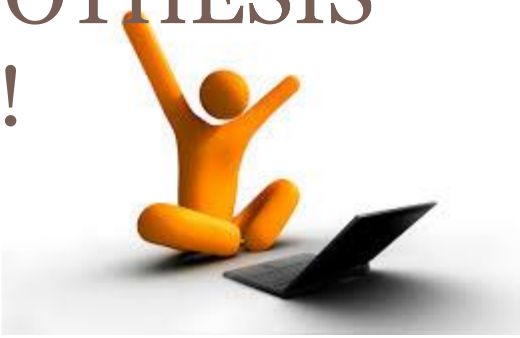
The registrar has made a claim that the mean hours worked "is more than 20" per week. The hypothesis will not be declared true unless the sample data strongly indicate that it is true. Thus, the burden of proof is placed on the registrar to justify her claim that the mean is greater than 20 hours.

3. **Formulate the null and the alternative hypothesis**

$$H_0 \leq 20 \text{ hours}$$

$$H_A > 20 \text{ hours (claim)}$$

FORMULATING THE HYPOTHESIS
LETS PRACTICE!!



Example 1:
The Frito-Lay Company produces several snack and food products that are sold throughout the United States and around the world. The company uses an automatic filling machine to fill the sacks with the desired weight. For instance, when the company is running potato chips on the fill line, the machine is set to fill the sacks with 20 ounces. Thus, if the machine is working properly, the mean fill will be 20 ounces. Each hour, a sample of sacks is collected and weighed, and the technicians determine whether the machine is still operating correctly or whether it needs adjustment.

Example 2:
The director of a state agency believes that the average starting salary for clerical employees in the state is less than $30,000 per year. To test her hypothesis, she has collected a simple random sample of 100 starting clerical salaries from across the state and found that the sample mean is $29,750. State the appropriate null and alternative hypotheses.

STATISTICAL HYPOTHESIS TESTING

If for;

- Single pop. mean, the sample mean ($\bar{x}$) is used to test the hypothesis: either σ known or σ unknown
- Single pop. proportion, the sample statistics ($\bar{p}$) is used to test the hypothesis

- Two pop. mean (with independent sample), the sample mean ($\bar{x}$) is used to test the hypothesis: either (σ_1 and σ_2) known or (σ_1 and σ_2) unknown
- Two pop. mean (with paired sample), the sample mean ($\bar{x}$) is used to test the hypothesis: either (σ_1 and σ_2) known or (σ_1 and σ_2) unknown
- Two pop. proportion, the sample statistics ($\bar{p}$) is used to test the hypothesis

STATISTICAL HYPOTHESIS TESTING: SIGNIFICANCE LEVEL AND CRITICAL VALUE

- We need to select the *cutoff* point that is as the separation between rejecting and not rejecting the null hypothesis.
- Significance level:
  - The maximum allowable probability of committing a Type I statistical error.
  - The probability is denoted by the symbol α.
- Critical value:
  - Having chosen a significance level, α, the decision maker then must calculate the corresponding cutoff point.
    - The value corresponding to a significance level that determines those test statistics that lead to rejecting the null hypothesis and those that lead to a decision not to reject the null hypothesis.

STATISTICAL HYPOTHESIS TESTING: DECISION RULES AND TEST STATISTICS

- Test statistics:
  - A function of the sampled observations that provides a basis for testing a statistical hypothesis.
    - The calculated *z*-value is an example of test statistics.

- Decision rules: You can use three equivalent approaches: ** does not make no difference as each method yields the same conclusion
  1. calculate a *z*-value and compare it to the critical value, z_α
  2. calculate a sample mean, $\bar{x}$ and compare it to the critical value, $\bar{x}_\alpha$
  3. use the method of p-value approach

z-Test Statistic for Hypothesis Tests for μ,σ Known

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where:

$\bar{x}$ = Sample mean
μ = Hypothesized value for the population mean
σ = Population standard deviation
n = Sample size

$\bar{x}_\alpha$ for Hypothesis Tests for μ,σ Known

$$\bar{x}_\alpha = \mu + z_\alpha \frac{\sigma}{\sqrt{n}}$$

where:

μ = Hypothesized value for the population mean
z_α = Critical value from the standard normal distribution
σ = Population standard deviation
n = Sample size

STATISTICAL HYPOTHESIS TESTING: ONE TAILED OR TWO TAILED

Two type of hypothesis tests:

- **one-tailed test**: the entire rejection region is located in one tail (either upper tail or lower tail) of sampling distribution.

Example:

$H_0 : \mu \leq 25$ days

$H_A : \mu > 25$ days

the entire region is located in the upper tail & the null hypothesis will be rejected only when the sample mean falls in the extreme upper tail of the sample distribution

- **two-tailed test**: the entire rejection region is split into two tails (upper tail and lower tail) of sampling distribution.

Example:

$H_0 : \mu = 25$ days

$H_A : \mu \neq 25$ days

The null hypothesis will be rejected only when the sample mean falls in the extreme upper tail or extremely small (lower tail) of the sample distribution

STATISTICAL HYPOTHESIS TESTING: DECISION RULES AND TEST STATISTICS

one-tailed test with lower tail

- **Approach 1**: calculate a *z*-value and compare it to the critical value, z_α
  - if the -*z*-value ≥ critical value, $-z_\alpha$ therefore accept H_o
  - if the -*z*-value < critical value, $-z_\alpha$ therefore reject H_o

- **Approach 2**: calculate a sample mean, $\bar{x}$ and compare it to the critical value, $\bar{x}_\alpha$
  - if the -$\bar{x}$ ≥ critical value, -$\bar{x}_\alpha$ therefore accept H_o
  - if the -$\bar{x}$ < critical value, -$\bar{x}_\alpha$ therefore reject H_o

- **Approach 3**: use the method of p-value approach and compare it to the probability in the rejection region, α
  - if the calculated –(p-value) ≥ probability in the rejection region, α therefore accept (do not reject) H_o
  - if the calculated – (p-value) < probability in the rejection region, α therefore reject H_o

STATISTICAL HYPOTHESIS TESTING: DECISION RULES AND TEST STATISTICS

one-tailed test with upper tail

- **Approach 1**: calculate a z-value and compare it to the critical value, z_α
  - if the z-value $\geq$ critical value, z_α therefore reject H_o
  - if the z-value $<$ critical value, z_α therefore accept H_o

- **Approach 2**: calculate a sample mean, $\bar{x}$ and compare it to the critical value, $\bar{x}_\alpha$
  - if the $\bar{x} \geq$ critical value, $\bar{x}_\alpha$ therefore reject H_o
  - if the $\bar{x}$ $<$ critical value,$\bar{x}_\alpha$ therefore accept H_o

- **Approach 3**: use the method of p-value approach and compare it to the probability in the rejection region, α
  - if the calculated p-value $\geq$ probability in the rejection region, α therefore accept (do not reject) H_o
  - if the calculated p-value $<$ probability in the rejection region, α therefore reject H_o

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

Example 1 (one-tailed hypothesis test for μ, σ known)
The ABX Specialist Hospital in United State, performs many knee replacement surgery procedures each year. Recently, research physicians at that hospital have developed a surgery process they believe will reduce the average patient recovery time. The hospital board will not recommend the new procedure unless there is substantial evidence to suggest that it is better than the existing procedure. Records indicate that the current mean recovery rate for the standard procedure is 142 days, with a standard deviation of 15 days. To test whether the new procedure actually results in a lower mean recovery time, the procedure was performed on a random sample of 36 patients and the $\bar{x}$ is assume 140.2 days ~ 140 days. The researchers wish to test the hypothesis using a 0.05 level of significance.

Steps to solution:
Hypothesis claim: new procedure of knee replacement gives lower mean recovery time (μ < 142 days)
Formulate the null and the alternative hypothesis

$$H_0 : \mu \geq 142 \text{ days}$$
$$H_A : \mu < 142 \text{ days (claim)}$$

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

σ = 15 days

Construct the rejection region and decision rule
This is one-tailed test with lower tail (left hand) of sampling distribution. With 0.05 level of significance, the critical value, $-z_{0.05} = -1.645$

Compute the test statistic
This problem will use z-test statistics.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{140.2 - 142}{\frac{15}{\sqrt{36}}} = -0.72$$

The decision rule: If $z < -1.645$, then reject H_o, otherwise accept H_o
Since $-0.72 > -1.645$, therefore H_o is accepted.

Draw a conclusion
There is no sufficient evidence to conclude that the new knee replacement procedure results in a shorter average recovery period. Thus, ABX Specialist Hospital will not be able to recommend the new procedure on the grounds that it reduces recovery time.

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM (LETS PRACTICE)

Example 1 (one-tailed hypothesis test for μ, σ known)

The Testing Center in Southern California creates standardized exams for a variety of quantitative disciplines, including business statistics. Recently the Testing Center received complaints from faculty who have used its latest business statistics test saying the mean time required to complete the exam exceeds the advertised mean of 40 minutes. Before responding, employees at the Testing Center plan to test this claim using an alpha level equal to 0.05 and a random sample size of n 100 business statistics students. Suppose that the sample of 100 students produced a sample mean of 43.5 minutes. Based on previous studies, suppose that the population standard deviation is known to be $\sigma = 8$ minutes.

Find the Answer:

a) **Hypothesis claim:**
b) **Formulate the null and the alternative hypothesis**
c) **Construct the rejection region and decision rule**
d) **Compute the test statistic**
e) **Identify the decision rule**
f) **Draw a conclusion**

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM (LETS PRACTICE)

Example 1 (one-tailed hypothesis test for μ, σ known)

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The Answer:

Hypothesis claim: The new claim states the mean time required to complete the exam exceeds (>) the advertised mean of 40 minutes

Formulate the null and the alternative hypothesis

$$H_0 : \mu \leq 40 \text{ minutes}$$

$$H_A : \mu > 40 \text{ minutes (claim)}$$

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

σ = *8 minutes*

Construct the rejection region and decision rule
This is one-tailed test with upper tail (right hand) of sampling distribution. With 0.05 level of significance, the critical value, $z_{0.05}$= 1.645

Compute the test statistic
This problem will use $\bar{x}$statistics.

$$\bar{x}_\alpha = \mu + z_\alpha \frac{\sigma}{\sqrt{n}} = 40 + 1.645 \frac{8}{\sqrt{100}} = 41.32$$

The decision rule: if the $\bar{x}$ < critical value, $\bar{x}_\alpha$ therefore accept H_o, otherwise reject

Since 43.5 > 41.32, therefore H_o is rejected.

Draw a conclusion
There is sufficient evidence to conclude that the mean time required to complete the exam exceeds the advertised time of 40 minutes. The Testing Center will likely want to modify the exam to shorten the average completion time.

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM (LETS PRACTICE)

Example 2 (two-tailed hypothesis test for μ, σ known)

The ABA Corporation is a wood products company with lumber, plywood, and paper plants in several areas of Malaysia. At its ABX subsidiaries, plywood plant, the company makes plywood used in residential and commercial building. One product made at the plant is 3/8-inch plywood which must have a mean thickness of 0.375 inches. The standard deviation, σ, is known to be 0.05 inch. Before sending a shipment to customers, the managers test whether they are meeting the 0.375 inch requirements by selecting a random sample of n 100 sheets of plywood and collecting thickness measurements. Suppose the sample mean for the random sample of 100 measurement is 0.378 inch. Construct the statistical hypothesis test with significance level, $\alpha = 0.05$ and help the manager to state the decision.

Find the Answer:

- **a) Hypothesis claim:**
- **b) Formulate the null and the alternative hypothesis**
- **c) Construct the rejection region and decision rule**
- **d) Compute the test statistic**
- **e) Identify the decision rule**
- **f) Draw a conclusion**

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM (LETS PRACTICE)

Example 2 (two-tailed hypothesis test for μ, σ known)
The ABA Corporation is a wood products company with lumber, plywood, and paper plants in several areas of Malaysia. At its ABX subsidiaries, plywood plant, the company makes plywood used in residential and commercial building. One product made at the plant is 3/8-inch plywood which must have a mean thickness of 0.375 inches. The standard deviation, σ, is known to be 0.05 inch. Before sending a shipment to customers, the managers test whether they are meeting the 0.375 inch requirements by selecting a random sample of n 100 sheets of plywood and collecting thickness measurements. Suppose the sample mean for the random sample of 100 measurement is 0.378 inch. Construct the statistical hypothesis test with significance level, $\alpha = 0.05$ and help the manager to state the decision.

The Answer:

Hypothesis claim: The 3/8-inch plywood which must have a mean thickness of 0.375 inches.

Formulate the null and the alternative hypothesis

$$H_0 : \mu = 0.375 \text{ inches (claim)}$$

$$H_A : \mu \neq 0.375 \text{ inches}$$

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

σ = *0.05 inches*

Construct the rejection region and decision rule

This is two-tailed test. With 0.05 level of significance, the critical value, for upper tail $z_{\alpha/2} = z_{0.05/2} = z_{0.025} = 1.96$ and for lower tail $-z_{\alpha/2} = -z_{0.05/2} = -z_{0.025} = -1.96$

Compute the test statistic

This problem will use z-test statistics.

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{0.378 - 0.375}{\frac{0.05}{\sqrt{100}}} = 0.6$$

The decision rule: if the z_{cal} < critical value, z_{α} therefore accept H_o, otherwise reject

Since $-1.96 < z = 0.60 < 1.96$ (two-tailed test), therefore H_o is accepted.

Draw a conclusion

There is sufficient evidence that the product met the 0.375 inch requirements. Therefore, they can continue with the shipment of the plywood.

t-Test Statistic for Hypothesis Tests for μ,σ unknown

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where:

$\bar{x}$ = Sample mean
μ = Hypothesized value for the population mean
s = Sample standard deviation, $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$
n = Sample size

with degree of freedom: *n-1*

to employ t-distribution, we must follow the assumption:
"the population is normally distributed"

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

Example 1 (hypothesis test for μ, σ unknown)

The Dairy Fresh Ice Cream uses a filling machine for its 64-ounce cartons. There is some variation in the actual amount of ice cream that goes into the carton. The machine can go out of adjustment and put a mean amount either less or more than 64 ounces in the cartons. To monitor the filling process, the production manager selects a simple random sample of 16 filled ice cream cartons each day. The test is conducted using $\alpha = 0.05$From the hypothesis test, the manager wants to know either the machine is in adjustment or not. Since, the standard deviation is unknown, the manager have run the pilot sample and the data is as follows:

62.7	64.7	64.0	64.5	64.6	65.0	64.4	64.2
64.6	65.5	63.6	64.7	64.0	64.2	63.0	63.6

Steps to solution:

Hypothesis claim: The machine can go out of adjustment and put a mean amount either less or more than 64 ounces in the cartons.

Formulate the null and the alternative hypothesis

$H_0 : \mu = 64$ ounce (machine is in adjustment)

$H_A : \mu \neq 64$ ounce (machine is out of adjustment)

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

Construct the rejection region and decision rule
This is two-tailed test. With 0.05 level of significance. The degree of freedom for t-distribution is: n-1= 16-1=15 degree of freedom. Therefore, $t = \pm 2.131$
Decision rule: If -$t < -2.131$ *or* $t > 2.131$, reject the H_o.

Compute the test statistic
This problem will use t-test statistics.

The sample mean is

$$\bar{x} = \frac{\sum x}{n} = \frac{1{,}027.3}{16} = 64.2$$

The sample standard deviation is

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n-1}} = 0.72$$

The *t*-test statistic, using Equation 9.3, is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{64.2 - 64}{\frac{0.72}{\sqrt{16}}} = 1.11$$

The decision rule: because -t= -1.11 is greater than -2.131 and 1.11 is not greater than 2.131, therefore H_o is accepted.

STATISTICAL HYPOTHESIS TESTING: EXAMPLE OF PROBLEM

Draw a conclusion

Based on these sample data, the company does not have sufficient evidence to conclude that the filling machine is out of adjustment.

HYPOTHESIS TEST FOR SINGLE POPULATION PROPORTION

- Previously: hypothesis testing about single population mean.
- Why need hypothesis test for proportion?
  - Example: A production manager might consider the proportion of defective items produced on an assembly line to determine whether the line should be restructured.
  - Example: A life insurance salesperson's performance assessment might include the proportion of existing clients who renew their policies.
- The basic concepts of hypothesis testing for proportions are the same as for means.

HYPOTHESIS TEST FOR SINGLE POPULATION PROPORTION

$\bar{p}_\alpha$- critical value for Proportions (approach 2)

$$\bar{p}_\alpha = p + z_\alpha \sqrt{\frac{p(1-p)}{n}}$$

z test-statistics for Proportions (approach 1)

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

where:
$\bar{p}$ = sample proportion
n = sample size
p = hypothesized population proportion
Z_α= z critical value

Approach 2 : calculate $\bar{p}$ value and compare it to the critical value, $\bar{p}_\alpha$

- if the $\bar{p}$ value $\geq$ critical value,$\bar{p}_\alpha$therefore reject H_o
- if the $\bar{p}$ value < critical value,$\bar{p}_\alpha$therefore accept (do not reject) H_o

Approach 3: p-value (same as testing for population mean)

STATISTICAL HYPOTHESIS TESTING: DECISION RULES AND TEST STATISTICS

one-tailed test with lower tail

- **Approach 1**: calculate a z-value and compare it to the critical value, z_α
  - if the -z-value ≥ critical value, $-z_\alpha$ therefore accept H_o
  - if the -z-value < critical value, $-z_\alpha$ therefore reject H_o

- **Approach 2** : calculate $\bar{p}$ value and compare it to the critical value, $\bar{p}_\alpha$
  - if the -$\bar{p}$ value ≥ critical value,$\bar{p}_\alpha$therefore accept H_o
  - if the -$\bar{p}$ value < critical value,$\bar{p}_\alpha$therefore reject H_o

- **Approach 3**: use the method of p-value approach and compare it to the probability in the rejection region, α
  - if the calculated –(p-value) ≥ probability in the rejection region, α therefore accept (do not reject) H_o
  - if the calculated – (p-value) < probability in the rejection region, α therefore reject H_o

STATISTICAL HYPOTHESIS TESTING: DECISION RULES AND TEST STATISTICS

one-tailed test with upper tail

- **Approach 1**: calculate a z-value and compare it to the critical value, z_α
  - if the z-value $\geq$ critical value, z_α therefore reject H_o
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  - if the calculated p-value $\geq$ probability in the rejection region, α therefore accept (do not reject) H_o
  - if the calculated p-value < probability in the rejection region, α therefore reject H_o

HYPOTHESIS TEST FOR SINGLE POPULATION PROPORTION
EXAMPLE OF PROBLEM (LETS PRACTICE)

Example (hypothesis testing for proportion)
The NBA started a new professional basketball league called the Developmental League (D League), where players who were not on NBA rosters could fine-tune their skills in hopes of getting called up to the NBA. The teams in this league are privately owned but connected to NBA teams. One of the D-League's teams is considering increasing the season ticket prices for basketball games. The marketing manager is **concerned** that some people will terminate their ticket orders if this change occurs. If more than 10% of the season ticket orders would be terminated, the marketing manager does not want to implement the price increase. To test this, a random sample of ticket holders is surveyed and asked what they would do if the prices were increased. The random sample of $n = 100$ season ticket holders showed that 14 would cancel their ticket orders if the price change were implemented. The hypothesis testing is carried with significance level, $\alpha = 0.05$

The Solution:

Hypothesis claim: population of proportion that season ticket holder would terminate their ticket orders if price increasing

Formulate the null and the alternative hypothesis

$$H_0 : p \leq 0.10$$
$$H_A : p > 0.10 \text{ (claim)}$$

Construct the rejection region and decision rule

This is one-tailed test. With 0.05 level of significance, the critical value, for upper tail $z_{\alpha} = z_{0.05} = 1.645$

Compute the test statistic

This problem will use z-test statistics for population proportion.

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

need to calculate $\bar{p} = \frac{x}{n}$, where the random sample of n = 100 season ticket holders showed that 14 would cancel their ticket orders if the price change were implemented.

Therefore, $\bar{p} = \frac{14}{100} = 0.14$

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.14 - 0.10}{\sqrt{\frac{0.10(1-0.10)}{100}}} = \frac{0.04}{0.03} = 1.33$$

The decision rule: if the z_{cal} < critical value, z_{α} therefore accept H_o , otherwise reject

Since, 1.33< 1.645, therefore accept H_o

Draw a conclusion

Based on the sample data, the marketing manager does not have sufficient evidence to conclude that more than 10% of the season ticket holders will cancel their ticket orders

What happen if the decision rule is based from $\bar{p}_{\alpha}$- critical value (p-value) for proportions. Will it gives the same conclusion?

STATISTICAL ERRORS

- Because of the potential for extreme sampling error, two possible errors can occur when a hypothesis is tested:
  - Type I: Rejecting the null hypothesis when it is, in fact, true.
  - Type II: Failing to reject the null hypothesis when it is, in fact, false.

Figure: The relationship between Decision and State of Nature

		State of Nature	
		Null Hypothesis True	Null Hypothesis False
Decision	Conclude Null True (Don't Reject H_0)	Correct Decision	Type II Error
	Conclude Null False (Reject H_0)	Type I Error	Correct Decision

STATISTICAL ERRORS

- The probability of a Type I error is controlled by the decision maker by the choice of α.
- In most business applications, there are adverse consequences associated with each type of error.

Example:

If a drug company's hypothesis tests for a new drug incorrectly conclude that the drug is safe when in fact it is not (Type II error), the company's customers may become ill or even die as a result.

On the other hand, a Type I error would mean that a potentially useful and safe drug would most likely not be made available to people who need it if the hypothesis tests incorrectly determined that the drug was not safe.

STATISTICAL ERRORS

As a decision maker using hypothesis testing, you need to be aware of the potential costs associated with both Type I and Type II statistical errors and conduct your tests accordingly.

STATISTICAL ERRORS

- Type II error: failing to reject or "accepting" a false null hypothesis
  - Therefore, we would like beta to be as small as possible.
  - How? to have a "test" which shows that the hypothesis test have a high probability of rejecting a false null hypothesis. ➔ Calculating β and Power of Test
- Power of Test: The probability that the hypothesis test will correctly reject the null hypothesis when the null hypothesis is false.
  - When the alternative hypothesis is true, the power of the test is computed using Equation: Power = $1 - \beta$

CALCULATING β

- To calculate beta, we must first specify a "what-if " value for a true population parameter taken from the alternative hypothesis.

- Then, β is computed conditional on that parameter being true.

STATISTICAL HYPOTHESIS TESTING: CALCULATING β AND POWER OF TEST

Example

American Tax Services, has claimed its clients save an average of more than $200 each by using the company's services. A consumer's group plans to randomly sample 64 customers to test this claim. The standard deviation of the amount saved is assumed to be $100. Before testing, the consumer's group is interested in knowing the probability that it will mistakenly conclude that the mean savings is less than or equal to $200 when, in fact, it does exceed $200, as the company claims. The test will be conducted using $\alpha = 0.05$. Determine the beta if the true population mean is $210 and give your summary.

The Answer:

Population parameter of interest

The consumer group is interested in the mean savings of American Tax Services' clients, μ.

Formulate the null and the alternative hypothesis

$$H_0 : \mu \leq \$200$$
$$H_A : \mu > \$200 \text{ (claim)}$$

STATISTICAL HYPOTHESIS TESTING: CALCULATING β AND POWER OF TEST

$\sigma = \$100$

Construct the rejection region and decision rule

This is one-tailed (upper tail) test. With 0.05 level of significance, the critical value, for upper tail $z_\alpha = z_{0.05} = 1.645$

Determine the critical value for $\bar{x}_\alpha$ $\bar{x}_{0.05} = \mu + z_{0.05}\frac{\sigma}{\sqrt{n}} = 200 + 1.645\frac{100}{\sqrt{64}} = 220.56$

Thus, the null hypothesis will be rejected if $\bar{x} > 220.56$

Specify the stipulated value for μ

The null hypothesis is false for all values greater than $200. What is beta if the stipulated mean is $210?

Compute the z test statistic on the stipulated population mean, μ

$$z = \frac{\bar{x}_{0.05} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{220.56 - 210}{\frac{100}{\sqrt{64}}} = 0.84$$

Draw a conclusion

In standard normal table for z = 0.84 = 0.7996, Thus β = 0.7996. There is 0.7996 probability that the hypothesis test will lead the consumer agency to mistakenly believe that the mean tax savings is less than or equal to $200 when, in fact, the mean savings is $210.

STATISTICAL HYPOTHESIS TESTING: CALCULATING β AND POWER OF TEST

Power of test

Power = 1 - β

Power = 1 – 0.7996 = 0.2004

Thus, in this situation there is only 0.2004 chance that the hypothesis test will correctly reject the null hypothesis that the mean is \$200 or fewer ($\mu \leq \$200$) when in fact, the mean savings is \$210.

What happen if the ‘true’ mean is \$250 and \$150?

STATISTICAL INFERENCE

HYPOTHESIS TESTING FOR TWO POPULATION MEANS:

Independent Samples

Paired Samples

STATISTICAL INFERENCE

HYPOTHESIS TESTING FOR TWO POPULATION MEANS:

Independent Samples

HYPOTHESIS TESTS FOR TWO POPULATION MEANS USING INDEPENDENT SAMPLES

- Why need to know? - require to test whether two populations have equal means, or whether one population mean is larger (smaller) than another.
- The hypothesis testing between the means will cover:
  1. The population standard deviation are known and the samples are independent
  2. The population standard deviation are unknown and the samples are independent

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE KNOWN, USING INDEPENDENT SAMPLES

z- test statistic for $\mu_1 - \mu_2$ when σ_1 and σ_2 are known, using independent samples

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where

$(\mu_1 - \mu_2)$ = Hypothesized difference in population means

Decision rule:

- if the z-value $\geq$ critical value, z_α therefore accept (do not reject) H_o
- if the z-value < critical value, z_α therefore reject H_o

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE KNOWN, USING INDEPENDENT SAMPLES

Problem:

AMCO Drilling Equipment, Inc., is a company that makes drilling equipment for the oil and gas industry. One item that the company makes is a coupling for use on natural gas drills. AMCO has two machines that make these couplings. It is well established that the standard deviation for the coupling's diameter made by machine 1 is 0.025 inches and the standard deviation for machine 2 is 0.034 inches. These are known values. However, the company is interested in determining whether there is a difference in the average diameters made by these machine. The company wishes to know whether machine 2 also provides couplings with higher average diameters than the average diameter produced by machine 1. If the test determines that machine 2 has a larger average diameter than machine 1, the managers will have maintenance attempt to adjust the diameters downward or they will replace the machine. The test will be conducted using $\alpha = 0.05$.

The manager have select simple random samples of 100 couplings from the two populations (machine 1 and machine 2 productions) and compute the sample means. The means computed from the samples are $\bar{x}_1 = 0.501$ inches and $\bar{x}_2 = 0.509$ inches.

Steps to solution:

1. Specify the population parameter of interest.

This μ_1-μ_2 the difference in the two population means.
The samples are independent because the diameters of couplings made by one machine can in no way influence the diameter of the couplings made by the other machine.

2. Formulate the appropriate null and alternative hypotheses.

We are interested in determining whether the mean diameter for machine 2 exceeds that for machine 1.

$$H_0 : \mu_1 \geq \mu_2$$
$$H_A : : \mu_1 < \mu_2 \text{ (claim)}$$

3. Determine critical value based from significance level and state the decision rule.

This is one-tailed test. With 0.05 level of significance, the critical value,
for upper tail $-z_\alpha$= -$z_{0.05} = -1.645$
The decision rule: if the z < critical value, z_α therefore reject H_0, otherwise accept
Alternatively, you can state the decision rule in terms of a *p*-value,
if the *p-value* < α therefore reject H_0, otherwise accept H_0

4. Compute the test statistic

This problem will use zstatistics.

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{(0.501 - 0.509) - 0}{\sqrt{\frac{0.025^2}{100} + \frac{0.034^2}{100}}} = -1.90$$

The decision rule: Since -1.90 < - 1.645, therefore H_0 is rejected.

5. Identify the summary for the problem studied

There is statistical evidence to conclude that the couplings made by machine 2 have a larger mean diameter than those made by machine 1. Thus, AMCO managers need to take action to modify the mean diameters from machine 2 or replace it.

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

- When testing the two population mean when the population standard deviation is unknown and the sample sizes are small, the critical value is a *t*-value from the *t*-distribution.

- The following assumptions hold:
  - Each populations has a normal distributed.
  - The two population variance are equal $(\sigma_1{}^2, \sigma_2{}^2)$.
  - The samples are independent.

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \qquad df = n_1 + n_2 - 2$$

where:

$\bar{x}_1$ and $\bar{x}_2$ = Sample means from populations 1 and 2

$\mu_1 - \mu_2$ = Hypothesized difference between population means

n_1 and n_2 = Sample sizes from the two populations

s_p = Pooled standard deviation (see Equation 10.4)

***t*-Test Statistic for $\mu_1 - \mu_2$ When σ_1 and σ_2 Are Unknown and Assumed Equal, Independent Samples**

$$s_p = \sqrt{\frac{(n_1 - 1)s^2{}_1 + (n_2 - 1)s^2{}_2}{n_1 + n_2 - 2}}$$

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

Problem:

A recent Associated Press news story out of Brussels, Belgium, indicated the European Union was considering a probe of computer makers after consumers complained that they were being overcharged for ink cartridges. Companies such as Canon, Hewlett-Packard, and Epson are the printer market leaders and make most of their printer related profits by selling replacement ink cartridges. Suppose an independent test agency wishes to conduct a test to determine whether name-brand ink cartridges generate more color pages on average than competing generic ink cartridges. The test is conducted with $\alpha = 0.05$. A simple random sample of 10 users was selected, and the users were given a name-brand cartridge. A second sample of 8 users was given generic cartridges. Both groups used their printers until the ink ran out. The number of pages printed was recorded. The samples are independent because the pages printed by users in one group did not in any way influence the pages printed by users in the second group. The means computed from the samples are

Name-brand cartridge	Generic cartridge
$\bar{x}_1 = 322.5$ pages	$\bar{x}_1 = 298.3$pages
$S_1 = 48.3$ pages	$S_2 = 53.3$ pages

Steps to solution:

1. **Define the population parameter of interest**

We are interested in determining whether the mean number of pages printed by name-brand cartridges (population 1) exceeds the mean pages printed by generic cartridges (population 2).

2. **Formulate the hypothesis null and alternative**

$$H_0 : \mu_1 \leq \mu_2$$
$$H_A : : \mu_1 > \mu_2 \text{ (claim)}$$

3. **Construct the rejection region**

This is one-tailed test. With 0.05 level of significance, the critical value of *t*-value from the t distribution, $t = (n_1+n_2-2)$ *degree of freedom*, thus $10 + 8 - 2 = 16$ degree of freedom. From the t-table, the critical value t is for upper tail $t_\alpha = t_{0.05}$ =1.7459

The decision rule: if the t > critical value, t_α therefore reject H_o, otherwise accept H_o

Alternatively, you can state the decision rule in terms of a *p*-value, if the *p-value* > α therefore reject H_o, otherwise accept H_o

Steps to solution:

4. Determine the test statistics

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

The pooled standard deviation is

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(10 - 1)48.3^2 + (8 - 1)53.3^2}{10 + 8 - 2}} = 50.55$$

Then the t-test statistic is

$$t = \frac{(322.5 - 298.3) - 0.0}{50.55\sqrt{\frac{1}{10} + \frac{1}{8}}} = 1.0093$$

5. Reach the decision

t = 1.0093 < $t_{0.05}$= 1.7459 . Therefore, acept H_o

6. Identify the summary for the problem studied

Based on these sample data, there is insufficient evidence to conclude that the mean number of pages produced by name-brand ink cartridges exceeds the mean for generic cartridges.

STATISTICAL INFERENCE

HYPOTHESIS TESTING FOR TWO POPULATION MEANS:

Paired Samples

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE UNKNOWN, USING PAIRED SAMPLES

- Recap:
  - What is paired samples? : Samples that are selected in such a way that values in one sample are matched with the values in the second sample for the purpose of controlling for extraneous factors.
  - Another term for paired samples is dependent samples.
  - Why paired samples? – to control for any variation in a sample. Example: different cars (and drivers), different painter and etc

TESTING FOR $\mu_1 - \mu_2$ WHEN σ_1 AND σ_2 ARE UNKNOWN, USING PAIRED SAMPLES

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}, \qquad df = (n-1)$$

where:

$\bar{d}$ = Mean paired difference = $\dfrac{\sum d}{n}$

μ_d = Hypothesized population mean paired difference

s_d = Sample standard deviation for paired differences = $\sqrt{\dfrac{\sum (d - \bar{d})^2}{n-1}}$

n = Number of paired values in the sample

***t*-Test Statistic for Paired Sample Test**

ESTIMATING THE DIFFERENCE BETWEEN TWO POPULATION MEANS WHEN σ_1 AND σ_2 ARE UNKNOWN, USING INDEPENDENT SAMPLES

Problem:

Referring to previous example, suppose the experiment regarding ink cartridges is conducted differently. Instead of having different samples of people use name-brand and generic cartridges, the test is done using paired samples. This means that the same people will use both types of cartridges, and the pages printed in each case will be recorded. The test is conducted with α = 0.01. Six randomly selected people have agreed to participate and will be tested on the name-brand and generic cartridges. The following data and paired differences were observed:

Printer User	Name-brand cartridge	Generic cartridge
1	306	300
2	256	260
3	402	357
4	299	286
5	306	290
6	257	260

Steps to solution:

1. **Define the population parameter of interest**

We are interested in determining whether name-brand cartridges produce more printed pages, on average, than generic cartridges, so we would expect the paired difference to be positive. We assume that the paired differences are normally distributed..

2. **Formulate the hypothesis null and alternative**

$$H_o : \mu_d \leq 0.0$$
$$H_A : : \mu_d > 0.0 \text{ (claim)}$$

3. **Construct the rejection region**

This is one-tailed test. With 0.01 level of significance, the critical value of *t*-value from the t distribution, $t = (n_1\text{-}1)$ *degree of freedom*, thus $6 - 1 = 5$ degree of freedom. From the t-table, the critical value t is for upper tail $t_\alpha = t_{0.01} = 3.3649$

The decision rule: if the t > critical value, t_α therefore reject H_o, otherwise accept H_o

Alternatively, you can state the decision rule in terms of a *p*-value, if the *p-value* > α therefore reject H_o, otherwise accept H_o

Steps to solution:

4. Determine the test statistics

The mean paired difference is:

$\bar{d} = \frac{\sum d}{n} = 12.17$

The standard deviation for the paired difference is:

$$s_d = \sqrt{\frac{\sum(d-\bar{d})^2}{n-1}} = 18.02$$

The test statistics:

$$t = \frac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{12.17 - 0.00}{\frac{18.02}{\sqrt{6}}} = 1.6543$$

Printer User	Name-brand cartridge	Generic cartridge	d
1	306	300	6
2	256	260	-4
3	402	357	45
4	299	286	13
5	306	290	16
6	257	260	-3

5. Reach the decision

t = $1.6543 < t_{0.01} = 3.3649$. Therefore, accept H_o

6. Identify the summary for the problem studied

Based on these sample data, there is insufficient evidence to conclude that name-brand ink cartridges produce more pages on average than generic brands.

THE END OF ITEM 2