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theory Problem-1
  imports Complex-Main
begin

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0.1 Problem 1

Let a, b be real numbers. Let's assume that, for all real numbers x, y the inequality $|(ax + by)(ay + bx)| \leq x^2 + y^2$ is satisfied. Show that $a^2 + b^2 \leq 2$.

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theorem problem1:
  fixes a b :: real
  assumes given:  $\bigwedge x\ y :: \text{real. } |(a*x + b*y)*(a*y + b*x)| \leq x^2 + y^2$ 
  shows  $a^2 + b^2 \leq 2$ 
proof -
  from given [where  $x=1$  and  $y=1$ ] have  $(a+b)^2 \leq 2$ 
    by (simp add: power2-eq-square)
  moreover from given [where  $x=1$  and  $y=-1$ ] have  $(a-b)^2 \leq 2$ 
    by (simp add: power2-eq-square right-diff-distrib')
  ultimately have  $(a+b)^2 + (a-b)^2 \leq 4$  by auto
  moreover have  $(a+b)^2 + (a-b)^2 = 2*(a^2 + b^2)$  by algebra
  ultimately show  $a^2 + b^2 \leq 2$  by auto
qed

end

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