```
theory Problem-1B
  imports HOL−Algebra.Algebra
begin
```

Problem 1B asks for a special case of Lagrange's theorem, thus we avoid using the general variant.

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theorem (in comm-group) problem1B:
  assumes finite: finite (carrier G)
  assumes closed: g ∈ carrier G
  shows g [^] order G = 1
proof −
  let ?f = λx. g ⊗ x
  have [simp]: ?f ‘ carrier G = carrier G
    by (simp add: closed group.surj-const-mult)
  have inj-on ?f (carrier G)
    by (simp add: closed group.inj-on-cmult)
  hence (⨂x ∈ carrier G. x) = (⨂x ∈ carrier G. g ⊗ x)
   using finprod-reindex[where h=?f and A=carrier G and f=λx. x, symmetric]
    by simp
  also have ... = (⨂x ∈ carrier G. g) ⊗ (⨂x ∈ carrier G. x)
    using closed by (intro finprod-multf) auto
  finally have (⨂x ∈ carrier G. g) = 1
    using closed by (intro r-cancel-one'[THEN iffD1]) auto
  thus ?thesis
    using closed unfolding order-def by (simp add: finprod-const)
qed

end
```