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theory Warmup-Problem-C
imports
  Complex-Main
  HOL-Library.Quadratic-Discriminant
begin

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## 0.1 Warmup problem C

This one is a straight-forward equation:

**theorem** *warmup3*:

$$\begin{aligned}
 &|x-1|*|x+2|*|x-3|*|x+4| = |x+1|*|x-2|*|x+3|*|x-4| \\
 &\longleftrightarrow x \in \{0, \sqrt{7}, -\sqrt{7}, \\
 &\quad \sqrt{(13 + \sqrt{73}) / 2}, \\
 &\quad -\sqrt{(13 + \sqrt{73}) / 2}, \\
 &\quad \sqrt{(13 - \sqrt{73}) / 2}, \\
 &\quad -\sqrt{(13 - \sqrt{73}) / 2}\}
 \end{aligned}$$

(**is** *?eqn*  $\longleftrightarrow$  *?sols*)

**proof** –

**have** *?eqn*  $\longleftrightarrow |(x-1)*(x+2)*(x-3)*(x+4)| = |(x+1)*(x-2)*(x+3)*(x-4)|$  (**is** -  $\longleftrightarrow$   $|?lhs| = |?rhs|$ )

**by** (*simp add: abs-mult*)

**also have** ...  $\longleftrightarrow ?lhs - ?rhs = 0 \vee ?lhs + ?rhs = 0$  **by** (*auto simp add: abs-eq-iff*)

**also have** ...  $\longleftrightarrow x*(x^2 - 7) = 0 \vee x^4 - 13*x^2 + 24 = 0$  **by** *algebra*

**also have**  $x*(x^2 - 7) = 0 \longleftrightarrow x \in \{0, \sqrt{7}, -\sqrt{7}\}$  **using** *plus-or-minus-sqrt* **by** *auto*

**also have**  $x^4 - 13*x^2 + 24 = 0 \longleftrightarrow x^2 \in \{(13 + \sqrt{73}) / 2, (13 - \sqrt{73}) / 2\}$

**using** *discriminant-nonneg* [**where**  $x=x^2$ , of 1 -13 24]

**by** (*auto simp add: algebra-simps discrim-def*)

**also have** ...  $\longleftrightarrow x \in \{\sqrt{(13 + \sqrt{73}) / 2},$   
 $-\sqrt{(13 + \sqrt{73}) / 2},$   
 $\sqrt{(13 - \sqrt{73}) / 2},$   
 $-\sqrt{(13 - \sqrt{73}) / 2}\}$

**proof** –

**have**  $0 \leq (13 - \sqrt{73}) / 2$  **by** (*auto simp add: real-le-lsqrt*)

**hence**  $x^2 = (13 - \sqrt{73}) / 2$

$$\longleftrightarrow x \in \{\sqrt{(13 - \sqrt{73}) / 2}, \\
 -\sqrt{(13 - \sqrt{73}) / 2}\}$$

**using** *plus-or-minus-sqrt*

**by** *blast*

**moreover have**  $x^2 = (13 + \sqrt{73}) / 2$

$$\longleftrightarrow x \in \{\sqrt{(13 + \sqrt{73}) / 2}, \\
 -\sqrt{(13 + \sqrt{73}) / 2}\}$$

**by** (*smt insert-iff power2-minus power-divide real-sqrt-abs real-sqrt-divide real-sqrt-pow2 singletonD*)

**ultimately show** *?thesis* **by** *blast*

**qed**

**ultimately show** *?thesis* **by** *blast*

**qed**

**end**