```
theory Problem-1
  imports Complex-Main
begin
```

0.1 Problem 1

Let a, b be real numbers. Let's assume that, for all real numbers x, y the inequality $|(ax+by)(ay+bx)| \leq x^2+y^2$ is satisfied. Show that $a^2+b^2 \leq 2$.

```
theorem problem1:
  fixes a b :: real
  assumes given: ⋀x y :: real. |(a*x + b*y)*(a*y + b*x)| ≤ x² + y²
  shows a² + b² ≤ 2
proof -
  from given [where x=1 and y=1] have (a+b)² ≤ 2
    by (simp add: power2-eq-square)
  moreover from given [where x=1 and y=-1] have (a-b)² ≤ 2
    by (simp add: power2-eq-square right-diff-distrib')
  ultimately have (a+b)² + (a-b)² ≤ 4 by auto
  moreover have (a+b)² + (a-b)² = 2*(a² + b²) by algebra
  ultimately show a² + b² ≤ 2 by auto
qed

end
```