OM 2020 — Stage 1

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1 Series I (September)

1.1 Problem 1

Let a, b be real numbers. Let's assume that, for all real numbers x, y the inequality $|(ax + by)(ay + bx)| \leq x^2 + y^2$ is satisfied. Show that $a^2 + b^2 \leq 2$.

theorem *OM1*:
  fixes *a b* :: *real*
  assumes *given*: $\bigwedge x\ y :: real.\ |(a*x + b*y)*(a*y + b*x)| \leq x^2 + y^2$
  shows $a^2 + b^2 \leq 2$
proof –
  from *given* [**where** $x=1$ **and** $y=1$] **have** $(a+b)^2 \leq 2$
    by (*simp add*: *power2-eq-square*)
  moreover from *given* [**where** $x=1$ **and** $y=-1$] **have** $(a-b)^2 \leq 2$
    by (*simp add*: *power2-eq-square right-diff-distrib'*)
  ultimately have $(a+b)^2 + (a-b)^2 \leq 4$ **by** *auto*
  moreover have $(a+b)^2 + (a-b)^2 = 2*(a^2 + b^2)$ **by** *algebra*
  ultimately show $a^2 + b^2 \leq 2$ **by** *auto*
qed