theory *Problem-1*
imports *HOL−Number-Theory.Number-Theory*
begin

Find all positive integers n such that $4^n + 2007$ is a perfect square.

theorem *problem1*:
assumes $n > 0$
shows $\nexists k::nat.\ k^2 = 4\hat{}n + 2007$
proof
assume $\exists k::nat.\ k^2 = 4\hat{}n + 2007$
then obtain $k::nat$ **where** *kk*: $k^2 = 4\hat{}n + 2007$ **by** *blast*
with ‹$n > 0$› **have** $[k^2 = 3]\ (mod\ 4)$
by (*simp add*: *cong-def flip*: *mod-add-eq*)
hence $(k\ mod\ 4) * (k\ mod\ 4)\ mod\ 4 = 3$
by (*simp add*: *cong-def mod-mult-eq power2-eq-square*)
thus *False*
using *mod-exhaust-less-4*[*of k*] **by** *auto*
qed

end