```
theory Warmup-Problem-C
  imports
    Complex-Main
    HOL−Library.Quadratic-Discriminant
begin
```

0.1 Warmup problem C

This one is a straight-forward equation:

```
theorem warmup3:
  |x−1|∗|x+2|∗|x−3|∗|x+4| = |x+1|∗|x−2|∗|x+3|∗|x−4|
    ⟷ x ∈ {0, sqrt 7, −sqrt 7,
             sqrt ((13 + sqrt 73) / 2),
            −sqrt ((13 + sqrt 73) / 2),
             sqrt ((13 − sqrt 73) / 2),
            −sqrt ((13 − sqrt 73) / 2)}
  (is ?eqn ⟷ ?sols)
proof −
  have ?eqn ⟷ |(x−1)∗(x+2)∗(x−3)∗(x+4)| = |(x+1)∗(x−2)∗(x+3)∗(x−4)| (is - ⟷ |?lhs| = |?rhs|)
    by (simp add: abs-mult)
  also have ... ⟷ ?lhs − ?rhs = 0 ∨ ?lhs + ?rhs = 0 by (auto simp add: abs-eq-iff)
  also have ... ⟷ x∗(x^2 − 7) = 0 ∨ x^4 − 13∗x^2 + 24 = 0 by algebra
  also have x∗(x^2 − 7) = 0 ⟷ x ∈ {0, sqrt 7, −sqrt 7} using plus-or-minus-sqrt by auto
  also have x^4 − 13∗x^2 + 24 = 0 ⟷ x² ∈ {(13 + sqrt 73) / 2, (13 − sqrt 73) / 2}
    using discriminant-nonneg [where x=x^2, of 1 −13 24]
    by (auto simp add: algebra-simps discrim-def)
  also have ... ⟷ x ∈ {sqrt ((13 + sqrt 73) / 2),
                        −sqrt ((13 + sqrt 73) / 2),
                         sqrt ((13 − sqrt 73) / 2),
                        −sqrt ((13 − sqrt 73) / 2)}
  proof −
    have 0 ≤ (13 − sqrt 73) / 2 by (auto simp add: real-le-lsqrt)
    hence x² = (13 − sqrt 73) / 2
          ⟷ x ∈ {sqrt ((13 − sqrt 73) / 2),
                 −sqrt ((13 − sqrt 73) / 2)}
      using plus-or-minus-sqrt
      by blast
    moreover have x² = (13 + sqrt 73) / 2
      ⟷ x ∈ {sqrt ((13 + sqrt 73) / 2),
            −sqrt ((13 + sqrt 73) / 2)}
       by (smt insert-iff power2-minus power-divide real-sqrt-abs real-sqrt-divide real-sqrt-pow2 singletonD)
    ultimately show ?thesis by blast
  qed
  ultimately show ?thesis by blast
qed

end
```