OM 2020 — Stage 1

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1   Series I (September)

1.1   Problem 1

Let a, b be real numbers. Let's assume that, for all real numbers x, y the inequality $|(ax + by)(ay + bx)| \leq x^2 + y^2$ is satisfied. Show that $a^2 + b^2 \leq 2$.

theorem *OM1*:
   fixes *a b* :: *real*
   assumes *given*: $\bigwedge x\ y$ :: *real*. $|(a{*}x\ +\ b{*}y){*}(a{*}y\ +\ b{*}x)| \leq x^2\ +\ y^2$
   shows $a^2\ +\ b^2 \leq 2$
proof −
   from *given* [**where** *x=1* **and** *y=1*] **have** $(a{+}b)^2 \leq 2$
     by (*simp add*: *power2-eq-square*)
   moreover from *given* [**where** *x=1* **and** *y=−1*] **have** $(a{-}b)^2 \leq 2$
     by (*simp add*: *power2-eq-square right-diff-distrib′*)
   ultimately have $(a{+}b)^2\ +\ (a{-}b)^2 \leq 4$ **by** *auto*
   moreover have $(a{+}b)^2\ +\ (a{-}b)^2 = 2{*}(a^2\ +\ b^2)$ **by** *algebra*
   ultimately show $a^2\ +\ b^2 \leq 2$ **by** *auto*
qed