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theory Warmup-Problem-C
imports
  Complex-Main
  HOL-Library.Quadratic-Discriminant
begin

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0.1 Warmup problem C

This one is a straight-forward equation:

theorem *warmup3*:

$$\begin{aligned}
& |x-1|*|x+2|*|x-3|*|x+4| = |x+1|*|x-2|*|x+3|*|x-4| \\
& \longleftrightarrow x \in \{0, \text{sqrt } 7, -\text{sqrt } 7, \\
& \quad \text{sqrt } ((13 + \text{sqrt } 73) / 2), \\
& \quad -\text{sqrt } ((13 + \text{sqrt } 73) / 2), \\
& \quad \text{sqrt } ((13 - \text{sqrt } 73) / 2), \\
& \quad -\text{sqrt } ((13 - \text{sqrt } 73) / 2)\} \\
& (\text{is } ?eqn \longleftrightarrow ?sols)
\end{aligned}$$

proof –

have *?eqn* $\longleftrightarrow |(x-1)*(x+2)*(x-3)*(x+4)| = |(x+1)*(x-2)*(x+3)*(x-4)|$ (**is** - $\longleftrightarrow |?lhs| = |?rhs|$)

by (*simp add: abs-mult*)

also have ... $\longleftrightarrow ?lhs - ?rhs = 0 \vee ?lhs + ?rhs = 0$ **by** (*auto simp add: abs-eq-iff*)

also have ... $\longleftrightarrow x*(x^2 - 7) = 0 \vee x^4 - 13*x^2 + 24 = 0$ **by** *algebra*

also have $x*(x^2 - 7) = 0 \longleftrightarrow x \in \{0, \text{sqrt } 7, -\text{sqrt } 7\}$ **using** *plus-or-minus-sqrt* **by** *auto*

also have $x^4 - 13*x^2 + 24 = 0 \longleftrightarrow x^2 \in \{(13 + \text{sqrt } 73) / 2, (13 - \text{sqrt } 73) / 2\}$

using *discriminant-nonneg* [**where** $x=x^2$, *of 1 -13 24*]

by (*auto simp add: algebra-simps discrim-def*)

also have ... $\longleftrightarrow x \in \{\text{sqrt } ((13 + \text{sqrt } 73) / 2),$
 $-\text{sqrt } ((13 + \text{sqrt } 73) / 2),$
 $\text{sqrt } ((13 - \text{sqrt } 73) / 2),$
 $-\text{sqrt } ((13 - \text{sqrt } 73) / 2)\}$

proof –

have $0 \leq (13 - \text{sqrt } 73) / 2$ **by** (*auto simp add: real-le-lsqrt*)

hence $x^2 = (13 - \text{sqrt } 73) / 2$

$$\begin{aligned}
& \longleftrightarrow x \in \{\text{sqrt } ((13 - \text{sqrt } 73) / 2), \\
& \quad -\text{sqrt } ((13 - \text{sqrt } 73) / 2)\}
\end{aligned}$$

using *plus-or-minus-sqrt*

by *blast*

moreover have $x^2 = (13 + \text{sqrt } 73) / 2$

$$\begin{aligned}
& \longleftrightarrow x \in \{\text{sqrt } ((13 + \text{sqrt } 73) / 2), \\
& \quad -\text{sqrt } ((13 + \text{sqrt } 73) / 2)\}
\end{aligned}$$

by (*smt insert-iff power2-minus power-divide real-sqrt-abs real-sqrt-divide real-sqrt-pow2 singletonD*)

ultimately show *?thesis* **by** *blast*

qed

ultimately show *?thesis* **by** *blast*

qed

end