theory *Problem-1*
imports *Complex-Main*
begin

0.1 Problem 1

Let a, b be real numbers. Let's assume that, for all real numbers x, y the inequality $|(ax + by)(ay + bx)| \leq x^2 + y^2$ is satisfied. Show that $a^2 + b^2 \leq 2$.

theorem *problem1*:
fixes a b :: *real*
assumes *given*: $\bigwedge x\ y$ :: *real*. $|(a*x + b*y)*(a*y + b*x)| \leq x^2 + y^2$
shows $a^2 + b^2 \leq 2$
proof −
from *given* [**where** $x=1$ **and** $y=1$] **have** $(a+b)^2 \leq 2$
by (*simp add*: *power2-eq-square*)
moreover from *given* [**where** $x=1$ **and** $y=-1$] **have** $(a-b)^2 \leq 2$
by (*simp add*: *power2-eq-square right-diff-distrib'*)
ultimately have $(a+b)^2 + (a-b)^2 \leq 4$ **by** *auto*
moreover have $(a+b)^2 + (a-b)^2 = 2*(a^2 + b^2)$ **by** *algebra*
ultimately show $a^2 + b^2 \leq 2$ **by** *auto*
qed

end