theory *Warmup-Problem-C*
imports
Complex-Main
HOL−Library.Quadratic-Discriminant
begin

0.1 Warmup problem C

This one is a straight-forward equation:

theorem *warmup3*:
$|x-1|*|x+2|*|x-3|*|x+4| = |x+1|*|x-2|*|x+3|*|x-4|$
$\longleftrightarrow x \in \{0, sqrt\ 7, -sqrt\ 7,$
$sqrt\ ((13 + sqrt\ 73) / 2),$
$-sqrt\ ((13 + sqrt\ 73) / 2),$
$sqrt\ ((13 - sqrt\ 73) / 2),$
$-sqrt\ ((13 - sqrt\ 73) / 2)\}$
(**is** *?eqn* $\longleftrightarrow$ *?sols*)
proof −
have *?eqn* $\longleftrightarrow |(x-1)*(x+2)*(x-3)*(x+4)| = |(x+1)*(x-2)*(x+3)*(x-4)|$
(**is** - $\longleftrightarrow |?lhs| = |?rhs|$)
by (*simp add*: *abs-mult*)
also have ... $\longleftrightarrow$ *?lhs − ?rhs = 0 ∨ ?lhs + ?rhs = 0* **by** (*auto simp add*: *abs-eq-iff*)
also have ... $\longleftrightarrow x*(x\hat{}2 - 7) = 0 \vee x\hat{}4 - 13*x\hat{}2 + 24 = 0$ **by** *algebra*
also have $x*(x\hat{}2 - 7) = 0 \longleftrightarrow x \in \{0, sqrt\ 7, -sqrt\ 7\}$ **using** *plus-or-minus-sqrt* **by** *auto*
also have $x\hat{}4 - 13*x\hat{}2 + 24 = 0 \longleftrightarrow x^2 \in \{(13 + sqrt\ 73) / 2, (13 - sqrt\ 73) / 2\}$
using *discriminant-nonneg* [**where** *x=x^2*, *of 1 −13 24*]
by (*auto simp add*: *algebra-simps discrim-def*)
also have ... $\longleftrightarrow x \in \{sqrt\ ((13 + sqrt\ 73) / 2),$
$-sqrt\ ((13 + sqrt\ 73) / 2),$
$sqrt\ ((13 - sqrt\ 73) / 2),$
$-sqrt\ ((13 - sqrt\ 73) / 2)\}$
proof −
have $0 \leq (13 - sqrt\ 73) / 2$ **by** (*auto simp add*: *real-le-lsqrt*)
hence $x^2 = (13 - sqrt\ 73) / 2$
$\longleftrightarrow x \in \{sqrt\ ((13 - sqrt\ 73) / 2),$
$-sqrt\ ((13 - sqrt\ 73) / 2)\}$
using *plus-or-minus-sqrt*
by *blast*
moreover have $x^2 = (13 + sqrt\ 73) / 2$
$\longleftrightarrow x \in \{sqrt\ ((13 + sqrt\ 73) / 2),$
$-sqrt\ ((13 + sqrt\ 73) / 2)\}$
by (*smt insert-iff power2-minus power-divide real-sqrt-abs real-sqrt-divide real-sqrt-pow2 singletonD*)
ultimately show *?thesis* **by** *blast*
qed
ultimately show *?thesis* **by** *blast*

qed

end