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theory Problem-1
  imports HOL-Number-Theory.Number-Theory
begin

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Find all positive integers n such that $4^n + 2007$ is a perfect square.

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theorem problem1:
  assumes  $n > 0$ 
  shows  $\exists k::nat. k^2 = 4^n + 2007$ 
proof
  assume  $\exists k::nat. k^2 = 4^n + 2007$ 
  then obtain  $k::nat$  where  $kk: k^2 = 4^n + 2007$  by blast
  with  $\langle n > 0 \rangle$  have  $[k^2 = 3] \pmod{4}$ 
    by (simp add: cong-def flip: mod-add-eq)
  hence  $(k \pmod{4}) * (k \pmod{4}) \pmod{4} = 3$ 
    by (simp add: cong-def mod-mult-eq power2-eq-square)
  thus False
    using mod-exhaust-less-4 [of  $k$ ] by auto
qed

end

```