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theory Problem-1A
  imports HOL-Algebra.Algebra
begin

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It seems that one cannot use  $\heartsuit$  as variable. My disappointment is immeasurable and my day is ruined.

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theorem problem1A:
  assumes group: group  $G$ 
  assumes subgrp: subgroup  $H$   $G$ 
  assumes proper:  $H \subset \text{carrier } G$ 
  assumes iso:  $G \cong G(\text{carrier} := H)$ 
  shows infinite (carrier  $G$ )
proof
  from iso obtain  $h$  where bij: bij-betw  $h$  (carrier  $G$ )  $H$ 
    unfolding is-iso-def iso-def by auto

  assume finite: finite (carrier  $G$ )
  with proper have  $\text{card } H < \text{card } (\text{carrier } G)$ 
    by (simp add: psubset-card-mono)
  moreover from finite and bij have  $\text{card } H = \text{card } (\text{carrier } G)$ 
    using bij-betw-same-card by fastforce
  ultimately show False by auto
qed
end

```