theory *Problem-1A*
  imports *HOL−Algebra.Algebra*
begin

It seems that one cannot use ♡ as variable. My disappointment is immeasurable and my day is ruined.

theorem *problem1A*:
  assumes *group*: *group G*
  assumes *subgrp*: *subgroup H G*
  assumes *proper*: $H \subset$ *carrier G*
  assumes *iso*: $G \cong G(\!|\mathit{carrier} := H|\!)$
  shows *infinite* (*carrier G*)
proof
  from *iso* **obtain** *h* **where** *bij*: *bij-betw h* (*carrier G*) *H*
    unfolding *is-iso-def iso-def* **by** *auto*

  assume *finite*: *finite* (*carrier G*)
  with *proper* **have** *card H* < *card* (*carrier G*)
    by (*simp add*: *psubset-card-mono*)
  moreover from *finite* **and** *bij* **have** *card H* = *card* (*carrier G*)
    using *bij-betw-same-card* **by** *fastforce*
  ultimately show *False* **by** *auto*
qed
end