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theory Warmup-Problem-C
imports
  Complex-Main
  HOL-Library.Quadratic-Discriminant
begin

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0.1 Warmup problem C

This one is a straight-forward equation:

theorem *warmup3*:

$$\begin{aligned}
 &|x-1|*|x+2|*|x-3|*|x+4| = |x+1|*|x-2|*|x+3|*|x-4| \\
 &\longleftrightarrow x \in \{0, \sqrt{7}, -\sqrt{7}, \\
 &\quad \sqrt{(13 + \sqrt{73}) / 2}, \\
 &\quad -\sqrt{(13 + \sqrt{73}) / 2}, \\
 &\quad \sqrt{(13 - \sqrt{73}) / 2}, \\
 &\quad -\sqrt{(13 - \sqrt{73}) / 2}\} \\
 &(\text{is } ?eqn \longleftrightarrow ?sols)
 \end{aligned}$$

proof –

have *?eqn* $\longleftrightarrow |(x-1)*(x+2)*(x-3)*(x+4)| = |(x+1)*(x-2)*(x+3)*(x-4)|$ (**is** - $\longleftrightarrow |?lhs| = |?rhs|$)

by (*simp add: abs-mult*)

also have ... $\longleftrightarrow ?lhs - ?rhs = 0 \vee ?lhs + ?rhs = 0$ **by** (*auto simp add: abs-eq-iff*)

also have ... $\longleftrightarrow x*(x^2 - 7) = 0 \vee x^4 - 13*x^2 + 24 = 0$ **by** *algebra*

also have $x*(x^2 - 7) = 0 \longleftrightarrow x \in \{0, \sqrt{7}, -\sqrt{7}\}$ **using** *plus-or-minus-sqrt* **by** *auto*

also have $x^4 - 13*x^2 + 24 = 0 \longleftrightarrow x^2 \in \{(13 + \sqrt{73}) / 2, (13 - \sqrt{73}) / 2\}$

using *discriminant-nonneg* [**where** $x=x^2$, *of* $1 - 13 \ 24$]

by (*auto simp add: algebra-simps discrim-def*)

also have ... $\longleftrightarrow x \in \{\sqrt{(13 + \sqrt{73}) / 2},$
 $-\sqrt{(13 + \sqrt{73}) / 2},$
 $\sqrt{(13 - \sqrt{73}) / 2},$
 $-\sqrt{(13 - \sqrt{73}) / 2}\}$

proof –

have $0 \leq (13 - \sqrt{73}) / 2$ **by** (*auto simp add: real-le-lsqrt*)

hence $x^2 = (13 - \sqrt{73}) / 2$

$$\longleftrightarrow x \in \{\sqrt{(13 - \sqrt{73}) / 2}, \\
 -\sqrt{(13 - \sqrt{73}) / 2}\}$$

using *plus-or-minus-sqrt*

by *blast*

moreover have $x^2 = (13 + \sqrt{73}) / 2$

$$\longleftrightarrow x \in \{\sqrt{(13 + \sqrt{73}) / 2}, \\
 -\sqrt{(13 + \sqrt{73}) / 2}\}$$

by (*smt insert-iff power2-minus power-divide real-sqrt-abs real-sqrt-divide real-sqrt-pow2 singletonD*)

ultimately show *?thesis* **by** *blast*

qed

ultimately show *?thesis* **by** *blast*

qed

end