

Modelling Infectious Diseases and Health Economic Evaluation of Vaccines

Class exercises

Deterministic SIR model

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Package deSolve

- Calling the library

```
library(deSolve)
```

- Set up the input arguments:

- `time`: vector of the timesteps that we want to solve the model at
- `initial_states`: vector storing the number of people in each compartment at the first step t_0
- `parameters`: vector containing the name and values of the parameters included in the model

Package deSolve

To solve the model, we need to define it as a function. The model function takes into arguments in the following order:

```
model_function <- function(time, state, parameters) {  
  with(as.list(c(state, parameters)),{  
  
    N = S + I + R  
  
    # Differential equations  
    dS <- mu*N - beta*S*I/N - mu*S  
    dI <- beta*S*I/N - nu*I - mu*I  
    dR <- nu*I - mu*R  
  
    # Output  
    return(list(c(dS, dI, dR)))  
  })  
}
```

Package deSolve

Solving the model equations using the `ode()` function and saving them as a dataframe object:

```
output <- as.data.frame(ode(y = initial_states,  
                           times = times,  
                           func = model_function,  
                           parms = parameters))
```

Class exercises

The .R codes for the class exercises are available at GitHub repository.
<https://github.com/NielHens/SIMIDcourse-CMcode>

The basic deterministic SIR model

Construct a deterministic SIR (Susceptible-Infectious-Recovered) model for the following scenario:

- In a closed population of 1,000,000 individuals, a single person is initially infected with a novel pathogen at time t_0 , with no individuals initially recovered.
- The basic reproduction number R_0 of this pathogen is 2, and the recovery rate ν corresponds to an average infectious period of 7 days.
- We aim to analyze the progression of the epidemic over a period of 730 days (i.e., 2 years).

Implement the necessary code, representing the population in proportions.

The basic deterministic SIR model

From the model output:

- ① *Determine how the proportions of susceptible, infectious, and recovered individuals evolve over time by plotting the trajectories of the S , I , and R compartments over the 730-day period.*
- ② *What is the proportion of the population have recovered at the end of the observed period?*
- ③ *Identify the peak proportion of infectious individuals and the day on which this peak occurs.*
- ④ *(Optional) How would the epidemic dynamics change if we keep the recovery rate (i.e., $\nu = 1/7$) constant but vary the basic reproduction number from 1 to 6?*

The SIRS and SIS models

This time, we want to incorporate demographic processes into the model.

Using the same materials from the previous exercise, extend your SIR model to SIRS and SIS with the demographic process, knowing that the transition rate from R compartment to S compartment is $\sigma = 1/7$, and the life expectancy is 80 years.

Implement the necessary codes, representing the population by proportions.

The MSEIR model

- In a constant population of 1,000,000 individuals, a single person is initially infected with a novel pathogen at time t_0 , with no individuals initially recovered.
- Some parameters: life expectancy is 80 years, duration of maternal immunity is 3 months, duration of the latent period is 2 days, duration of the infectious period is 1 week, and the basic reproduction number is 2.

Develop an MSEIR model and run it in the period of 1000 years (the step size is 0.01), using both absolute numbers and proportions.

The MSEIR model

- ⑤ *(Optional) What would happen if one switches to a resolution of days for the time scale?*

The basic SIR model with vaccination

In this part, we will extend the SIR model to incorporate vaccination and analyze its impact on the population.

- Reuse the code from the first exercise with the following parameters: recovery rate $\nu = 1/7$, mortality rate $\mu = 1/80$, the basic reproduction number $R_0 = 3$, and the vaccination coverage $p_{vac} = 0.4$.
- Assuming that the vaccine has 100% efficacy, we aim to analyze the progression of the epidemic over a period of 730 days (i.e., two years).

Write the codes in both numbers and proportions.

The basic SIR model with vaccination

- 6 *What is the vaccination coverage that should prevent the population from the epidemic?*
- 7 *If the efficacy of this vaccine is only 70%, what proportion of the population would have to be vaccinated to prevent an epidemic?*

The basic SIR model with multiple sub-populations

In this task, you will learn how to construct an age-structure SIR model for two groups of children (group 1) and adults (group 2).

- For simplicity, we assume that there is no aging within the population, meaning that individuals do not transit from children group (represent for 20% of the total population) to the adult group.
- We also assume that both groups share the same rate of recovery $\nu = 1/7$, the mortality rate $\mu = 1/80$, the total population size is 1 million.
- Given the assumption of age-specific mixing patterns, the force of infection differs between age groups, we assume the probability of getting infection per contact is $b = 0.05$, whereas the average number of contacts per day, retrieved from Belgian social contact study 2006 (SOCRATES tool) are: $c_{11} = 5.43$, $c_{12} = 6$, $c_{21} = 1.57$, and $c_{22} = 10.05$.

Using these information, write code to simulate the spread of infection over a 100-day period and plot the model output.

The basic SIR model with multiple sub-populations

- ⑧ *What are the proportions of children and adults the became infected?*
- ⑨ *Extend your model to three (or more) subgroups. The social contact matrix can be obtained from the above-mentioned SOCRATES tool.*