Full State Pose Estimation Using a Satellite Imager

by

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To God, my Wife and my Mother

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- Pinkmatter for your financial support for taking the risk of investing in me and my future. I will be enternally grateful. Thank you for letting me feel like part of the company and taking such good care of me during the visits.
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"*Above all else remember the friends you made along the way,*
because it is not your journey that defines you, it is the people you help
and help you along the way."
– Mr. Niel Theron

Declaration

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Date: 1 September 2025

Abstract

Pose estimation on nanosatellites is still an on going topic of interest. It is important for satellite to know there position and attitude to do accurate target tracking. Traditional solutions to the pose estimation problem is mainly star trackers, which looks at the constalations of stars to determine the attitude and GPS to determine the position of the satellite along with other sensors like magnetometers and coarse sun sensors.

In this thesis, a sensor is developed that utilises the onboard satellite imager, to estimate the position and the attitude of the satellite. The sensor uses a camera model to take pictures of the Earth surface, a feature detector is ran on the image using scale invariant feature transform (SIFT) to identify and establish corrospondance of features. A full state kinematic estimator using the extended Kalman Filter (EKF) based on the simultanous localisation and mapping (SLAM) approach. The filter makes used of feature vectors and feature discripters detected on the image. This is used to estimate attitude and position of the satellite.

An simulation environment in MATLAB is developed to propagate a satellite and determine the ground truth pose. Several traditional sensors like the star tracker and magnetometer and GPS to be able to compare the Earth Tracker and create the possiblity to fuse the sensors and determine the accuracy. Results show that the filter estimates the system states successfully. It is concluded that ...

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Nomenclature

Variables and Functions

Constants

ω_e	Rotation speed of the Earth
c	A constant.

Functions

f	A function.

Variables

$\mathbf{x}$	A variable.

Acronyms and Abbreviations

ADCS Attitude Determinination and Control System

BRF Body Refrence Frame

CRF Camera Reference Frame

DCM Direction Cosine Matrix

ECEF Earth Centred Earth Fixed

ECI Earth Centred Inertial

EKF Extended Kalman Filter

GPS Global Positioning System

LLA Lattitude Longitude and Altitude Reference Frame

LVLH Local Vertical Local Horizon

SLAM Simultanous Localisation and Mapping

Definitions

A

Attitude The orientation of a satellite in space.

P

Pose The combination of a satellite's position and attitude.

S

State Estimation The ability to determine a state of a system using mathematical models.

Student is an entity needing a thesis to transcend the state of being a student.

Variables and Functions

Constants

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Chapter 1

Introduction

1.1 Problem Background

- Satellites are getting smaller - Because this leads to stallites having reduced costs and timelines - This is enables by the miniraturisation of electronics

- One of the big industires in satellites is remote sensing - Remote Sening is the application where satellites are used to monitor the Earth - One of the applications is to take images of the Earth

- This leads to the problem that high accuracy is needed to take images of the targets on the Earth's surface - COTS components which is mainly used on small satellites lack the accuracy needed - Magnetometers is to low of an accuracy - Star Trackers have the right accuracy, but is expensive

1.2 Proposed Solution

- Proposed solution is to develop an estimation algorithm that can estimate the full state of the satellite - The Full State of a Satellite is its postition in Space and its attitude or its orientation in space. - The satellite uses the imager itself to determine position and attitude. - This can lead to reduce costs as the satellite is using an instrument which is already onboard the satellite. - Utilising the components when it is idle - Observing the target directly

1.3 Document Outline

- Chapter 2: Wil investigate previous sensors that is being used to determine Propose - Previous techniques estimating the pose - Some light touching on feature detection as this is crucial to the pose estimation system

- Chapter 3: Wil introduce the modelling of the system - Rigid Body Kinematics - Position Kinematics - Attitude Kinematics - Kalman Filters - Extended Kalman Filters

- Chapter 4: Measurement Generation - Feature detection - PinHole Camera Model. - The Plant - The Plant Model - The Measurment Model

- Chapter 5: State estimation - The Extended Kalman Filter - Update Step - Prediction Step - Simulator

- Chapter 6 is results

- Chapter 7 is Conclusion - Future Work

Chapter 2

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Chapter 3

Modelling

3.1 Introduction

This project focuses on the pose estimation of a satellite using an satellite image. This is essentially a localistion problem and requires a realistic description of the system. The aim of this chapter is to sufficiently define the problemand the proposed solution. Estimation algorithms is discussed and an estimator is chosen to solve the localisation problem. Further, attitude representations of a rigid body is introduced along with the dynamic and kinematic models used to describe a satellite in inertial space. Attention is given to quaternion attitude representations along with their propagation using angular rates.

3.2 Problem Defintion

The satellite is orbiting in an inertial refrence frame (ECI). It has different sensors to estimate the satellites pose. What is my problem I want to solve. So I have a satellite that is orbiting the earth. This satellite has a camera connected to the body refrence frame (BRF) which takes satellite imagary. This image in then passed through a feature detector to determiine the features in the system. This features is used to create an internal catalogue which is used for the attitude and position estimation. The feature vector is determined the the camera characteritics which is in the BRF. Therefore the attitude and position dynamics, must be described.

The problem of localising a robot in an unkown environment is often solved using simultanous localisation and mapping (SLAM). SLAM is a method used used by a robot to map an unknown environement and simultanously locate itself in the map. The sensor, with a pose x_t, receives measurements z_t, at a given time step t. Given the measurements, the aim is to estimate the sensors location relative to the detected features.

The sensor with the refrence frame $\mathcal{B}$, shown in Figure 3.1

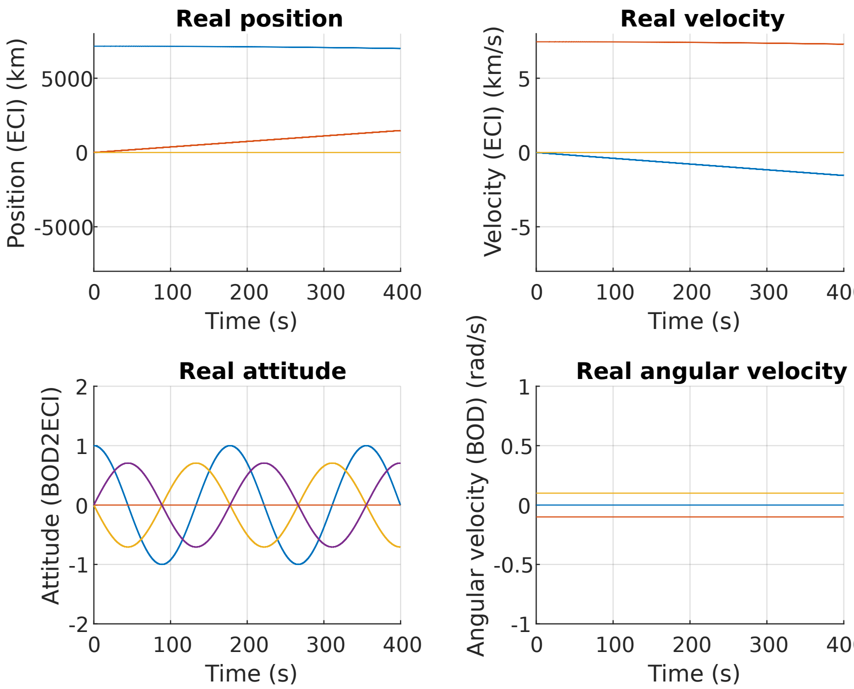

Figure 3.1: Your figure caption.

3.3 Refrence Frame Transsformations

In this masters we are going to encounter a few different refrence frames. To accurateley to create the measurement model we should have an understanding of all the different reference models and how to transform from one to another

3.3.1 Lattitude, longitude and altitude

The lattitude, longitude of a feature or the position of the satellite is donated with the $\mathcal{L}$. The lattitude of a feature is the position of how high or low it above the equator, having a range of $-90° to 90°$. The longitude is based of the greenwich maridian, a longitude line that pases through the north- and south pole, it has a range of $-180° to 180°$. The altitude is measured form the the "WGS84" elliptical globe.

Insert Figure

3.3.2 Earth Ceneterd Earth Fixed

The Earth Ceneterd Earth Fixed refrence frame is represented by the $\mathcal{F}$ and is very simular to the $\mathcal{L}$ reference frame with the z-axis alligned with the northpole and the x-axis points at the crossing of the Prime Maridian an the Equator, where the y-axis completes the right hand rule. The x,y and z-axis is defined in kilometers. To covert from $\mathcal{L}$ to $\mathcal{F}$ is to use a "WGS84" transfomr. Where WGS84 stands for World Geodetic System 1984, which is the standard coordinate system used for Global Positioning System (GPS). The WGS84 transformation uses a reference ellipsoid that uses a semi-major axis of 6,378 km and a flatting of 1/298.2

$$\mathbf{A}_{\mathcal{L}}^{\mathcal{F}} = f(WGS84) \tag{3.1}$$

Insert Figure Here

3.3.3 Earth Ceneterd Inertial

The Earth Centered Inertial refrence fream (ECI) refrenced by $\mathcal{I}$ shares a refrence frame axis with the ECEF, but is rotated about the z-axis. This rotation is governed by the rotation speed of the earth ω_e which is 7.2921×10^{-5} rad/s and time t.

$$\mathbf{A}_{\mathcal{F}}^{\mathcal{I}} = R(\omega_e t) = \begin{bmatrix} \cos(\omega_e t) & -\sin(\omega_e t) & 0 \\ \sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.2}$$

Insert Figure Here

3.3.4 Orbital reference frame

The orbital reference frame used is the Local Vertical Local Horizon (LVLH) denoted by $\mathcal{O}$. The LVLH frame is a rotating, orbit-attached corrdinate system commonly used in spacecraft dynamics. It moves with the satellite and is defined relative to its orbit around Earth. The x-axis is the "Local Horizon" also called "along track" pointing forward it is tangent to the orbit and points in the direction of motion. The z-axis is the local vertical and is also called the Nadir direction, it points to the barycenter of the system, in this case the center of the Earth. The y-axis is called the cross track it completes the right handed system. It points out of the orbital plane, typically the angular momentum vector direction (normal to the orbit plane).

if $\mathbf{r}$ is the position vector of the satellite and $\mathbf{v}$ is the velocity vector of the satellite. The equation for the reference frame is:

$$\bar{z}_{\mathcal{O}} = -\frac{\mathbf{r}}{||\mathbf{r}||} \tag{3.3}$$

$$\bar{y}_{\mathcal{O}} = \frac{\mathbf{r} \times \mathbf{v}}{||\mathbf{r} \times \mathbf{v}||} \tag{3.4}$$

$$\bar{x}_{\mathcal{O}} = \bar{y}_{\mathcal{O}} \times \bar{z}_{\mathcal{O}} \tag{3.5}$$

For this reference frame there should also be a refrence frame translation introduced. Which is done by substracing $\mathbf{r}$ from the vector

$$\mathbf{f}_{\mathcal{O}} = \mathbf{A}_{\mathcal{I}}^{\mathcal{O}} \times (\mathbf{f}_{\mathcal{I}} - \mathbf{r}_{\mathcal{I}}) \tag{3.6}$$

Insert Figure Here. This is unfinished explain a bit more. Actually want to change it to the 4x4 transformation matrix

3.3.5 Camera Reference Frame

The camera refrence frame denoted by $\mathcal{C}$ and the body reference frame $\mathcal{B}$ in this thesis is the same reference frame. This reference frame is transformed by using your standard quaternion rotaion matrix.

$$\mathbf{f}_{\mathcal{C}} = \mathbf{A}_{\mathcal{O}}^{\mathcal{C}} \times \mathbf{f}_{\mathcal{O}} \tag{3.7}$$

3.4 Rigid Body Mechanics

3.4.1 Kinematics

The pose of a rigid body in a refrence frame consists of the position and attitude of the body. The attitude, or orientation of a body-fixed reference frame to a known reference frame. This is usualy represented by a rotation matrix, often referred to as a direction cosine Matrix (DCM). A rotation about a single coordinate axis is referred to as a coordinate rotation. A coordinate rotation about the x-,y- and z-axes with angles ϕ, θ and ψ, of the body can be respectivley describes as, [Willem de Jong p.23]

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \tag{3.8}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \tag{3.9}$$

$$R_z(\psi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.10}$$

Any rotation in 3D space can be described by three coordinate rotations. The DCM describing the attitude of the target in the camera reference frame (CRF), $\mathbf{A}_{\mathcal{C}}^{\mathcal{B}}$, can be represented by three Eular angles. Each of the angles corresponds to one coordinate rotation. The order of the Eular 1-2-3 rotation, shown in Figure 3.5, is expressed as

$$\boldsymbol{A}_{\mathcal{C}}^{\mathcal{B}} = R_x(\phi)R_y(\theta)R_z(\psi) \tag{3.11}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \tag{3.12}$$

$$\begin{bmatrix} C\theta C\psi & C\theta S\psi & -S\theta \\ S\phi S\theta C\psi - C\phi S\psi & S\phi S\theta S\psi + C\phi C\psi & S\phi C\theta \\ C\phi S\theta C\psi + S\phi S\psi & C\phi S\theta S\psi - S\phi C\psi & C\phi C\theta \end{bmatrix} \tag{3.13}$$

Where S is the sine function and C is the cosine function. The Eular angles are calculated as follows

$$\phi = \arctan 2\left(\frac{a_{2,3}}{a_{3,3}}\right) \tag{3.14}$$

$$\theta = \arctan 2\left(\frac{-a_{1,3}}{\sqrt{a_{1,1}^2} + \sqrt{a_{1,2}^2}}\right) \tag{3.15}$$

$$\psi = \arctan 2\left(\frac{a_{1,2}}{a_{1,1}}\right) \tag{3.16}$$

mathematicl singularities occur when using Eular angles to represent large rotations. When both $a_{1,1}$ and $a_{1,2}$ in Equation 3.11 are zero, the expressions for ψ and θ ar undefined. This is known as *gimbal lock*, where the changes in the first and third Eular angles are indistinguishable when the second angle nears a criticual value. Alternatively, the DCM can be described using quaternions, which do not have these singularities. The quaternion rotation is Figure ?? is expressed by the Eular axis $\bar{\mathbf{e}} = [e_x, e_y, e_z]^T$ and the angle θ

$$\mathbf{q} = \begin{bmatrix} q_s \\ q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} \cos(\theta/2) \\ e_x \sin(\theta/2) \\ e_y \sin(\theta/2) \\ e_z \sin(\theta/2) \end{bmatrix} \tag{3.17}$$

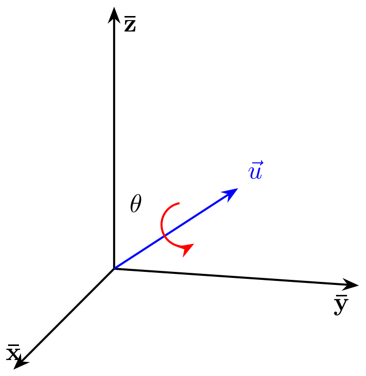

Figure 3.2: Quaternion Rotation

The DCM as a function of Quaternion set is expressed as,

$$\mathbf{A}_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} q_s^2 + q_x^2 - q_y^2 - q_z^2 & 2(q_x q_y - q_s q_z) & 2(q_x q_z + q_s q_y) \\ 2(q_x q_y + q_s q_z) & q_s^2 - q_x^2 + q_y^2 - q_z^2 & 2(q_y q_z - q_s q_x) \\ 2(q_x q_z - q_s q_y) & 2(q_y q_z + q_s q_x) & q_s^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix} \tag{3.18}$$

Using the normalisation constraint, $q_s^2 + q_x^2 + q_y^2 + q_z^2 = 1$, the DCM Simplifies to,

$$\mathbf{A}_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} 1 - 2(q_y^2 + q_z^2) & 2(q_x q_y - q_s q_z) & 2(q_x q_z + q_s q_y) \\ 2(q_x q_y + q_s q_z) & 1 - 2(q_x^2 + q_z^2) & 2(q_y q_z - q_s q_x) \\ 2(q_x q_z - q_s q_y) & 2(q_y q_z + q_s q_x) & 1 - 2(q_x^2 + q_y^2) \end{bmatrix} \tag{3.19}$$

The body-fixed angular rates of the satellite in CRF, $\omega_{\mathcal{C}}^{\mathcal{B}}$, is expressed as a function of qauternions by,

$$\omega_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = 2 \begin{bmatrix} -q_x & q_s & -q_z & q_y \\ -q_3 & q_4 & q_1 & -q_2 \\ -q_4 & -q_3 & q_2 & q_s \end{bmatrix} \begin{bmatrix} \dot{q}_s \\ \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix} \tag{3.20}$$

Inversly the quaternion rates as a function of the body rates are,

$$\begin{bmatrix} \dot{q}_s \\ \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_{bx} & -\omega_{by} & -\omega_{bz} \\ \omega_{bx} & 0 & \omega_{bz} & -\omega_{by} \\ \omega_{by} & -\omega_{bz} & 0 & \omega_{bx} \\ \omega_{bz} & \omega_{by} & -\omega_{bx} & 0 \end{bmatrix} \begin{bmatrix} q_s \\ q_x \\ q_y \\ q_z \end{bmatrix} \tag{3.21}$$

Quaternions will be used throughout this thesis for attitude representations. Quaternions do not have ambiguity regarding the order of rotations and the rotation is around a well-defined axis. The sin and cosine elements of the rotation matrix are already encodedin the quaternion form of the DCM. Therfore, only one matrix operation is required for attitude transforms, where Eular angles reguire three.

3.4.2 Dynamics

The rotational dynamics of a rigid body satellite can be described using the Newton-Euler equations, which are applicable to all rigid inertial bodies [?]. The angular momentum of the satellite is expressed as:

$$\dot{\mathbf{H}} = \frac{d\mathbf{H}}{dt} = \mathbf{I}\dot{\boldsymbol{\omega}} \tag{3.22}$$

where $\mathbf{H}$ represents the angular momentum vector and $\mathbf{I}$ is the diagonalized moment of inertia tensor about the satellite's principal axes. In the absence of external torques, the rotational kinematics of a rigid satellite about its center of mass can be described by Euler's rotational equations:

$$I_{xx}\dot{\omega}_x = \omega_y\omega_z(I_{yy} - I_{zz}) \tag{3.23}$$

$$I_{yy}\dot{\omega}_y = \omega_x\omega_z(I_{zz} - I_{xx}) \tag{3.24}$$

$$I_{zz}\dot{\omega}_z = \omega_x\omega_y(I_{xx} - I_{yy}) \tag{3.25}$$

where I_{xx}, I_{yy}, and I_{zz} are the principal moments of inertia, which remain constant and depend on the satellite's mass distribution and geometric configuration.

The stability characteristics of the satellite's rotational motion are governed by its mass distribution. According to Marsden and Ratiu [?], rotation about the major and minor principal axes is inherently stable, while rotation about the intermediate axis exhibits unstable behavior. Under constant energy conditions, any initial rotation about the intermediate axis will gradually redistribute energy to the major and minor axes through nutation effects.

For the translational dynamics, Newton's second law governs the linear motion of the satellite with mass m. The discrete-time position and velocity propagation equations are:

$$\mathbf{r}_t = \mathbf{r}_{t-1} + \mathbf{v}_t\Delta t + \frac{1}{2m}\mathbf{F}(t)\Delta t^2 \tag{3.26}$$

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \frac{1}{m}\mathbf{F}(t)\Delta t \tag{3.27}$$

where $\mathbf{F}(t)$ represents the net external force acting on the satellite. For the orbital environment considered in this work, where external perturbations are negligible compared to gravitational forces, and given that precise mass properties may not be available, the translational motion can be approximated using kinematic models where the current velocity depends primarily on the previous velocity state.

To propagate the quaternion representing the satellite's attitude over time, the quaternion derivative must first be computed. The time derivative of the quaternion $\mathbf{q}_{B/I}$, which describes the rotation from the inertial frame to the body frame, is calculated using quaternion multiplication with the angular velocity vector:

$$\dot{\mathbf{q}}_{B/I} = \frac{1}{2}(\mathbf{q}_{B/I} \otimes \boldsymbol{\omega}) \tag{3.28}$$

where $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ is the angular velocity vector expressed in the body frame. Expanding this quaternion multiplication yields:

$$\dot{\mathbf{q}}_{B/I} = \frac{1}{2}\begin{bmatrix} q_{B/I,0}\omega_x - q_{B/I,3}\omega_y + q_{B/I,2}\omega_z \\ q_{B/I,3}\omega_x + q_{B/I,0}\omega_y - q_{B/I,1}\omega_z \\ -q_{B/I,2}\omega_x + q_{B/I,1}\omega_y + q_{B/I,0}\omega_z \\ -q_{B/I,1}\omega_x - q_{B/I,2}\omega_y - q_{B/I,3}\omega_z \end{bmatrix} \tag{3.29}$$

where $q_{B/I,0}$, $q_{B/I,1}$, $q_{B/I,2}$, and $q_{B/I,3}$ are the scalar and vector components of the quaternion, respectively.

The quaternion integration is performed using a simple Euler integration scheme. First, the quaternion is propagated forward in time using:

$$\bar{\mathbf{q}}_{B/I}(t + \Delta t) = \mathbf{q}_{B/I}(t) + \dot{\mathbf{q}}_{B/I}\Delta t \tag{3.30}$$

where $\bar{\mathbf{q}}_{B/I}(t + \Delta t)$ represents the unnormalized quaternion after integration. Since quaternion integration may introduce numerical errors that violate the unit quaternion constraint, the result must be renormalized:

$$\mathbf{q}_{B/I}(t + \Delta t) = \frac{\bar{\mathbf{q}}_{B/I}(t + \Delta t)}{||\bar{\mathbf{q}}_{B/I}(t + \Delta t)||} \tag{3.31}$$

This normalization step ensures that the quaternion maintains its unit magnitude, preserving the validity of the attitude representation.

3.5 Conclusion

Chapter 4

Image Processing

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Chapter 5

State Estimation

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Chapter 7

Conclusion and Future Work

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7.2 Future Work

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Appendix A

Appendix title goes here