Stellenbosch
UNIVERSITY
IYUNIVESITHI
UNIVERSITEIT

Full State Pose Estimation Using a Satellite Imager

by

D.P. Theron

Dissertation presented for the degree of Masters of (Electronic Engineering) in the Faculty of Engineering at Stellenbosch University

Supervisor: Prof. H.W. Jordaan

2025

To God, my Wife and my Mother

Acknowledgements

- Prof. H.W. Jordaan, thank you for all your patience and guidance trough this journey.
- Pinkmatter for your financial support for taking the risk of investing in me and my future. I will be enternally grateful. Thank you for letting me feel like part of the company and taking such good care of me during the visits.
- Clarissa, my wife, thank you for all your love and support during the difficult times.
- My Mother, words cannot describe how much you meant to me during my masters degree.
- My friend and collogues in the ESL, especially Brandon Chetty, Dane Groves and Mark Msonko, who put up with my 'new productivity hacks' and 'crazy ideas', but also keeping me on track.
- God, for all your guidance, support, love and faith in me, even though I didn't deserve it.

“ *Above all else remember the friends you made along the way, because it is not your journey that defines you, it is the people you help and help you along the way.* ”
– Mr. Niel Theron

Declaration

By submitting this dissertation electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

Date: 1 September 2025

Abstract

Pose estimation on nanosatellites is still an on going topic of interest. It is important for satellite to know there position and attitude to do accurate target tracking. Traditional solutions to the pose estimation problem is mainly star trackers, which looks at the constalations of stars to determine the attitude and GPS to determine the position of the satellite along with other sensors like magnetometers and coarse sun sensors.

In this thesis, a sensor is developed that utilises the onboard satellite imager, to estimate the position and the attitude of the satellite. The sensor uses a camera model to take pictures of the Earth surface, a feature detector is ran on the image using scale invariant feature transform (SIFT) to identify and establish corrospondance of features. A full state kinematic estimator using the extended Kalman Filter (EKF) based on the simultanous localisation and mapping (SLAM) approach. The filter makes used of feature vectors and feature discripters detected on the image. This is used to estimate attitude and position of the satellite.

An simulation environment in MATLAB is developed to propagate a satellite and determine the ground truth pose. Several traditional sensors like the star tracker and magnetometer and GPS to be able to compare the Earth Tracker and create the possiblity to fuse the sensors and determine the accuracy. Results show that the filter estimates the system states successfully. It is concluded that ...

Uittreksel

Table of Contents

List of Figures

List of Tables

Nomenclature

Variables and Functions

Constants

ω_e	Rotation speed of the Earth
c	A constant.

Functions

f	A function.

Variables

$\mathbf{x}$	A variable.

Acronyms and Abbreviations

ADCS Attitude Determinination and Control System

BRF Body Refrence Frame

CRF Camera Reference Frame

DCM Direction Cosine Matrix

ECEF Earth Centred Earth Fixed

ECI Earth Centred Inertial

EKF Extended Kalman Filter

GPS Global Positioning System

LLA Lattitude Longitude and Altitude Reference Frame

LVLH Local Vertical Local Horizon

SLAM Simultanous Localisation and Mapping

Definitions

A

Attitude The orientation of a satellite in space.

P

Pose The combination of a satellite's position and attitude.

S

State Estimation The ability to determine a state of a system using mathematical models.

Student is an entity needing a thesis to transcend the state of being a student.

Variables and Functions

Constants

ω_e	Rotation speed of the Earth
c	A constant.

Functions

f	A function.

Variables

$\mathbf{x}$	A variable.

Chapter 1

Introduction

1.1 Problem Background

- Satellites are getting smaller - Because this leads to stallites having reduced costs and timelines - This is enables by the miniraturisation of electronics

- One of the big industires in satellites is remote sensing - Remote Sening is the application where satellites are used to monitor the Earth - One of the applications is to take images of the Earth

- This leads to the problem that high accuracy is needed to take images of the targets on the Earth's surface - COTS components which is mainly used on small satellites lack the accuracy needed - Magnetometers is to low of an accuracy - Star Trackers have the right accuracy, but is expensive

1.2 Proposed Solution

- Proposed solution is to develop an estimation algorithm that can estimate the full state of the satellite - The Full State of a Satellite is its postition in Space and its attitude or its orientation in space. - The satellite uses the imager itself to determine position and attitude. - This can lead to reduce costs as the satellite is using an instrument which is already onboard the satellite. - Utilising the components when it is idle - Observing the target directly

1.3 Document Outline

- Chapter 2: Wil investigate previous sensors that is being used to determine Propose - Previous techniques estimating the pose - Some light touching on feature detection as this is crucial to the pose estimation system

- Chapter 3: Wil introduce the modelling of the system - Rigid Body Kinematics - Position Kinematics - Attitude Kinematics - Kalman Filters - Extended Kalman Filters

- Chapter 4: Measurement Generation - Feature detection - PinHole Camera Model. - The Plant - The Plant Model - The Measurment Model

- Chapter 5: State estimation - The Extended Kalman Filter - Update Step - Prediction Step - Simulator

- Chapter 6 is results

- Chapter 7 is Conclusion - Future Work

Chapter 2

Literature

2.1 Introduction

2.2 Satellite Position and Attitude Determination Systems

2.2.1 Position Determination Methods

2.2.2 Attitude Determination Systems

2.3 Earth Observation Satellite Systems and Imaging Technologies

2.3.1 Heritage Earth Observation Missions

2.3.2 Commercial Earth Observation Satellites

2.3.3 Camera Technologies in Earth Observation

2.3.4 Emerging Satellite Constellations

2.4 Computer Vision for Satellite Applications

2.4.1 Classical Feature Detection Methods

2.4.2 Earth Feature Tracking and Landmark Recognition

2.5 Vision-Based Pose Estimation Techniques

2.5.1 Camera-Based Navigation Systems

2.5.2 Geometric Pose Estimation Methods

2.6 State Estimation and Sensor Fusion

2.6.1 Filtering Techniques for Satellite Applications

2.6.2 Multi-Sensor Fusion Architectures

2.6.3 Robustness and Reliability Techniques

2.7 Earth-Tracking Systems for Satellite Pose Estimation

2.7.1 Ground Feature Databases and Maps

Chapter 3

Modelling

3.1 Introduction

This project focuses on the pose estimation of a satellite using satellite images. This is essentially a localistion problem and requires a realistic description of the system. The aim of this chapter is to sufficiently define the problemand the proposed solution. Estimation algorithms is discussed and an estimator is chosen to solve the localisation problem. Further, attitude representations of a rigid body is introduced along with the dynamic and kinematic models used to describe a satellite in inertial space. Attention is given to quaternion attitude representations along with their propagation using angular rates.

3.2 Problem Defintion

A satellite orbiting Earth in the Earth-Centered Inertial (ECI) reference frame performs Earth observation missions, continuously capturing high-resolution imagery of the planet's surface for scientific, commercial, or operational purposes. To fulfill mission objectives effectively, the satellite must provide not only high-quality imagery but also precise geographic information about observed areas. This requires accurate knowledge of the satellite's six-degree-of-freedom pose (three-dimensional position and three-dimensional attitude) relative to the ECI frame at the moment each image is captured.

Traditional satellite pose determination relies on external systems such as Global Navigation Satellite Systems (GNSS) and ground-based tracking networks. However, this thesis investigates an autonomous approach where the satellite performs "visual navigation" by identifying known ground features in its imagery and using these observations to determine its orbital state. The satellite essentially performs "reverse GPS" - instead of receiving position signals from space, it observes recognizable landmarks on Earth's surface and computes its pose from these visual references.

The core technical challenge lies in the transformation from raw imagery to precise pose estimates. This involves several interdependent problems: **(1) Feature Detection** - identifying which pixels in the imagery correspond to cataloged landmarks among millions of pixel observations; **(2) Geometric Inversion** - solving the complex inverse problem of determining six-dimensional pose from two-dimensional image projections of three-dimensional landmarks with known geographic coordinates; and **(3) Uncertainty Management** - handling measurement noise, feature detection errors, and dynamic orbital motion in real-time. This thesis assumes the availability of a pre-established catalog of ground features with precisely known geographic coordinates in the ECI frame. The feature matching problem - associating detected image features with specific catalog entries - is considered solved through prior knowledge of the observed terrain and existing geographic databases.

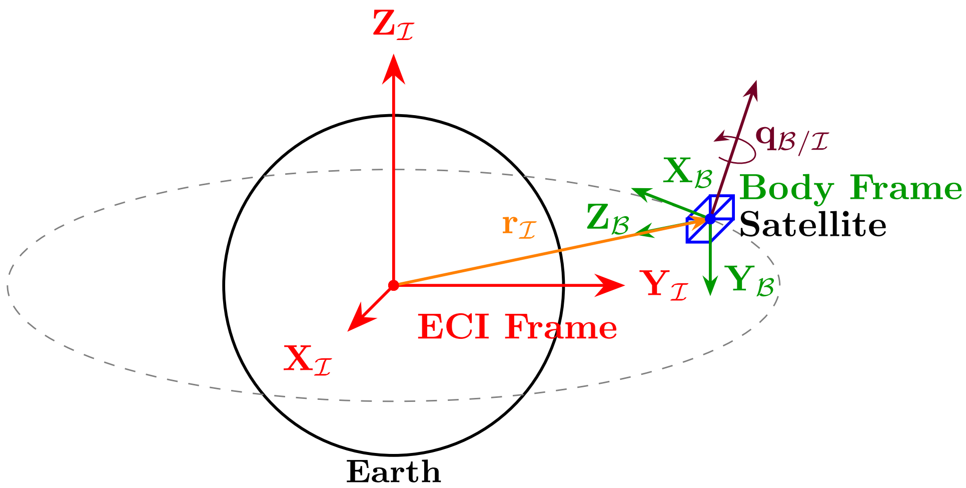

Figure 3.1: Satellite pose estimation concept showing orbital geometry, reference frames

This problem structure aligns with a simplified version of the Simultaneous Localization and Mapping (SLAM) framework. In this context, **localization** corresponds to determining the satellite's pose relative to the ECI frame using observations of cataloged features, while the **mapping** component is reduced to feature catalog utilization rather than creation. Since the geographic locations of observable features are assumed known a priori, the primary focus becomes the pose estimation problem given established feature correspondences.

The satellite's pose estimation system must account for the dynamic nature of orbital motion, the geometric relationship between the camera frame and satellite body frame, and the projection characteristics of the imaging system, while maintaining computational efficiency suitable for real-time onboard processing.

3.3 Refrence Frame Transsformations

In this masters we are going to encounter a few different refrence frames. To accurateley to create the measurement model we should have an understanding of all the different reference models and how to transform from one to another

3.3.1 Lattitude, longitude and altitude

The lattitude, longitude of a feature or the position of the satellite is donated with the $\mathcal{L}$. The lattitude of a feature is the position of how high or low it above the equator, having a range of $-90°$ to $90°$. The longitude is based of the greenwich maridian, a longitude line that pases through the north- and south pole, it has a range of $-180°$ to $180°$. The altitude is measured form the the "WGS84" elliptical globe.

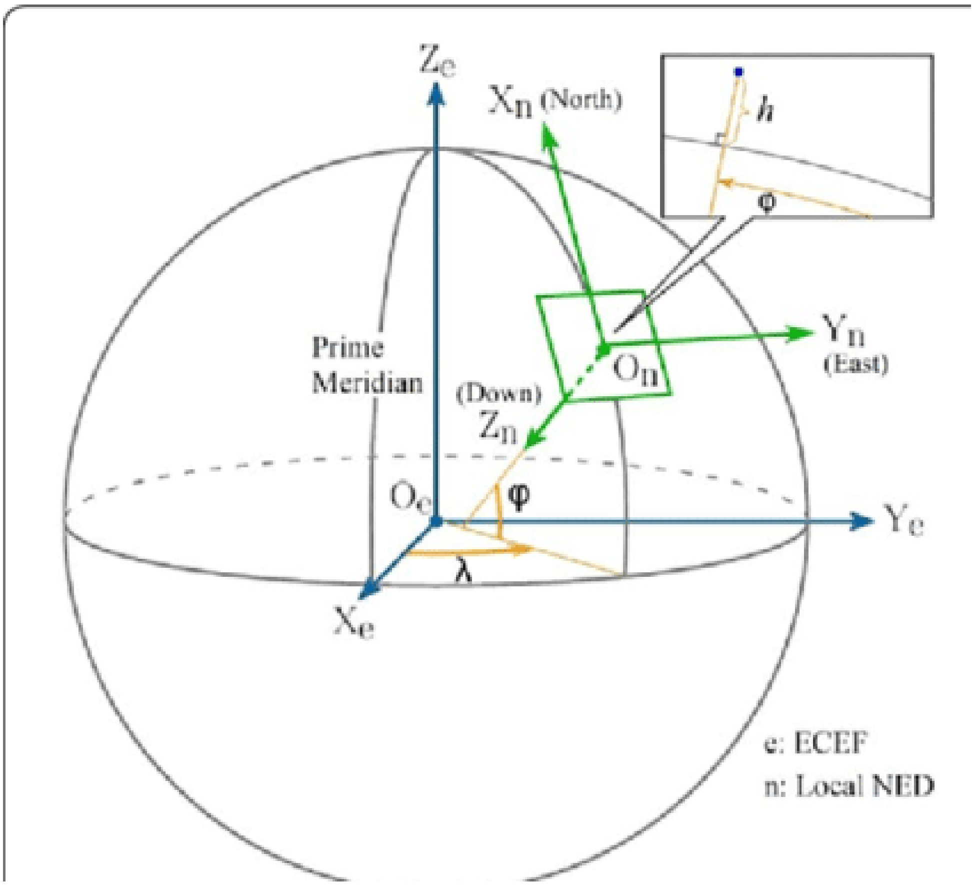

Figure 3.2: The Lattitude, Longitude, Altitutde refercne frame in yellow with the ECEF refernce frame in bluem and the NED reference frame in green.

$$\mathbf{r}_{\mathcal{L}} = \begin{bmatrix} \lambda \\ \phi \\ h \end{bmatrix} \tag{3.1}$$

3.3.2 Earth Ceneterd Earth Fixed

The Earth Ceneterd Earth Fixed refrence frame is represented by the $\mathcal{F}$ and is very simular to the $\mathcal{L}$ reference frame with the z-axis alligned with the northpole and the x-axis points at the crossing of the Prime Maridian an the Equator, where the y-axis completes the right hand rule. The x,y and z-axis is defined in kilometers. To covert from $\mathcal{L}$ to $\mathcal{F}$ is to use a "WGS84" transfomr. Where WGS84 stands for World Geodetic System 1984, which is the standard coordinate system used for Global Positioning System (GPS). The WGS84 transformation uses a reference ellipsoid that uses a semi-major axis of 6,378 km and a flatting of 1/298.2

$$\mathbf{A}_{\mathcal{L}}^{\mathcal{F}} = f(WGS84) \tag{3.2}$$

Insert Figure Here

3.3.3 Earth Ceneterd Inertial

The Earth Centered Inertial refrence fream (ECI) refrenced by $\mathcal{I}$ shares a refrence frame axis with the ECEF, but is rotated about the z-axis. This rotation is governed by the rotation speed of the earth ω_e which is 7.2921×10^{-5} rad/s and time t.

$$\mathbf{A}_{\mathcal{F}}^{\mathcal{I}} = R(\omega_e t) = \begin{bmatrix} \cos(\omega_e t) & -\sin(\omega_e t) & 0 \\ \sin(\omega_e t) & \cos(\omega_e t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.3}$$

Insert Figure Here

3.3.4 Orbital reference frame

The orbital reference frame used is the Local Vertical Local Horizon (LVLH) denoted by $\mathcal{O}$. The LVLH frame is a rotating, orbit-attached corrdinate system commonly used in spacecraft dynamics. It moves with the satellite and is defined relative to its orbit around Earth. The x-axis is the "Local Horizon" also called "along track" pointing forward it is tangent to the orbit and points in the direction of motion. The z-axis is the local vertical and is also called the Nadir direction, it points to the barycenter of the system, in this case the center of the Earth. The y-axis is called the cross track it completes the right handed system. It points out of the orbital plane, typically the angular momentum vector direction (normal to the orbit plane).

if $\mathbf{r}$ is the position vector of the satellite and $\mathbf{v}$ is the velocity vector of the satellite. The equation for the reference frame is:

$$\bar{z}_{\mathcal{O}} = -\frac{\mathbf{r}}{||\mathbf{r}||} \tag{3.4}$$

$$\bar{y}_{\mathcal{O}} = \frac{\mathbf{r} \times \mathbf{v}}{||\mathbf{r} \times \mathbf{v}||} \tag{3.5}$$

$$\bar{x}_{\mathcal{O}} = \bar{y}_{\mathcal{O}} \times \bar{z}_{\mathcal{O}} \tag{3.6}$$

For this reference frame there should also be a refrence frame translation introduced. Which is done by substracing $\mathbf{r}$ from the vector

$$\mathbf{f}_{\mathcal{O}} = \mathbf{A}_{\mathcal{I}}^{\mathcal{O}} \times (\mathbf{f}_{\mathcal{I}} - \mathbf{r}_{\mathcal{I}}) \tag{3.7}$$

Insert Figure Here. This is unfinished explain a bit more. Actually want to change it to the 4x4 transformation matrix

3.3.5 Camera Reference Frame

The camera refrence frame denoted by $\mathcal{C}$ and the body reference frame $\mathcal{B}$ in this thesis is the same reference frame. This reference frame is transformed by using your standard quaternion rotaion matrix.

$$\mathbf{f}_{\mathcal{C}} = \mathbf{A}_{\mathcal{O}}^{\mathcal{C}} \times \mathbf{f}_{\mathcal{O}} \tag{3.8}$$

3.3.6 Image Reference Frame

3.4 Rigid Body Mechanics

3.4.1 Kinematics

The pose of a rigid body in a refrence frame consists of the position and attitude of the body. The attitude, or orientation of a body-fixed reference frame to a known reference frame. This is usualy represented by a rotation matrix, often referred to as a direction cosine Matrix (DCM). A rotation about a single coordinate axis is referred to as a coordinate rotation. A coordinate rotation about the x-,y- and z-axes with angles ϕ, θ and ψ, of the body can be respectivley describes as, [Willem de Jong p.23]

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \tag{3.9}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \tag{3.10}$$

$$R_z(\psi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.11}$$

Any rotation in 3D space can be described by three coordinate rotations. The DCM describing the attitude of the target in the camera reference frame (CRF), $\mathbf{A}_{\mathcal{C}}^{\mathcal{B}}$, can be represented by three Eular angles. Each of the angles corresponds to one coordinate rotation. The order of the Eular 1-2-3 rotation, shown in Figure 3.5, is expressed as

$$\boldsymbol{A}_{\mathcal{C}}^{\mathcal{B}} = R_x(\phi)R_y(\theta)R_z(\psi) \tag{3.12}$$

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} \tag{3.13}$$

$$\begin{bmatrix} C\theta C\psi & C\theta S\psi & -S\theta \\ S\phi S\theta C\psi - C\phi S\psi & S\phi S\theta S\psi + C\phi C\psi & S\phi C\theta \\ C\phi S\theta C\psi + S\phi S\psi & C\phi S\theta S\psi - S\phi C\psi & C\phi C\theta \end{bmatrix} \tag{3.14}$$

Where S is the sine function and C is the cosine function. The Eular angles are calculated as follows

$$\phi = \arctan 2\left(\frac{a_{2,3}}{a_{3,3}}\right) \tag{3.15}$$

$$\theta = \arctan 2\left(\frac{-a_{1,3}}{\sqrt{a_{1,1}^2} + \sqrt{a_{1,2}^2}}\right) \tag{3.16}$$

$$\psi = \arctan 2\left(\frac{a_{1,2}}{a_{1,1}}\right) \tag{3.17}$$

mathematicl singularities occur when using Eular angles to represent large rotations. When both $a_{1,1}$ and $a_{1,2}$ in Equation 3.12 are zero, the expressions for ψ and θ ar undefined. This is known as *gimbal lock*, where the changes in the first and third Eular angles are indistinguishable when the second angle nears a criticual value. Alternatively, the DCM can be described using quaternions, which do not have these singularities. The quaternion rotation is Figure ?? is expressed by the Eular axis $\bar{\mathbf{e}} = [e_x, e_y, e_z]^T$ and the angle θ

$$\mathbf{q} = \begin{bmatrix} q_s \\ q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} \cos(\theta/2) \\ e_x \sin(\theta/2) \\ e_y \sin(\theta/2) \\ e_z \sin(\theta/2) \end{bmatrix} \tag{3.18}$$

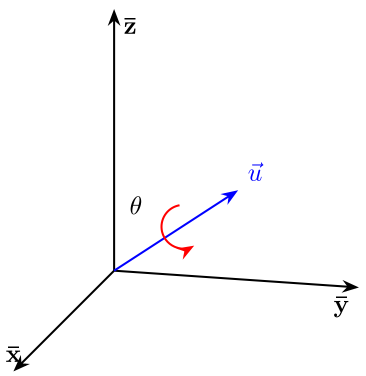

Figure 3.3: Quaternion Rotation

The DCM as a function of Quaternion set is expressed as,

$$\mathbf{A}_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} q_s^2 + q_x^2 - q_y^2 - q_z^2 & 2(q_x q_y - q_s q_z) & 2(q_x q_z + q_s q_y) \\ 2(q_x q_y + q_s q_z) & q_s^2 - q_x^2 + q_y^2 - q_z^2 & 2(q_y q_z - q_s q_x) \\ 2(q_x q_z - q_s q_y) & 2(q_y q_z + q_s q_x) & q_s^2 - q_x^2 - q_y^2 + q_z^2 \end{bmatrix} \tag{3.19}$$

Using the normalisation constraint, $q_s^2 + q_x^2 + q_y^2 + q_z^2 = 1$, the DCM Simplifies to,

$$\mathbf{A}_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} 1 - 2(q_y^2 + q_z^2) & 2(q_x q_y - q_s q_z) & 2(q_x q_z + q_s q_y) \\ 2(q_x q_y + q_s q_z) & 1 - 2(q_x^2 + q_z^2) & 2(q_y q_z - q_s q_x) \\ 2(q_x q_z - q_s q_y) & 2(q_y q_z + q_s q_x) & 1 - 2(q_x^2 + q_y^2) \end{bmatrix} \tag{3.20}$$

The body-fixed angular rates of the satellite in CRF, $\omega_{\mathcal{C}}^{\mathcal{B}}$, is expressed as a function of qauternions by,

$$\omega_{\mathcal{C}}^{\mathcal{B}} = \begin{bmatrix} \omega_{bx} \\ \omega_{by} \\ \omega_{bz} \end{bmatrix} = 2 \begin{bmatrix} -q_x & q_s & -q_z & q_y \\ -q_3 & q_4 & q_1 & -q_2 \\ -q_4 & -q_3 & q_2 & q_s \end{bmatrix} \begin{bmatrix} \dot{q}_s \\ \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix} \tag{3.21}$$

Inversly the quaternion rates as a function of the body rates are,

$$\begin{bmatrix} \dot{q}_s \\ \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -\omega_{bx} & -\omega_{by} & -\omega_{bz} \\ \omega_{bx} & 0 & \omega_{bz} & -\omega_{by} \\ \omega_{by} & -\omega_{bz} & 0 & \omega_{bx} \\ \omega_{bz} & \omega_{by} & -\omega_{bx} & 0 \end{bmatrix} \begin{bmatrix} q_s \\ q_x \\ q_y \\ q_z \end{bmatrix} \tag{3.22}$$

Quaternions will be used throughout this thesis for attitude representations. Quaternions do not have ambiguity regarding the order of rotations and the rotation is around a well-defined axis. The sin and cosine elements of the rotation matrix are already encodedin the quaternion form of the DCM. Therfore, only one matrix operation is required for attitude transforms, where Eular angles reguire three.

3.4.2 Dynamics

The rotational dynamics of a rigid body satellite can be described using the Newton-Euler equations, which are applicable to all rigid inertial bodies [?]. The angular momentum of the satellite is expressed as:

$$\dot{\mathbf{H}} = \frac{d\mathbf{H}}{dt} = \mathbf{I}\dot{\boldsymbol{\omega}} \tag{3.23}$$

where $\mathbf{H}$ represents the angular momentum vector and $\mathbf{I}$ is the diagonalized moment of inertia tensor about the satellite's principal axes. In the absence of external torques, the rotational kinematics of a rigid satellite about its center of mass can be described by Euler's rotational equations:

$$I_{xx}\dot{\omega}_x = \omega_y\omega_z(I_{yy} - I_{zz}) \tag{3.24}$$

$$I_{yy}\dot{\omega}_y = \omega_x\omega_z(I_{zz} - I_{xx}) \tag{3.25}$$

$$I_{zz}\dot{\omega}_z = \omega_x\omega_y(I_{xx} - I_{yy}) \tag{3.26}$$

where I_{xx}, I_{yy}, and I_{zz} are the principal moments of inertia, which remain constant and depend on the satellite's mass distribution and geometric configuration.

The stability characteristics of the satellite's rotational motion are governed by its mass distribution. According to Marsden and Ratiu [?], rotation about the major and minor principal axes is inherently stable, while rotation about the intermediate axis exhibits unstable behavior. Under constant energy conditions, any initial rotation about the intermediate axis will gradually redistribute energy to the major and minor axes through nutation effects.

For the translational dynamics, Newton's second law governs the linear motion of the satellite with mass m. The discrete-time position and velocity propagation equations are:

$$\mathbf{r}_t = \mathbf{r}_{t-1} + \mathbf{v}_t \Delta t + \frac{1}{2m}\mathbf{F}(t)\Delta t^2 \tag{3.27}$$

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \frac{1}{m}\mathbf{F}(t)\Delta t \tag{3.28}$$

where $\mathbf{F}(t)$ represents the net external force acting on the satellite. For the orbital environment considered in this work, where external perturbations are negligible compared to gravitational forces, and given that precise mass properties may not be available, the translational motion can be approximated using kinematic models where the current velocity depends primarily on the previous velocity state.

To propagate the quaternion representing the satellite's attitude over time, the quaternion derivative must first be computed. The time derivative of the quaternion $\mathbf{q}_{B/I}$, which describes the rotation from the inertial frame to the body frame, is calculated using quaternion multiplication with the angular velocity vector:

$$\dot{\mathbf{q}}_{B/I} = \frac{1}{2}(\mathbf{q}_{B/I} \otimes \boldsymbol{\omega}) \tag{3.29}$$

where $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ is the angular velocity vector expressed in the body frame. Expanding this quaternion multiplication yields:

$$\dot{\mathbf{q}}_{B/I} = \frac{1}{2}\begin{bmatrix} q_{B/I,0}\omega_x - q_{B/I,3}\omega_y + q_{B/I,2}\omega_z \\ q_{B/I,3}\omega_x + q_{B/I,0}\omega_y - q_{B/I,1}\omega_z \\ -q_{B/I,2}\omega_x + q_{B/I,1}\omega_y + q_{B/I,0}\omega_z \\ -q_{B/I,1}\omega_x - q_{B/I,2}\omega_y - q_{B/I,3}\omega_z \end{bmatrix} \tag{3.30}$$

where $q_{B/I,0}$, $q_{B/I,1}$, $q_{B/I,2}$, and $q_{B/I,3}$ are the scalar and vector components of the quaternion, respectively.

The quaternion integration is performed using a simple Euler integration scheme. First, the quaternion is propagated forward in time using:

$$\bar{\mathbf{q}}_{B/I}(t + \Delta t) = \mathbf{q}_{B/I}(t) + \dot{\mathbf{q}}_{B/I}\Delta t \tag{3.31}$$

where $\bar{\mathbf{q}}_{B/I}(t + \Delta t)$ represents the unnormalized quaternion after integration. Since quaternion integration may introduce numerical errors that violate the unit quaternion constraint, the result must be renormalized:

$$\mathbf{q}_{B/I}(t + \Delta t) = \frac{\bar{\mathbf{q}}_{B/I}(t + \Delta t)}{||\bar{\mathbf{q}}_{B/I}(t + \Delta t)||} \tag{3.32}$$

This normalization step ensures that the quaternion maintains its unit magnitude, preserving the validity of the attitude representation.

3.5 Conclusion

Chapter 4

Image Processing

4.1 Introduction

4.2 Pinhole Camera Model

The Ideal Pinhole Camera model can be seen as a dark box with a hole on the side of the box, this is called the optical centre. If a point passes through the optical centre, it will be reflected on the inside of the box. The two sides of the box being defined as the focal length f and the projection side of the box is defined as the projection of the plane.

4.3 Measurement Extraction

Chapter 5

State Estimation

5.1 Introduction

5.2 Extended Kalman Filter

5.3 System Modelling

5.3.1 Motion Model

5.3.2 Measurement Model

5.4 Simulation

5.5 Practical Consideration

5.5.1 Number of Features

5.5.2 Outliers

5.6 Conclusion

Chapter 6

Experiments

6.1 Introduction

6.2 Conclusion

Chapter 7

Conclusion and Future Work

7.1 Conclusion

7.2 Future Work

References

Appendix A

Appendix title goes here