EEEN313/ECEN405

Three Phase Power 2 with examples

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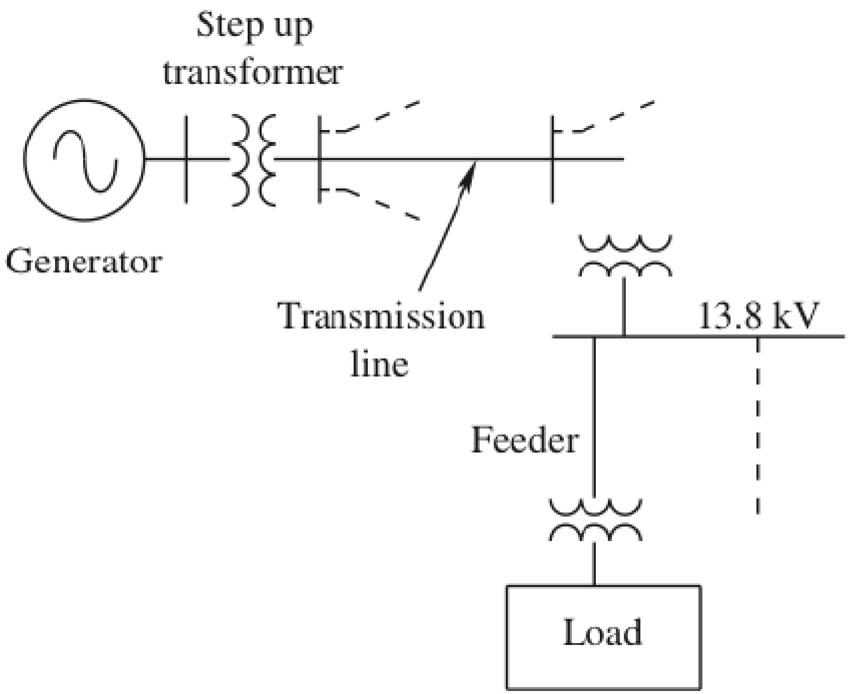
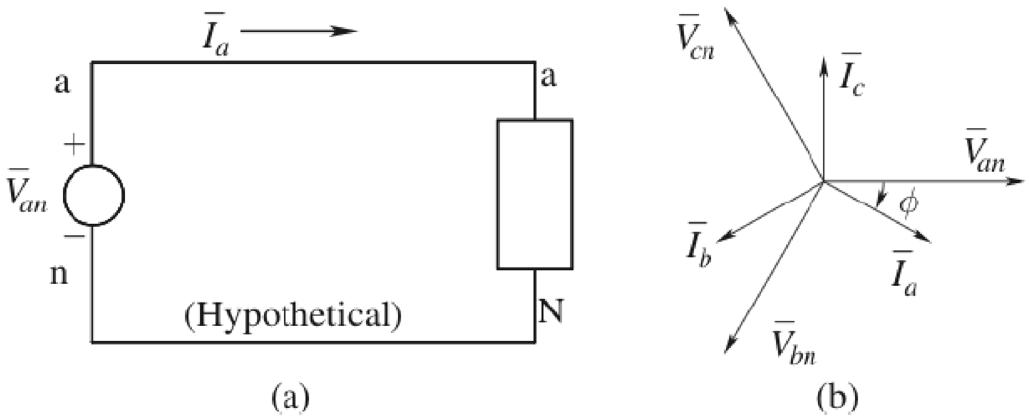
3 phase single line diagram

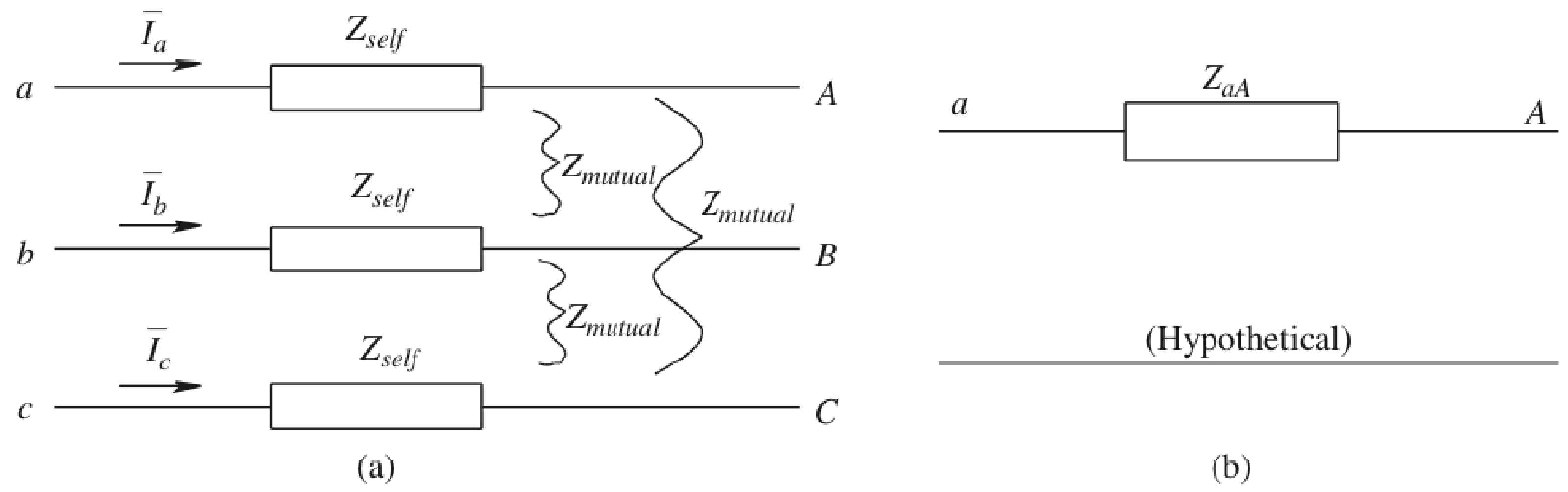


Per-phase circuit



The total real and reactive powers in a balanced three-phase circuit can be obtained by multiplying the per-phase values by a factor of three.
The power factor is same as its per-phase value.

Mutual coupled phases

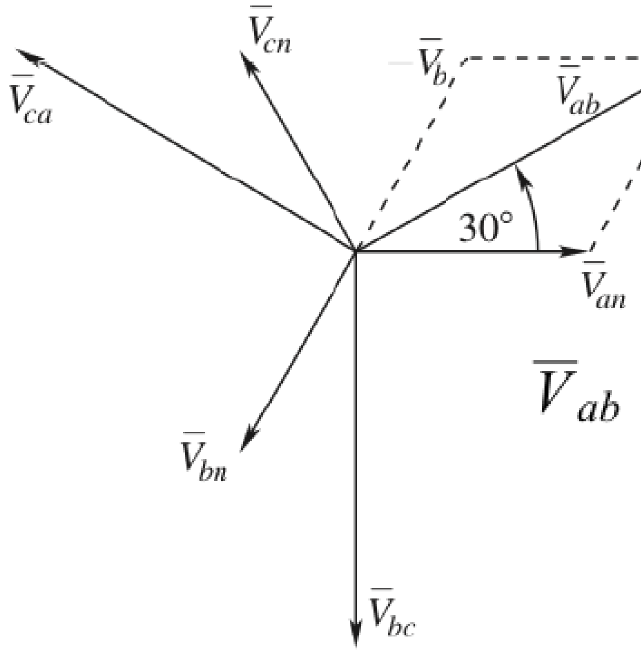


$$\overline{V}_{aA} = Z_{self}\bar{I}_a + Z_{mutual}\bar{I}_b + Z_{mutual}\bar{I}_c$$

$$\bar{I}_a + \bar{I}_b + \bar{I}_c = 0. \quad \overline{V}_{an} + \overline{V}_{bn} + \overline{V}_{cn} = 0 \quad \text{and} \quad v_{an}(t) + v_{bn}(t) + v_{cn}(t) = 0$$

$$\overline{V}_{aA} = (Z_{self} - Z_{mutual})\bar{I}_a = Z_{aA}\bar{I}_a$$ where $$Z_{aA} = Z_{self} - Z_{mutual}$$

Line-Line voltages



$$\overline{V}_{ab} = \sqrt{3}V_{ph}\angle\frac{\pi}{6}$$

$$\overline{V}_{bc} = \sqrt{3}V_{ph}\angle\left(\frac{\pi}{6}-\frac{2\pi}{3}\right) = \sqrt{3}V_{ph}\angle-\frac{\pi}{2}$$

$$\overline{V}_{ca} = \sqrt{3}V_{ph}\angle\left(\frac{\pi}{6}-\frac{4\pi}{3}\right) = \sqrt{3}V_{ph}\angle-\frac{7\pi}{6}$$

$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn}$, $\overline{V}_{bc} = \overline{V}_{bn} - \overline{V}_{cn}$, and $\overline{V}_{ca} = \overline{V}_{cn} - \overline{V}_{an}$

$V_{LL} = \sqrt{3}V_{ph}$ $\overline{V}_{ab}$ leads $\overline{V}_{an}$ by $\pi/6$ radians (30°).

Example 1

- A 460-volt, 50-hertz three-phase Y-connected load draws a line current of $65\angle -26^{\circ}$ A.
- Find P, Q, and S for this three-phase load

$$P = \sqrt{3}\left|V_{line}\right|\left|I_{line}\right| pf = \sqrt{3}(460)(65)\cos\left(0^{\circ} - (-26^{\circ})\right)$$
$$= 46.55 \ \text{kW}$$
$$Q = \sqrt{3}\left|V_{line}\right|\left|I_{1ine}\right| \sin\left(0^{\circ} - (-26^{\circ})\right)$$
$$= 22.70 \ \text{kVAR}$$

I’ve chosen 0° as the angle for the specified load voltage.

$$S = P + jQ = 46.5 + j22.70 \ \ kVA$$
$$= 51.79\angle 26^{\circ} \ \ kVA$$

Example 2

Another three-phase system operates at 8,100 volts and 50 hertz. The connected load absorbs 285 kW at 0.85 power factor lagging. Find the line current.

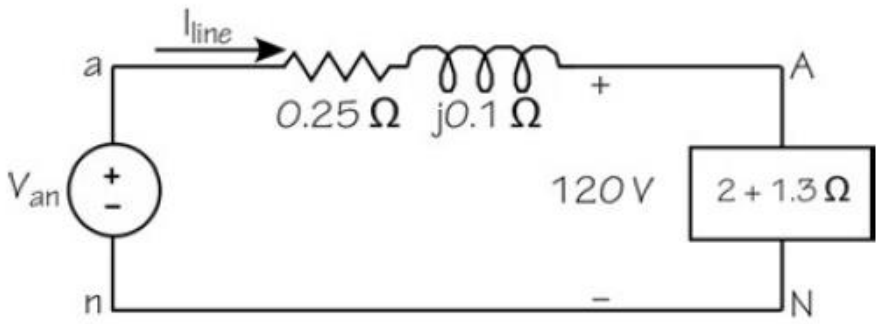
$$P = \sqrt{3}\left|V_{line}\right|\left|I_{line}\right| pf$$

$$\begin{aligned} I_{line} &= \frac{P}{\sqrt{3}\left|V_{line}\right| pf} \angle -\cos^{-1}(pf) \\ &= \frac{285 \times 10^3}{\sqrt{3}(8100)(0.85)} = 23.9\angle -31.8^\circ \text{ A} \end{aligned}$$

We have to consciously insert the correct sign into the phase angle.
Here, the power factor is *lagging* so the load must be inductive. That means that the current *lags* the voltage so its angle must be negative relative to the angle of the voltage

Example 3 – Y connected

A 208-V, 60-Hz, 3Ø, Y-connected load has a per-phase impedance of 2 + j1.3 /Ø. The wire impedance is 0.25 + j0.1 /Ø. Find the line current, the power factor of the load, and the percent efficiency of the system.

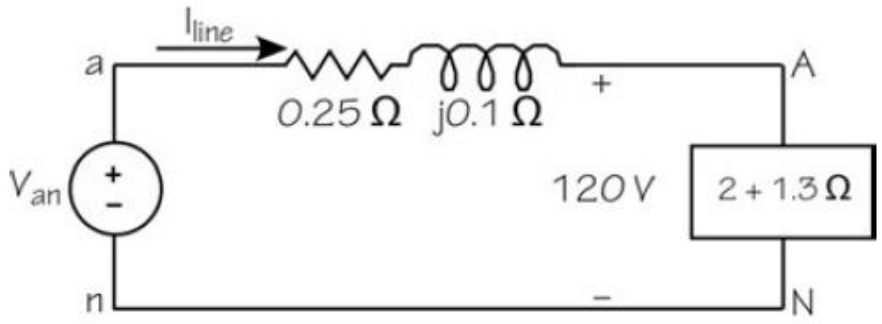


The line current, in both the single-phase equivalent and the three-phase system, is the load voltage (taken with a phase angle of 0°) divided by the load impedance:

$$I_{line} = \frac{120}{2 + j1.3} = 50.3\angle -33.0^\circ \quad A$$

The power factor of the load comes from the phase angle of the load voltage (chosen as 0°) and the phase angle of the line current

$$pf = \cos(-33.0^\circ) = 0.839 \quad \text{lagging}$$



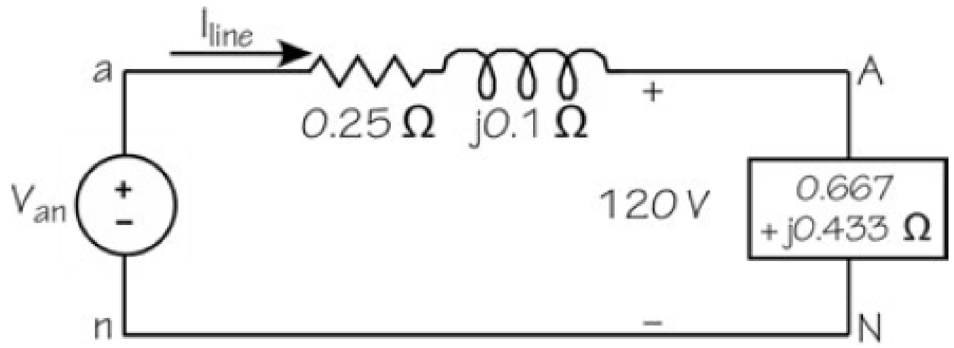
The power absorbed by the single-phase load is

$$P_{load} = (120)(50.3)(0.839) = 5.06 \ \text{kW}$$

$$P_{loss} = (0.25)(50.3)^2 = 632.5 \ \text{W}$$

$$\%\eta = 100 \frac{5.06 \times 10^3}{5.06 \times 10^3 + 632.5} = 88.9\%$$

Example 4 – Delta connected Load



$$I_{line} = \frac{120}{0.667 + j0.433} = 150.9\angle -33.0^\circ \text{ A}$$

$$P_{load} = (120)(150.9)(\cos(-33.0^\circ)) = 15.19 \text{ kW}$$

This result is exactly three times that for the Y-connected impedance.

$$P_{loss} = (0.25)(150.9)^2 = 5.69 \text{ kW}$$

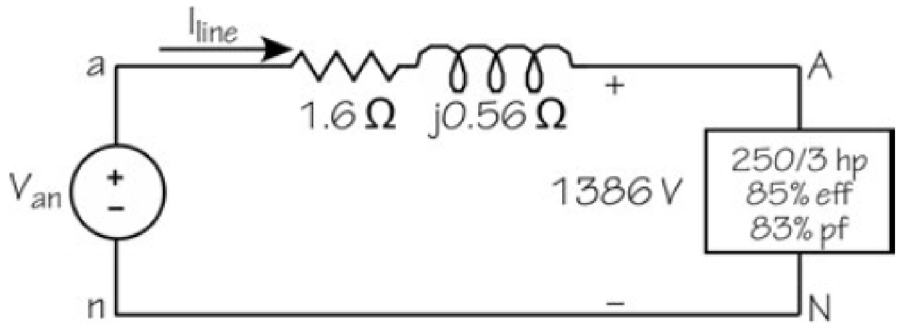
$$\%\eta = 100\frac{15.19}{15.19 + 5.69} = 72.7\%$$

The efficiency is low because the wiring is not large enough to efficiently carry the large current.

Example 5 – Three Phase Motor

A three-phase 2400-volt, 250-horsepower induction motor is 85% efficient. Its rated power factor is 83%. The line impedance is 1.6 + j0.56 ohms per phase. Find the overall efficiency of the system

$$P_{motor} = \frac{250 \times 746}{0.85} = 219.4 \ \text{kW}$$



$$V_{load} = \frac{2400}{\sqrt{3}} = 1386 \ \text{V}$$

(single phase equivalent)

$$P_{motor/phase} = 219.4/3 = 73.14 \ \text{kW} \ @0.83 \text{ lagging}$$

$$I_{line} = \frac{73.14 \times 10^3}{(1386)(0.83)} \angle -\cos^{-1}(0.83) = 63.58\angle -33.9^\circ \ \text{A}$$

$$P_{loss/phase} = (1.6)(63.58)^2 = 6.47 \ \text{kW}$$

$$\%\eta = 100 \frac{73.14}{73.14 + 6.47} = 91.9\%$$

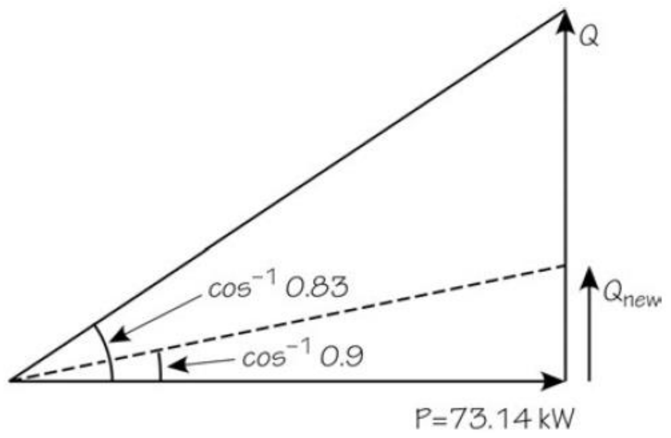
(electrical)

$$\text{overall } \%\eta = 0.919 \times 0.85 = 78.1\%$$

(electrical and motor)

Example 6 – Power Factor Support for Three Phase Motor

How much reactive power do we need to maintain a power factor of 0.9?



$$Q_{load/phase} = 73.14\tan\left(\cos^{-1}(0.83)\right) = 49.15 \quad kVAR$$

By using the triangle, the new reactive power must be (to achieve 0.9)

$$Q_{new/phase} = 73.14\tan\left(\cos^{-1}(0.9)\right) = 35.42 \quad kVAR$$

Since the motor is inductive, PF is lagging. So we need capacitors.

$$Q_{C/phase} = 35.42 - 49.15 = -13.73 \ \ \mathrm{kVAR}/\phi$$