ENGR222 Assignment 4

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2. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a linear transformation, and

$$T(2,0,-1) = (1,0,2,1),\ T(0,1,1) = (-4,3,1,0),\ T(-3,1,2) = (0,1,-2,0)$$

Find the matrix A so that $T\mathbf{v} = A\mathbf{v}$ for all $\mathbf{v} \in \mathbb{R}^3$

$$A\begin{bmatrix}2\\0\\-1\end{bmatrix} = \begin{bmatrix}1\\0\\2\\1\end{bmatrix},\ A\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}-4\\3\\1\\0\end{bmatrix},\ A\begin{bmatrix}-3\\1\\2\end{bmatrix} = \begin{bmatrix}0\\1\\-2\\0\end{bmatrix}$$

$$A\begin{bmatrix}2 & 0 & -3\\0 & 1 & 1\\-1 & 1 & 2\end{bmatrix} = \begin{bmatrix}1 & -4 & 0\\0 & 3 & 1\\2 & 1 & -2\\1 & 0 & 0\end{bmatrix}$$

$$A = \begin{bmatrix}1 & -4 & 0\\0 & 3 & 1\\2 & 1 & -2\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & -3\\0 & 1 & 1\\-1 & 1 & 2\end{bmatrix}^{-1}$$

$$A = \begin{bmatrix}-5 & 7 & -11\\2 & -1 & 4\\1 & 1 & 0\\-1 & 3 & -3\end{bmatrix}$$

3. Find the matrix for the projection $\mathbb{R}^3 \to \mathbb{R}^2$ onto yz-plane (i.e., $(x,y,z) \to (y,z)$)

$$x : \begin{bmatrix}1\\0\\0\end{bmatrix} \to \begin{bmatrix}0\\0\end{bmatrix}$$

$$y : \begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix}$$

$$z : \begin{bmatrix}0\\0\\1\end{bmatrix} \to \begin{bmatrix}0\\1\end{bmatrix}$$

$$A \to \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

4. Find the eigenvalues and eigenvectors for each eigenvalue of the following:

$$\begin{bmatrix}-2 & 1 & -1\\19 & -5 & 4\\43 & -13 & 12\end{bmatrix}$$

$$\text{Eigenvalues} : \lambda_1 = -1, \quad \lambda_2 = 2, \qquad \lambda_3 = 4$$

$$\text{Eigenvectors} : \mathbf{v}_1 = \begin{pmatrix}0\\1\\1\end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix}-1\\-1\\3\end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix}-1\\1\\7\end{pmatrix}$$