EEEN313/ECEN405

Power (Electric)

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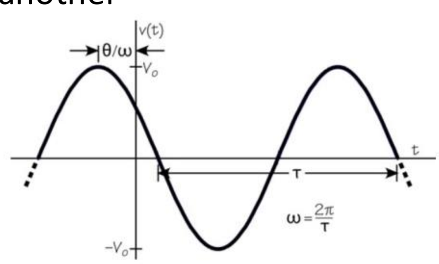
Sine and Cosine

- Alternating Current (AC) – Sines and Cosines that move large blocks of energy from one place to another

Cosine starts at t=0
The period of cosine is τ
We say that this is a *leading* phase angle ϑ.



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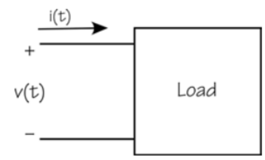
AC Power



- We can treat AC power as a function of time (in sine and cosine)

$$v(t) = V_{peak}\cos(2\pi f t + \theta_v)$$
$$i(t) = I_{peak}\cos(2\pi f t + \theta_i)$$

$$p(t) = v(t)i(t)$$

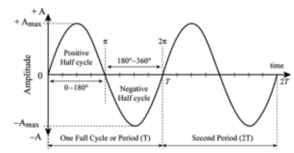


Figure 1

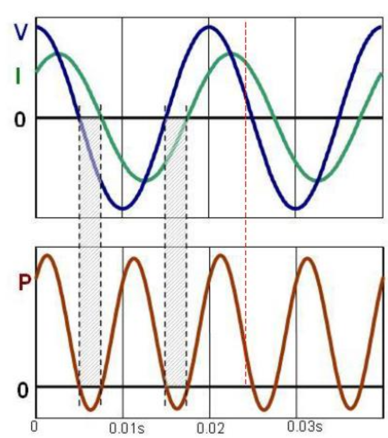
P(t) is instantaneous power
But this isn't very useful, because we really need to know how much power is being delivered on the average.
More important, how much energy is being moved (this is what power companies want us to pay for)

Instantaneous Power

- Instantaneous Power – power at instant in time

$$p(t) = v(t)\ i(t)$$

- If *p(t)* has positive value (current in same direction as **positive** voltage polarity) it is **source** of power
- if **negative** then it is **sink** (dissipater) of power



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Energy

- is the time integral of power
  - often use symbol *W* (but units are joules)
  - (do not confuse with unit for power Watts, W)

$$W = \int_{t_1}^{t_2} p(t)dt$$

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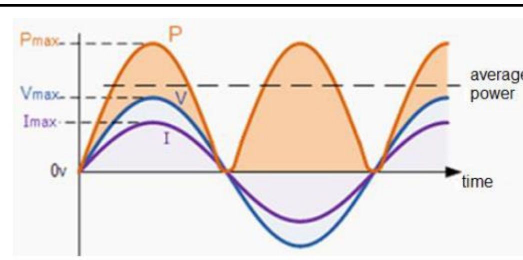
Average Power



- Power Averaged over entire cycle
- Also known as **real** power or **active** power

$$P_{av} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt = \frac{W}{T}$$

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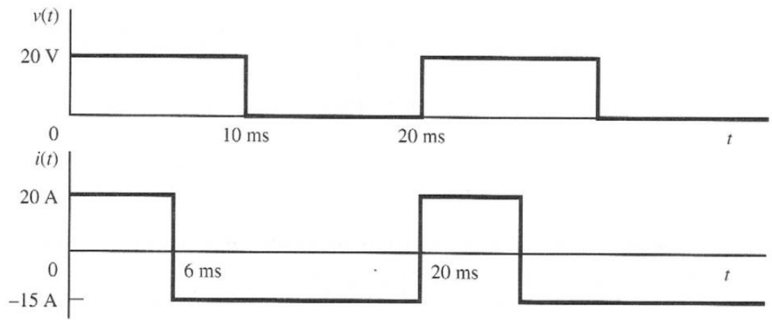
Example

- Find:
  - The instantaneous power
  - Energy absorbed by the device in one period
  - Average power absorbed by the device



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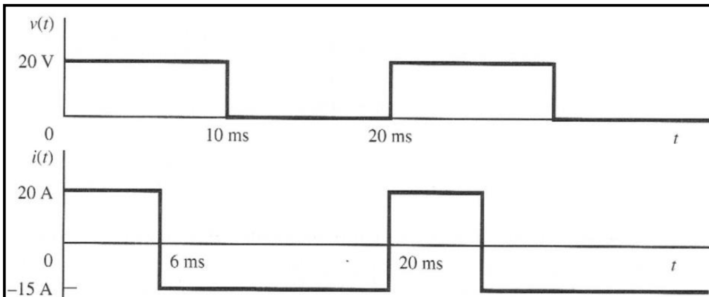

$$v(t) = \begin{cases} 20\text{ V} & 0 < t < 10\text{ ms} \\ 0\text{ V} & 10\text{ ms} < t < 20\text{ ms} \end{cases} \qquad i(t) = \begin{cases} 20\text{ A} & 0 < t < 6\text{ ms} \\ -15\text{ A} & 6\text{ ms} < t < 20\text{ ms} \end{cases}$$

(a) Instantaneous power is simply:

$$p(t) = \begin{cases} 400\ \text{W} & 0 < t < 6\text{ ms} \\ -300\ \text{W} & 6\text{ ms} < t < 10\text{ ms} \\ 0 & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

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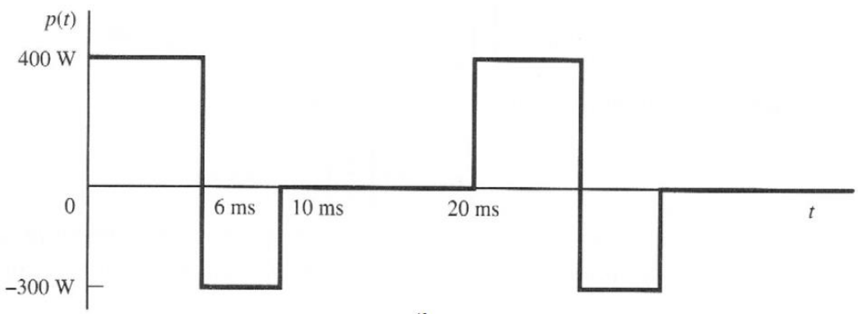
(a) Instantaneous power is simply:

$$p(t) = \begin{cases} 400 \;\; \text{W} & 0 < t < 6 \text{ ms} \\ -300 \;\; \text{W} & 6 \text{ ms} < t < 10 \text{ ms} \\ 0 & 10 \text{ ms} < t < 20 \text{ ms} \end{cases}$$



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(b) Energy absorbed by the device in one period is:

$$W = \int_0^T p(t)dt$$

$$= \int_0^{0.006} 400dt + \int_{0.006}^{0.010} (-300)dt + \int_{0.010}^{0.020} (0)dt$$

$$= 2.4 - 1.2 = 1.2 \text{ J}$$

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(a) Instantaneous power is simply:

$$p(t) = \begin{cases} 400 \text{ W} & 0 < t < 6 \text{ ms} \\ -300 \text{ W} & 6 \text{ ms} < t < 10 \text{ ms} \\ 0 & 10 \text{ ms} < t < 20 \text{ ms} \end{cases}$$

(c) Average power is

$$P_{av} = \frac{1}{T}\int_0^T p(t)dt$$

$$= \frac{1}{0.020}\left[\int_0^{0.006} 400dt + \int_{0.006}^{0.010} (-300)dt + \int_{0.010}^{0.020} (0)dt\right]$$

$$= 60 \text{ W} \quad \left(= \frac{W}{T}\right)$$

DC Source

- DC voltage with average current

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\, i(t)\,dt$$

- If the circuit is purely resistive v=Ri

$$P_{av} = \frac{R}{T}\int_0^T i^2(t)dt$$

RMS – Root Mean Square

- For the rms current I $$P_{av} = RI_{RMS}^2$$

- Hence the term 'RMS'

$$P_{av} = \frac{R}{T}\int_0^T i^2(t)dt \qquad I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

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AC Power Quantities - Averages

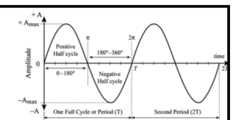

Figure 1

- Average power comes from averaging the instantaneous power p(t) over one period of the ac waveform

$$p(t) = \left[V_{peak}\cos(2\pi ft + \theta_v)\right]\left[I_{peak}\cos(2\pi ft + \theta_i)\right]$$

$$p(t) = \frac{V_{peak}I_{peak}}{2}\left[\cos(\theta_v - \theta_i) + \cos\left(4\pi ft + (\theta_v - \theta_i)\right)\right]$$

$$P_{av} = \frac{V_{peak}I_{peak}}{2}\cos(\theta_v - \theta_i)$$ Good for Mathematicians

We don't use the peak values in AC systems; we use RMS value. For sine wave $V_{peak}/\sqrt{2} = 0.707\ V_{peak}$

$$P = V_{RMS}I_{RMS}\cos\left(\theta_v - \theta_i\right)$$

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RMS from Peak Values

Rectangular pulse (duty cycle D)	$V_{RMS} = V_p \sqrt{D}$
Sinusoids	$V_{RMS} = V_p / \sqrt{2}$
Full-wave rectified sinusoids	$V_{RMS} = V_p / \sqrt{2}$
Half-wave rectified sinusoids	$V_{RMS} = V_p / 2$
Sum of different frequency sinusoids	$V_{RMS} = \sqrt{V_{1,RMS}^2 + V_{2,RMS}^2 + \ldots}$
Triangular wave	$V_{RMS} = V_p / \sqrt{3}$

(substitute I for V to determine currents)

Sum of periodic waveforms

- If periodic voltage is the sum of two periodic voltage waveforms $v_1(t) + v_2(t)$

$$V_{RMS}^2 = \frac{1}{T}\int_0^T (v_1 + v_2)^2\, dt$$

$$= \frac{1}{T}\int_0^T v_1^2 dt \frac{1}{T}\int_0^T 2v_1 v_2 dt + \frac{1}{T}\int_0^T v_2^2 dt$$

- If the voltages are of two different frequencies (orthogonal), then the term $v_1 v_2$ is zero

Clicker Quiz

The rms voltage of a rectangular wave is dependent on duty cycle D.

A. True
B. False

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Clicker Quiz

The Average Power and Energy are related by time only

- True
- False

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