EEEN313/ECEN405

Electromagnetics

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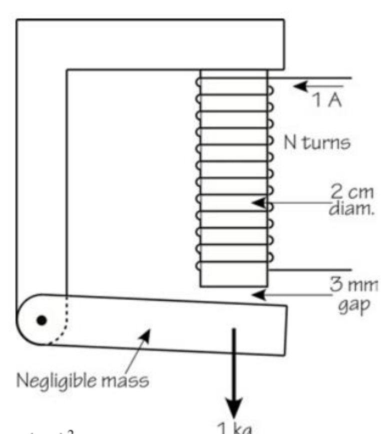
Design Example - Actuator

- Design to pull 1 kg mass upward
- We need to find the number of turns of wire we need to lift it.



$$\mathfrak{F} = Ni = 1N \text{ A-turns}$$

$$A_{gap} = \pi\left(\frac{2\times10^{-2}}{2}\right)^2 = 10^{-4}\pi \text{ m}^2$$

Force in the airgap

$$F = \frac{(1)\left(4\pi\times10^{-7}\right)\left(10^{-4}\pi\right)N^2}{(2)\left(3\times10^{-3}\right)^2} = 21.93\times10^{-6}N^2 \text{ newtons} = \frac{\mu_r\,\mu_0 A(Ni)^2}{2x^2} \text{ newtons (N)}$$

2

Design Example

The force necessary to just move the mass is

$$F = 1 \times 9.8 \ \text{ newtons}$$

Equating the required force to the available force and solving for the number of turns:

$$9.8 = 21.93 \times 10^{-6} N^2$$
$$N = \sqrt{\frac{9.8}{21.93 \times 10^{-6}}} = 668 \ \text{ turns}$$

Notice that the force equation contains the term Ni. If we want to use just half the current, we need twice the turns, and so on.

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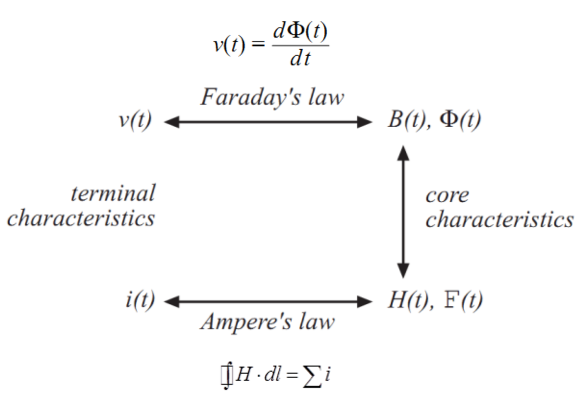
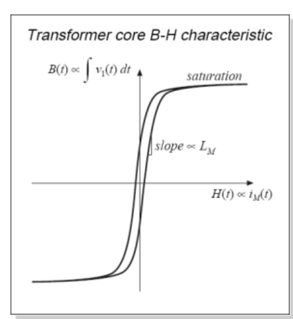
- Basic relations





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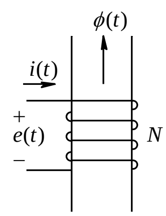
FARADAY'S LAW: INDUCED VOLTAGE IN A COIL DUE TO TIME-RATE OF CHANGE OF FLUX LINKAGE



$$e(t) = \frac{d}{dt}\lambda(t) = N\frac{d}{dt}\phi(t)$$

$$\phi(t) = \phi(0) + \frac{1}{N}\int_0^t e(\tau)\cdot d\tau$$

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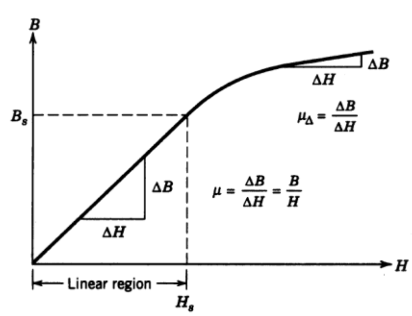
Some Background

- B-H curve



$B = \mu H$

where, B: *flux density* $[Wb/m^2]$ *or* $[T]$

μ: *permeability* $[H/m]$

where, $\mu = \mu_0 \cdot \mu_r$

$(\mu_0 = 4\pi \times 10^{-7}, \mu_r = 500 \sim 3000,$ *iron*$)$

- H: Magnetic Field Strength
- Permeability : degree of magnetization that a material obtains in response to an applied magnetic field

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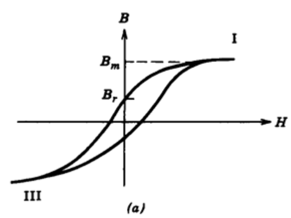
Types

- Various types of converters with electrical isolation. Two main types based on the way they utilise the transformer core:
  - Unidirectional core excitation where only positive part (Q1) of B-H loop is used
  - Bidirectional core excitation where both the positive (Q1) and the negative (Q3) part of B-H loop is utilised



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Types

- Unidirectional:
  - Based on the non-isolation topologies
  - Flyback Converter (from Buck-Boost)
  - Forward Converter (from Step-Down)
- Bidirectional:
  - Inverter topologies
  - Push-pull
  - Half Bridge
  - Full Bridge

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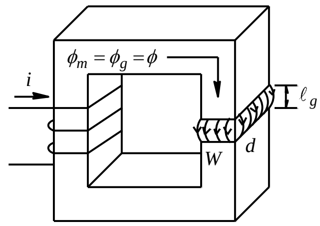
AMPERE-TURNS AND FLUX



$$H_m \ell_m + H_g \ell_g = Ni$$

$$H = \frac{B}{\mu}$$

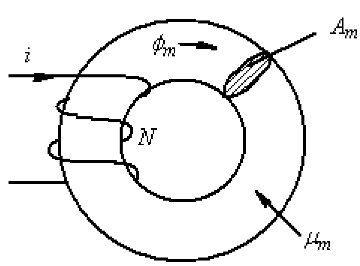
$$\phi\left(\frac{\ell_m}{A_m \mu_m} + \frac{\ell_g}{A_g \mu_o}\right) = Ni$$

$$B = \frac{\Phi}{A}$$

$\Re_m$ $\Re_g$

$$\phi = \frac{Ni}{\Re} \qquad \Re = \Re_m + \Re_g$$

INDUCTANCE



$$\lambda_m = N\phi_m = L_m i$$

Flux Linkage Flux

$$i \xrightarrow{\times\left(\frac{N}{\ell_m}\right)} H_m \xrightarrow{\times(\mu_m)} B_m \xrightarrow{\times(A_m)} \phi_m \xrightarrow{\times(N)} \lambda_m$$

$$L_m = \frac{N^2}{\frac{\ell_m}{\mu_m A_m}}$$

Design of High-Frequency Inductors and Transformers

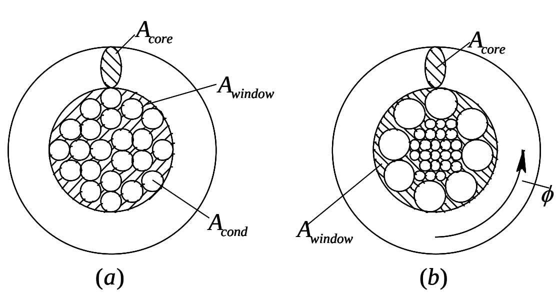
BASICS OF MAGNETIC DESIGN

- The peak flux density $B_{\max}$ in the magnetic core to limit core losses, and
- The peak current density $J_{\max}$ in the winding conductors to limit conduction losses

Popular Materials

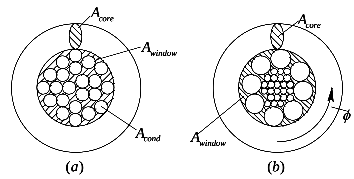
Material	Permeability (relative)	Bsat (tesla)	Loss @ 0.1 T, 100 kHz (mW/cm3)	Usage
Ferrite (Mag. Inc. P)	2500	0.5	80	Power Transformers Filter Inductors (gapped) PFC Inductors (gapped)
Ferrite (Mag. Inc. W)	10,000	0.42	250	EMI Filters (common-mode only)
Molypermalloy (Mag. Inc. MPP)	60	0.75	340	Filter Inductors PFC Inductors
Sendust (Mag. Inc. Kool-Mu)	60	1	850	Filter Inductors PFC Inductors
Powdered iron (Micrometals 52)	75	1.4	3200	Filter Inductors PFC Inductors
80% Cobalt tape (Honeywell 2714A)	100,000	0.55	90	Mag. Amps

INDUCTOR AND TRANSFORMER CONSTRUCTION



AREA-PRODUCT METHOD

Core Window Area A_{window}



$$A_{window} = \frac{1}{k_w} \sum_y \left(N_y A_{cond,y} \right)$$

$$A_{cond,y} = \frac{I_{rms,y}}{J_{\max}}$$

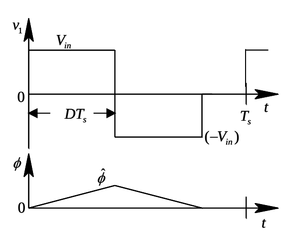
$$A_{window} = \frac{\sum_y \left(N_y I_{rms,y} \right)}{k_w J_{\max}}$$

Core Cross-Sectional Area A_{core}

$$A_{core} = \frac{\hat{\phi}}{B_{\max}}$$

inductor: $\hat{\phi} = \frac{L\hat{I}}{N}$ $\quad A_{core} = \frac{L\hat{I}}{NB_{\max}}$

transformer:



$$\hat{\phi} = \frac{k_{conv} V_{in}}{N_1 f_s}$$

$$A_{core} = \frac{k_{conv} V_y}{N_y f_s B_{\max}}$$

Core Area-Product $A_p = A_{core} A_{window}$

inductor: $$A_p = \frac{L \hat{I} I_{rms}}{k_w J_{\max} B_{\max}}$$

transformer: $$A_p = \frac{k_{conv} \sum V_y I_{y,rms}}{k_w B_{\max} J_{\max} f_s}$$ note: $\frac{V_1}{N_1} = \frac{V_2}{N_2} = etc$

Design Procedure Based on Area-Product A_p

inductor: $$N = \frac{L\hat{I}}{B_{\max} A_{core}} \qquad L = \frac{N^2}{\Re_g} \qquad \Re_g = \frac{\ell_g}{\mu_o A_{core}} \qquad \ell_g = \frac{N^2 \mu_o A_{core}}{L}$$

transformer: $$N_y = \frac{k_{conv} V_y}{A_{core} f_s B_{\max}}$$

Databases

Core No.	Material	AP = $A_W A_{core}$	R_θ ΔT=60 °C	P_{sp} @ ΔT=60 °C	J_{rms} @ ΔT=60 °C & P_{sp}	B_{ac} @ ΔT=60 °C & 100 kHz	$k_{cu} J_{rms} \hat{B} \cdot A_W A_{core}$
• 8 •	• 3F3 •	• 2.1 cm^4 •	• 9.8 °C/W •	• 237 mW/cm^3 •	• $3.3/\sqrt{k_{cu}}$ •	• 170 mT •	• $.0125\sqrt{k_{cu}}$ •

Core No.	Material	AP = $A_W A_c$	R_θ ΔT=60 °C	P_{sp} @ T_s=100 °C	J_{rms} @ T_s=100 °C & P_{sp}	$\hat{B}_{rated}$ @ T_s=100 °C & 100 kHz	$2.22\, k_{cu}\, f\, J_{rms}\, \hat{B}\, AP$ (f = 100kHz)
• 8 •	• 3F3 •	• 2.1 cm^4 •	• 9.8 °C/W •	• 237 mW/cm^3 •	• $(3.3/\sqrt{k_{cu}}) \cdot \sqrt{\frac{R_{dc}}{R_{ac}}}$ A/mm^2 •	• 170 mT •	• $2.6 \times 10^3 \cdot \sqrt{\frac{k_{cu} R_{dc}}{R_{ac}}}$ [V-A] •

Some Points

- This design ignores eddy current losses in windings
  - Can be substantial due to proximity
  - To minimise loss, inductors with single layer construction is suggested
- Temperature rise is an another important factor
- Core saturation

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Thermal considerations – A glimpse

- Winding and core losses rise core temperature – limit to 100-125C
  - Core losses increase above 100C
  - Saturation flux decreases with temp increase
  - Effect on other components
- Copper resistivity changes with temperature

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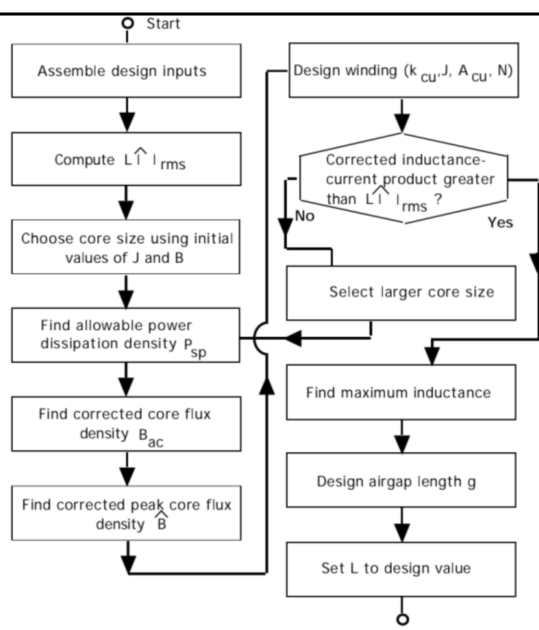
Design Problem Summary

- Challenge - conversion of component operating specs in converter circuit into component design parameters.
- Goal - simple, easy-to-use procedure that produces component design specs that result in an acceptable design having a minimum size, weight, and cost.
- Inductor electrical (e.g.converter circuit) specifications.
  - Inductance value L
  - Inductor currents rated peak current I, rated rms current I_{rms}, and rated dc current (if any) I_{dc}
  - Operating frequency f.
  - Allowable power dissipation in inductor or equivalently maximum surface temperature of the inductor T_s and maximum ambient temperature T_a.
- Transformer electrical (converter circuit) specifications.
  - Rated rms primary voltage V_{pri}
  - Rated rms primary current I_{pri}
  - Turns ratio N_{pri}/N_{sec}
  - Operating frequency f
  - Allowable power dissipation in transformer or equivalently maximum temperatures T_s and T_a
- Design procedure outputs.
  - Core geometry and material.
  - Core size (A_{core} , A_w)
  - Number of turns in windings.
  - Conductor type and area A_{cu}.
  - Air gap size (if needed).
- Three impediments to a simple design procedure.

1. Dependence of J_{rms} and B on core size.
2. How to chose a core from a wide range of materials and geometries.
3. How to design low loss windings at high operating frequencies.

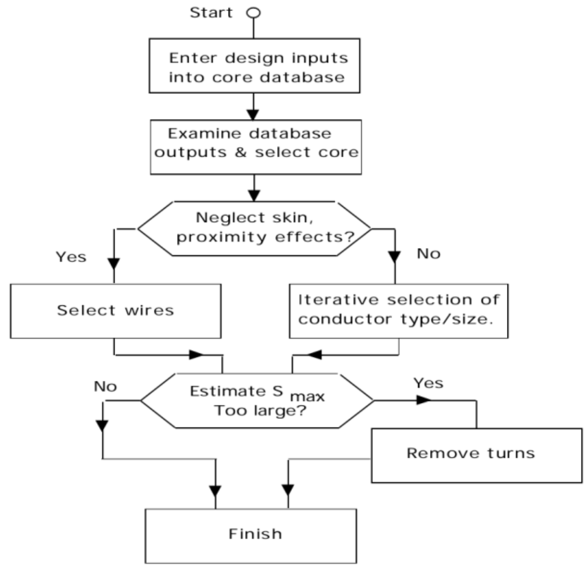
- Detailed consideration of core losses, winding losses, high frequency effects (skin and proximity effects), heat transfer mechanisms required for good design procedures.

Iterative design procedure - Inductor



- Iterative design procedure essentially consists of constructing the core database until a suitable core is found.
- Choose core material and shape and conductor type as usual.
- Use stored energy relation to find an initial area product $A_w A_c$ and thus an initial core size.
  - Use initial values of J_{rms} = 2-4 A/mm^2 and B_{ac} = 50-100 mT.
- Use initial core size estimate (value of a in double-E core example) to find corrected values of J_{rms} and B_{ac} and thus corrected value of $k_{cu} J_{rms} \hat{B} \; A_w A_{core}$.
- Compare $k_{cu} J_{rms} \hat{B} \; A_w A_{core}$ with $L I I_{rms}$ and iterate as needed into proper size is found.

Single Pass Tx Design



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Design Example

- Design inputs
  - V_{pri} = 300 V rms ; I_{rms} = 4 A rms
  - Turns ratio n = 4
  - Operating frequency f = 100 kHz
  - T_s = 100 °C and T_a = 40 °C
- V - I rating S = (300 V rms)(4 A rms) = 1200 watts
- Core material, shape, and size.
  - Use 3F3 ferrite because it has largest performance factor at 100 kHz.
  - Use double-E core. Relatively easy to fabricate winding.
- Core volt-amp rating = $2{,}600 \sqrt{k_{cu}} \sqrt{\frac{R_{dc}}{R_{ac}}}$
  - Use solid rectangular conductor for windings because of high frequency. Thus k_{cu} = 0.6 and R_{ac}/R_{dc} = 1.5.
  - Core volt-amp capability = $2{,}600 \sqrt{\frac{0.6}{1.5}}$ = 1644 watts. > 1200 watt transformer rating. Size is adequate.
- Using core database, R_θ = 9.8 °C/W and P_{sp} = 240 mW/cm^3.
- Flux density and number of primary and secondary turns.
  - From core database, B_{ac} = 170 mT.
  - $N_{pri} = \frac{300\sqrt{2}}{(1.5\text{x}10^{-4}\text{m}^2)(2\pi)(10^5\text{Hz})(0.17\text{ T})}$ = 26.5 ≈ 24. Rounded down to 24 to increase flexibility in designing sectionalized transformer winding.
  - $N_{sec} = \frac{24}{6} = 6$.
- From core database $J_{rms} = \frac{3.3}{\sqrt{(0.6)(1.5)}}$ = 3.5 A/mm^2.
  - $A_{cu,pri} = \frac{4\text{ A rms}}{3.5\text{ A rms/mm}^2}$ = 1.15 mm^2
  - $A_{cu,sec}$ = (4)(1.15 mm^2) = 4.6 mm^2

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- Primary and secondary conductor areas - proximity effect/eddy currents included. Assume rectangular (foil) conductors with $k_{cu} = 0.6$ and layer factor $F_l = 0.9$.
  - Iterate to find compatible foil thicknesses and number of winding sections.
  - 1st iteration - assume a single primary section and a single secondary section and each section having single turn per layer. Primary has 24 layers and secondary has 6 layers.
- Primary layer height $h_{pri} = \frac{A_{cu,pri}}{F_l h_w}$

$$= \frac{1.15\ \mathrm{mm}^2}{(0.9)(20\ \mathrm{mm})} = 0.064\ \mathrm{mm}$$

  - Normalized primary conductor height

$$\phi = \frac{\sqrt{F_l}\, h_{pri}}{d} = \frac{\sqrt{0.9}\,(0.064\ \mathrm{mm})}{(0.24\ \mathrm{mm})} = 0.25\ ;$$

$\delta = 0.24$ mm in copper at100 kHz and 100 °C.

  - Optimum normalized primary conductor height $\phi = 0.3$ so primary winding design is satisfactory.

- Secondary layer height $h_{sec} = \frac{A_{cu,sec}}{F_l h_w}$

$$= \frac{4.6\ \mathrm{mm}^2}{(0.9)(20\ \mathrm{mm})} \approx 0.26\ \mathrm{mm}.$$

  - Normalized secondary conductor height

$$\phi = \frac{\sqrt{F_l}\, h_{sec}}{d} = \frac{\sqrt{0.9}\,(0.26\ \mathrm{mm})}{(0.24\ \mathrm{mm})} = 1$$

  - However a six layer section has an optimum $\phi = 0.6$. A two layer section has an optimum $\phi = 1$. 2nd iteration needed.

- 2nd iteration - sectionalize the windings.
  - Use a secondary of 3 sections, each having two layers, of height $h_{sec} = 0.26$ mm.
  - Secondary must have single turn per layer. Two turns per layer would require $h_{sec} = 0.52$ mm and thus $\phi = 2$. Examination of normalized power dissipation curves shows no optimum $\phi = 2$.

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Summary

- Magnetics - Basics
- Design of High-Frequency Inductors and Transformers

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Concept Quiz

In inductors and transformers discussed here, the window fill-factor is generally as follows:

A. Close to 0.5

B. Close to 1.0