# EEEN313/ECEN405

Power Calculations 2

Engineering Quotes:

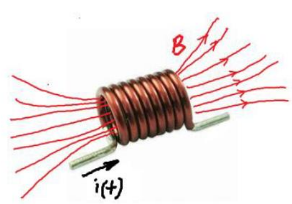
**"A good engineer is someone who can do for a dime what any darn fool can do for a dollar."**



1

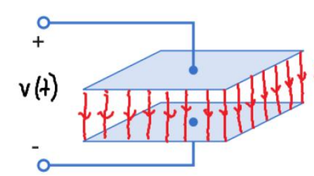
## Inductors and Capacitors

- **Ideal** inductors and capacitors are solely energy storage circuit elements
- On average they do not dissipate power
- They simply absorb (store) energy and subsequently release it




Inductors store energy in magnetic field

$$w(t) = \frac{1}{2} L i^2(t)$$




Capacitors store energy in electric field

$$w(t) = \frac{1}{2} C v^2(t)$$

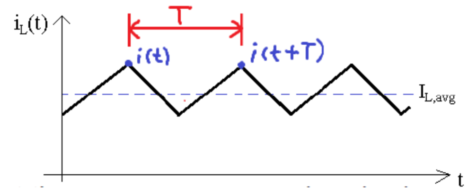


2

# Inductor Average Power

- If the inductor is periodic, the stored energy at the end of one period is same as at the beginning

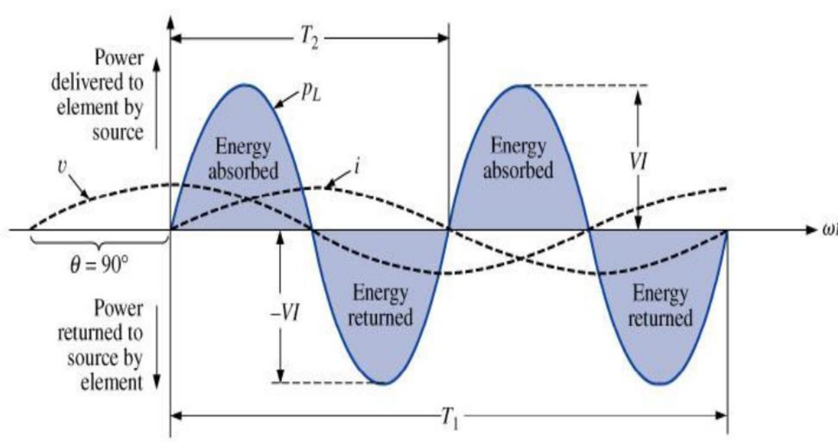
$$i(t+T) = i(t)$$






3

# V, I, P for an Inductor






5

# Inductor Average Voltage

- From the V-I relationship for an inductor

$$i(t_0 + T) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\, dt + i(t_0)$$

- As starting and ending values are same for periodic currents

$$i(t_0 + T) - i(t_0) = \frac{1}{L}\int_{t_0}^{t_0+T} v_L(t)\, dt = 0$$

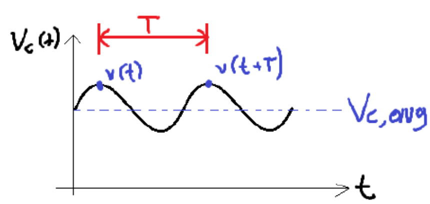
$$\boxed{\text{avg}[v_L(t)] = V_L = \frac{1}{T}\int_{t_0}^{t_0+T} v_L(t)\, dt = 0}$$

This is very important conclusion which is used in the analysis of switching converters



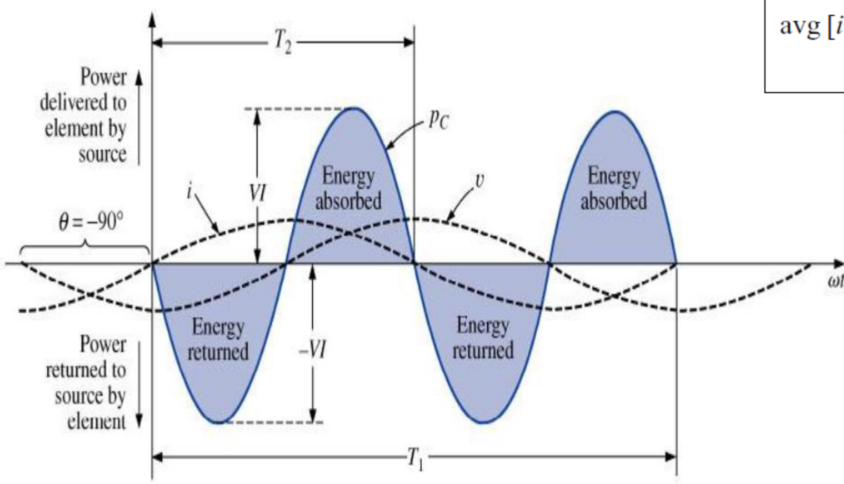
# Same for Capacitor

$$v(t + T) = v(t)$$




$$P_{c,avg} = \frac{1}{2}C\left[\underbrace{v^2(t+T) - v^2(t)}_{0}\right] = 0$$

## V, I, P of Capacitor



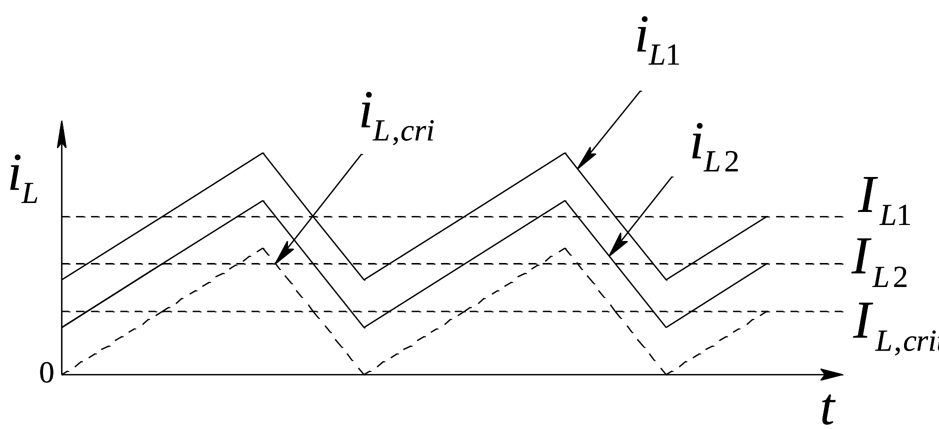

$$\text{avg}\,[i_C(t)] = I_C = \frac{1}{T}\int_{t_0}^{t_0+T} i_C(t)\,dt = 0$$

This is very important conclusion which is used in the analysis of switching converters



8

## Border of CCM and DCM of an Inductor






9

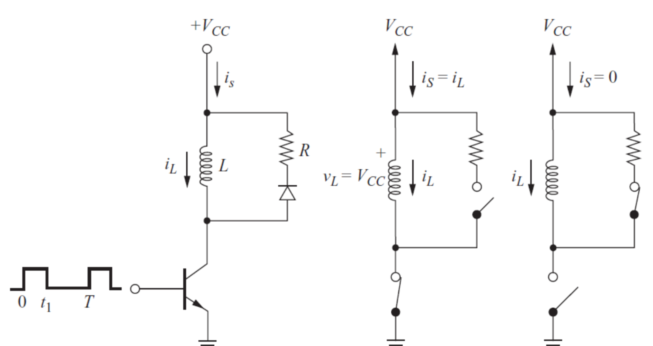
# Energy Recovery

- For periodic operations, net energy in L and C must be zero in steady state condition, otherwise, leading to voltage and current ramp-ups
- Circuit efficiency can be improved if energy can be transferred **to the load (or supply)** rather than dissipated in circuit



13

# Energy Recovery




$$i_L(t)=\frac{1}{L}\int_0^t v_L(t)dt+i_L(0)=\frac{1}{L}\int_0^t V_{CC}dt+0=\frac{V_{CC}t}{L}$$

$$i_L(t)=\left(\frac{V_{CC}t_1}{L}\right)e^{-(t-t_1)/\tau}$$

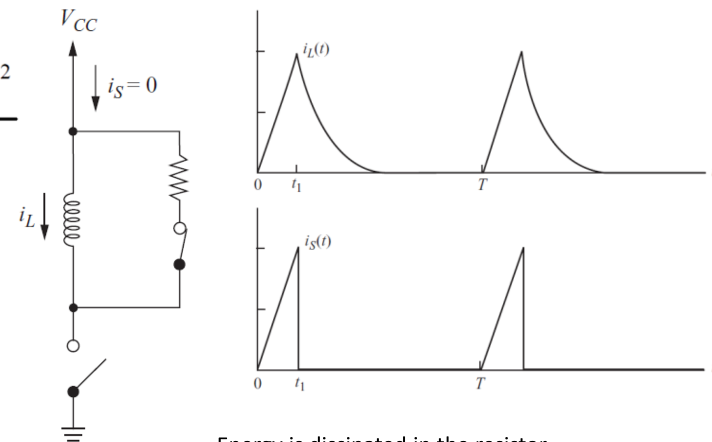


14

# Energy Recovery - NOT

$$i_L(t) = \left(\frac{V_{CC}t_1}{L}\right)e^{-(t-t_1)/\tau}$$

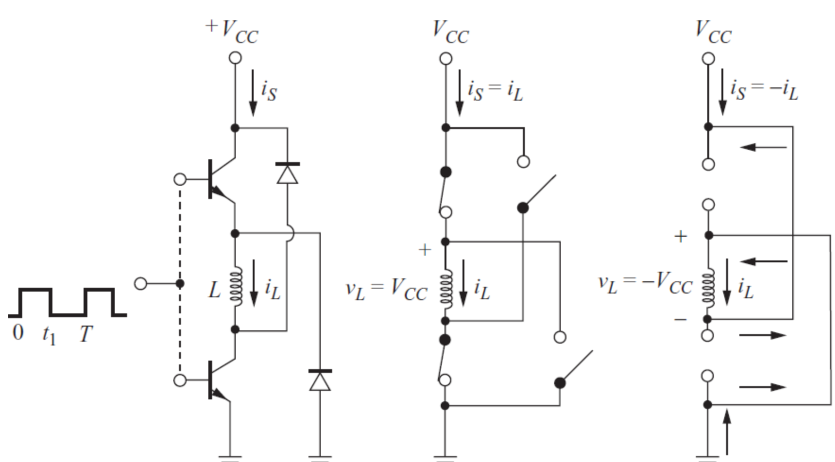
$$P_R = P_s = \frac{(V_{CC}t_1)^2}{2LT}$$




Energy is dissipated in the resistor

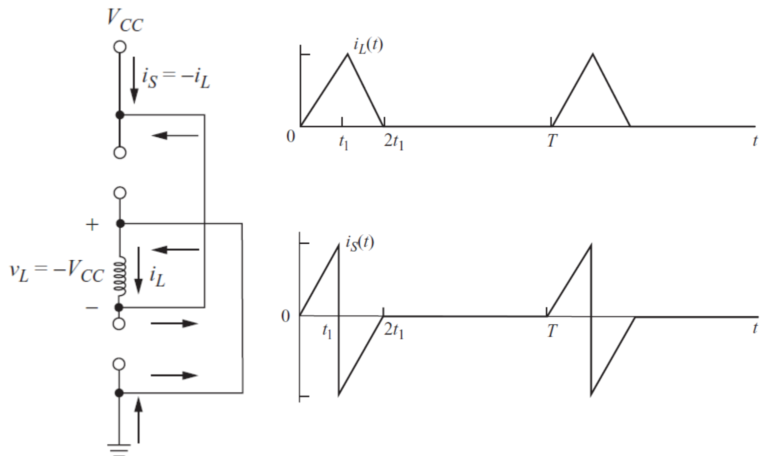


# Energy Recovery



# Energy Recovery

$$i_L(t) = \left(\frac{V_{CC}}{L}\right)(2t_1 - t) \qquad t_1 < t < 2t_1$$

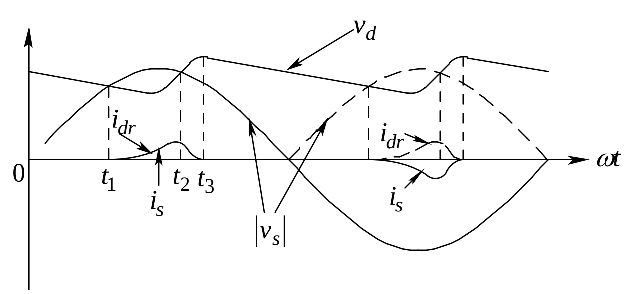





17

# Sinusoidal AC Circuits

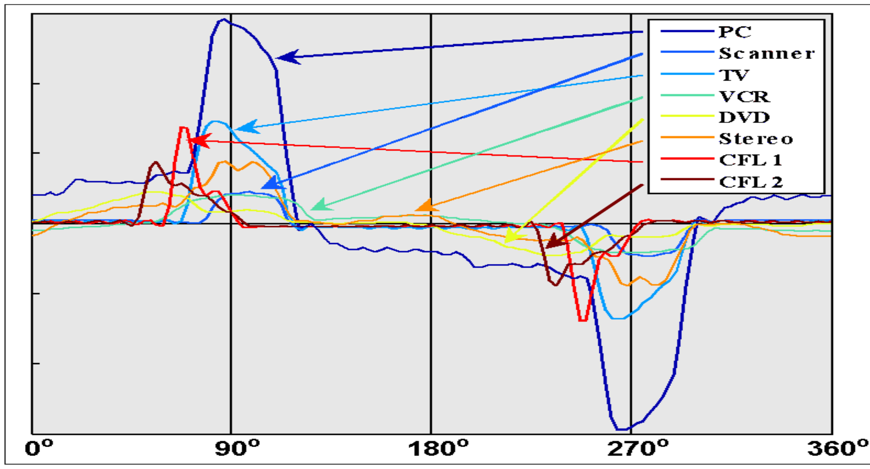
- Voltages and Currents in PE converters are non-sinusoidal
- A non-sinusoidal periodic waveform can be represented by a Fourier series of sinusoids






18

# Current Waveform Examples








# Average Power Calculations

- If periodic voltage and current waveforms represented by the Fourier series as
- Then the average power is computed as

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n) \qquad i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos(n\omega_0 t + \phi_n)$$

$$P = \frac{1}{T}\int_0^T v(t)i(t)\,dt \qquad P = \sum_{n=0}^{\infty} P_n = V_0 I_0 + \sum_{n=1}^{\infty} V_{n,\,\mathrm{rms}} I_{n,\,\mathrm{rms}} \cos(\theta_n - \phi_n)$$

- Note that total average power is the sum of the powers at the frequencies in the Fourier series

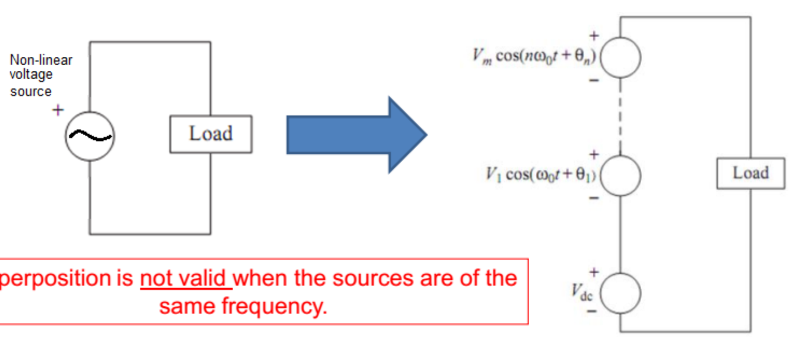
$$P = V_0 I_0 + \sum_{n=1}^{\infty} \left( \frac{V_{n,\,\max} I_{n,\,\max}}{2} \right) \cos(\theta_n - \phi_n)$$

# Non-sinusoidal Source – Linear Load

- If a non-sinusoidal periodic voltage is applied to a linear load the power absorbed by the load can be determined by using superposition
- A non-sinusoidal periodic voltage is equivalent to the series combination of the Fourier series voltages



- The current in the load can be determined using superposition



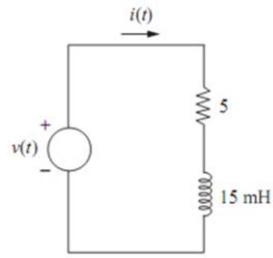

Superposition is <u>not valid</u> when the sources are of the same frequency.

# Example

- A non-sinusoidal voltage source has a fourier series of

$$v(t) = 10 + 20\cos(2\pi 60t - 25^\circ) + 30\cos(4\pi 60t + 20^\circ)\ \text{V}$$

- This voltage is connected to a load that is 5 ohm resistor and 15 mH inductor in series.
- Determine the power absorbed by the load.






# Solution

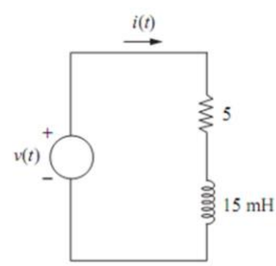
- The dc term is

$$I_0 = \frac{V_0}{R} = \frac{10}{5} = 2\ \text{A}$$

- AC current terms are computed from phasor analysis:

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{R + j\omega_1 L} = \frac{20\angle(-25^\circ)}{5 + j(2\pi 60)(0.015)} = 2.65\angle(-73.5^\circ)\ \text{A}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{R + j\omega_2 L} = \frac{30\angle 20^\circ}{5 + j(4\pi 60)(0.015)} = 2.43\angle(-46.2^\circ)\ \text{A}$$




- Load current then can be calculated as

$$i(t) = 2 + 2.65\cos(2\pi 60t - 73.5^\circ) + 2.43\cos(4\pi 60t - 46.2^\circ)\ \text{A}$$

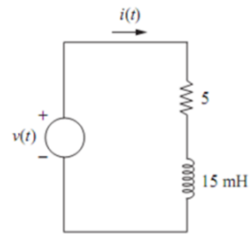
# Power Absorbed

- The power at each frequency in the Fourier series can be determined as follows,

dc term: $P_0 = (10\text{ V})(2\text{ A}) = 20\text{ W}$

$\omega = 2\pi 60$: $P_1 = \dfrac{(20)(2.65)}{2}\cos(-25° + 73.5°) = 17.4\text{ W}$

$\omega = 4\pi 60$: $P_2 = \dfrac{(30)(2.43)}{2}\cos(20° + 46°) = 14.8\text{ W}$




- Total power is then

$$P = 20 + 17.4 + 14.8 = 52.2\text{ W}$$

- **Alternative Method:** Since the average power of inductor is zero, the power absorbed by the load can be calculated using rms current as follows

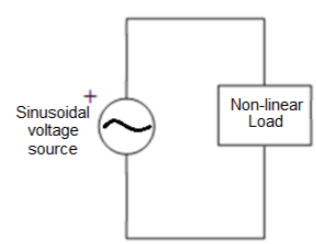
$$P = I_{\text{rms}}^2 R = \left[2^2 + \left(\frac{2.65}{\sqrt{2}}\right)^2 + \left(\frac{2.43}{\sqrt{2}}\right)^2\right]5 = 52.2\text{ W}$$



# Sinusoidal Source and Nonlinear Load

- If a sinusoidal voltage source is applied to a nonlinear load, the current waveform will not be sinusoidal but can be represented as a Fourier series
- Voltage source is linear

$$v(t) = V_1 \sin(\omega_0 t + \theta_1)$$




- Current is represented by Fourier

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \sin(n\omega_0 t + \phi_n)$$

- Average power absorbed by the load is computed as

$$P = V_0 I_0 + \sum_{n=1}^{\infty}\left(\frac{V_{n,\max} I_{n,\max}}{2}\right)\cos(\theta_n - \phi_n)$$

$$= (0)(I_0) + \left(\frac{V_1 I_1}{2}\right)\cos(\theta_1 - \phi_1) + \sum_{n=2}^{\infty}\frac{(0)(I_{n,\max})}{2}\cos(\theta_n - \phi_n)$$

$$= \left(\frac{V_1 I_1}{2}\right)\cos(\theta_1 - \phi_1) = V_{1,\text{rms}} I_{1,\text{rms}}\cos(\theta_1 - \phi_1)$$

**Note that the only nonzero power term is at the frequency of the applied voltage!!!**



31

- Power factor of the load

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{\text{rms}} I_{\text{rms}}}$$

$$\text{pf} = \frac{V_{1,\text{rms}} I_{1,\text{rms}}\cos(\theta_1 - \phi_1)}{V_{1,\text{rms}} I_{\text{rms}}} = \left(\frac{I_{1,\text{rms}}}{I_{\text{rms}}}\right)\cos(\theta_1 - \phi_1)$$

- RMS current is computed

$$I_{\text{rms}} = \sqrt{\sum_{n=0}^{\infty} I_{n,\text{rms}}^2} = \sqrt{I_0^2 + \sum_{n=1}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}$$



32

## Power Indices

- Note that the power factor term commonly used in linear circuits is called the ***displacement power factor***

$$\text{pf} = \cos(\theta_1 - \phi_1)$$

- The **distortion factor** (DF) describes the reduction in power factor as a result of the nonlinear distortion, and from the *PF* expression for distorted voltage:

$$\text{DF} = \frac{I_{1,\,\text{rms}}}{I_{\text{rms}}} \qquad \text{pf} = [\cos(\theta_1 - \phi_1)]\,\text{DF}$$





## THD

- THD is the ratio of the rms value of all the non-fundamental frequency terms to the rms value of the fundamental frequency term

$$\text{THD} = \sqrt{\frac{\sum_{n \neq 1} I^2_{n,\,\text{rms}}}{I^2_{1,\,\text{rms}}}} = \frac{\sqrt{\sum_{n \neq 1} I^2_{n,\,\text{rms}}}}{I_{1,\,\text{rms}}}$$

- or

$$\text{THD} = \sqrt{\frac{I^2_{\text{rms}} - I^2_{1,\,\text{rms}}}{I^2_{1,\,\text{rms}}}} \qquad \text{DF} = \sqrt{\frac{1}{1 + (\text{THD})^2}}$$

# Apparent Power

- Since only non-zero term for reactive power is at the frequency of voltage, the reactive power can be expressed as

$$Q = \frac{V_1 I_1}{2} \sin(\theta_1 - \phi_1)$$

- With P and Q defined for the non-sinusoidal case, apparent power S must include a term to account for the current at frequencies which are different from the voltage frequency



# S

- The term ***distortion volt-amps*** **D** is traditionally used in the computation of S

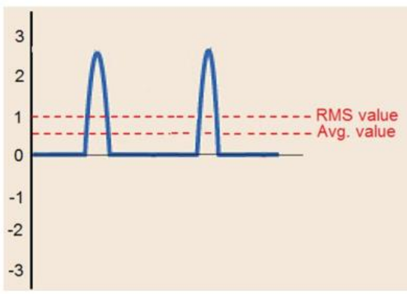
$$S = \sqrt{P^2 + Q^2 + D^2}$$

$$D = V_{1,\mathrm{rms}} \sqrt{\sum_{n \neq 1}^{\infty} I_{n,\mathrm{rms}}^2} = \frac{V_1}{2} \sqrt{\sum_{n \neq 1}^{\infty} I_n}$$

# Factors

- Other terms that are sometimes used for non-sinusoidal current (or voltages) are **form factor** and **crest factor**

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{avg}}} \qquad \text{Crest factor} = \frac{I_{\text{peak}}}{I_{\text{rms}}}$$





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37

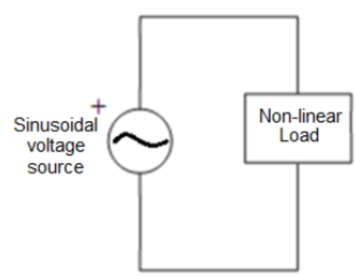
# Example

- A sinusoidal voltage source of *v(t)=100 cos(377t)* V is applied to a nonlinear load, resulting in a non-sinusoidal current which is expressed in Fourier series form as

$$i(t) = 8 + 15\cos(377t + 30°) + 6\cos[2(377)t + 45°] + 2\cos[3(377)t + 60°]$$

- Determine,

a) The power absorbed by the load
b) The power factor of the load
c) The distortion factor of the load current
d) The total harmonic distortion of the load current

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38

# Solution

a) The power absorbed by the load is determined by computing the power absorbed at each frequency in the Fourier series

$$P = (0)(8) + \left(\frac{100}{\sqrt{2}}\right)\left(\frac{15}{\sqrt{2}}\right)\cos 30° + (0)\left(\frac{6}{\sqrt{2}}\right)\cos 45° + (0)\left(\frac{2}{\sqrt{2}}\right)\cos 60°$$

$$P = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{15}{\sqrt{2}}\right)\cos 30° = 650 \text{ W}$$

b) The rms voltage and rms current are

$$V_{rms} = \frac{100}{\sqrt{2}} = 70.7 \text{ V} \qquad I_{rms} = \sqrt{8^2 + \left(\frac{15}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 14.0 \text{ A}$$

then the power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{rms}I_{rms}} = \frac{650}{(70.7)(14.0)} = 0.66$$



# Solution

c) The distortion factor is computed as

$$\text{DF} = \frac{I_{1,\,rms}}{I_{rms}} = \frac{\frac{15}{\sqrt{2}}}{14.0} = 0.76$$

d) The total harmonic distortion of the load current is obtained as

$$\text{THD} = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \sqrt{\frac{14^2 - \left(\frac{15}{\sqrt{2}}\right)^2}{\left(\frac{15}{\sqrt{2}}\right)^2}} = 0.86 = 86\%.$$

41