

Test 2

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1. Question 1

$$aug = \left[\begin{array}{cccc|c} 3 & 3 & -1 & -1 & 0 \\ 2 & 2 & & 1 & 2 \\ 1 & 1 & & 0 & 0 \end{array} \right] \begin{array}{c} 5 \\ 1 \\ 1 \end{array}$$

Solved using the python sympy library:

```
x,y,z,w = symbols("x,y,z,w")
M = Matrix(( [3,3,-1,-2,0],
              [2,2,1,2,5],
              [1,1,0,0,1]))
linsolve(M,x,y,z,w)
```

$$(x,y,z,w) = (1-y,y,3-2w,w)$$

2. Question 2

$$T(1,0,0) \rightarrow \begin{bmatrix} 2 \\ 0 \end{bmatrix}, T(0,1,0) \rightarrow \begin{bmatrix} 3 \\ 1 \end{bmatrix}, T(0,0,1) \rightarrow \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & -4 \end{bmatrix}$$

3. Question 3

4. Question 4

$$X : \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}, Y : \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}, Z : \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}, W : \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

5. Question 5

$$A = \begin{bmatrix} 3 & 2 & -6 \\ -2 & 5 & 4 \\ 4 & -3 & -8 \end{bmatrix}, V = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$[A^T A \mid A^T V]$$

$$A^T A = \begin{bmatrix} 29 & -16 & -58 \\ -16 & 38 & 32 \\ -58 & 32 & 116 \end{bmatrix}$$

$$A^T V = \begin{bmatrix} -9 \\ 27 \\ 18 \end{bmatrix}$$

$$[A^T A \mid A^T V] \rightarrow \left[\begin{array}{ccc|c} 29 & -16 & -58 & -9 \\ -16 & 38 & 32 & 27 \\ -58 & 32 & 116 & 18 \end{array} \right]$$

Row Reduce:

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & \frac{5}{47} \\ 0 & 1 & 0 & \frac{71}{94} \\ 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2z + \frac{5}{47} \\ \frac{71}{94} \\ z \end{bmatrix}$$

6. Question 6

7. Question 7

$$x = A^t \cdot v \rightarrow \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, Ax = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

8. Question 8

$$A(n+1) = 0.7A(n) + 0.1B(n)$$

$$B(n+1) = 0.9B(n) + 0.3A(n)$$

$$\begin{bmatrix} A(n+1) \\ B(n+1) \end{bmatrix} = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{bmatrix} A(n) \\ B(n) \end{bmatrix}$$

Calculate eigenvalues:

$$\lambda_1 = 1, \lambda_2 = \frac{3}{2}$$

$$\lambda_1 \text{ has eigenvector of } \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

The tanks have 4000 gallons, therefore:

$$A = 1000 \text{ Gallons}$$

$$B = 3000 \text{ Gallons}$$