EEEN313/ECEN405

Power Factor

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P and Q

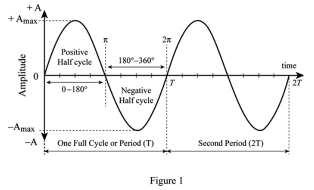

Figure 1

- **P** has the unit of watts and the unit symbol **W**. P is sometimes referred to as **average power** or **real power**, but much more commonly, it's just plain power. If someone says simply, "power," it means P.

$$p(t) = \frac{V_{peak} I_{peak}}{2}\left[\cos(\theta_v - \theta_i) + \cos\left(4\pi f t + (\theta_v - \theta_i)\right)\right]$$

- This second term is still power, even though its average is zero.
- This is the power that flows one way for a quarter of a cycle, then the other way for a quarter of a cycle.
- From this second term we define a term called **Reactive power Q**.

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Reactive Power Q

- Q delivers no energy on average
- Represents the energy that flows forwards and backwards
- No net flow of energy but accounts for part of the current flowing in the system – hence we need to include in our calculations
- Q is positive for loads that are primarily inductive; negative if the loads are primarily capacitive.
- Units are VAR (volt-amp-reactive)

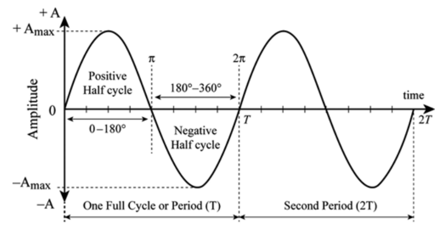

Figure 1

$$Q = \frac{V_{peak} I_{peak}}{2} \sin(\theta_v - \theta_i)$$
$$= V_{RMS} I_{RMS} \sin(\theta_v - \theta_i)$$

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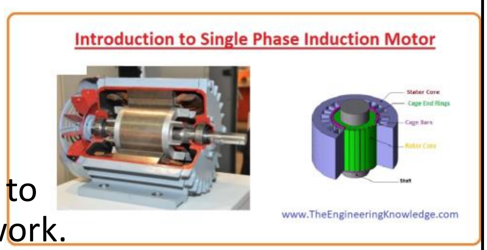
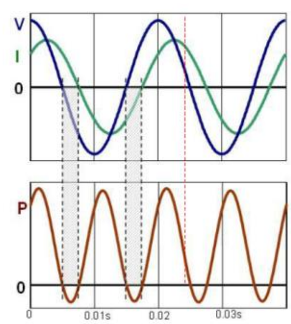
Why Q is important?



- Take induction motor for example: Needs current to supply the magnetic field that makes the motor work.
- Just P cant supply this current; hence we need Q.
- The increase in current will increase losses. Hence we need to quantify Q.
- If you include an inductor or a capacitor, your circuit is consuming Q (through the tracks/wires and coils)



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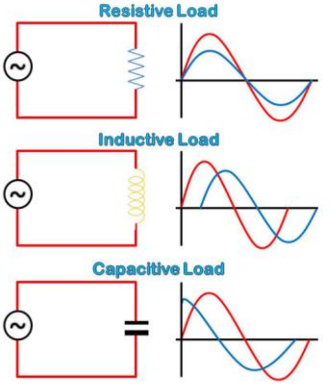
Power Factor

- Defined as the cosine of the angle between the voltage and the current.

$$\textit{power factor} = \cos(\theta_v - \theta_i)$$

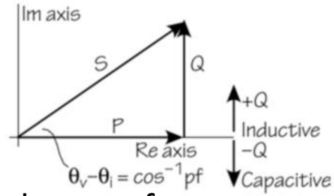
- Usually in the form of 'leading' or 'lagging' as the cosine value will always produce a positive value (0 to 1).
- How do we know lead or lag?
  - If the load is inductive, current lags the voltage in time. So angle of current will be less than voltage angle. So current lags voltage. So power factor is lagging.
  - So if load is capacitive, the power factor is leading as current leads.



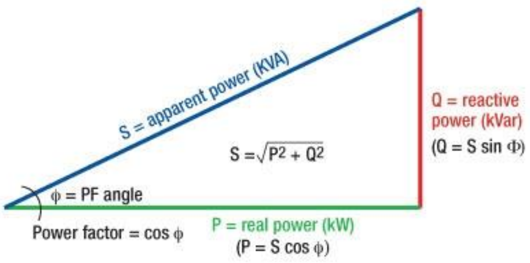
Complex Power or Apparent Power

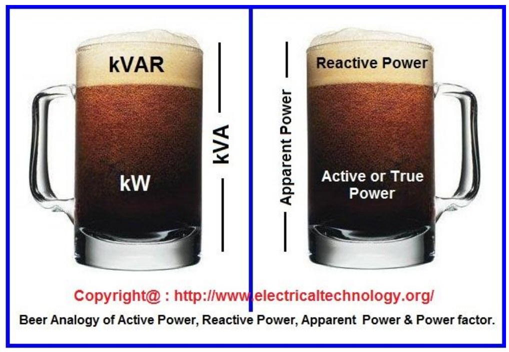
- Another power quantity that is useful is complex power S: $S = P + jQ$
- Unit is volt-ampere (VA)
- S is a complex number with a real part of P and an imaginary part of Q
- S is also the product of phasor voltage and phasor current $S = VI^*$
- Asterisk indicates the complex conjugate
- Power Factor (pf) is defined as the ratio of average power to apparent power: $\textit{power factor} = \dfrac{P}{|S|}$

Power Triangle



- Easy way to visualize the relationship among P,Q,S and power factor





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Apparent Power S - Summary

- Apparent Power S is magnitude of the complex power

$$\boldsymbol{S} = P + jQ \qquad \text{and} \qquad S = |\boldsymbol{S}| = \sqrt{P^2 + Q^2}$$

$$\boxed{S = V_{\text{rms}} I_{\text{rms}}}$$

Unit of S is VA

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Reactive Power - Summary

- Reactive power is characterized by energy stored during the one-half cycle and the energy retrieval during the other half

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} \sin(\theta - \phi)$$

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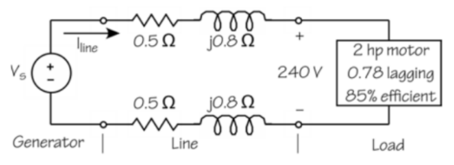
Motor Load



Find the power delivered to the motor, the line current, the power lost in the line, the total P, Q, and S, the power factor at the generator, the percent voltage regulation, and the percent efficiency.

1 HP is 746 W. The motor is 85% efficient, so motor power input is

$$P_{load} = \frac{2 \times 746}{0.85} = 1.755 \ \text{kW}$$

the reactive power input to the motor is $Q_{load} = 1.755 \tan \cos^{-1} 0.78 = 1.408 \ \text{kVAR}$

We can get the line current from

$$P_{load} = |V||I_{line}|\, pf_{load}$$

$$I_{line} = \frac{1.755 \times 10^3}{240 \times 0.78} \angle -\cos^{-1} 0.78 = 9.375 \angle -38.7° \ \text{A}$$

Power loss is mainly resistive part

$$P_{line} = 2R_{line}|I_{line}|^2 = 87.9 \ \text{W}$$

Generator voltage using KVL

$$\begin{aligned} V_s &= 240 + V_{line} \\ &= 240 + (0.5 + j0.8 + j0.8 + 0.5)(9.375\angle -38.7°) \\ &= 256.8 \angle 1.3° \ \text{V} \end{aligned}$$

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Example Continued…

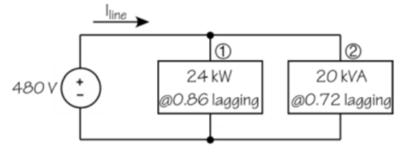
Beginning with Apparent Power $S = VI^*$

$$
\begin{aligned}
S_s &= (256.8\angle 1.3°)(9.375\angle +38.7°) = 2.408\angle 40.0° \ \text{kVA} \\
P_s &= \text{Re}[S_s] = 1.844 \ \text{kW} \\
Q_s &= \text{Im}[S_s] = 1.548 \ \text{kVAR} \\
pf_s &= \cos(40.0°) = 0.766 \ \text{lagging}
\end{aligned}
$$

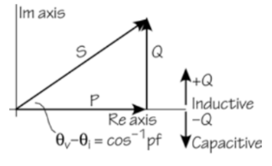
Now Voltage Regulation (Voltage raise due to sudden loss of load) $\%VR = \dfrac{|V_s| - |V_{load}|}{|V_{load}|}100 = \dfrac{256.8 - 240}{240}100 = 7.0\%$

Percent Efficiency $\%\eta = \dfrac{P_{load}}{P_s}100 = \dfrac{1.755}{1.844}100 = 95.2\%$

Adding Powers



Find the total Power, Q, S, power factor and line current



From power triangle $Q_1 = P_1 \tan\cos^{-1} pf = 24 \tan\cos^{-1} 0.86 = 14.24 \ \text{kVAR}$

Complex Power $S_1 = (24 + j14.24) \ \text{kVA}$

For second load
$$
\begin{aligned}
P_2 &= |S_2|\,pf_2 = 20 \times 0.72 = 14.4 \ \text{kW} \\
Q_2 &= |S_2|\sin\cos^{-1} pf_2 = 13.88 \ \text{kVAR}
\end{aligned}
$$

The current is calculated from $S = V \times I^*$:

$$
I^*_{line} = \frac{S_{total}}{V} = \frac{47.6 \times 10^3 \angle 36.2°}{480} = 99.2\angle 36.2° \ \text{A}
$$
$I_{line} = 99.2\angle -36.2°$ A (note the sign)

P is conserved; so is Q and S
$$
\begin{aligned}
P_{total} &= P_1 + P_2 = 24 + 14.4 = 38.4 \ \text{kW} \\
Q_{total} &= Q_1 + Q_2 = 14.24 + 13.88 = 28.12 \ \text{kVAR} \\
S_{total} &= P_{total} + jQ_{total} = 38.4 + j28.12 = 47.6\angle 36.2° \ \text{kVA} \\
pf_{total} &= \frac{P_{total}}{|S_{total}|} = \frac{38.4}{47.6} = 0.807 \ \text{lagging}
\end{aligned}
$$