ENGR222 Assignment 2

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1. The following questions are concerned with the function

$$f(x,y) = -2x^3 + 3x^2y + 2y^3 - 9y + 5$$

   (a) Determine the first order partial derivative of $f(x,y)$

$$f_x = -6x^2 + 6xy$$
$$f_y = 6y^2 + 3x^2 - 9$$

   (b) Determine the second order partial derivatives of $f(x,y)$

$$f_{xx} = -12x + 6y$$
$$f_{yy} = 12y$$
$$f_{xy} = 6x$$

   (c) Find all of the critical points of $f(x,y)$

   By inspection we know $(x = y = -1, 1)$

   Let $x = 0$

$$f_x = 0$$
$$f_y = 6y^2 - 9 = 0$$
$$\therefore y = \sqrt{\frac{9}{6}} = \sqrt{\frac{3}{2}}$$

   Let $y = 0$

$$f_x = -6x^2 = 0$$
$$f_y = 3x^2 - 9$$

No solution for x when $y = 0$

   The critical points are $\rightarrow [(1,1),\ (-1,-1),\ (0, \sqrt{\tfrac{3}{2}})]$

(d) Classify the critical point $(0, \sqrt{\frac{3}{2}})$

$$D = f_{xx}(0, \sqrt{\frac{3}{2}}) \times f_{yy}(0, \sqrt{\frac{3}{2}}) - f_{xy}^2(0, \sqrt{\frac{3}{2}})$$

$$f_{xx}(0, \sqrt{\frac{3}{2}}) = 3\sqrt{6}$$
$$f_{yy}(0, \sqrt{\frac{3}{2}}) = 6\sqrt{6}$$
$$f_{xy}(0, \sqrt{\frac{3}{2}}) = 0$$

$$D = 3\sqrt{6} \times 6\sqrt{6} - 0^2 = 108$$

Since $D > 0$ and $f_{xx} > 0$ we know this critical point is a local minimum

2. Quick questions

(a) Determine the directional derivative of $f(x, y, z) = e^x \cdot \cos(y) \cdot (1 - z)^2$ in direction $\bar{u} = (0.36, 0.48, 0.8)$ from the origin:

$$D_u f(x_0, y_0, z_0) = f_x(x_0, y_0, z_0)\bar{u}_1 + f_y(x_0, y_0, z_0)\bar{u}_2 + f_z(x_0, y_0, z_0)\bar{u}_3$$

$$f_x = e^x cos(y)(1 - z)^2$$
$$f_y = -e^x sin(y)(1 - z)^2$$
$$f_z = e^x cos(y)(2z - 2)$$

$$f_x(0, 0, 0) = 1 \cdot 1 \cdot 1 = 1$$
$$f_y(0, 0, 0) = -1 \cdot 0 \cdot 1 = 0$$
$$f_z(0, 0, 0) = 1 \cdot 1 \cdot -2 = -2$$

$$D_u f(0, 0, 0) = 0.36 - 1.6 = -1.24$$

(b) Determine the local linear approximation of $f(x,y,z) = (1+x)(1-y^2)(1-z)^2$ at the point $(1,2,3)$

$$L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0)$$

$$\begin{aligned} f_x &= (1-y^2)(1-z)^2 \\ f_y &= (1+x)(-2y)(1-z)^2 \\ f_z &= 2(1+x)(1-y^2)(z-1) \end{aligned}$$

$$\begin{aligned} f(1,2,3) &= (1+1)(1-2^2)(1-3)^2 = -24 \\ f_x(1,2,3) &= (1-2^2)(1-3)^2 = -12 \\ f_y(1,2,3) &= (1+1)(-2(2))(1-3)^2 = -32 \\ f_z(1,2,3) &= 2(1+1)(1-2^2)(3-1) = -24 \end{aligned}$$

$$\begin{aligned} L(1,2,3) &= -24 - 12(x-1) - 32(y-2) - 24(z-3) \\ L(1,2,3) &= 124 - 12x - 32y - 24z \end{aligned}$$

(c) Determine the 2nd degree Taylor polynomial of $f(x,y) = e^{-x^2}e^{-y^2}$ as the point (1,1)

$$\begin{aligned} L(x,y) &= f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) \\ p_2(x,y) &= L(x,y) + \frac{1}{2}\left[f_{xx}(x_0,y_0)(x-x_0)^2 + 2f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)(y-y_0)^2\right] \end{aligned}$$

$$\begin{aligned} f_x &= (-2x)e^{-x^2}e^{-y^2} \\ f_y &= (-2y)e^{-x^2}e^{-y^2} \end{aligned}$$

$$\begin{aligned} f_{xx} &= (4x^2-2)e^{-x^2-y^2} \\ f_{yy} &= (4y^2-2)e^{-x^2-y^2} \\ f_{xy} &= (4xy)e^{-x^2}e^{-y^2} \end{aligned}$$

$$\begin{aligned} L(1,1) &= e^{-2} - 2e^{-2}(x-1) - 2e^{-2}(y-1) \\ p_2(1,1) &= e^{-2} - 2e^{-2}(x-1) - 2e^{-2}(y-1) \\ &\quad + \frac{1}{2}\left[2e^{-2}(x-2)^2 + 2e^{-2}(y-2)^2 + 8e^{-2}(x-1)(y-1)\right] \end{aligned}$$

(d)

3. Double Integrals

(a)
(b)
(c)
(d)