1.) $\int \frac{6t+3}{2t^2-5t+2}\,dt = \int \frac{A}{(t-2)} + \frac{B}{(2t-1)}\,dt \quad \therefore \quad A=5 \quad B=-4$

a.)

$$\int \frac{5}{(t-2)} + \frac{-4}{(2t-1)}\,dt = 5\ln|t-2| - 2\ln|2t-1|$$

$$\int_1^2 \frac{3-3x}{2x^2+6x}\,dx = \int_1^2 \frac{A}{2x} + \frac{B}{x+3}\,dx \quad \therefore \quad A=1 \quad B=-2$$

b.)

$$\int_1^2 \frac{1}{2x} - \frac{2}{x+3}\,dx = \left[\tfrac{1}{2}\ln|2x| - 2\ln|x+3|\right]_1^2 = -0.0997$$

2.) $\frac{dy}{dx} = 3y(11-y) \quad \therefore \quad dy\left(\frac{1}{3y(11-y)}\right) = dx$

a.)

$$\frac{1}{3y(11-y)} = \frac{A}{3y} + \frac{B}{11-y} \quad \therefore \quad 1 = A(11-y) + B(3y)$$

$$A = \tfrac{1}{11} \qquad B = \tfrac{1}{33}$$

$$\frac{1}{3y(11-y)} = \frac{1}{33y} + \frac{1}{33(11-y)} = \frac{1}{33}\left(\frac{1}{y} + \frac{1}{11-y}\right)$$

$$\frac{1}{33}\int\left(\frac{1}{y} + \frac{1}{11-y}\right)dy = \int dx$$

$$\frac{1}{33}\left[\ln|y| - \ln|11-y|\right] = x + C$$

$$\frac{1}{33}\ln\left|\frac{y}{11-y}\right| = x + C \quad \therefore \quad \frac{y}{11-y} = Ae^{33x} \quad \therefore \quad y = \frac{11Ae^{33x}}{Ae^{33x}-1}$$

$$y(0) = 5 \quad \therefore \quad \frac{11Ae^0}{Ae^0-1} \qquad A = -\frac{5}{6}$$

b.) as $x \to -\infty$, $y \to \infty$.

c.) as $x \to \infty$, $y \to 0$.

3.)

a.)

$$\frac{dx}{dt} = \frac{\cos(3t)}{x^2} \quad \therefore \int x^2\,dx = \int \cos(3t)\,dt$$

$$\frac{x^3}{3} = \frac{\sin(3t)}{3} + C \quad \therefore \quad x^3 = \sin(3t) \quad \therefore \quad x = \sqrt[3]{\sin(3t)} + C$$

$$\text{at } x(0) = 1 \quad 1 = \sqrt[3]{\sin(0)} + C \quad \therefore \quad C = 1$$

$$x = \sqrt[3]{\sin(3t)} + 1$$

b.)

$$\frac{dy}{dx} = \frac{x\ln x}{e^y} \quad \therefore \int e^y\,dy = \int x\ln x\,dx$$

$$e^y = \int x\ln x\,dx \qquad u = \ln x \qquad u' = \frac{1}{x}$$

$$v\,\frac{x^2}{2} \qquad v' = x$$

$$e^y = \frac{x^2\ln x}{2} - \int \frac{x^2}{x}\,dx \quad \therefore \quad e^y = \frac{x^2\ln x - x^2}{2} + C$$

$$y = \ln\left|\frac{x^2\ln x - x^2}{2}\right| + C$$

c.)

$$\frac{dy}{dt} = e^{y+t} = e^y e^t \quad \therefore \int \frac{dy}{e^y} = \int e^t\,dt$$

$$-e^{-y} = e^t \quad \therefore \quad e^{-y} = -e^t \quad \therefore \quad -y = \ln(-e^t) + C$$

$$y = -\ln(-e^t) + C$$

$\mu = e^{\int P(t)\,dt}$

4.)

a.) $\frac{dy}{dx} + \frac{1}{x}y = 1$ where $P(x) = \frac{1}{x}$ $Q(x) = 1$

$\mu = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$

$\frac{d}{dx}(\mu y) = \mu Q \therefore \frac{d}{dx}(xy) = x$

$xy = \int x\,dx \therefore xy = \frac{x^2}{2}$ $y = \frac{x^2}{2x}$

b.) $x^2 \frac{dy}{dx} + xy - x^2 e^x = 0 \therefore \frac{dy}{dx} + \frac{1}{x}y = e^x$

$\mu = e^{\int \frac{1}{x}\,dx} = x$

$\frac{d}{dx}(\mu y) = \mu Q \therefore \frac{d}{dx}(xy) = xe^x$

$xy = \int xe^x$ $u = x$ $u' = 1$

$v = e^x$ $v' = e^x$

$xy = xe^x - \int e^x\,dx \therefore xy = xe^x - e^x + C$

$y = \frac{xe^x - e^x}{x} + C$

c.) $\frac{dx}{dt} + \frac{2}{t}x = \sin(t)$

$$\mu = e^{\int \frac{2}{t} dt} = e^{2\ln t} = t^2$$

$$\frac{d}{dx}(\mu x) = \mu Q \quad \therefore \quad t^2 x = \int t^2 \sin(t)\, dt$$

$$u = t^2 \qquad u' \ 2t$$

$$v = -\cos(t) \qquad v' \ \sin(t)$$

$$\int t^2 \sin(t)\, dt = -t^2\cos(t) + \int \cos(t)\, 2t\, dt$$

$$u = 2t \qquad u' = 2$$

$$v = \sin(t) \qquad v' = \cos(t)$$

$$\int 2t\cos(t) = -2t\sin(t) - \int 2\cos(t)\, dt = -2t\sin(t) - 2\sin(t)$$

$$\int t^2 \sin(t)\, dt = -t^2\cos(t) - 2t\sin(t) - 2\sin(t) + c$$

$$t^2 x = -t^2\cos(t) - 2t\sin(t) - 2\sin(t) + c$$

$$\boxed{x = \frac{-t^2\cos(t) - 2t\sin(t) - 2\sin(t)}{t^2}}$$