# Solutions to ENGR 122 Assignment 6

1. $f(x) = e^{x/2} - 5x, \quad x_1 = 6$

$f'(x) = \frac{1}{2} e^{x/2} - 5$

$$x_{n+1} = x_n = \frac{f(x_n)}{f'(x_n)}$$

$f(x_1) = -9.914 \quad f'(x_1) = 5.043$

$$x_2 = 6 - \frac{(-9.914)}{5.043} = 7.966$$

$f(x_2) = 13.848, \quad f'(x_2) = 21.839$

$$x_3 = 7.966 - \frac{13.848}{21.839} = 7.332$$

$f(x_3) = 2.435, \quad f'(x_3) = 14.548$

$$x_4 = 7.332 - \frac{2435}{14.548} = 7.165$$

$f(x_4) = 0.138, \quad f'(x_4) = 12.982$

$$x_5 = 7.165 - \frac{0.138}{12.982} = 7.154$$

$f(x_5) = -0.004 \quad f'(x_5) = 12.883$

$$x_6 = 7.154 - \frac{(-0.004)}{12.883} = 7.154$$

The root with 2 decimal digits correct:

$$x^* = 7.15$$

2.(a) $y' = 6t^2 - 42t + 60 = 6(t-2)(t-5)$

$y'' = 12t - 42$

when $t=2$, $y=61$, $y'=0$, $y''<0$. Hence $(2,61)$ is a maximum point

when $t=5$, $y=34$, $y'=0$, $y''>0$. Hence $(5,34)$ is a minimum point

when $t<3.5$, $y''<0$ and $y'$ is decreasing

when $t>3.5$, $y''>0$ and $y'$ is increasing

The concavity changes at $t=3.5$. When $t=3.5$, $y=47.5$ so $(3.5, 47.5)$ is a point of inflexion.

| $t$ | | 2 | | 3.5 | | 5 | |
|---|---|---|---|---|---|---|---|
| $y'$ | + | 0 | − | | − | 0 | + |
| $y''$ | − | | − | 0 | + | | + |
| | | max | | | | min | |

(b) $y = t^3 - t$

$y' = 3t^2 - 1$

$y'' = 6t$

When $t = 1/\sqrt{3}$, $y = -2/3\sqrt{3}$, $y' = 0$, $y'' > 0$, so $(1/\sqrt{3}, -2/3\sqrt{3})$ is a minimum point

When $t = -1/\sqrt{3}$, $y = 2/3\sqrt{3}$, $y' = 0$, $y'' < 0$ so $(-1/\sqrt{3}, 2/3\sqrt{3})$ is a maximum point

When $t < 0$, $y'' < 0$ and $y'$ is decreasing

When $t > 0$, $y'' > 0$ and $y'$ is increasing

The concavity changes at $t = 0$, so $(0, 0)$ is a point of inflexion

| $t$ | | $-1/\sqrt{3}$ | | $0$ | | $1/\sqrt{3}$ | |
|---|---|---|---|---|---|---|---|
| $y'$ | $+$ | $0$ | $-$ | | $-$ | $0$ | $+$ |
| $y''$ | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $y$ | ↗ | | ↘ | | ↘ | | ↗ |
| | | max | | | | min. | |

(c) Similar

$(0, 2)$ minimum

3. (a) $y'' = x^3 - y^2 \qquad y''(0) = -1$

$y''' = 3x^2 - 2yy' \quad y'''(0) = -2(1)(-1) = 2$

$P_3(x) = 1 - x - \frac{x^2}{2} + \frac{2x^3}{3!}$

$P_3(0.25) = 0.7240$

(b) $y^{(4)} = 6x - 2yy'' - 2(y')^2$

$y^{(4)}(0) = (-2)(1)(-1) - 2(-1)^2 = 0$

$P_4(0.25) = 0.7240$ as in (a).

-4- (a) $P_3(x) = 1 - \frac{k^2 x^2}{2}$

(b) For example

$P_3(x) = 1 - 2x^2$ (by setting $k=2$)