1.) $\int t \ln(t)\, dt$ $\qquad v' = t \quad \therefore \quad v = \frac{t^2}{2}$

a) $\qquad u = \ln(t) \quad \therefore \quad u' = \frac{1}{t}$

$$\frac{1}{2}t^2 \ln(t) - \int \frac{t}{2} = \underline{\frac{1}{2}t^2 \ln(t) - \frac{t^2}{4}} + C$$

b.)

$\int x^2 e^x \cdot dx$ $\qquad v' = e^x \qquad v = e^x$

$\qquad u = x^2 \qquad u' = 2x$

$x^2 e^x - \int e^x 2x$ $\qquad v' = e^x \qquad v = e^x$

$\qquad u = 2x \qquad u' = 2$

$2xe^x - \int 2e^x = 2xe^x - 2e^x$ (sub in there)

$$\int x^2 e^x\, dx = \underline{x^2 e^x - 2xe^x - 2e^x} = \underline{e^x(x^2 - 2x - 2) + C}$$

c.)

$\int_{-\infty}^{3} xe^x\, dx$ $\qquad v' = e^x \qquad v = e^x$

$\qquad u = x \qquad u' = 1$

$$\int_{-\infty}^{3} xe^x\, dx = \left[xe^x - \int e^x\right]_{-\infty}^{3} = \left[xe^x - e^x\right]_{-\infty}^{3}$$

$$\underline{(3e^3 - e^3) - (-\infty e^{-\infty} - e^{-\infty}) = (60.26 - 20.09) - (0 - 0) = 40.17}$$

2.)

a.) $\int (1-x)^{\frac{1}{3}}\,dx$ set $u = (1-x)$

$\frac{du}{dx} = -1 \quad \therefore \quad du = -dx$

$-\int u^{\frac{1}{3}}\,du = \frac{-3u^{\frac{4}{3}}}{4} + C$

b.) $\int \frac{1}{x\ln(x)}\,dx$ let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x} \quad \therefore \quad du = \frac{dx}{x} = \frac{1}{x}dx$

$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{x}\cdot\frac{1}{\ln x}\,dx = \int \frac{1}{u}\,du$

$\int \frac{1}{u}\,du = \ln(u) = \ln(\ln(x)) + C$

c.) $\cancel{\int \cos(x)\sin^{1/2}(x)\cdot dx}$ $\int \cos(x)\sqrt{\sin(x)}\cdot dx$ let $u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\cdot dx$

$\int \sqrt{u}\,du = \int u^{1/2}\,du$

$= \frac{2u^{3/2}}{3} = \frac{2\sin^{3/2}(x)}{3}$

d.) $\int \frac{2x+1}{x^2+x+1}\,dx$ let $u = x^2+x+1$

$\frac{du}{dx} = 2x+1 \quad \therefore \quad du = 2x+1\cdot dx$

$\int \frac{du}{u} = \ln(u) = \ln(x^2+x+1) + C$

3.) $\int \sqrt{x-1}\,dx = \frac{2(x-1)^{3/2}}{3} + c$

a.) average $= \frac{1}{3}\left(\left[\frac{2(4-1)^{3/2}}{3}\right] - \left[\frac{2(1-1)^{3/4}}{3}\right]\right) = \frac{2\sqrt{3}}{3}$

$\int \frac{1}{(x+1)^2}\,dx = -(x+1)^{-1} + c$

b.) average $= \frac{1}{3}\left(\left[-(3+1)^{-1}\right] - \left[-(0+1)^{-1}\right]\right) = 0.25$

$\int \frac{1}{x-1}\,dx = \ln|x-1| + c$

c.) average $= \frac{1}{1}\left(\left[\ln|3-1|\right] - \left[\ln|2-1|\right]\right) = 0.693$

4.) r.m.s value $= \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx}$

a.) $\sqrt{\frac{1}{4-1}\int_1^4 \left(\sqrt{x-1}\right)^2 dx} = \sqrt{\frac{1}{3}\int_1^4 x-1\,dx}$

r.m.s $= \sqrt{\frac{1}{3}\left[\frac{x^2}{2} - x\right]_1^4} = \sqrt{\frac{1}{3}\left(\left[\frac{4^2}{2}-4\right] - \left[\frac{1^2}{2}-1\right]\right)} = \frac{\sqrt{6}}{2}$

b.) r.m.s $= \sqrt{\frac{1}{3}\int_0^3 \left(\frac{1}{(x+1)^2}\right)^2 dx} = \sqrt{\frac{1}{3}\int_0^3 \left(\frac{1}{(x+1)^4}\right) dx}$

r.m.s $= \sqrt{\frac{1}{3}\left[\frac{-(x+1)^{-3}}{3}\right]_0^3} = \sqrt{\frac{1}{3}\left(\left[\frac{-(4)^{-3}}{3}\right] - \left[\frac{-(1)^{-3}}{3}\right]\right)} = \frac{\sqrt{7}}{8}$

c.) r.m.s $= \sqrt{\frac{1}{3-2}\int_2^3 \left(\frac{1}{x-1}\right)^2 dx} = \sqrt{\int_2^3 \frac{1}{(x-1)^2}\,dx}$

r.m.s $= \sqrt{\left[-(x-1)^{-1}\right]_2^3} = \sqrt{\left[-(3-1)^{-1}\right] - \left[-(2-1)^{-1}\right]} = \frac{\sqrt{2}}{2}$

5.) $\int_0^1 e^{-x^2}\,dx$

a.)

$$\begin{aligned}
0 &\longrightarrow y_0 = 1\\
0.25 &\longrightarrow y_1 = 0.9394\\
0.5 &\longrightarrow y_2 = 0.7788\\
0.75 &\longrightarrow y_3 = 0.5698\\
1 &\longrightarrow y_4 = 0.3679
\end{aligned}$$

$$\int_0^1 e^{-x^2}\,dx = \frac{0.25}{2}\left(1 + 2(0.9394) + 2(0.7788) + 2(0.5698) + 0.3679\right)$$

$$= 0.7430$$

error:

$$M = 4(1)^2 e^{-(1)^2} = 1.472$$

$$\frac{dy}{dx} = e^{-x^2} = -2xe^{-x^2}$$

$$\frac{d^2y}{dx^2} = -2xe^{-x^2} = 4x^2e^{-x^2}$$

$$\frac{h^2}{12}(b-a)M = \frac{0.25^2}{12}(1 \times 1.472) = 7.664 \times 10^{-3}$$

$$\text{error}(100) : = \frac{0.01^2}{12}(1.472) = 1.226 \times 10^{-5}$$

b.) $\int_0^1 \cos(\pi x^2)\,dx$

$$\begin{aligned}
0 &\longrightarrow y_0 = 1\\
0.25 &\longrightarrow y_1 = 0.9809\\
0.5 &\longrightarrow y_2 = \frac{\sqrt{2}}{2}\\
0.75 &\longrightarrow y_3 = -0.1951\\
1 &\longrightarrow y_4 = -1
\end{aligned}$$

$$\int_0^1 \cos(\pi x^2)\,dx = \frac{0.25}{2}\left(1 + 2(0.9809) + 2\left(\frac{\sqrt{2}}{2}\right) + 2(-0.1951) - 1\right)$$

$$= 0.3732$$

$$\frac{dy}{dx} = -2\pi x \sin(\pi x^2)$$

$$\frac{d^2y}{dx^2} = -4\pi^2 x^2 \cos(\pi x^2)$$

$$\max(f''(x)) = 39.478$$

$$\text{error}(4) = \frac{0.25^2}{12}(39.478) = 0.2056$$

$$\text{error}(100) = \frac{0.01^2}{12}(39.478) = 3.29 \times 10^{-4}$$

6.) $\int_0^1 e^{-x^2}\,dx$

a.)

$0 \longrightarrow y_0 = 1$

$0.25 \longrightarrow y_1 = 0.9394$

$0.5 \longrightarrow y_2 = 0.7788$

$0.75 \longrightarrow y_3 = 0.5698$

$1 \longrightarrow y_4 = 0.3679$

$$\int_0^1 e^{-x^2}\,dx = \frac{0.25}{3}\Big(1 + 2(0.9394 + 0.5698) + 4(0.7788) + 0.3679\Big)$$

$$= 0.6251$$

$f^{(3)}(x) = -8x^3 e^{-x^2}$

$f^{(4)}(x) = 16x^4 e^{-x^2}$

$M = 5.886$

$\text{error}(4) = \frac{0.25^4}{180}(1-0)\,5.886 = 1.2773\times10^{-4}$

$\text{error}(100) = \frac{0.01^4}{180}(5.886) = 3.27\times10^{-10}$

b.) $\int_0^1 \cos(\pi x^2)\,dx$

$0 \longrightarrow y_0 = 1$

$0.25 \longrightarrow y_1 = 0.9809$

$0.5 \longrightarrow y_2 = \frac{\sqrt{2}}{2}$

$0.75 \longrightarrow y_3 = {}^{-}0.1951$

$1 \longrightarrow y_4 = {}^{-}1$

$$\int_0^1 \cos(\pi x^2)\,dx = \frac{0.25}{3}\Big(1 + 2(0.9809 + {}^{-}0.1951) + 4\big(\tfrac{\sqrt{2}}{2}\big) + {}^{-}1\Big)$$

$$= 0.3667$$

$f^{(3)}(x) = 8\pi^3 x^3 \sin(\pi x^2)$

$f^{(4)}(x) = 16\pi^4 x^4 \cos(\pi x^2)$

$M = 16\pi^4$

$\text{error}(4) = \frac{0.25^4}{180}(16\pi^4) = 0.0338$

$\text{error}(100) = \frac{0.01^4}{180}(16\pi^4) = 8.6586\times10^{-8}$