ENGR122 Assignment 10

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DUE: 1pm on 19 October 2018 online

You have two weeks to complete this final assignment.

1. Suppose the effect of friction on a spring is proportional to its velocity.

$$\frac{d^2x}{dt^2} = \overbrace{-2x}^{\text{standard spring}} - \overbrace{2\frac{dx}{dt}}^{\text{friction}}$$

   (a) Solve the equation.

   (b) Explain how the solution differs from when there is **negative** friction.

2. Consider the linear differential equation

$$\frac{dy}{dx} = \frac{x-y}{x} \qquad \text{with } y(1) = 1.$$

   (a) Find the exact solution.

   (b) Estimate $y(2)$ using Euler's method with $h = 0.5$.

   (c) Estimate $y(2)$ using Euler's **improved** method with $h = 0.5$.

   (d) Compare the value of $y(2)$ for the three approaches (exact, and the two approximations).

3. Find the first and second partial derivatives (that is $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial x \partial y}$) of

   (a) $f(x, y) = \pi x^2 y$.

   (b) $f(x, y) = \cos(x^2 + y^2)$

   (c) $f(x, y) = e^{2x} \cos y$

4. You are at position $(1, 2)$. For each function in Q3,

   (a) What is the steepest direction?

   i.e. write down the normalized gradient $\frac{\nabla f(1,2)}{\|\nabla f(1,2)\|}$.

   (b) Find the directions $\mathbf{u}$ that cause you to "walk along the side of the mountain without going up or down."

   That is, find $\mathbf{u}$ that solves $\mathbf{u} \cdot \nabla f(1, 2) = 0$, subject to $\|\mathbf{u}\| = 1$.

5. Find the stationary points of the following functions, and figure out whether they are local maxima, minima, or saddle points.

   (a) $f(x, y) = x^2 + x + y^2$

   (b) $f(x, y) = y + 3y^2 + xe^x$

6. Compute the first-order Taylor approximation at $(1, 2)$ to each of the functions in Q5.

7. Compute the second-order Taylor approximation at $(1, 2)$ to Q5b.

Tutorial Questions for 8–17 Oct 2018, ENGR122

1. Solve the following 2nd order ODEs

   (a)
   $$\frac{d^2y}{dx^2} + 11y = 0$$

   (b)
   $$\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + 100y = 0$$

   (c)
   $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

2. Suppose that a spring has **negative friction**

   $$\frac{d^2x}{dt^2} = \overbrace{-2x}^{\text{standard spring}} + 2\overbrace{\frac{dx}{dt}}^{???}$$

   (a) Solve the equation.

   (b) What happens as $t \to \infty$?

3. Consider the linear differential equation

   $$\frac{dy}{dx} = \frac{x^2 + y^2}{x} \quad \text{with } y(2) = 1.$$

   (a) Estimate $y(3)$ using Euler's method with $h = 0.5$.

   (b) Estimate $y(3)$ using Euler's **improved** method with $h = 0.5$.

Consider the following functions

(a) $f(x, y) = ax + by + c + dx^2 + exy + fy^2$ for a, b, c, d, e, f real numbers.

(b) $f(x, y) = \frac{1}{x^2+y^2}$

(c) $f(x, y) = \ln(x + y - 5)$

For each of the above functions,

- What is the steepest direction at the point $(-1, 1)$? I.e. compute $\nabla f(-1, 1)$.
- Find the directions, at $(-1, 1)$ that cause you to walk along the contour lines. I.e. find $\mathbf{u}$ with $\|\mathbf{u}\| = 1$ such that $D_{\mathbf{u}} f(-1, 1) = 0$.

4. Find the stationary points of the following function, and figure out whether they are local maxima, minima, or saddle points.

   $$f(x, y) = x^2 + y - xy$$

5. Compute the first-order Taylor approximation at $(1, 2)$ to the function in Q.4 above.

6. Compute the second-order Taylor approximation at $(1, 2)$ to the function in Q.4 above.