ENGR122 Assignment 7

Mark McGuinness, SMS, Victoria University of Wellington

DUE: 1pm 21 September 2018, submit online.

1. Show using geometry that if A is the area between $y = x^2$ and the x-interval [0,1], and if B is the area between $y = \sqrt{x}$ and the same x-interval, then A+B = 1. *Hint: since the two functions are inverses of each other in the first quadrant, if you reflect one about the line $y = x$, you get the other function.*

2. On Mars, gravity causes objects to accelerate downwards at $3.9m/s^2$. Robin drops a laptop from a bridge on Mars. It hits the ground after $20s$.

   (a) How far did it fall?

   (b) How fast was the laptop going when it hit the ground?

   (c) If Robin threw the laptop downwards at $12m/s$ (from the same height as in part a), how long would it take to to hit the ground? What would its speed be when it hit?

3. Suppose you are on Jupiter, and that you want to drop a laptop off a building so that it hits the ground at Usain Bolt's top speed. Gravitational acceleration on Jupiter is $23.6m/s^2$. Usain Bolt's top running speed is $12m/s$.

   (a) How long will it take for a falling laptop to reach Usain Bolt's speed?

   (b) How high should the building be?

4. The velocity of an object is described by

   (a) $v(t) = e^{-2t}$

   (b) $v(t) = \frac{t}{2} - 3t^2$

   (c) $v(t) = 2t - e^t$

   In each case:

   - Write down the function describing the distance traveled by the object as a function of time.

     That is, compute the *indefinite integral* $\int v(t)dt$.

   - Calculate the distance traveled between $t = 1$ and $t = 2$.

     That is, compute the *definite integral* $\int_1^2 v(t)dt$.

   - Similarly, calculate the distance traveled between $t = 2$ and $t = 3$.

5. A car rolls along a street. The **vertical** velocity of a point on the wheel is given by

$$v(t) = \pi \sin\left(\frac{\pi t}{4}\right) m/s.$$

   (a) Compute the *indefinite integral*

$$\int \pi \sin\left(\frac{\pi t}{4}\right) dt.$$

   (b) What is the value of the constant of integration, c, if the object's height at time 0 is $0m$?

   (c) What is the object's height after $2s$?

   (d) And after $4s$?

   (e) What is the wheel's diameter?

6. Compute the following indefinite integrals:

   (a)

$$\int \Big[\sin(3x - 1) - x\Big] dx$$

   (b)

$$\int \cos^2(4x) \cdot dx$$

   Hint: Recall that $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

Tutorial Questions for week ending 21 September 2018, ENGR122

1. Show using geometry that the area between the graph of $y = 1/x$ and the x-interval [1,2] is the same as the area between the graph of $y = 1/x$ and the x-interval [0.5,1] *Hint: the function $1/x$ is its own inverse in the first quadrant, so it is symmetrical upon reflection about the line $y = x$.*

2. On the moon, gravity causes objects to accelerate downwards at $1.6m/s^2$. Alex drops a laptop from a bridge on the moon. It hits the ground after $20s$.

   (a) How far did it fall?

   (b) How fast was the laptop going when it hit the ground?

3. Suppose you are on moon, and that you want to drop a laptop off a building so that it hits the ground at Usain Bolt's top speed. Gravitational acceleration on the moon is $1.6m/s^2$. Usain Bolt's top running speed is $12m/s$.

   (a) How long will it take for a falling laptop to reach Usain Bolt's speed?

   (b) How high should the building be?

4. The velocity of an object is described by

   (a) $v(t) = 2t^3$

   (b) $v(t) = 3t^{-3}$

   (c) $v(t) = e^{-\frac{t}{3}}$

   (d) $v(t) = e^{4t}$

   (e) $v(t) = 3t - \frac{t^2}{5}$

   (f) $v(t) = 2t - e^{-t}$

   In each case:

   - Write down the function describing the distance traveled by the object as a function of time.

     That is, compute the *indefinite integral* $\int v(t)dt$.

   - Calculate the distance traveled between $t = 0$ and $t = 1$.

     That is, compute the *definite integral* $\int_0^1 v(t)dt$.

   - Similarly, calculate the distance traveled between $t = 1$ and $t = 2$.

5. A car rolls along a street. The **vertical** velocity of a point on the wheel is given by

$$v(t) = \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right) m/s.$$

   (a) Compute the *indefinite integral*

$$\int \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right) dt.$$

   (b) What is the value of the constant of integration, c, if the object's height at time 0 is $0m$?

   (c) What is the object's height after $2s$?

   (d) And after $4s$?

   (e) What is the wheel's diameter?

6. Compute the following indefinite integrals:

   (a)

$$\int \Big[\cos(2x+1)+1\Big] dx$$

   (b)

$$\int \sin^2(3x) \cdot dx$$

   Hint: Recall that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.