Solution to ENGR122 Assignment 1

1 (a) $x^2+4x-21=0$

$a=1,\ b=4,\ c=-21$

$D=b^2-4ac = 16-4\cdot(1)\cdot(-21) = 100 > 0$, so the solutions

are: $x_{1,2} = \dfrac{-b\pm\sqrt{D}}{2a} = \dfrac{-4\pm 10}{2} = \begin{cases} 3 \\ -7 \end{cases}$

(b) $x=\pm 1$, (c) $x_1=-\frac{1}{2},\ x_2=1$ (d) $x_{1,2}=\dfrac{1\pm\sqrt{33}}{8}$

(e) $x=\pm i$ (f) $x_{1,2}=\dfrac{3\pm i\sqrt{7}}{2}$

2. We write the polynomial in the following form:

$3x^3-11x^2+16x-12 = (x-2)(ax^2+bx+c)$

$= ax^3+(b-2a)x^2+(c-2b)x-2c$

Comparing the coefficients of these polynomials we get:

$a=3,\ b=-5,\ c=6$

So $3x^3-11x^2+16x-12=0$ is equivalent to:

$(x-2)\cdot(3x^2-5x+6)=0$

So either $x-2=0$ or $3x^2-5x+6=0$

$x=2$ or $x=\dfrac{5}{6}\pm\dfrac{\sqrt{47}}{6}i$

3. $x^2+2x-8 \leq 0$

First we compute the roots of the quadratic polynomial.

$x_1=-4$ and $x_2=2$. So we have:

$$x^2+2x-8 = (x+4)(x-2)$$

x		-4		2	
$x+4$	$-$	0	$+$		$+$
$x-2$	$-$		$-$	0	$+$
$(x+4)(x-2)$	$+$	0	$-$	0	$+$

From the table we observe that $x^2+2x-8 \leq 0$ wher $\boxed{-4 \leq x \leq 2}$

4. (a) $-11+8i$ (b) $5-3i$ (c) $-2i$

5. (a) $z_1+z_2 = 7-6i$

(b) $z_1-z_2 = -1+10i$

(c) $z_2-z_1 = 1-10i$ $(=-(z_1-z_2))$

(d) $z_1 z_2 = (3+2i)(4-8i) = 12-24i+8i-16i^2$

$= 12-16i+16$

$= 28-16i$

(e) $$\frac{z_1}{z_2} = \frac{(3+2i)}{(4-8i)} = \frac{(3+2i)(4+8i)}{(4-8i)(4+8i)} = \frac{-4+32i}{16+64} =$$

$$= -\frac{1}{20}+\frac{2}{5}i.$$

6. (a) $\frac{5+3i}{2+2i} = \frac{(5+3i)(2-2i)}{(2+2i)(2-2i)} = \frac{10-10i+6i-6i^2}{2^2+2^2} =$

$= \frac{10-4i+6}{4+4} = \frac{16-4i}{8} = 2-\frac{1}{2}i$

(b) $\frac{-2+3i}{i} = \frac{(-2+3i)(-i)}{i(-i)} = \frac{2i-3i^2}{1^2} = 3+2i$

(c) $(5+3i)(2-i)-(3+i) = 10$

(d) $(1-2i)^2 = 1 - 2\cdot 2i + (2i)^2 = -3-4i$

(e) $\frac{5-8i}{3-4i} = \frac{(5-8i)(3+4i)}{(3-4i)(3+4i)} = \frac{15+20i-24i-32i^2}{3^2+4^2} =$

$= \frac{15-4i+32}{9+16} = \frac{47-4i}{25} = \frac{47}{25} - \frac{4}{25}i.$

(f) $\frac{3}{3+2i} + \frac{1}{5-i} = \frac{3(3-2i)}{(3+2i)(3-2i)} + \frac{5+i}{(5-i)(5+i)} =$

$= \frac{9-6i}{9+4} + \frac{5+i}{25+1} = \frac{9-6i}{13} - \frac{5+i}{26} =$

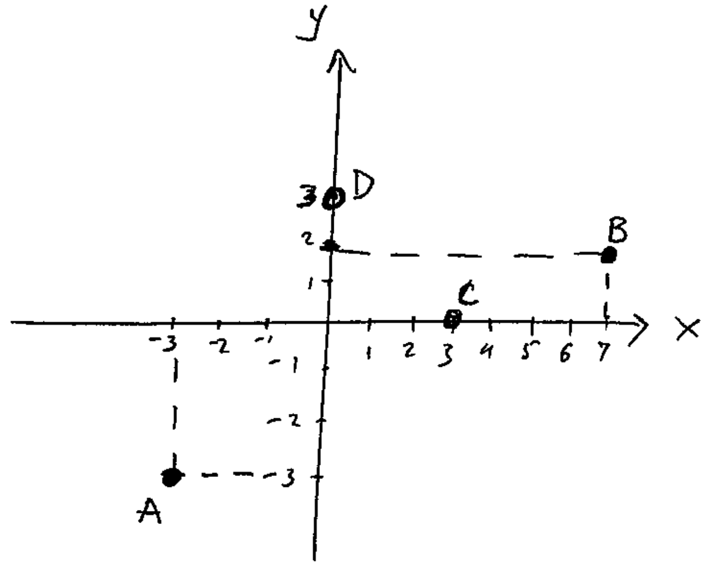
$= \frac{18-12i+5+i}{26} = \frac{23}{26} - \frac{11}{26}i$

7. (a) $A(-3,-3)$

(b) $B(7,2)$

(c) $C(3,0)$

(d) $D(0,3)$



8.

$$\frac{2+x-yi}{3x+yi} = 1+2i \implies$$

$$\implies 2+x-yi = (1+2i)(3x+yi)$$

$$\implies (2+x)-yi = 3x + yi + 6xi + (2y)(i^2)$$

$$\implies (2+x)-yi = (3x-2y) + (y+6x)i$$

The complex number on the left side is equal to the one on the right. That is:

$$\left.\begin{array}{l} 2+x = 3x-2y \\ -y = y+6x \end{array}\right\} \xrightarrow[\text{the system}]{\text{solving}} \left.\begin{array}{l} 2+x = 3x+2\cdot(3x) \\ y = -3x \end{array}\right\}$$

$$\implies \begin{array}{l} x = 2/8 \\ y = -3/4 \end{array}$$

9. $z = z_1 + \dfrac{z_2 z_3}{z_2+z_3} = z_1 + \dfrac{z_2 z_3 (\overline{z_2+z_3})}{(z_2+z_3)(\overline{z_2+z_3})} =$

$= z_1 + \dfrac{z_2 z_3(\overline{z_2}+\overline{z_3})}{(z_2+z_3)(\overline{z_2+z_3})} = z_1 + \dfrac{z_3 z_2 \overline{z_2} + z_2 z_3 \overline{z_3}}{(z_2+z_3)(\overline{z_2+z_3})}$

We have: $z_2+z_3 = -2+16i$

$\overline{z_2+z_3} = -2-16i$

$z_2\overline{z_2} = 3^2+4^2 = 25$

$z_3\overline{z_3} = (-5)^2+(12)^2 = 169$

So

$z = 2+3i + \dfrac{25(-5+12i)+169(3+4i)}{4+256} =$

$= 2+3i + \dfrac{-125+300i+507+676i}{260} =$

$= 2+3i + \dfrac{382+976i}{260} = \dfrac{451}{130} + \dfrac{439}{65} i.$

10. $\dfrac{1}{z_3} = \dfrac{1}{z_1} + \dfrac{1}{z_1 z_2} = \dfrac{z_2+1}{z_1 \cdot z_2} \Rightarrow z_3 = \dfrac{z_1 z_2}{z_2+1} =$

$= \dfrac{(3-4i)(5+2i)}{5+2i+1} = \dfrac{23-14i}{6+2i} = \dfrac{(23-14i)(6-2i)}{(6+2i)(6-2i)} =$

$= \dfrac{138-46i-84i+28i^2}{6^2+2^2} = \dfrac{110-130i}{40} = \dfrac{11}{4} - \dfrac{13}{4} i$