1.)

a.) $y = \sin^2(5+x) \therefore \dfrac{dy}{dx} = 2\sin(x+5)\cos(x+5)$

b.) $y = e^{2\sin(x)}$ $\qquad \dfrac{dy}{dx} = 2\cos\, e^{2\sin(x)}$

c.) $y = (4x+7)^5$ $\qquad \dfrac{dy}{dx} = 5(4x+7)^4 \times 4 = 20(4x+7)^4$

d.) $y = e^{-x}\cos(5x)$ $\qquad \dfrac{dy}{dx} = -e^{-x}\, 5\cos(5x) + \sin(5x)$

e.) $y = \ln\cos(4x)$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{\cos(4x)} \cdot -4\sin(4x)$

f.) $y = \dfrac{1}{x^2+1}$ $\qquad \dfrac{dy}{dx} = -\dfrac{2x}{(x^2+1)^2}$

g.) $y = \dfrac{x^3\sin(2x)}{\cos(x)}$ $\qquad \dfrac{dy}{dx} = \dfrac{3x^2 \cdot 2\cos(2x) \cdot -\sin(x)}{-(\cos(x))^2}$

h.) $y = x^3 e^{-x}\tan(x)$ $\qquad \dfrac{dy}{dx} = 3x^2 e^{-x}\tan(x) - x^3 e^{-x}\tan(x) + x^3 e^{-x}\sec^2(x)$

i) $y = \dfrac{xe^{5x}}{\sin(x)}$ $\qquad \dfrac{dy}{dx} = \dfrac{5xe^{5x}\sin(x) - xe^{5x}\cos(x)}{\sin^2(x)}$

2.) with parametric equations $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$y = \dfrac{2-t}{1-t} \therefore \dfrac{dy}{dt} = \dfrac{1}{(1-t)^2}$

$x = \dfrac{5+3t}{1-t} \therefore \dfrac{dx}{dt} = \dfrac{8}{(1-t)^2}$

$\therefore \dfrac{dy}{dx} = \dfrac{\frac{1}{(1-t)^2}}{\frac{8}{(1-t)^2}} = \dfrac{1}{8}$

$\therefore \dfrac{dy^2}{d^2x} = 0$

3.) if $\frac{dy}{dx} = 0$, it is a turning point or point of inflection.

if $\frac{dy^2}{d^2x} < 0$, max   if $\frac{dy^2}{d^2x} > 0$, min, and if $\frac{dy^2}{d^2x} = 0$, inflection.

a.) $y = x^2 - x + 6$

$\frac{dy}{dx} = 2x - 1$ $\therefore 2x - 1 = 0$ $x = 0.5$

$\frac{dy^2}{d^2x} = 2$ The turning point at 0.5 is a minimum

b.) $y = x - 1$

$\frac{dy}{dx} = 1$ Because $1 \neq 0$ there can be no turning point. The gradient is constant.

c.) $y = x^3 - 12x$

$\frac{dy}{dx} = 3x^2 - 12$ $\therefore 3x^2 - 12 = 0$

$\qquad \qquad \qquad \qquad \qquad \parallel$

$3(x+2)(x-2) = 0$ $x = 2$ or $^-2$

$\frac{dy^2}{d^2x} = 6x$ sub in 2 and -2

at $x = 2$, it is a minimum

at $x = ^-2$, it is a maximum

4.) a.) $y = 3x^2 + 6x - 1 \quad \therefore \quad \frac{dy}{dx} = 6x + 6$

$6x + 6 = 0 \qquad \underline{x = {}^-1} \qquad \frac{dy^2}{d^2x} = 6$

$x = {}^-1$ is a minimum.

b.) $y = {}^-x^2 - x + 4 \quad \therefore \quad \frac{dy}{dx} = -2x - 1$

$-2x - 1 = 0 \qquad \underline{x = \frac{-1}{2}} \qquad \frac{dy^2}{d^2x} = -2$

$x = \frac{-1}{2}$ is a maximum.

c.) $y = x^5 - \frac{5x^3}{3} \quad \therefore \quad \frac{dy}{dx} = 5x^4 - 5x^3$

$5x^4 - 5x^3 = 5x^3(x-1) \quad \therefore \quad \underline{x = 1 \text{ or } 0}$

$\frac{dy^2}{d^2y} = 20x^3 - 15x^2$, sub in 1 and 0

$\boxed{\begin{array}{l} x = 0 \text{ is a point of inflection} \\ x = 1 \text{ is a minimum} \end{array}}$

d.) $y = 2x^2 \ln x \quad \therefore \quad \frac{dy}{dx} = 4x \ln x + \frac{2x^2}{x}$

$4x \ln x + \frac{2x^2}{x} = 0 \qquad x = \frac{1}{\sqrt{e}}$

sub in values around $\frac{1}{\sqrt{e}}$ into original

0.1	$\frac{1}{\sqrt{e}}$	1
+	0	+

Since both sides are increasing, the root $x = \frac{1}{\sqrt{e}}$ must be a minimum.

5.) e^x about $x=0$ and $\frac{dy}{dx} = e^x$

$x=1$

$e^0 = 1$

$$P_1(x) = f(a) + \frac{df}{dx}(a)\frac{(x-a)^1}{1!}$$

$$P_1(0) = 1 + 1\frac{x^1}{1!} \quad = 1 + x$$

$$P_1(1) = e^1 + \cancel{1}^{\,e^1}\frac{(x-1)^1}{1!} \quad = \cancel{} \; e^1 + (e^1 x - 1)$$

b.) for $x=0$

$$P_1(0.1) = 1 + 0.1 \quad = \cancel{}\; 1.1$$

$$y(0.1) = e^{0.1} \quad = 1.105$$

for $x=1$

$$P_1(1) = e^1 + \cancel{}^{\,e^1 0.1} - 1 \quad = \cancel{}\; 0.2718$$

$$y(0.1) = e^{0.1} \quad = 1.105$$

c.) For a Taylor polynomial $P_1(x)$, it is only accurate at aproximating a function within a small distance of the point it is about. Because of this, for the function e^x, if we take $P_1(0.1)$ around the x value 0, it will be very close to the actual value $y(0.1)$.

However, if we take $P_1(0.1)$ around the x value 1, and compare it to $y(0.1)$, They will be very different as The point 0.1 is not very close to the point that P_1 is about.

6. $y = 3x^4 + 1$ around the point $\underline{x = 2}$

$$\frac{dy}{dx} = 12x^3 \qquad \frac{dy^2}{d^2x} = 36x^2$$

$$P_2(x) = 49 + \frac{96(x-2)^1}{1!} + \frac{144(x-2)^2}{2!}$$

b)

$$\left.\begin{aligned} y(1.8) &= 32.5 \\ P_2(1.8) &= 32.7 \end{aligned}\right\}$$

These two values are relativly close, as The point we are looking at (1.8) is close to the point that the Taylor polynomial is about.

7.) $y = \sin(x)$ about 0

$\underline{y(0) = \sin(0) = 0}$

$$\frac{dy}{dx} = \cos(0) = 1$$

$$\frac{dy^2}{d^2x} = -\sin(0) = 0$$

$$\frac{dy^3}{d^3y} = -\cos(0) = -1$$

$$P(x) = 0 + x + \frac{-1x^3}{3!} + \frac{x^5}{5!} + \frac{-x^7}{7!}$$

$\therefore$

$$P(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} \cdots$$