(a) **Solution:**

$$\begin{aligned} rms &= \sqrt{\frac{1}{3}\int_1^4 (x-1)\,dx} \\ &= \sqrt{\frac{1}{3}\left[\frac{x^2}{2}-x\right]_1^4} \\ &= \sqrt{\frac{1}{3}(8-4-\frac{1}{2}+1)} \end{aligned}$$

(b) **Solution:**

$$\begin{aligned} rms &= \sqrt{\frac{1}{3}\int_0^3 \frac{1}{(x+1)^4}\,dx} \\ &= \sqrt{\frac{1}{3}\int_0^3 (x+1)^{-4}\,dx} \\ &= \sqrt{\frac{1}{9}\left[-(x+1)^{-3}\right]_0^3} \\ &= \frac{\sqrt{-(3+1)^{-3}+1}}{3} = \frac{1}{3}\sqrt{1-\frac{1}{4^3}} \end{aligned}$$

(c) **Solution:**

$$\begin{aligned} rms &= \sqrt{\int_2^3 (x-1)^{-2}\,dx} \\ &= \sqrt{\left[-(x-1)^{-1}\right]_2^3} \\ &= \sqrt{\left[\frac{-1}{3-1}+\frac{1}{2-1}\right]} = \sqrt{1-\frac{1}{2}} \end{aligned}$$

5. Use the trapezium rule, with 4 strips, to estimate

   (a) $\int e^{-x^2} dx$ on $[0,1]$.

   Solution:

$$\int_0^1 e^{-x^2}\,dx \approx \frac{1}{4}\frac{1}{2}\left(1+2e^{-(\frac{1}{4})^2}+2e^{-(\frac{1}{2})^2}+2e^{-(\frac{3}{4})^2}+e^{-1}\right)$$

   Compute that $f'(x) = -2xe^{-x^2}$ and $f''(x) = (-2+4x^2)e^{-x^2}$. It then follows (why? check) that $|f''(x)|] \leq 2$ on $[0,1]$, the

$$|\text{error}| \leq (\frac{1}{4})^2 \cdot \frac{1}{12} \cdot 2$$