VICTORIA
UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA
O TE ŪPOKO O TE IKA A MĀUI

ENGR142 2018, 2nd Trimester

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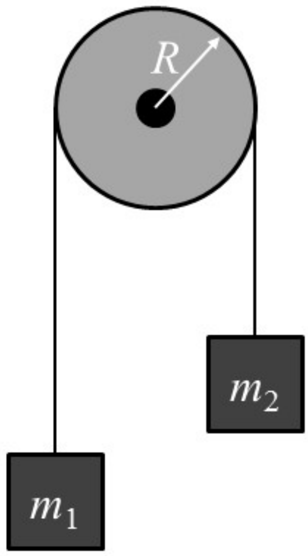
Assignment 5

Due date: 11:59 PM, Friday 31st August, 2018

Problem 1 : Pulley problem *(8 Marks)*

The figure to the right shows a pulley with mass M, radius R, and moment of inertia $I = \frac{1}{2}MR^2$ connected to two masses m_1 and m_2 via ideal ropes (massless, inextensible). The pulley can rotate without friction about its central rotation axis.



(a) By considering the net torque on the pulley show that it has angular acceleration given by

$$\alpha = \frac{2(T_1 - T_2)}{MR},$$

where T_1 and T_2 are the tensions in the strings.

(b) Hence show that the downward acceleration of the mass m_1 is given by

$$a_1 = \frac{(m_1 - m_2)g}{\frac{1}{2}M + m_1 + m_2},$$

Hint: *Use the net force on m_1 to find an expression for T_1, and do a similar thing for m_2 and T_2. Then use $a_1 = -a_2$ and $a_1 = \alpha R$. For both masses choose the downward direction as positive.*

Problem 2 : Driven oscillations *(12 Marks)*

Use the code you have developed in assignments 3 and 4 to investigate driven oscillations of a mass on a spring. As before choose $m = 1$ kg, $k = 4$ N/m, $b = 0.4$ kg/s, and take as initial conditions $x_i = x(0) = 2$ m and $v_i = v(0) = 0$ m/s.

(a) Add an extra force term to represent a driving force of form $F_{\text{driving}} = F_0 \cos(\omega_{dr} t)$, where ω_{dr} is the driving frequency and F_0 gives the strength of the driving force. Choose $F_0 = 2$ N and $\omega_{dr} = 3\,\text{rad}\,\text{s}^{-1}$.

(a.1) Calculate $x(t)$ between $t_i = 0$ and $t_f = 80$ seconds. Use a large number of data points (about 30,000) to ensure an accurate description. Plot the results (i.e., plot $x(t)$ vs t).

(a.2) After about 20 seconds $x(t)$ should exhibit steady oscillations. What is the angular frequency of these oscillations? What is the amplitude of these oscillations? (It is fine to just read the amplitude from the graph, but you can use some more elaborate code to find the maximum value of $x(t)$ for $t \gtrsim 20$ s if you wish.)

(b) (b.1) Repeat the calculation above for a range of values of ω_{dr} between 0 and 6 rad s^{-1}, measuring the amplitude of the steady oscillations in each case. Plot the amplitude as a function of ω_{dr}.
About 10 different values of ω_{dr} should do. You can use more if you want, although this will likely require you to write code to loop through the ω_{dr} values and to calculate the amplitude in the steady oscillation region.

(b.2) Which driving frequency gives the maximum amplitude of oscillation? Is this the expected value? Very briefly explain.

Hint: Example Python code is available under assignment 4, which you can use if you wish. It should be very easy to change this to include a driving force.

Problem 3 : Driven oscillations extension *(5 Marks)*

This is an extension problem: you do not need to complete this, but you can score bonus marks if you do (you could get 25/20 for the assignment!!).

(a) Repeat the calculation of part 2(b.1) above, but use different damping coefficients b. Try $b = 1$ kg/s and $b = 3$ kg/s, and plot the amplitude as a function of ω_{dr} in each case (ideally put the graphs for $b = 0.4$, 1, and 3 kg/s on the same axes).

(b) Add a curve to the plot above showing the expected behaviour of the amplitude as a function of ω_{dr}. This is given by (see lecture notes)

$$A = \frac{F_0}{m} \frac{1}{\sqrt{(\omega_{dr}^2 - \omega_0^2)^2 + \frac{b^2\omega_{dr}^2}{m^2}}}, \tag{1}$$

where $\omega_0 = \sqrt{\frac{k}{m}}$.