VICTORIA
UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA
O TE ŪPOKO O TE IKA A MĀUI

ENGR142 2018, 2nd Trimester

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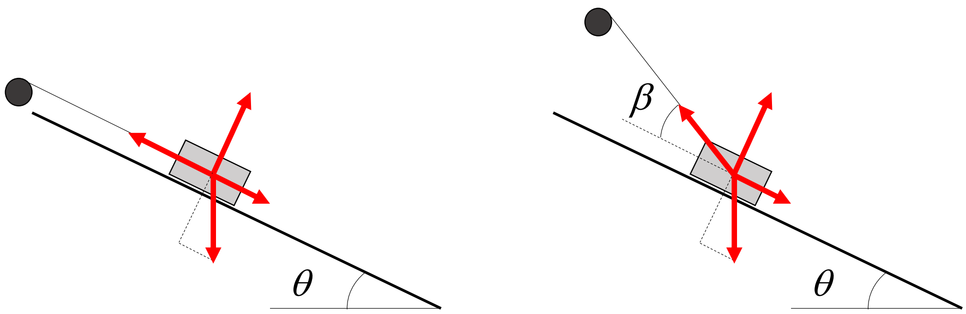
Assignment 3

Due date: 11:59 PM, Thursday 16th August, 2018

Problem 1 : Lego crane *(8 Marks)*



The diagram above illustrates two situations where a cart is pulled by a string up a slope of angle θ. In the first case the string pulls the cart directly up the slope, while in the second case it pulls at an angle β to the slope. The tension force $\vec{F}_T$, the weight force, the normal force, and the friction force are all shown in a free-body diagram for both cases. The friction force between the cart and the slope is $\vec{F}_f = \mu \vec{N}$, where μ is the coefficient of friction and $\vec{N}$ is the normal force.

(a) By applying Newton's second law along the slope and perpendicular to the slope show that the acceleration of the cart along the slope is given by

$$a = \frac{F_T \left[\cos(\beta) + \mu \sin(\beta)\right] - mg \left[\sin(\theta) + \mu \cos(\theta)\right]}{m}$$

(b) If $F_T = 4$ N, $\mu = 0.5$, the cart has mass $m = 0.25$ kg, and $\theta = 30°$ show that more rapid acceleration is achieved pulling at an angle $\beta = 20°$ than pulling directly up the slope $\beta = 0°$. Explain in one or two sentences why this is the case.

Problem 2 : Oscillating mass *(12 Marks)*

Consider a mass m connected to a spring with spring constant k. In addition to the spring force the mass experiences a drag force given by $F_d = -bv$, where b is a constant and v is the velocity. The resulting oscillations are described by the formula $x(t) = Ae^{-\alpha t}\cos(\omega t + \phi)$, where the constants α and ω are given by $\alpha = b/2m$ and $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$. In the following take $m = 1$ kg and $k = 4$ N/m, and for the initial amplitude and phase use $A = 2$ m and $\phi = 0$.

(a) Consider the case $b = 0.4$ kg/s. Calculate ω and α and plot $x(t)$ between $t_i = 0$ and $t_f =$ 12 seconds. Use as the initial position $x(0) = 1$ m.

(b) Now perform a numerical calculation of $x(t)$ for the same values $b = 0.4$ kg/s and $x(0) =$ 2 m, and use the same start and finish times. First calculate for a total of 100 data points between t_i and t_f, then repeat the calculation using 10000 points (i.e., use a much smaller time step in the second case).

   (b.1) Plot the results on the same graph as you used for part (a) above.

   (b.2) The two calculations will not be the same. Explain, using a basic physics principle, why the result of the first calculation (100 data points) cannot be correct (one or two sentences only).

(c) Perform numerical calculations of $x(t)$ using 10000 data points for $b = 0$, 0.5, 2, 4, and 12 kg/s. Plot $x(t)$ in each case (use a single graph to show all results). Use the values for t_i and t_f given above.