

1 a.) $A+D$ is not possible,

$$C-A = \begin{bmatrix} -7 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ -3 & 0 \end{bmatrix}$$

$D-E$ is not possible

$$b.) A \times B ~~not possible~~ = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \text{not possible}$$

$$C \times A = \begin{bmatrix} -7 & 1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ 12 & 16 \end{bmatrix}$$

$$A \times C = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -7 & 1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -7 & 5 \\ -21 & 19 \end{bmatrix}$$

$D \times A = \text{not possible}$

$D \times B = \text{not possible}$

$B \times D = \text{not possible}$

$E \times B = \text{not possible}$

$B \times E = \text{not possible}$

$$A \times E = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 3 \\ 10 & 17 & 8 \end{bmatrix}$$

$$c.) 7C = 7 \begin{bmatrix} -7 & 1 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -49 & 7 \\ 0 & 28 \end{bmatrix}$$

$$-3D = -3 \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -6 & -3 \end{bmatrix}$$

$$KE = k \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 2k & 3k & 4k \\ 1k & 2k & -1k \end{bmatrix}$$

$$2.) A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 18 & 14 \\ 7 & 11 \end{bmatrix} \cdot \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 86 & 78 \\ 39 & 47 \end{bmatrix}$$

$$3.) \quad AB = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & 4 \\ 5 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 0 & 3 & 4 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 \times 5 + 2 \times 1 & 2 + 9 + 6 & 1 + 12 + 10 \\ -5 + 4 & -2 + 12 & -1 + 20 \\ 25 - 1 & 10 + 3 - 3 & 5 + 4 - 5 \end{bmatrix} = \begin{bmatrix} 7 & 17 & 23 \\ -1 & 10 & 19 \\ 24 & 10 & 4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & 4 \\ 5 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 0 & 3 & 4 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 5 + 6 + 2 & 2 + 9 + 6 & 1 + 12 + 10 \\ -5 + 4 & -2 + 12 & -1 + 20 \\ 25 - 1 & 10 + 3 - 3 & 5 + 4 - 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 & 1 \\ 0 & 3 & 4 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ -1 & 0 & 4 \\ 5 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 16 & 17 \\ 17 & 4 & 8 \\ 23 & 8 & 9 \end{bmatrix}$$

Same process as above
just done in head so
less space needed.

$$4.) \quad A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \\ 5 & 1 & -1 \end{bmatrix} \therefore A^T = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 0 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -7 & 0 \\ 0 & 2 & 5 \\ 3 & 4 & 5 \end{bmatrix} \therefore B^T = \begin{bmatrix} 1 & 0 & 3 \\ -7 & 2 & 4 \\ 0 & 5 & 5 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 2 & -1 & 5 \\ 1 & 0 & 4 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ -7 & 2 & 4 \\ 0 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 9 & 23 & 27 \\ 1 & 20 & 23 \\ -4 & -3 & 8 \end{bmatrix}$$

$$5.) \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Det } A = 4 - 6 = -2$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

$$A + aA^{-1} = bI$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + a \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} \therefore \begin{array}{l} 2 + a = 0 \\ 3 + 1.5a = 0 \end{array} \quad \underline{a = -2}$$

$$1 + (-2 \times -2) = b = 5$$

$$a = -2, \quad b = 5$$

$$6.) \quad A = \begin{bmatrix} 3 & 7 & 6 \\ -2 & 1 & 0 \\ 4 & 2 & 5 \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 7 & 6 \\ -2 & 1 & 0 \\ 4 & 2 & 5 \end{vmatrix} = 6 \begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} + 5 \begin{vmatrix} 3 & 7 \\ -2 & 1 \end{vmatrix} = 6 \times -8 + 5 \times 17$$

$$|A| = 37$$

7.)

~~$$D = \begin{vmatrix} 2 & 1 & -3 \\ 1 & -1 & 2 \\ 2 & -2 & -1 \end{vmatrix} = -3 \begin{vmatrix} 1 & -1 \\ 2 & -2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 2 & -2 \end{vmatrix} + (-1) \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 15$$~~

~~$$D_x = \begin{vmatrix} 0 & 1 & -3 \\ 10 & -1 & 2 \\ -1 & -2 & -1 \end{vmatrix} = -10 \begin{vmatrix} 1 & -3 \\ -2 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & -3 \\ -1 & 2 \end{vmatrix} =$$~~

$$D = \begin{vmatrix} 2 & -3 & 1 \\ 5 & 4 & 1 \\ 2 & -2 & -1 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} 2 & -3 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = -43$$

$$D_x = \begin{vmatrix} 0 & -3 & 1 \\ 10 & 4 & 1 \\ -1 & -2 & -1 \end{vmatrix} = \begin{vmatrix} 10 & 4 \\ -1 & -2 \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ -1 & -2 \end{vmatrix} - \begin{vmatrix} 0 & -3 \\ 10 & 4 \end{vmatrix} = -43$$

$$D_y = \begin{vmatrix} 2 & 0 & 1 \\ 5 & 10 & 1 \\ 2 & -1 & -1 \end{vmatrix} = \begin{vmatrix} 5 & 10 \\ 2 & -1 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 2 & -1 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 5 & 10 \end{vmatrix} = -43$$

$$D_z = \begin{vmatrix} 2 & -3 & 0 \\ 5 & 4 & 10 \\ 2 & -2 & -1 \end{vmatrix} = 10 \begin{vmatrix} 2 & -3 \\ 2 & -2 \end{vmatrix} - \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = -43$$

$$x = \frac{D_x}{D} = \frac{-43}{-43} = 1$$

$$x = 1$$

$$y = 1$$

$$y = \frac{D_y}{D} = \frac{-43}{-43} = 1$$

$$z = 1$$

$$z = \frac{D_z}{D} = \frac{-43}{-43} = 1$$

8.)

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & -5 \\ 1 & -1 & 2 & 12 \\ 7 & -2 & 3 & 37 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -3 & -5 \\ 0 & -3 & 7 & 29 \\ 0 & 5.5 & -13.5 & -54.5 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -3 & -5 \\ 0 & -3 & 7 & 29 \\ 0 & 0 & -\frac{2}{3} & -\frac{4}{3} \end{array} \right]$$

$$-\frac{2}{3}z = -\frac{4}{3} \quad z = 2$$

$$-3y + 14 = 29 \quad y = -5$$

$$2x - 5 - 6 = -5 \quad x = 3$$

$$x = 3$$

$$y = -5$$

$$z = 2$$

9.)