ENGR122 Assignment 9

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DUE: 1pm on 5 October 2018 online

1. Use partial fractions to find
   (a) $\int \frac{6t+3}{2t^2-5t+2}dt$
   (b) $\int_1^2 \frac{3-3x}{2x^2+6x}dx$
2. Consider the DE
   $$\frac{dy}{dx} = 3y(11-y) \quad \text{subject to } y(0) = 5.$$
   (a) Solve the equation.
   Hint: Solve $\frac{1}{3y(11-y)} = \frac{A}{3y} + \frac{B}{11-y}$ for A and B and then integrate the RHS.
   (b) As $x \to -\infty$, what happens to $y(x)$?
   (c) As $x \to \infty$, what happens to $y(x)$?
3. Solve the following separable – or, nearly separable – ODEs
   (a)
   $$\frac{dx}{dt} = \frac{\cos 3t}{x^2} \quad \text{where } x(0) = 1$$
   (b) *[integrate by parts along the way]*
   $$\frac{dy}{dx} = \frac{x \ln x}{e^y}$$
   (c)
   $$\frac{dy}{dt} = e^{y+t}$$
   (d)
   $$\frac{dy}{dx} = \frac{y}{x} + \frac{y^4}{x^4}$$
   Hint: Let $z = \frac{y}{x}$.
4. Solve the following first order linear ODEs using integrating factors
   [***General hint:*** *Convert 1st order linear DEs into standard form* $\frac{dy}{dx}+P(x)y = Q(x)$ *as an initial step.]*
   (a)
   $$\frac{dy}{dx} + \frac{y}{x} = 1$$
   (b) *[parts along the way]*
   $$x^2\frac{dy}{dx} + xy - x^2e^x = 0$$
   (c) *[use parts twice]*
   $$\frac{dx}{dt} + \frac{2x}{t} = \sin t$$
   (d) *[parts]*
   $$\frac{dx}{dt} + 6t^2x = t^2 + 2t^5$$

Tutorial Questions for 1–4 Oct 2018, ENGR122

1. Use partial fractions to compute $\int_0^1 \frac{4x+7}{4x^2+8x+3}dx$

2. Solve the following separable (or nearly separable) ODEs:

   (a)
   $$\frac{dy}{dx} = 2y(7-y)$$
   Hint: Solve $\frac{1}{2y(7-y)} = \frac{A}{2y} + \frac{B}{7-y}$ for A and B and then integrate the RHS.

   (b)
   $$\frac{dy}{dx} = y(e^x + \sin 8x)$$

   (c)
   $$\frac{dy}{dx} = \frac{x}{e^{3x}}$$

   (d)
   $$x\frac{dy}{dx} + y = e^x$$
   Hint: write the LHS as $\frac{d}{dx}(xy)$.

3. Solve the following 1st order linear ODEs

   (a) *[use integration by substitution]*
   $$\frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^2}$$

   (b)
   $$x^2\frac{dy}{dx} + xy = 1$$

   (c)
   $$\frac{dx}{dt} + \frac{2x}{t} = t^2$$

   (d)
   $$\frac{dx}{dt} + tx = 3t$$