1.) The constant of integration has no affect on the integrating factor because The integrating factor is multiplied through both sides of the equation. Because of this, and that c is a constant, we are able to cancle it out without affecting the solution.

2.)

a.) $F = ma$ where $a = g = 9.81$

$\frac{dv}{dt} = g$ and $\frac{dx}{dt} = v = gt \quad \therefore x = \frac{1}{2}gt^2 + V_0$ where $V_0 = 0$

$t = \sqrt{\frac{2x}{g}}$ where $t = \frac{V}{g}$ and $x = h \quad \therefore \frac{V}{g} = \sqrt{\frac{2h}{g}}$

$V = \sqrt{2gh}$

b.) Rate of outflow : $va = a\sqrt{2gh}$

Rate of change of volume : $v\,A(h) = A(h)\frac{dh}{dt}$

since $\frac{dh}{dt} < 0$, $\therefore a\sqrt{2gh}$ must be negative so that liquid leaving = rate of valum

$A(h)\frac{dh}{dt} = -a\sqrt{2gh}$

c.) $h = 3$, $r_{Tank} = 1 \therefore A = \pi$, $r_{out} = 0.1 \therefore a = \pi(0.1^2) = 0.01\pi$

$A\frac{dh}{dt} = -a\sqrt{2gh} = \pi\frac{dh}{dt} = -0.01\pi\sqrt{2gh}$

$dh\frac{1}{\sqrt{h}} = -0.01\sqrt{2g}\,dt \quad \therefore \int h^{-\frac{1}{2}}\,dh = \int -0.01\sqrt{2g}\,dt$

$2\sqrt{h} = -0.01\sqrt{19.6}\,t + C \quad [\text{at } t=0,\ h=3] \quad \therefore C = 2\sqrt{3}$

$\boxed{t = \frac{2\sqrt{h} - 2\sqrt{3}}{-0.01\sqrt{19.6}}}$

sub in $h=0$ to find time to empty the tank.

$t = 78.25$ seconds

3 a.) $s(t+\Delta t) = s(t) + r\Delta t\, s(t) + k\Delta t$

$$\frac{s(t+\Delta t) - s(t)}{\Delta t} = r s(t) + k \quad \therefore \quad \frac{ds}{dt} = r s(t) + k$$

$$\boxed{S'(t) - r s(t) = k} \qquad \mu = e^{\int -r\, dt} = e^{-rt}$$

$$\frac{d}{dt}\left[e^{-rt} s(t)\right] = \int k e^{-rt} dt \quad \therefore \quad e^{-rt} s(t) = \frac{-ke^{-rt}}{r} + C$$

$$s(t) = \frac{C}{e^{-rt}} - \frac{k}{r} = \boxed{Ae^{rt} - \frac{k}{r}}$$

where $A - \frac{k}{r} = 0$

$\therefore A = \frac{k}{r}$

b.) if $r = 0.075$, $t = 40$, $s(t) = 1{,}000{,}000$, $A = k/r$

$$s(t) = \frac{k}{r} e^{rt} - \frac{k}{r} \quad \therefore \quad s(t) = k\left(\frac{1}{r}e^{rt} - \frac{1}{r}\right)$$

$$k = \frac{s(t)}{\left(\frac{1}{r}e^{rt} - \frac{1}{r}\right)} \qquad k = 3929.68$$

c.)

if $t = 40$, $s(t) = 1{,}000{,}000$, $k = 2000$, $A = k/r$

$\boxed{\text{rate} = 9.8\% \text{ interest rate}}$

4) a.) $(t-3)y' + (\ln t)y = 2t \quad \therefore \quad y' + \frac{\ln t}{t-3}y = \frac{2t}{t-3}$

This equation is undefined at values $t=0$ and $t=3$

$-\infty < t < 0 \;, \quad 0 < t < 3 \;, \quad 3 < t < \infty$

since the IVP $y(1) = 2$; The interval is $(0, 3)$

b.) $y' + (\tan t)y = \sin t$

This is undefined every $\frac{\pi}{2} + \pi n$ where $n \in \mathbb{Z}$

The interval is $\frac{\pi}{2} < t < \frac{3\pi}{2}$ or $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ as the initial value π is within this range.

5.)

$y' = \frac{t-y}{2t+3y}$ This equation is undefined when $2t + 3y = 0$

from this we know that $y = \frac{-2}{3}t$ is a line of discontinuity, meaning that the solution is defined everywhere other than $y = \frac{-2}{3}t$.

6.)

$t^2 \frac{dy}{dt} + 2ty = y^3 \quad \therefore \quad v = y^{-2}, \quad \frac{dv}{dt} = -2y^{-3}\frac{dy}{dt}$

$\frac{y^3}{-2}\frac{dv}{dt} = \frac{dy}{dt}$

$\frac{y^3}{-2} t^2 \frac{dv}{dt} + 2ty = y^3$

$\frac{t^2}{-2}\frac{dv}{dt} + 2ty^{-2} = 1 \quad \therefore \quad \frac{t^2}{-2}\frac{dv}{dt} + 2tv = 1$

$\frac{dv}{dt} - \frac{4v}{t} = \frac{-2}{t^2} \quad \therefore \quad \mu = e^{\int \frac{-4}{t}\,dt} = e^{-4\log|t|} = \frac{1}{t^4}$

$\frac{d}{dt}\left[\frac{1}{t^4}v\right] = \int \frac{-2}{t^2} \times \frac{1}{t^4} = \int \frac{-2}{t^6} = \int -2t^{-6} = \frac{2}{5t^5} + C$

$v = \frac{2}{5t} + C \quad \therefore \quad \frac{1}{y^2} = \frac{2}{5t} + C \quad \therefore \quad \boxed{y = \sqrt{\frac{5t}{2} + \frac{1}{c}}}$