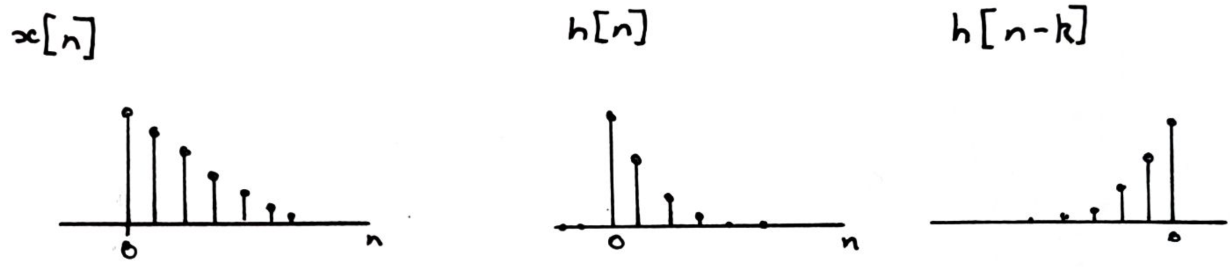
2.21 a.)

$x[n] = \alpha^n u[n]$, $h[n] = \beta^n u[n]$

$y[n] = x[n] * h[n]$



Region 1.) $n < 0$: $y[n] = 0$

Region 2.) $n \geq 0$:

$$y[n] = \sum_{k=0}^{n} \alpha^k \beta^{n-k}$$

$$= \sum_{k=0}^{n} \alpha^k \beta^n \beta^{-k} = \beta^n \sum_{k=0}^{n} \alpha^k \beta^{-k} = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k$$

$$\boxed{y[n] = \beta^n \frac{\alpha^{n+1} - \beta^{n+1}}{\beta^n(\alpha - \beta)} = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}} \cdot u[n]$$

b.)

$x[n] = h[n] = \alpha^n u[n]$

$y[n] = x[n] * h[n]$

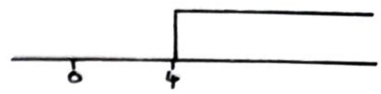
Region 1.) $n < 0$: $y[n] = 0$

Region 2.) $n \geq 0$:

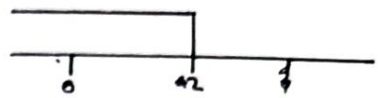
$$y[n] = \sum_{k=0}^{n} \alpha^k \alpha^{n-k}$$

$$= \alpha^n \sum_{k=0}^{n} \left(\frac{\alpha}{\alpha}\right)^k = \alpha^n \sum_{k=0}^{n} 1^k = \boxed{\alpha^n \cdot (n+1)\, u[n]}$$

c.) $x[n] = \left(\frac{-1}{2}\right)^{n} u[n-4]$: $u[n-4]$



$h[n] = 4^n u[-n+2]$: $u[-n+2]$



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Region 1.) $n < 6$:

$$y[n] = \sum_{k=4}^{\infty} \left(\frac{-1}{2}\right)^k 4^{n-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{-1}{2}\right)^k 4^{n-k} - \sum_{k=0}^{3} \left(\frac{-1}{2}\right)^k 4^{n-k}$$

$$= 4^n \left[\sum_{k=0}^{\infty} \left(\frac{-1}{8}\right)^k - \sum_{k=0}^{3} \left(\frac{-1}{8}\right)^k \right]$$

$$\boxed{= 4^n \left[\frac{8}{9} - \frac{455}{512} \right] = 4^n \left(\frac{1}{4608} \right)}$$

Region 2.) $n \geq 6$

$$y[n] = \sum_{k=n-2}^{\infty} \left(\frac{-1}{2}\right)^k 4^{n-k} = 4^n \left[\sum_{k=0}^{\infty} \left(\frac{-1}{8}\right)^k - \sum_{k=0}^{n-3} \left(\frac{-1}{8}\right)^k \right]$$

$$\cancel{= 4^n \left[\frac{8}{9} - \left(1 - \left(\frac{-1}{8}\right) \right. \right.}$$

$$\boxed{= 4^n \left[\frac{8}{9} - \left[\frac{1 + \frac{1}{8}^{n+2}}{1 + \frac{1}{8}} \right] \right]}$$

2.22 a.)

$$x(t) = e^{-\alpha t} u(t) \qquad h(t) = e^{-\beta t} u(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Region 1.) $t < 0$: $y(t) = 0$

Region 2.) $t \geq 0$:

$$y(t) = \int_0^t e^{-\alpha\tau} e^{-\beta(t-\tau)}\, d\tau$$

$$= \int_0^t e^{-\alpha\tau} e^{-\beta t} e^{\beta\tau}\, d\tau = e^{-\beta t} \int_0^t e^{-\alpha\tau} e^{\beta\tau}\, d\tau$$

$$= e^{-\beta t} \int_0^t e^{\tau(\beta-\alpha)}\, d\tau = e^{-\beta t} \left[\frac{e^{\tau(\beta-\alpha)}}{\beta-\alpha} \right]_0^t$$

$$\boxed{y(t) \quad \frac{e^{-\beta t}\left(e^{t(\beta-\alpha)} - 1\right)}{\beta - \alpha}\, u(t)}$$

$$x(t) = h(t) = e^{-\alpha t} u(t)$$

Region 1.) $t < 0$: $y = 0$

Region 2.) $t \geq 0$:

$$y(t) = \int_0^t e^{-\alpha\tau} e^{-\alpha(t-\tau)}\, d\tau$$

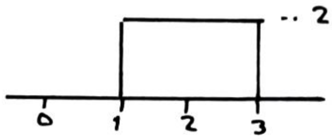
$$= e^{-\alpha t} \int_0^t e^{t(\alpha-\alpha)}\, d\tau = e^{-\alpha t} \int_0^t 1\, d\tau = e^{-\alpha t} \left[\tau \right]_0^t$$

$$\boxed{y(t) = t e^{-\alpha t} u(t)}$$

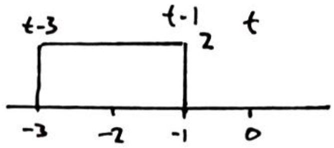
c.)

$x(t) = \sin(\pi t)$

$h(t)$



$h(t-\tau)$



Region 1: $t < 1$

$y(t) = 0$

Region 4: $t > 5$

$y(t) = 0$

Region 2: $1 < t < 3$

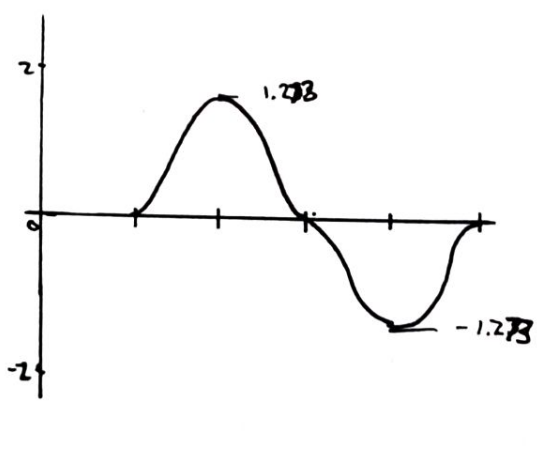
$$y(t) = \int_0^{t-1} \sin(\pi\tau) . 2 \, d\tau = \left[-2\cos(\pi\tau)\frac{1}{\pi} \right]_0^{t-1}$$

$$\boxed{= -2\cos(\pi(t-1))\frac{1}{\pi} + \frac{2}{\pi}}$$

Region 3: $3 < t < 5$

$$y(t) = \int_{t-3}^{2} \sin(\pi\tau) . 2 \, d\tau = \left[-2\cos(\pi\tau)\frac{1}{\pi} \right]_{t-3}^{2}$$

$$\boxed{= \frac{-2}{\pi} + 2\cos(\pi(t-3))\frac{1}{\pi}}$$



d.) $x(t) = at + b$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Region 1.) $-\infty < t < \infty$:

$$y(t) = \int_{t-1}^{t} \frac{4}{3}(a\tau + b)\, d\tau \quad = \left[\frac{2}{3}a\tau^2 + \frac{4}{3}b\tau\right]_{t-1}^{t}$$

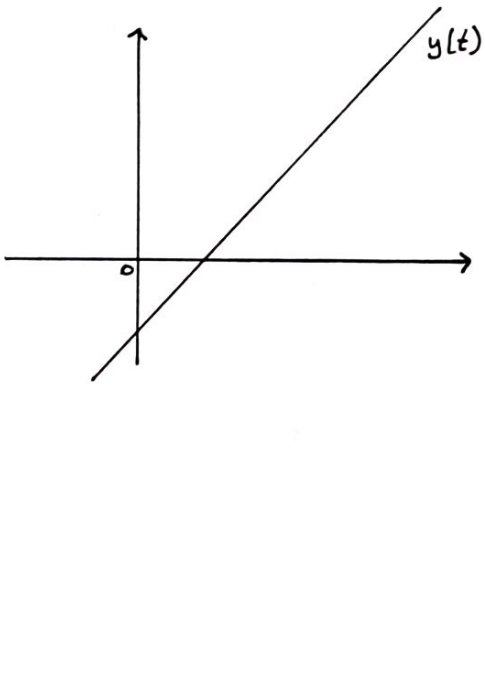
$$\boxed{y(t) = \left(\frac{2}{3}at^2 + \frac{4}{3}bt\right) - \left(\frac{2}{3}a(t-1)^2 + \frac{4}{3}b(t-1)\right)}$$

$$y(t) = x(t) * \delta(t-2) \cdot \frac{-1}{3}$$

$$= \frac{-1}{3}x(t-2)$$

$$= \frac{-1}{3}(a(t-2) + b)$$

$$\boxed{y(t) = \left(\frac{2}{3}at^2 + \frac{4}{3}bt\right) - \left(\frac{2}{3}a(t-1)^2 + \frac{4}{3}b(t-1)\right) + \frac{-1}{3}(a(t-2)+b)}$$



It's still a line.

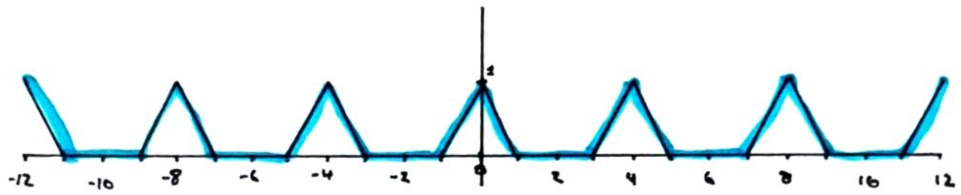
a affects gradient

b affects y intercept.

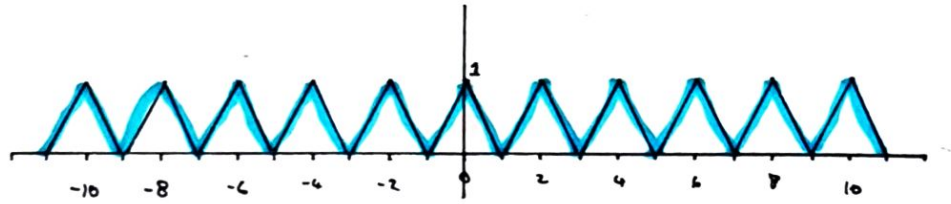
2.33

actual signal highlited in Blue

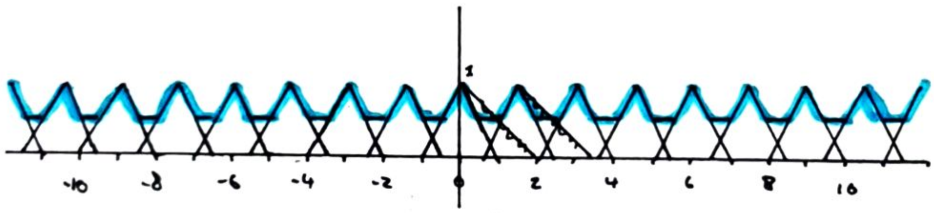
a.) $T = 4$



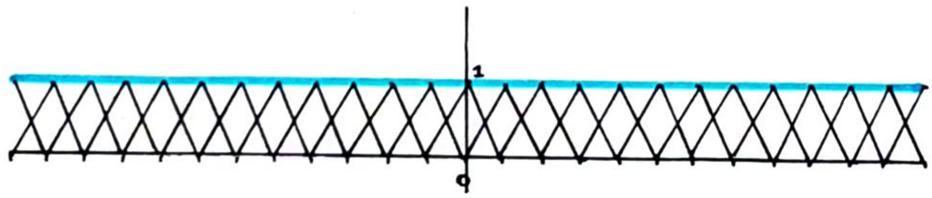
b.) $T = 2$



c.) $T = \frac{3}{2}$



d.) $T = 1$



2.27

$$A_y = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau \right] dt$$

and

$$A_y = \int_{-\infty}^{\infty} x(t) \, dt \cdot \int_{-\infty}^{\infty} h(t) \, dt$$

let $t - \tau = u$ $\therefore \frac{du}{dt} = 1$ $\therefore \boxed{dt = du}$

$$A_y = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau) h(u) \, d\tau \right] du$$

$$= \int_{-\infty}^{\infty} h(u) \left[\int_{-\infty}^{\infty} x(\tau) \, d\tau \right] du$$

$$\boxed{A_y = \int_{-\infty}^{\infty} h(u) du \cdot \int_{-\infty}^{\infty} x(\tau) \, d\tau \qquad A_y = A_x A_h}$$