1.)

a.) $\frac{dy}{dx} + 3x^2 y = x^2 \quad \therefore \mu = e^{\int 3x^2\,dx}$

$= e^{x^3}$

$\frac{d}{dx}\left[e^{x^3} y\right] = e^{x^3} x^2 \quad \therefore e^{x^3} y = \int e^{x^3} x^2\,dx$

Let $x^3 = u \quad \therefore \frac{du}{dx} = 3x^2$

$e^{x^3} y = \int \frac{1}{3} e^u\,du = \frac{1}{3} e^u + C \quad \therefore e^{x^3} y = \frac{1}{3} e^u + C$

$\boxed{y = \frac{1}{3} + \frac{C}{e^{x^3}} \quad \text{where } I = (-\infty, +\infty)}$

b.) $x\frac{dy}{dx} - y = x^2 \sin x \quad \therefore \frac{dy}{dx} - \frac{1}{x} y = x \sin x \quad \therefore \mu = e^{\int -\frac{1}{x}\,dx}$

$= e^{-\ln|x|}$

$= \frac{1}{x}$

$\frac{d}{dx}\left[\frac{1}{x} y\right] = \sin x \quad \therefore \frac{1}{x} y = \int \sin x\,dx$

$= -\cos x + C$

$\boxed{y = -x\cos x + xC = x(-\cos x + C) \quad \text{where } I = (-\infty, +\infty)}$

c.) $\cos x \frac{dy}{dx} + (\sin x) y = 1 \quad \therefore \frac{dy}{dx} + \frac{\sin x}{\cos x} y = \frac{1}{\cos x}$

$\mu = e^{\int \tan x\,dx} = e^{-\ln|\cos x|} = \frac{1}{\cos x}$

$\frac{1}{\cos x} y = \int \left[\frac{1}{\cos x}\right]^2 dx \quad \therefore \sec(x)\, y = \int \sec^2 x\,dx$

$\sec(x)\, y = \tan(x) + C = \frac{\tan(x)}{\sec(x)} + \frac{C}{\sec(x)}$

$y = \boxed{\sin(x) + C(\cos(x))} \quad \text{where } I = (-\infty, +\infty)$

d.) $\frac{dr}{d\theta} + r\sec(\theta) = \cos(\theta)$

$$\mu = e^{\int \sec(\theta)\, d\theta}$$

$$\mu = \tan(\theta) + \sec(\theta)$$

$$\frac{d}{dx}\left[(\tan(\theta) + \sec(\theta))r\right] = (\tan(\theta) + \sec(\theta))\cos\theta$$

$$(\tan(\theta) + \sec(\theta))r = \int \sin(\theta) + 1 \; d\theta$$

$$= -\cos(\theta) + \theta + C$$

$$\boxed{r = \frac{-\cos(\theta)}{\tan(\theta) + \sec(\theta)} + \frac{\theta}{\tan(\theta) + \sec(\theta)} + \frac{C}{\tan(\theta) + \sec(\theta)}}$$

2.)

a.) $(1+t^4)\frac{dy}{dt} = \frac{t^3}{y} \quad \therefore \quad y\, dy = \frac{t^3}{1+t^4}\, dt$

$$\int y\, dy = \int \frac{t^3}{1+t^4}\, dt \quad \therefore \quad \frac{y^2}{2} = \int \frac{t^3}{1+t^4}\, dt$$

let $1+t^4 = u \quad \therefore \quad \frac{du}{dt} = 4t^3 \quad \therefore \quad du = 4t^3\, dt$

$$\frac{y^2}{2} = \frac{1}{4}\int \frac{1}{u}\, du = \frac{1}{4}\ln|u| + C$$

$$\frac{y^2}{2} = \frac{1}{4}\ln|1+t^4| + C \quad \therefore \quad y = \pm\sqrt{\left[\frac{1}{2}\ln|1+t^4| + 2C\right]}$$

$$\boxed{y = \pm\sqrt{\left[\frac{1}{2}\ln|1+t^4| + 2C\right]}}$$

b.) $e^{-y}\sin(t) - \frac{dy}{dt}\cos^2 t = 0 \quad \therefore \quad e^{-y}\sin(t) = \frac{dy}{dt}\cos^2 t$

$$\frac{\sin(t)}{\cos^2(t)}\,dt = e^y\,dy$$

let $\cos(t) = u \quad \therefore \quad \frac{du}{dt} = -\sin(t)$

$$-\int \frac{1}{u^2}\,du = \int e^y \quad \therefore \quad \frac{1}{u} = e^y$$

$$e^y = \frac{1}{\cos(t)} \quad \therefore \quad e^y = \sec(t)$$

$$\boxed{y = \ln(\sec(t))}$$