

1.) a.) ~~My~~ $y' = -2 - y$

we know this because the slope at $y = -2$ must be zero

$y > -2$ must have a negative slope,

$y < -2$ must have a positive slope

$y = -2$	0
$y > -2$	-
$y < -2$	+

b.) $y' = 1 - 2y$

Slope at $y = 0.5$ must be zero.

$y > 0.5$ must be negative

$y < 0.5$ must be positive

$y = 0.5$	0
$y < 0.5$	+
$y > 0.5$	-

2.) a.) see attached figure

b.) as t becomes larger, solutions approach the general solution.

c.) $\frac{dy}{dx} + 2y = t \quad \therefore \mu = e^{\int 2 dt} = e^{2t}$

$\frac{d}{dx} (e^{2t} y) = e^{2t} t \quad \therefore u = t \quad v = e^{2t}/2$
 $du = 1 \quad dv = e^{2t}$

~~$\frac{d}{dx} (e^{2t} y) = \int e^{2t} t$~~ $\int e^{2t} t = \frac{e^{2t}}{2} t - \int \frac{e^{2t}}{2}$

~~$e^{2t} y = 2e^{2t} t - \int 2e^{2t}$~~
 ~~$e^{2t} y = 2e^{2t} t + \frac{1}{2} e^{2t}$~~

$= \frac{t}{2} e^{2t} - \frac{1}{4} e^{2t} + C$
 $= e^{2t} \left(\frac{t}{2} - \frac{1}{4} \right) + C$

$e^{2t} y = e^{2t} \left(\frac{t}{2} - \frac{1}{4} \right) + C$

$y = A e^{-2t} \left(\frac{t}{2} - \frac{1}{4} \right) + \frac{t}{2} - \frac{1}{4}$

d.) as $t \rightarrow +\infty$, Ae^{-2t} will vanish to zero, and the term $\frac{t}{2}$ will go to $+\infty$. This means all other solutions will move towards the general solution.

3.) $y' = 3y - 3$

a.) $y' = 3(y-1)$

$$\frac{1}{y-1} y' = 3$$

$$\frac{d}{dt} (\ln|y-1|) = 3$$

b.) $\frac{d}{dt} (\ln|y-1|) = 3$

$$\ln|y-1| = 3t + C$$

$$|y-1| = e^{3t+C}$$

$$y-1 = Ae^{3t}$$

$$y = 1 + Ae^{3t}$$

c.) The constant C must be arbitrary for us to be able to remove the absolute value sign around $|y-1|$, because if $C \in [0, +\infty)$, then $y-1$ could never be negative.

d.) Sub $y = 1 + Ae^{3t}$ into $y' = 3(y-1)$

$$y' = 3Ae^{3t} = 3(Ae^{3t} + 1 - 1) \therefore 3Ae^{3t} = 3Ae^{3t}$$

e.)

~~Sub in $y(0) = 3$~~

Sub in $y(0) = 3$

$$3 = 1 + Ae^{3 \times 0}$$

$$3 = 1 + A$$

$$A = 2$$

\therefore

$$y = 1 + 2e^{3t}$$

Figure 1.

