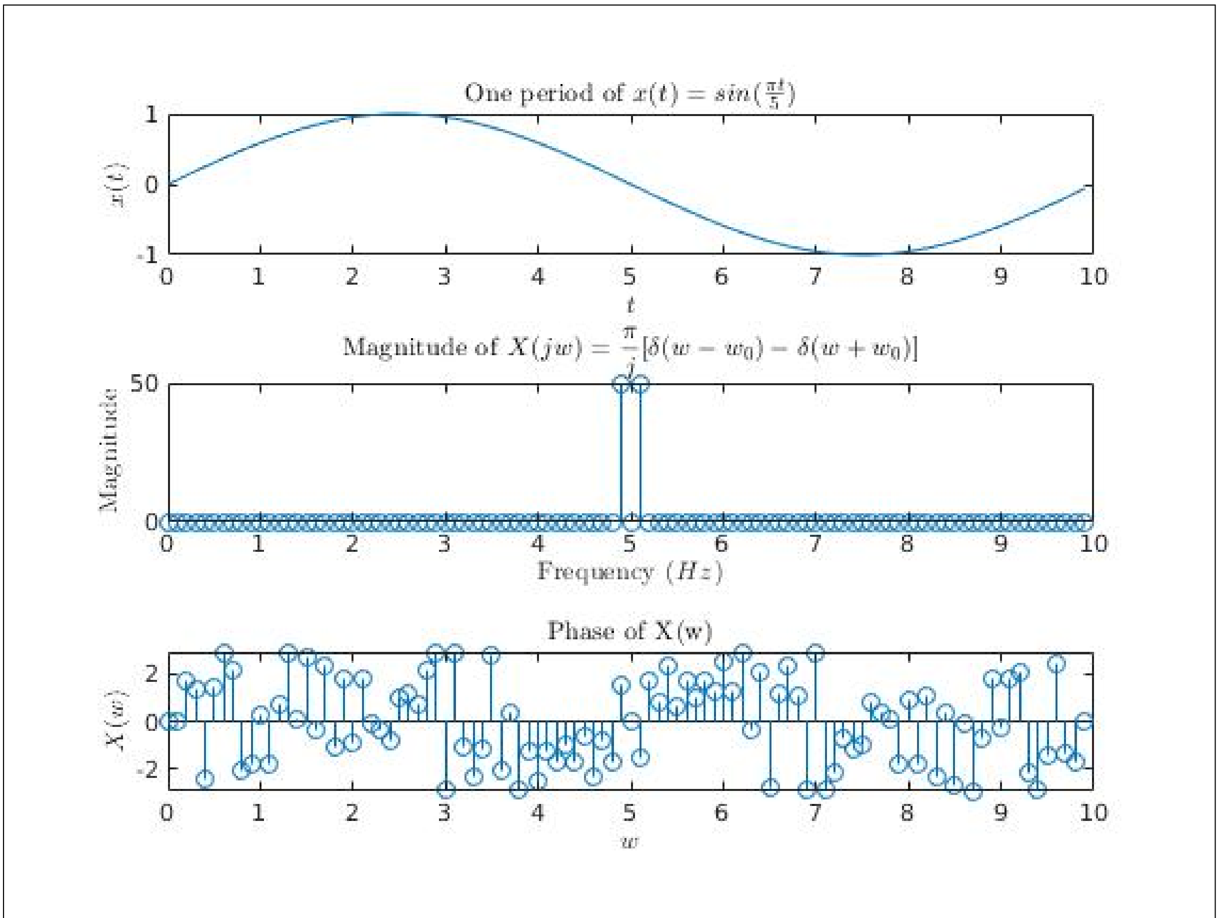
ECEN 220 Lab 3

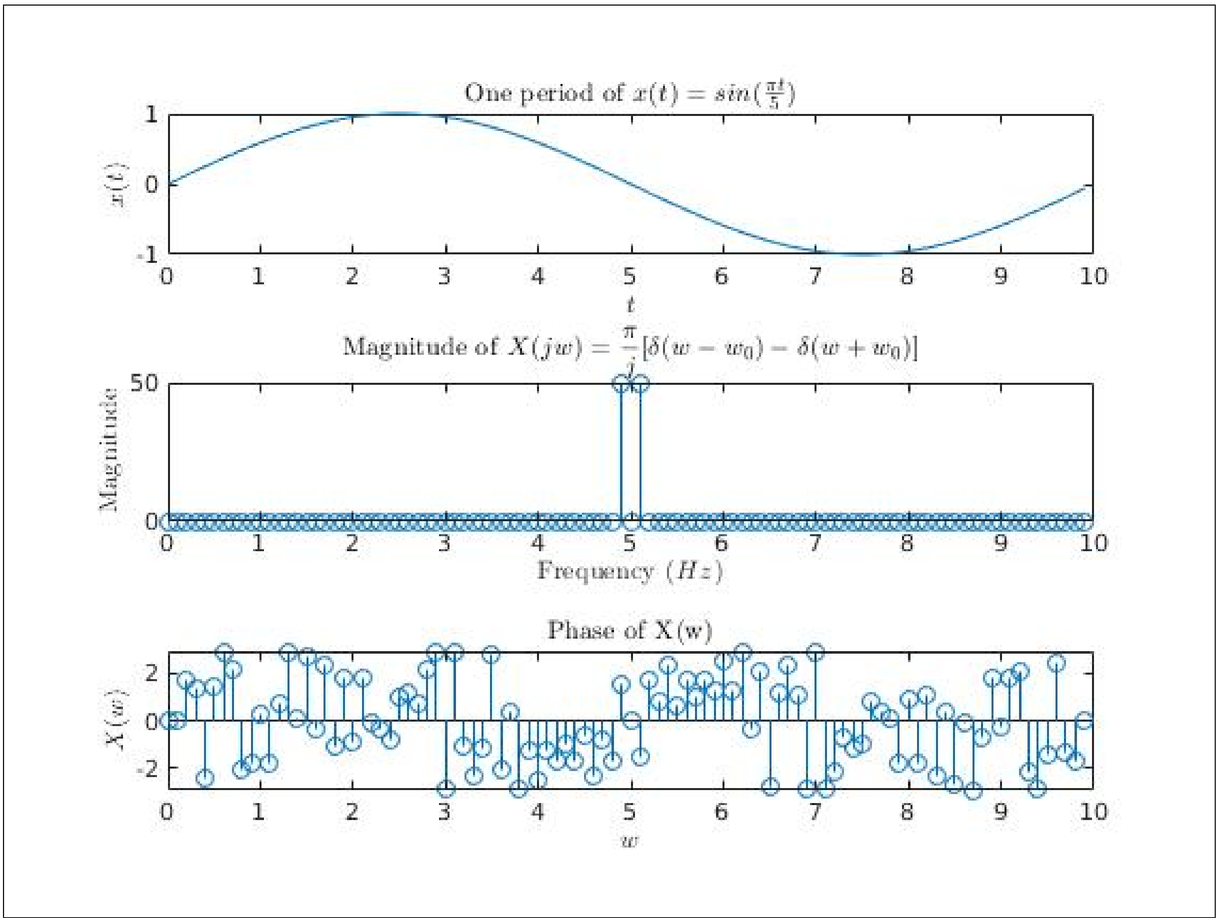
Niels Clayton
300437590

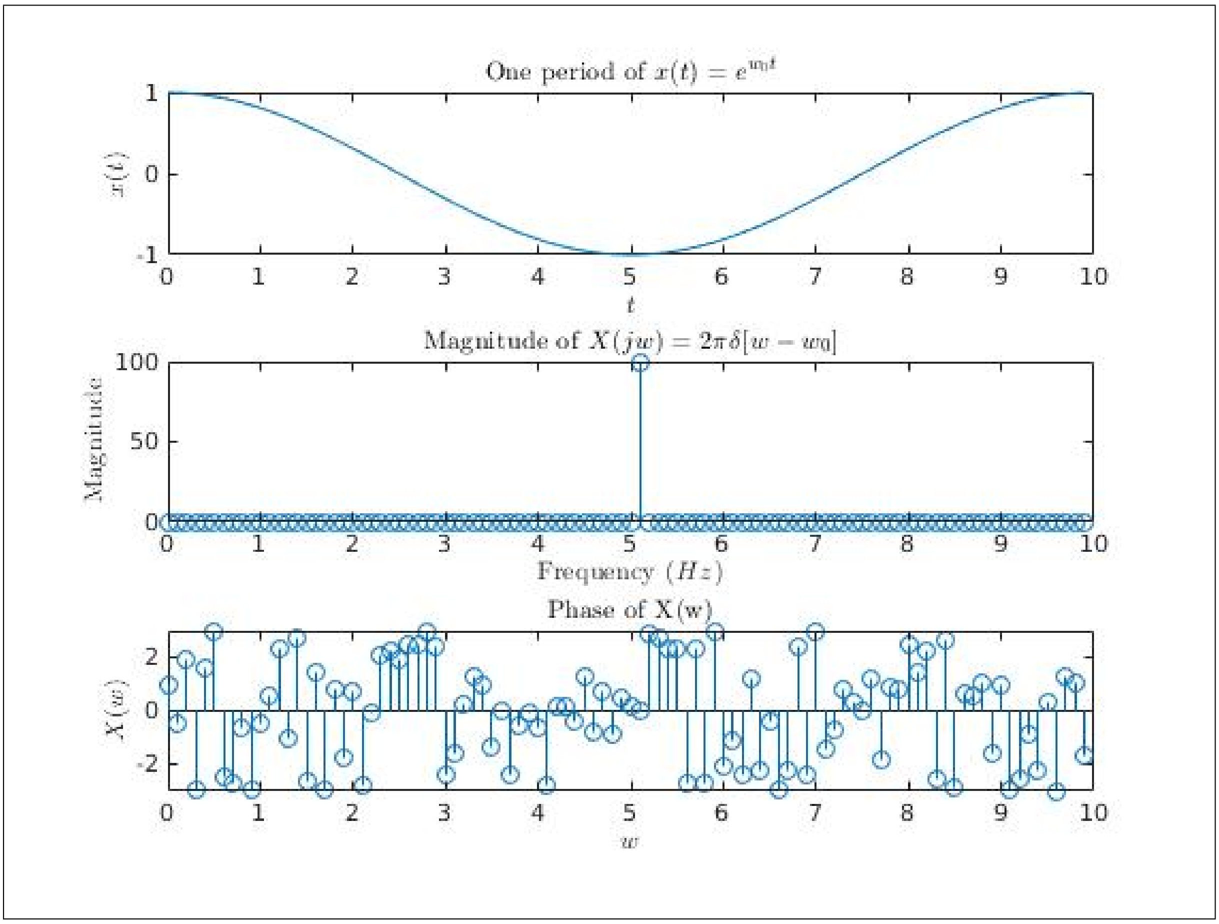
Part 1: Matlab Fourier Transform $cos(w_0 t)$

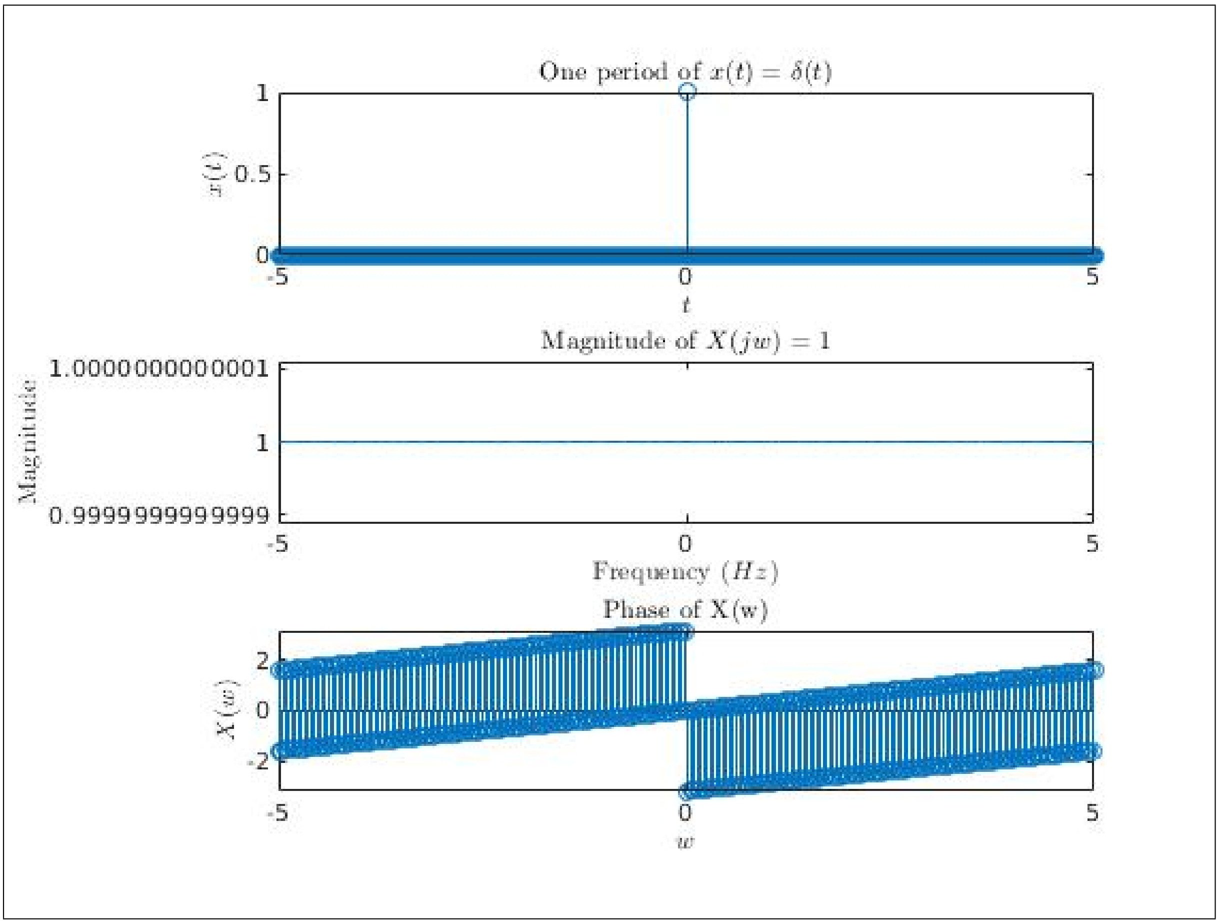


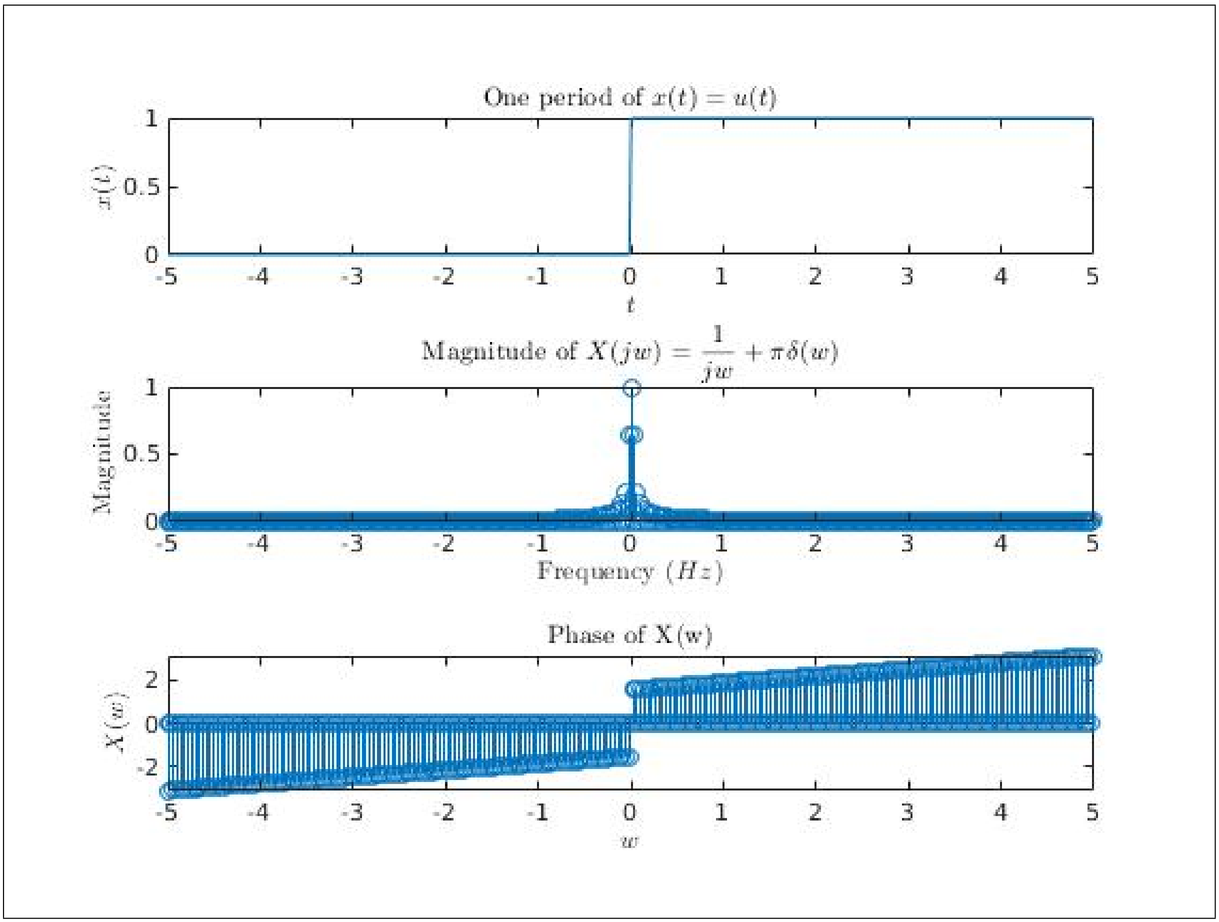
$cos(x)$ is an even function, this means that when taking the Fourier transform of it, the result should be purely real. This means that we expect to see two peaks on the frequency spectrum of w_0 and $-w_0$, and for there to be zero phase. We do observe the expected peaks in frequency, however we don't observe the expected phases. This is do to floating point errors within a computer. When you manually look at the values of $X(jw)$ we can see that they all contain an imaginary component of the magnitude of 1×10^{-17}. This imaginary component is almost non-existent, however it does still have a phase, leading to the noise seen.

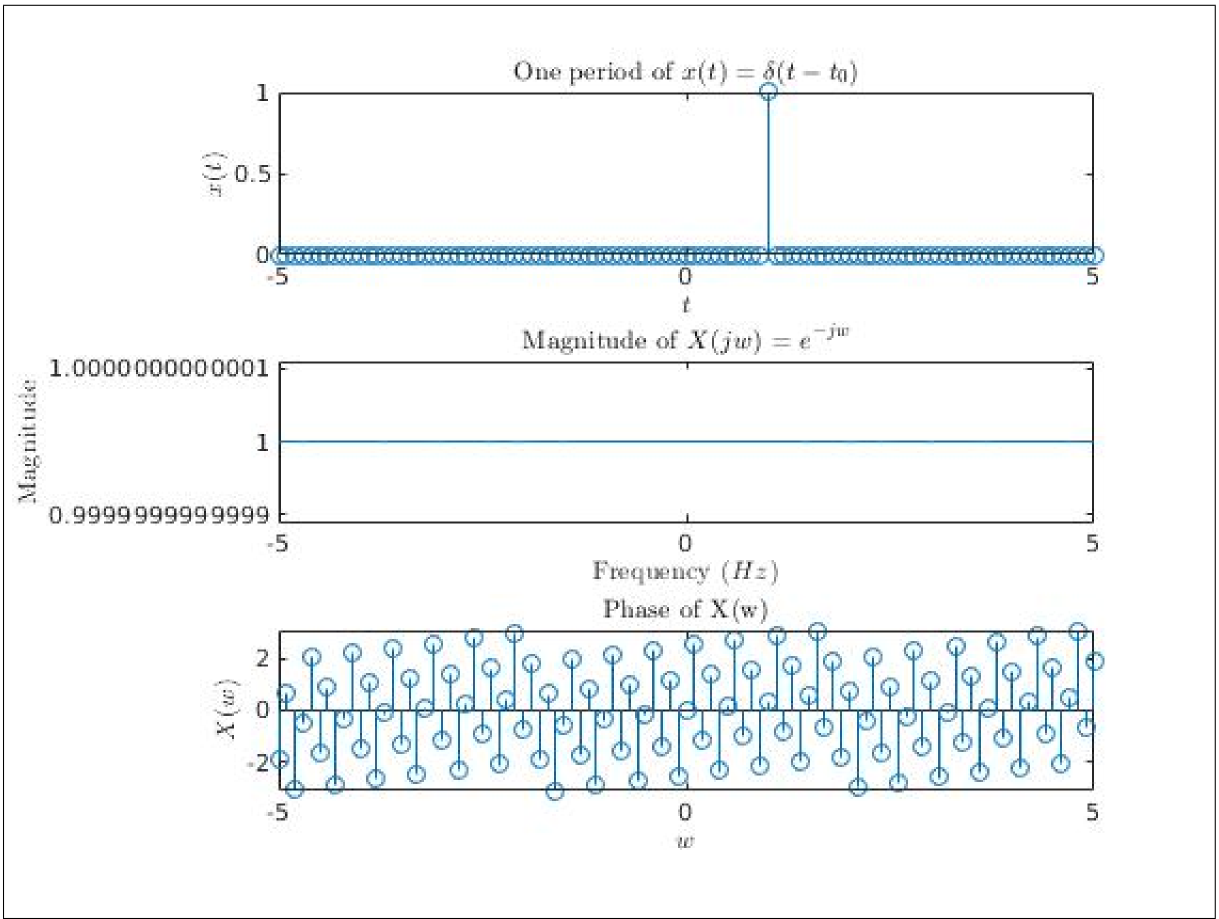
Part 2: Matlab Fourier Transform of other functions

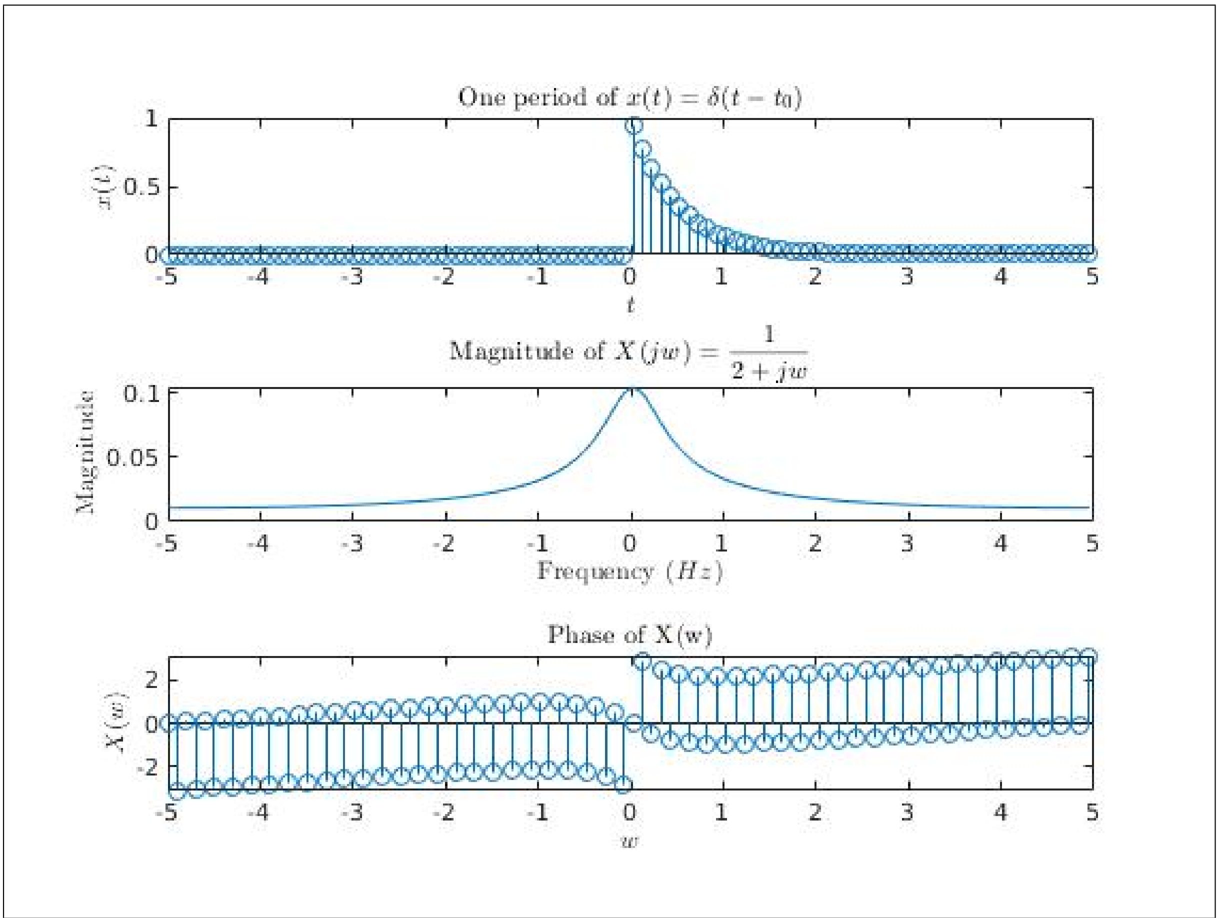



One period of $x(t) = e^{w_0 t}$
$x(t)$
t
Magnitude of $X(jw) = 2\pi\delta[w - w_0]$
Magnitude
Frequency (Hz)
Phase of X(w)
$X(w)$
w

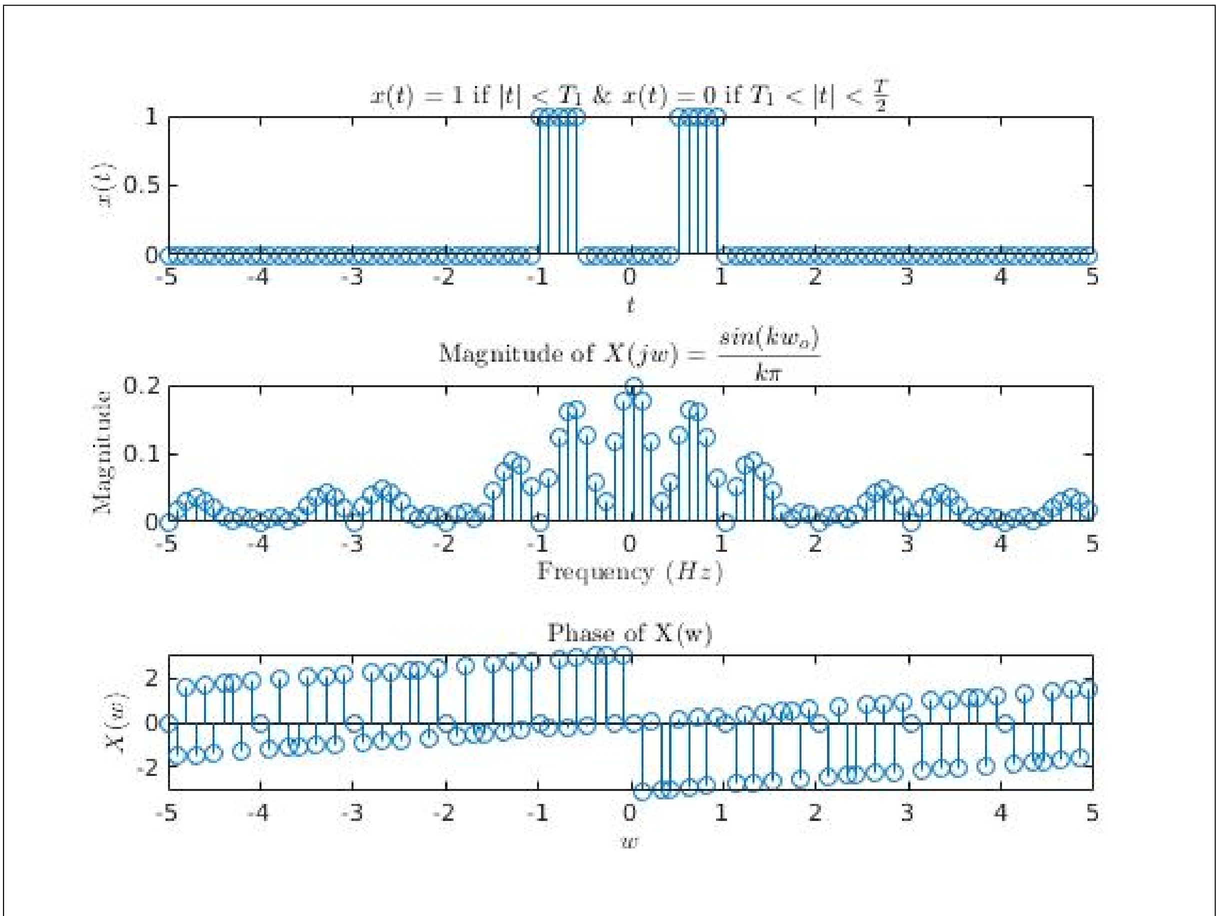

One period of $x(t) = \delta(t)$
$x(t)$
t
Magnitude of $X(jw) = 1$
Magnitude
1.0000000000001
0.9999999999999
Frequency (Hz)
Phase of X(w)
$X(w)$
w


One period of $x(t) = u(t)$
$x(t)$
t
Magnitude of $X(jw) = \frac{1}{jw} + \pi\delta(w)$
Magnitude
Frequency (Hz)
Phase of X(w)
$X(w)$
w


One period of $x(t) = \delta(t - t_0)$
$x(t)$
t
Magnitude of $X(jw) = e^{-jw}$
Magnitude
1.0000000000001
0.9999999999999
Frequency (Hz)
Phase of X(w)
$X(w)$
w



One period of $x(t) = \delta(t - t_0)$
$x(t)$
t
Magnitude of $X(jw) = \dfrac{1}{2 + jw}$
Magnitude
Frequency (Hz)
Phase of X(w)
$X(w)$
w



$x(t) = 1$ if $|t| < T_1$ & $x(t) = 0$ if $T_1 < |t| < \frac{T}{2}$
$x(t)$
t
Magnitude of $X(jw) = \dfrac{sin(kw_o)}{k\pi}$
Magnitude
Frequency (Hz)
Phase of X(w)
$X(w)$
w

Matlab Code

```matlab
%% Question 1
clc, clear

% generate one period of the signal

F_s = 10;                   % Sample rate
Ts = 1/F_s;                 % Period of a sample
T = 10;                     % period of the signal

N = T.*F_s;                 % Number of samples total
t = linspace(0, T-Ts, N);   % Values of t to sample at

w_0 = 2*pi/T;
x = cos(w_0*t);

subplot(3, 1, 1);
plot(t, x);
title('One period of $$x(t)=cos(\frac{\pi t}{5})$$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);   % The range on which the frequency will be plotted
X = fftshift(fft(x));       % Compute the fourier transform of x


subplot(3,1,2);
stem(t,abs(X/N)*2);
title("Magnitude of X(w)",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')


%% Question 2 sin(x)
clc, clear

% generate one period of the signal

F_s = 10;                   % Sample rate
Ts = 1/F_s;                 % Period of a sample
T = 10;                     % period of the signal

N = T.*F_s;                 % Number of samples total
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```matlab
t = linspace(0, T-Ts, N);   % Values of t to sample at

w_0 = 2*pi/T;
x = sin(w_0*t);

subplot(3, 1, 1);
plot(t, x);
title('One period of $x(t)=sin(\frac{\pi t}{5})$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);   % The range on which the frequency will be plotted
X = fftshift(fft(x));       % Compute the fourier transform of x


subplot(3,1,2);
stem(t,abs(X));
title("Magnitude of $$X(jw)=\frac{\pi}{j}[\delta(w-w_0)-\delta(w+w_0)]$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')

%% Question 2 exponental
clc, clear

% generate one period of the signal

F_s = 10;                   % Sample rate
Ts = 1/F_s;                 % Period of a sample
T = 10;                     % period of the signal

N = T.*F_s;                 % Number of samples total
t = linspace(0, T-Ts, N);   % Values of t to sample at

w_0 = 2*pi/T;
x = exp(1i.*w_0.*t);

subplot(3, 1, 1);
plot(t, x);
title('One period of $x(t)=e^{w_0 t}$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);   % The range on which the frequency will
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    be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x


subplot(3,1,2);
stem(t,abs(X));
title("Magnitude of $$X(jw)=2\pi \delta[w-w_0]$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')

%% Question 2 delta
clc, clear

% generate one period of the signal

F_s = 101;                         % Sample rate
Ts = 1/F_s;                        % Period of a sample
T = 5;                             % period of the signal

N = T.*F_s;                        % Number of samples total
t = linspace(-5, T, N);    % Values of t to sample at

x = (t==0);

subplot(3, 1, 1);
stem(t, x);
title('One period of $x(t)= \delta (t)$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);    % The range on which the frequency will
    be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x


subplot(3,1,2);
plot(t,abs(X));
title("Magnitude of $$X(jw)=1$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
```

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ylabel('$$X(w)$$','interpreter','latex')

%% Question 2 Unit step
clc, clear

% generate one period of the signal

F_s = 100;                    % Sample rate
Ts = 1/F_s;                   % Period of a sample
T = 5;                        % period of the signal

N = T.*F_s;                   % Number of samples total
t = linspace(-5, T-Ts, N);    % Values of t to sample at

w_0 = 2*pi/T;
x = (t>=0);

subplot(3, 1, 1);
plot(t, x);
title('One period of $x(t)= u(t)$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);    % The range on which the frequency will be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x


subplot(3,1,2);
stem(t,abs(X/N)*2);
title("Magnitude of $$X(jw)=\frac{1}{jw}+\pi \delta(w)$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')

%% Question 2 delta shifted
clc, clear

% generate one period of the signal

F_s = 10;                     % Sample rate
T = 10;                       % period of the signal

N = T.*F_s;                   % Number of samples total
t = linspace(-5, 5, 101);     % Values of t to sample at

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x = (t==1);

subplot(3, 1, 1);
stem(t, x);
title('One period of $x(t)= \delta (t - t_0)$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);    % The range on which the frequency will be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x

subplot(3,1,2);
plot(t,abs(X));
title("Magnitude of $$X(jw)=e^{-jw}$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')

%% Question 2 exp
clc, clear

% generate one period of the signal

F_s = 20;                    % Sample rate
Ts = 1/F_s;                  % Period of a sample
T = 5;                      % period of the signal

N = T.*F_s;                  % Number of samples total
t = linspace(-5, T-Ts, N);    % Values of t to sample at
a = 2;

x = exp(-(a.*t)).*(t>=0);

subplot(3, 1, 1);
stem(t, x);
title('One period of $x(t)= \delta (t - t_0)$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);    % The range on which the frequency will be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x

subplot(3,1,2);
plot(t,abs(X/N)*2);
title("Magnitude of $$X(jw)=\frac{1}{2+jw}$$",'interpreter','latex')
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xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')


%% Question 2 piece wise
clc, clear

% generate one period of the signal

F_s = 20;                    % Sample rate
Ts = 1/F_s;                  % Period of a sample
T = 5;                      % period of the signal
T1 = 1;

N = T.*F_s;                  % Number of samples total
t = linspace(-5, T-Ts, N);    % Values of t to sample at


x = (abs(t) < T1) - (abs(t) < T1/2);

subplot(3, 1, 1);
stem(t, x);
title('$x(t) = 1$ if $|t| < T_1$ \& $x(t) = 0$ if $T_1 < |t| < \frac{T}{2}$','interpreter','latex')
xlabel('$$t$$','interpreter','latex')
ylabel('$$x(t)$$','interpreter','latex')

w = (-N/2:N/2-1)*(F_s/N);    % The range on which the frequency will be plotted
X = fftshift(fft(x));        % Compute the fourier transform of x

subplot(3,1,2);
stem(t,abs(X/N)*2);
title("Magnitude of $$X(jw)=\frac{sin(kw_o)}{k\pi}$$",'interpreter','latex')
xlabel('Frequency $(Hz)$','interpreter','latex')
ylabel('Magnitude','interpreter','latex')

subplot(3,1,3);
stem(t,angle(X/N));
title("Phase of X(w)",'interpreter','latex')
xlabel('$$w$$','interpreter','latex')
ylabel('$$X(w)$$','interpreter','latex')
```