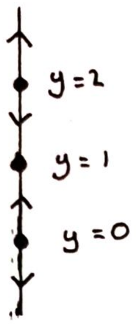
1.) $y(y-1)(y-2) = 0$ when $y = 0, 1,$ or 2



only the point $y=1$ is asymptotically stable. points $y=0$ and $y=2$ are not.

2.) The equation is exact if $M_y = N_x$

a.)

$$(\sin y - y\sin x)\,dx + (\cos x + x\cos y - y)\,dy = 0$$

$$\therefore \underbrace{(\sin y - y\sin x)}_{M} + \underbrace{(\cos x + x\cos y - y)}_{N}\frac{dy}{dx} = 0$$

$M_y = \cos y - \sin x$

$N_x = -\sin x + \cos y$

since $M_y = N_x$, DE is exact

$$\Psi_x = M, \quad \Psi = \int M\,dx = \int \sin y - y\sin x\,dx = x\sin y + y\cos x + h(y)$$

$$\Psi_y = \frac{d}{dy}\Psi = x\cos y + \cos x + h'(y)$$

$\therefore\ h'(y) = -y$

$h(y) = \frac{-y^2}{2} + C$

$$\boxed{\therefore f(x,y) = x\sin y + y\cos x - \frac{y^2}{2} + C}$$

b.) $x\dfrac{dy}{dx} = 2xe^{x} - y + 6x^2$

$$\underbrace{2xe^{x} - y + 6x^2}_{M}\ \underbrace{- x}_{N}\ \frac{dy}{dx} = 0$$

$M_y = -1$, $N_x = -1$ $\therefore$ since $M_y = N_x$ The DE is exact

$\Psi_x = M \therefore \Psi = \int M \, dx = \int 2xe^{x} - y + 6x^{2} \, dx$

$= 2xe^{x} - 2e^{x} - xy + 2x^{3} + h(y)$

$\Psi_y = \Psi\dfrac{d}{dy} = -x + h'(y) \therefore h'(y) = 0$

$h(y) = C$

$$\boxed{f(x,y) = 2xe^{x} - 2e^{x} - xy + 2x^{3} + C = 0}$$

3.) $(2y^2 + 3x)\,dx + (2xy)\,dy = 0 = \underbrace{(2y^2 + 3x)}_{M} + \underbrace{2xy}_{N}\dfrac{dy}{dx}$

$M_y = 4y$, $N_x = 2y$

$\mu = e^{\int \frac{M_y - N_x}{2xy} dx} = e^{\int \frac{2y}{2xy}} = e^{\int \frac{1}{x}} = x$

multiply through by $\mu = x$

$$\underbrace{2xy^2 + 3x^2}_{M}\ \underbrace{+ 2x^2y}_{N}\ \frac{dy}{dx} = 0$$

$\Psi_y = N \therefore \Psi = \int N \, dy \therefore \Psi = \int 2x^2y \, dy = x^2y^2 + h(x)$

$\Psi_x = \Psi\dfrac{d}{dx} = 2xy^2 + h'(x) \therefore h'(x) = 3x^2$

$h = x^3 + C$

$$\boxed{f(x,y) = x^2y^2 + x^3 + C}$$

5.)

a.) $y'' - 36y = 0$ The characteristic equation is $r^2 - 36 = 0$

$r^2 - 36 = (r+6)(r-6) \therefore \underline{r = 6, -6}$

$$\boxed{y(t) = C_1 e^{6t} + C_2 e^{-6t}}$$

b.) $y'' - 36y' = 0 \quad \therefore r^2 - 36r = 0 = r(r-36)$

$\underline{r = 36 \text{ or } 0}$

$$\boxed{y(t) = C_1 e^{36t} + C_2}$$ (since e^0 will always be 1)

c.) $y'' - 3y' + 2y = 0 \quad \therefore r^2 - 3r + 2 = 0 = (r-1)(r-2)$

$\underline{r = 1 \text{ or } 2}$

$$\boxed{y(t) = C_1 e^{t} + C_2 e^{2t}}$$

6.)

a.) $ty'' - y' = 0 \quad \therefore tr^2 - t = r(tr - 1) = 0$

$r = 0$ or $\frac{1}{t} \quad \therefore y = C_1 e^{0t} + C_2 e^{\frac{1}{t}t}$

$y = C_1 + C_2 e^{\frac{1}{t}t} = C_1 + C_2 t^2$

b.) $y = C_1 + C_2 t^2$ sub in $y(1) = 0 \quad \therefore \boxed{C_1 = C_2}$

$y' = 2C_2 t$ sub in $y'(1) = 1$

$\therefore C_2 = C_1 = 0.5$

$$\boxed{y = 0.5 + 0.5t^2}$$

c.)

$y = C_1 + C_2 t^2$ sub in $y(0)=0$ $\therefore$ $C_1 = 0$

$y' = 2C_2 t$ sub in $y'(0) = 1$ $\therefore$ - Not possible

This evaluates to $0 = 1$ which is not true.

8.) $y'' + p(t)y' + q(t)y = 0$

where $y = t^4$
$y' = 4t^3$
$y'' = 12t^2$

$y = t^4$ can be a solution, because at $t=0$, y, y', and y'' will all evaluate to 0. meaning that the equation will evaluate to 0 aswell.

9.)

a) $y'' - 10y' + 25y = 0$ $\therefore$ $r^2 - 10r + 25 = 0$

$(x-5)^2 = 0$

$$\boxed{y = C_1 e^{5t} + C_2 t e^{5t}}$$

b.) $2y'' + 2y' + y = 0$ $\therefore$ $2r^2 + 2r + 1 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \therefore \frac{-2 \pm \sqrt{4-8}}{4} = \frac{-2}{4} \pm \frac{\sqrt{-4}}{4}$$

$r = \frac{-1}{2} \pm \frac{i}{2}$

$$\boxed{y = e^{-\frac{1}{2}x}\left[C_1 \cos\left(\tfrac{1}{2}\right) + C_2 \sin\left(\tfrac{1}{2}\right)\right]}$$