Home Assignment no. 3

In this home assignment we consider the scattering of a time-harmonic plane wave by an infinite circular lossless magneto-dielectric cylinder in the case of transverse-magnetic polarization. A rectangular *xyz*-coordinate system, and the associated circular cylindrical $\rho\phi z$-coordinate system, is introduced with the *z*-axis coinciding with the axis of the cylinder which has radius a. The cylinder material has permittivity ε, permeability μ, wave number k, and intrinsic impedance η; the corresponding parameters of the surrounding free space are denoted by ε_0, μ_0, k_0, and η_0, respectively. With E_0 being a constant (of unit V/m), and other terms defined as in [Balanis, AEE] the incident electric field is

$$\boldsymbol{E}^i = \boldsymbol{a}_z E_0 e^{-jk_0 x} = \boldsymbol{a}_z E_0 \sum_{-\infty}^{\infty} j^{-n} J_n(k_0\rho) e^{jn\phi} \qquad (e^{j\omega t} \text{ time factor}) \qquad (1)$$

Task 1

Explain why the incident electric field can be expressed as the cylindrical wave expansion in (1). Furthermore, determine the corresponding expansion for incident magnetic field $\boldsymbol{H}^i$.

Task 2

For $0 \le \rho \le 5\lambda_0$ (λ_0 being free-space wavelength) and $\phi = 0$, plot $\mathrm{Re}(E_z^i)$ from the 2 expressions in (1) with the cylindrical wave expansion truncated at index N = 10, 20, 30, and 40, respectively (the summation can first be expressed using only non-negative indices as shown in [Balanis; (11-85a)]. Furthermore, analyse this plot and suggest a truncation rule; i.e. for a given distance ρ what is the sufficient truncation number N_ρ to ensure convergence?

The scattered electric field outside the cylinder $\boldsymbol{E}^s$ and the (total) electric field inside the cylinder $\boldsymbol{E}^t$ are

$$\boldsymbol{E}^s = \boldsymbol{a}_z \sum_{-\infty}^{\infty} B_n H_n^{(2)}(k_0\rho) e^{jn\phi},\ \rho > a \quad \text{and} \quad \boldsymbol{E}^t = \boldsymbol{a}_z \sum_{-\infty}^{\infty} C_n J_n(k\rho) e^{jn\phi},\ \rho < a \qquad (2)$$

Task 3

Explain why these field can be expressed as these cylindrical wave expansions. Furthermore, determine the corresponding expansions for the scattered and total magnetic fields; $\boldsymbol{H}^s$ and $\boldsymbol{H}^t$, respectively.

Task 4

Determine the constants B_n and C_n in (2).

Task 5

Plot the magnitude of the total field, inside as well as outside the cylinder for $\rho \le 3a$, for the 2 cases

a) $a = 5\lambda_0$, $\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$

b) $a = 5\lambda_0$, $\varepsilon = 5\varepsilon_0$, $\mu = 5\mu_0$

Furthermore, observe and analyse the two plots.

The End