Advanced Electromagnetics - Assignment 3

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Task 1

A usual solution to the wave equation that describes a TM-wave moving in the $+x$ direction takes the form as given in the assignment.

$$\boldsymbol{E}^i = \boldsymbol{a}_z E_0 e^{-jkx} \tag{1}$$

For a plane wave, we can rewrite the x-dependant phasor into a cylindrical dependency using simple trigonometry

$$e^{-jk_o x} = e^{-jk_o \cos(\phi)\rho} \tag{2}$$

This exponential can then be expanded by the so called Jacobi-Anger expansion[1]

$$e^{jz\cos\theta} = \sum_{n=-\infty}^{\infty} j^n J_n(z) e^{jn\theta} \tag{3}$$

A solution to a TM-wave moving in the x direction can then be expressed as an infinite sum of complexly weighted cylindrical Bessel function by combining (1),(2) and (3) noting that $J_n(-x) = (-1)^n J_n(x)$ and that between rectangular and cylindrical coordinates $\hat{a_z}$ remains unchanged we get

$$\boldsymbol{E}^i = \boldsymbol{a}_z E_0 e^{-jk_0 x} = \boldsymbol{a}_z E_0 \sum_{-\infty}^{\infty} j^{-n} J_n(k_0\rho) e^{jn\phi} \tag{4}$$

To obtain the Magnetic field $\boldsymbol{H^i}$ we then use the Maxwell-Faraday law, being situated in a source free region it can be written as

$$\boldsymbol{H^i} = -\frac{1}{j\omega\mu_0} \nabla \times \boldsymbol{E^i} \tag{5}$$

Here it is seen that we will obtain a ρ and a ϕ component of the H-field, since $\boldsymbol{E^i}$ only has a z component and ρ and ϕ dependence giving us the incident magnetic field as

[1] A. Erdélyi et al.,Higher Trancendental Functions Vol 2, sec. 7.2.4, eq. 27

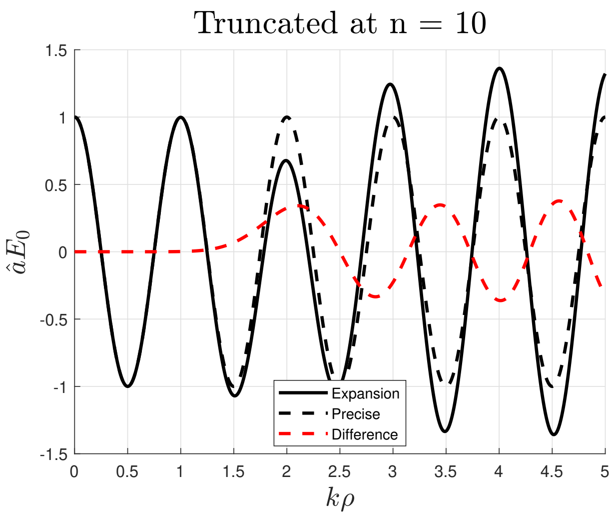
$$\boxed{H_\rho^i = -\frac{1}{j\omega\mu_0}\frac{1}{\rho}\frac{\partial E_z^i}{\partial \phi} = -\frac{E_0}{j\omega\mu_0\rho}\sum_{-\infty}^{\infty} n j^{-n+1} J_n\left(k_0\rho\right)e^{jn\phi}} \tag{6}$$

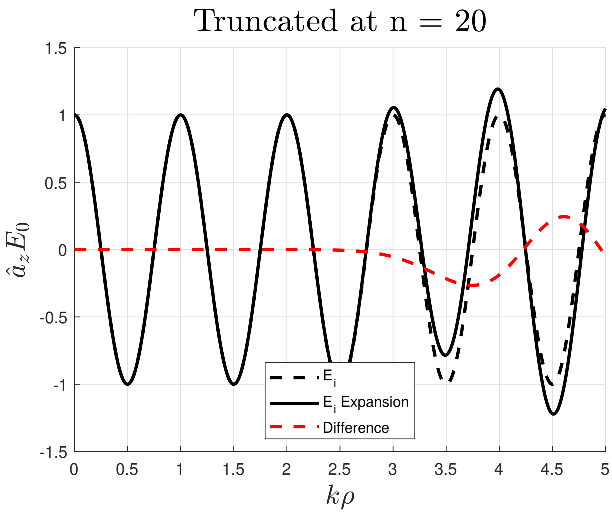
$$\boxed{H_\phi^i = \frac{1}{j\omega\mu_0}\frac{\partial E_z^i}{\partial \rho} = \frac{E_0 k_0}{j\omega\mu_0}\sum_{-\infty}^{\infty} j^{-n} J_n'\left(k_0\rho\right)e^{jn\phi}} \tag{7}$$

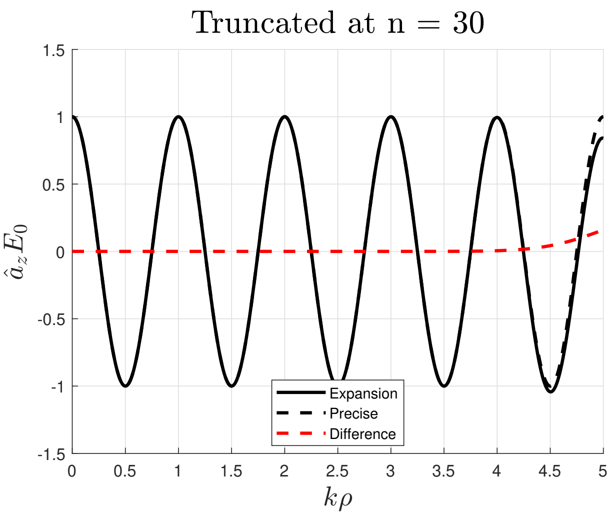
Where we, like in Balanis, denote the prime as the partial derivative of the entire argument of the Hankel and Bessel functions.

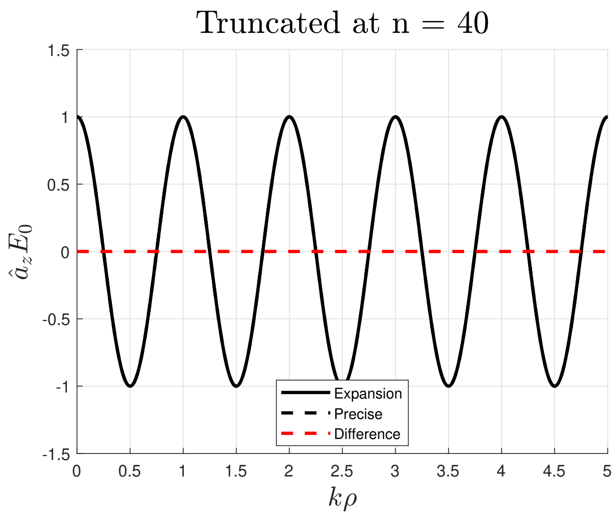
Task 2

Plotting the given expression for the real part of the given $\boldsymbol{E^i}$ field,









So in summary, the divergence seems to happen around $k\rho = 1.25$ for $N = 10$,$k\rho = 2.7$ for $N = 20$,$k\rho = 4.25$ for $N = 30$ and beyond the limit of the plot for $N = 40$

To try to get a larger overview, an additional script that find the divergence automatically has been developed. It simply calculates the precise and expanded expression and compares the two, noting when the expanded expression diverges by a certain threshold from the precise expression. This is then repeated for a number of N values, plotting the relation between N and the distance $k\rho$ to the divergence point then gives

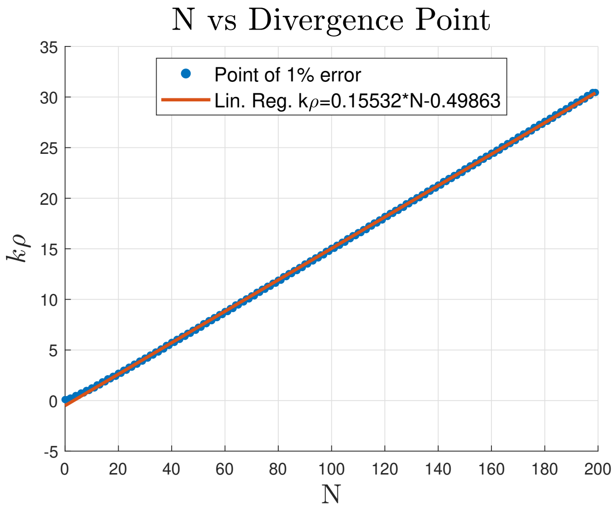


Figure 1: Divergence points plotted against number of Bessel functions taken into the expansion in equation (4). Threshold for divergence is set to 1% deviation from exact expression

Here we have also included a linear regression, since the dataset seemed to be at least locally extremely linear. The rule of thumb given in the slides states that $N_{\text{max}} = k\rho_{\text{max}} + 10$, which judging from this analysis seems very optimistic, from the regression we therefore suggest $N > 6.5(k\rho_{max} + 0.5)$ to ensure convergence.

Task 3

After a separation of variables, ρ dependant solutions to the wave equation in cylindrical coordinates can be expressed by one of the two functions.[2].

$$f_1(\rho) = A_1 J_m(\beta_\rho \rho) + B_1 Y_m(\beta_\rho \rho) \tag{8}$$

$$f_2(\rho) = C_1 H_m^{(1)}(\beta_\rho \rho) + D_1 H_m^{(2)}(\beta_\rho \rho) \tag{9}$$

Because of the cylindrical symmetry of the problem, we expect both the scattered and the total field inside of the cylinder to only have a ρ-component, and we therefore can write them in either of the two forms, depending on the domains of the fields.

The scattered field we expect to be a combination of traveling waves going in the $+\rho$ direction, because it will be the result of either a reflection at the boundary, a continuation of the field inside the magneto-dielectric cylinder going through the boundary back into free space, or a combination of both. We, therefore, express it as a Hankel function of the second kind, since these functions inherently describe traveling waves in the $+\rho$ direction, this is because they can be rewritten to contain a $e^{-jk\rho}$ relation. This Second order Hankel function can, like in a Fourier series, then be used to describe any periodic function with only ρ-dependance, making it possible to write the time-harmoic scattered field as.

[2] Balanis, Advanced Engineering Electromagnetics, Chap. 3. p. 113, eq 3-67

$$\boldsymbol{E}^s = \boldsymbol{a}_z \sum_{-\infty}^{\infty} B_n H_n^{(2)}(k_0\rho)\, e^{jn\phi} \tag{10}$$

The total field inside the cylinder is also expected to only have a ρ component, again because of symmetry. But this time our field has the demand that is should be finite within the entire area, including the origin of the coordinate system. Since we have a singularity in the Neuman function $Y_m(k\rho)- > \infty$ for $k\rho \to 0$ we will have to obtain a constant $B_1 = 0$ for our field to be finite, and since the Hankel functions are defined with the Neuman functions inside of them, thereby also having the singularity, C_1 and D_1 would also have to be zero. The total field inside the magneto-dielectric cylinder, therefore, has to be expressed by Bessel functions of the first kind, it being the only non-zero solution. With the Fourier-Bessel expansion, we can then write it as

$$\boldsymbol{E}^t = \boldsymbol{a}_z \sum_{-\infty}^{\infty} C_n J_n(k\rho) e^{jn\phi}, \rho < a \tag{11}$$

To determine the magnetic counterparts of these electric fields, we will follow a very similar procedure as in Task 1.

The scattered magnetic field can be found by

$$\boldsymbol{H^s} = -\frac{1}{j\omega\mu_0} \nabla \times \boldsymbol{E^s} \tag{12}$$

Since we have the same symmetry and dependencies as in Task 1 this can be split into

$$\boxed{H_\rho^s = -\frac{1}{j\omega\mu_0}\frac{1}{\rho}\frac{\partial E_z^s}{\partial \phi} = -\frac{1}{j\omega\mu_0\rho}\sum_{-\infty}^{\infty} nB_n H_n^{(2)}(k_0\rho)\, e^{jn\phi}} \tag{13}$$

$$\boxed{H_\phi^s = \frac{1}{j\omega\mu_0}\frac{\partial E_z^s}{\partial \rho} = \frac{1}{j\eta_0}\sum_{-\infty}^{\infty} B_n H_n^{(2)\prime}(k_0\rho)\, e^{jn\phi}} \tag{14}$$

And following exactly the same procedure for the magnetic field inside the dielectric, $\boldsymbol{H^t}$ yields

$$\boxed{H_\rho^t = -\frac{1}{j\omega\mu}\frac{1}{\rho}\frac{\partial E_z^t}{\partial \phi} = -\frac{1}{\omega\mu\rho}\sum_{-\infty}^{\infty} nC_n J_n(k\rho) e^{jn\phi}, \quad \rho < a} \tag{15}$$

$$\boxed{H_\phi^t = \frac{1}{j\omega\mu}\frac{\partial E_z^t}{\partial \rho} = \frac{1}{j\eta}\sum_{-\infty}^{\infty} C_n J_n'(k\rho) e^{jn\phi}, \quad \rho < a} \tag{16}$$

Task 4

The boundary conditions at the edge of our cylinder can be written as

$$\hat{a}_\rho \times ((\boldsymbol{E_i} + \boldsymbol{E_s}) - \boldsymbol{E_t})|_{\rho=a} = 0 \tag{17}$$

and

$$\hat{a}_\rho \times ((\boldsymbol{H_i} + \boldsymbol{H_s}) - \boldsymbol{H_t})|_{\rho=a} = 0 \tag{18}$$

Now, this cross-product only involves the components tangential to the boundary. So the boundary conditions simplify to

$$E^i_\phi|_{\rho=a} + E^s_\phi|_{\rho=a} = E^t_\phi|_{\rho=a} \tag{19}$$

and

$$H^i_\phi|_{\rho=a} + H^s_\phi|_{\rho=a} = H^t_\phi|_{\rho=a} \tag{20}$$

Inserting our given E-field and found H field then gives us

$$\boldsymbol{a}_z E_0 \sum_{-\infty}^{\infty} j^{-n} J_n\left(k_0 a\right) e^{jn\phi} + \boldsymbol{a}_z \sum_{-\infty}^{\infty} B_n H_n^{(2)}\left(k_0 a\right) e^{jn\phi} = \boldsymbol{a}_z \sum_{-\infty}^{\infty} C_n J_n(ka) e^{jn\phi} \tag{21}$$

and

$$\frac{E_0}{j\eta_0} \sum_{-\infty}^{\infty} j^{-n} J_n'\left(k_0 a\right) e^{jn\phi} + \frac{1}{j\eta_0} \sum_{-\infty}^{\infty} B_n H_n^{(2)'}\left(k_0 \rho\right) e^{jn\phi} = \frac{1}{j\eta} \sum_{-\infty}^{\infty} C_n J_n'(k\rho) e^{jn\phi} \tag{22}$$

Which then can be simplified to a boundary requirement of

$$j^{-n} J_n\left(k_0 a\right) + B_n H_n^{(2)}\left(k_0 a\right) = C_n J_n(ka) \tag{23}$$

and

$$\frac{E_0}{\eta_0} j^{-n} J_n'(k_0 a) + \frac{1}{\eta_0} B_n H_n^{(2)\prime}(k_0 a) = \frac{1}{\eta} C_n J_n'(ka) \tag{24}$$

Solving this system of equations (23) and (24) for B_n and C_n reveals the two constants to be

$$\boxed{B_n = j^{-n} \frac{E_0\left(J_n(ka) J_n'(k_0 a)\eta - J_n(k_0 a) J_n'(ka)\eta_0\right)}{J_n'(ka) H_n^{(2)}(k_0 a)\eta_0 - J_n(ka) H_n^{(2)\prime}(k_0 a)\eta}} \tag{25}$$

and

$$\boxed{C_n = j^{-n} \frac{E_0 \eta \left(H_n^{(2)}(k_0 a) J_n'(k_0 a) - J_n(k_0 a) H_n^{(2)}{}'(k_0 a) \right)}{J_n'(ka) H_n^{(2)}(k_0 a) \eta_0 - J_n(ka) H_n^{(2)}{}'(k_0 a) \eta}} \tag{26}$$

As a check of these expressions, we will inspect if they reduce to the expected free space solutions at when the material parameters are those of free space. In that case, $\mu = \mu_0$ and $\epsilon = \epsilon_0$, which means that $k = k_0$ and $\eta = \eta_0$ and it is quite easy to see that (25) and (26) reduce to

$$B_n = 0 \ , \ C_n = E_0 j^{-n} \tag{27}$$

This means that if the cylinder material parameters match those of free space, there will be no scattered field, and the total field inside of the cylinder will be exactly the same as the incident field! This is of course exactly as would be expected.

Task 5

A script was developed to calculate the electric field in a wavelength-normalized coordinate system. It works by creating a mesh of points and then calculating the distance from the origin of the coordinate system to every mesh-point, thereby obtaining ρ. Then the angle from the x-axis is calculated for every mesh-point, obtaining ϕ. The given formulas for the electric field are then calculated within their respective domains and summed. The Matlab script is appended as a listing at the end of this document.

Firstly we will consider the scenario with $\epsilon_r = 5$ and $\mu_r = 1$

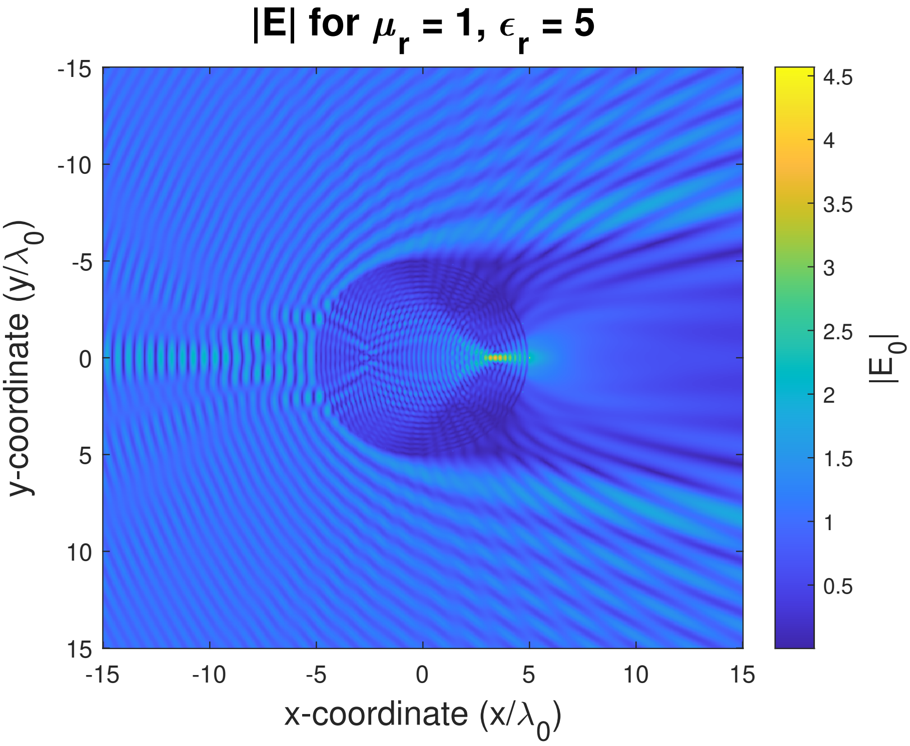


Figure 2: Amplitude plot of total field in and around dielectric cylinder with an plane-incidence field moving in the $+x$ direction, radius $= 5\lambda_0$, number of cylindrical functions is 150 for every field expression, 1500×1500 mesh resolution.

In the above plot, we see some interesting phenomenon, since the intrinsic impedance η is different between the two media, a reflection of the incident wave is happening at the left boundary of the cylinder. This creates a standing wave effect in the area where a sphere is approximately a normal plane to the incident wave. Also, as the ϵ_r also changes the index of refraction, we can see that the wave-front bends at non-normal incidence, as we would expect from ray-theory. The cylindrical shape therefore also ends up acting like a lens, focusing the light into a point. The plot also seems to uphold the continuity across the boundary, which is demanded by the boundary condition.

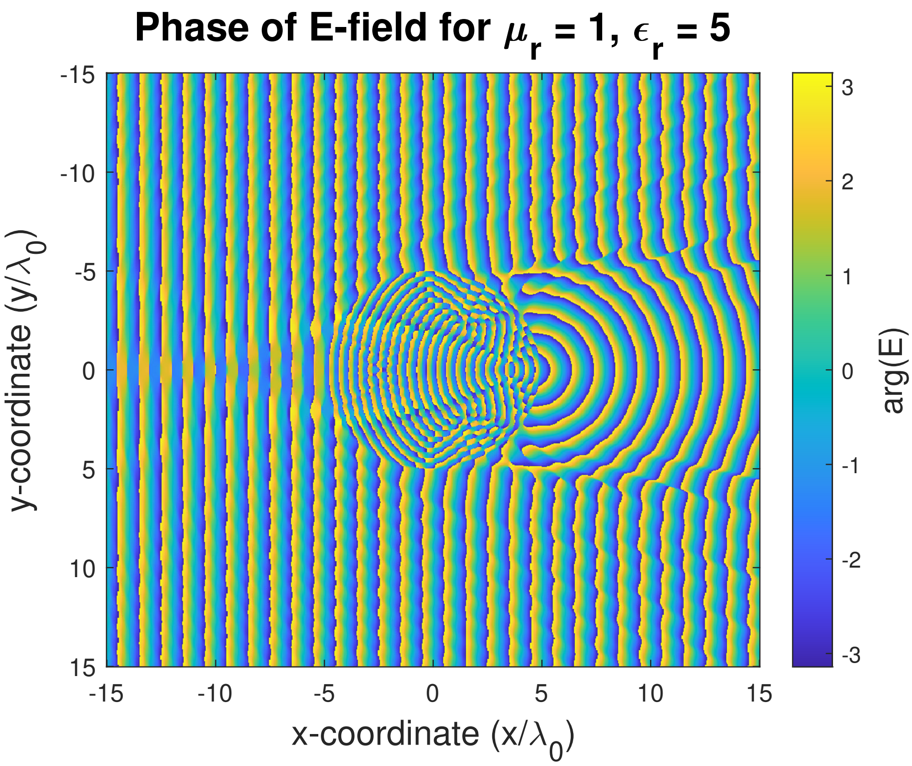
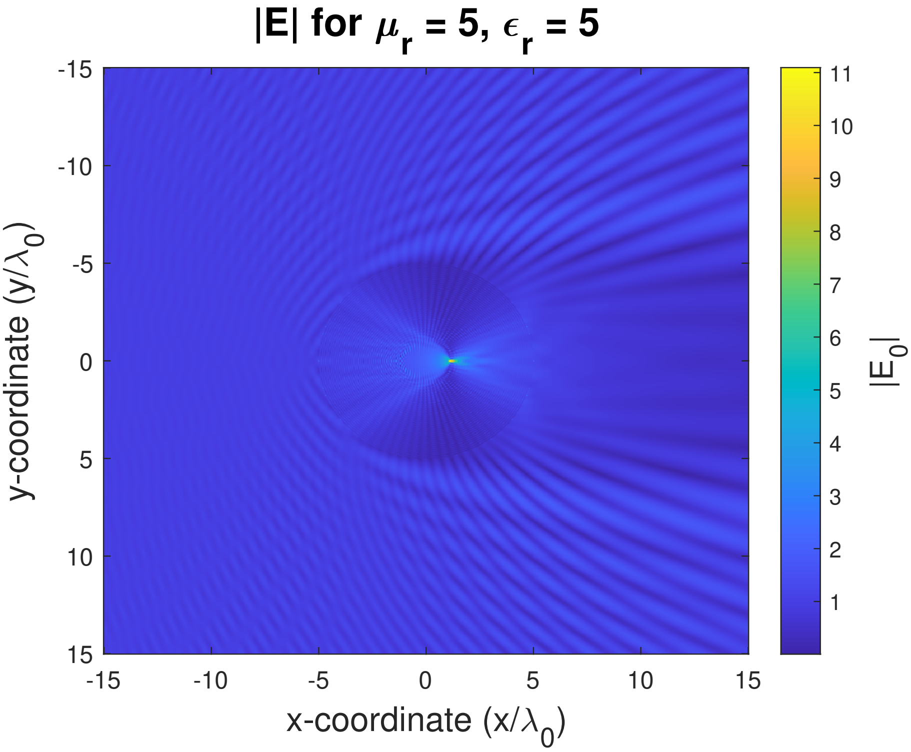


Figure 3: Phase plot same situation as figure (2)

The phase plots reveal some interesting things about the dominance of the different fields in different regions, and allow us to see the wavefront. As expected, the incident field is by far the largest contributor in the arrival direction, though the scattered field cancels and dominates the incident field in the 'shadow' of the cylinder. A resonance pattern inside the cylinder also emerges, arising from the continual reflection of the wave inside against the boundary, quite like in spherical laser resonators.



With $\mu_r = 5$ we are now in a situation where the intrinsic impedance inside of the cylinder is identical to the intrinsic impedance outside of the cylinder. This leads us to a boundary where no reflections occur, and the scattered field, therefore, becomes a refraction-only phenomenon. We also see that the focusing effect of our cylinder has become increased, again this fits well with the idea that there are no longer any reflections inside of the cylinder that might lead to interference at the focusing point, and the cylinder has therefore become an much better cylindrical lens. The focusing point has also been moved closer to the center because of the increased index of refraction.

Note the strong focusing point warps the color scheme, and notice that the field is still quite strong outside of the cylinder. A more precise plot of the wavefront can be seen in the phase plot.

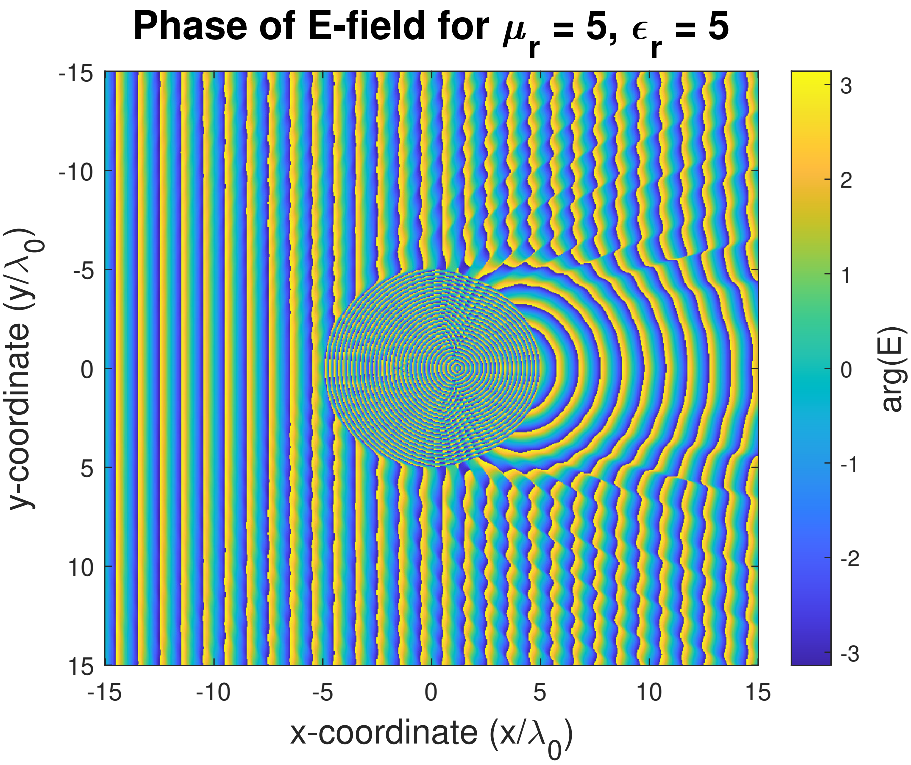


Figure 4: Caption

Here one can note how we see the wavenumber, k, inside of the cylinder has increased dramatically, since it is proportional to the index of refraction.

In addition to these plots, animated plots have also been developed. If the reader is interested they should be publicly accessible in this[3] Google Drive folder.

[3]https://drive.google.com/drive/folders/1ze3y$_5 - 43kUVbdT6ICrpq5oBhcEJ9Qox?usp = sharing$

Appendix

Task 2 plotting code

```
clear all; close all;

plotStart = 0;
plotEnd = 5;
resX = 1000;
x = plotStart:(plotEnd-plotStart)/resX:plotEnd;
y = ones(100,1);
lambda = 1;
Ei = cos(2*pi*x);
figure()
plot(Ei,'LineWidth',2)

%%
Ei_cyl = 0;
N = 10;
epsilon = [1,2*ones(1,N)];
phi =0;
for n = 0:N
   Ei_cyl = Ei_cyl + (1i).^(-n)*epsilon(n+1)*besselj(n,2*pi*x).*cos(n*phi);
end
Ei_cyl = real(Ei_cyl);
figure()
hold on;
plot(x,Ei_cyl,'k','linewidth',2)
plot(x,Ei,'--k','LineWidth',2)
plot(x,Ei-Ei_cyl,'--r','LineWidth',2)
grid on;
title('Truncated at n = '+string(N),'Interpreter','latex','FontSize',20)
ylabel('$\hat{a} E_0$','Interpreter','latex','FontSize',17)
xlabel('$k\rho$','Interpreter','latex','FontSize',17)
legend('Expansion','Exact','Difference','location','best')
xlim([0,5]);
ylim([-1.5,1.5]);
saveas(gca, 'Task2_trunc_10', 'epsc');
%%
Ei_cyl = 0
N = 20
epsilon = [1,2*ones(1,N)]

phi =0;
for n = 0:N
   Ei_cyl = Ei_cyl + (1i).^(-n)*epsilon(n+1)*besselj(n,2*pi*x).*cos(n*phi);
end
Ei_cyl = real(Ei_cyl);
figure()
hold on;
plot(x,Ei,'--k','LineWidth',2)
plot(x,Ei_cyl,'k','linewidth',2)
plot(x,Ei-Ei_cyl,'--r','LineWidth',2)
grid on;
title('Truncated at n = '+string(N),'Interpreter','latex','FontSize',20)
ylabel('$\hat{a}_z E_0$','Interpreter','latex','FontSize',17)
xlabel('$k\rho$','Interpreter','latex','FontSize',17)
```

```
legend('Expansion','Exact','Difference','location','best')
xlim([0,5]);
ylim([-1.5,1.5]);
legend('E_i','E_i Expansion','Difference', 'location','best')
saveas(gca, 'Task2_trunc_20', 'epsc');
%%
Ei_cyl = 0
N = 30
epsilon = [1,2*ones(1,N)]

phi =0;
for n = 0:N
   Ei_cyl = Ei_cyl + (1i).^(-n)*epsilon(n+1)*besselj(n,2*pi*x).*cos(n*phi);
end
Ei_cyl = real(Ei_cyl);
figure()
hold on;
plot(x,Ei_cyl,'k','linewidth',2)
plot(x,Ei,'--k','LineWidth',2)
plot(x,Ei-Ei_cyl,'--r','LineWidth',2)
grid on;
title('Truncated at n = '+string(N),'Interpreter','latex','FontSize',20)
ylabel('$\hat{a}_z E_0$','Interpreter','latex','FontSize',17)
xlabel('$k\rho$','Interpreter','latex','FontSize',17)
legend('Expansion','Exact','Difference','location','best')
xlim([0,5]);
ylim([-1.5,1.5]);
saveas(gca, 'Task2_trunc_30', 'epsc');
%%
Ei_cyl = 0
N = 40
epsilon = [1,2*ones(1,N)]

phi =0;
for n = 0:N
   Ei_cyl = Ei_cyl + (1i).^(-n)*epsilon(n+1)*besselj(n,2*pi*x).*cos(n*phi);
end
Ei_cyl = real(Ei_cyl);
figure()
hold on;
plot(x,Ei_cyl,'k','linewidth',2)
plot(x,Ei,'--k','LineWidth',2)
plot(x,Ei-Ei_cyl,'--r','LineWidth',2)
grid on;
title('Truncated at n = '+string(N),'Interpreter','latex','FontSize',20)
ylabel('$\hat{a}_z E_0$','Interpreter','latex','FontSize',17)
xlabel('$k\rho$','Interpreter','latex','FontSize',17)
legend('Expansion','Exact','Difference','location','best')
xlim([0,5]);
ylim([-1.5,1.5]);
saveas(gca, 'Task2_trunc_40', 'epsc');

%%
plotStart = 0;
plotEnd = 50;
resX = 1000;
x = plotStart:(plotEnd-plotStart)/resX:plotEnd;
y = ones(100,1);
```

```
lambda = 1;
Ei = cos(2*pi*x);


Ei_cyl = 0;
M = 200;
epsilon = [1,2*ones(1,M)];

phi =0;
thresh = 0.01;
diffPoints = 0;
for m = 1:M
    Ei_cyl = 0;
    for n = 0:m-1
       Ei_cyl = Ei_cyl + (1i).^(-n)*epsilon(n+1)*besselj(n,2*pi*x);
    end
    diff = abs(Ei-real(Ei_cyl));
    divPoint = diff(diff>thresh);
    diffPoints(m) = find(diff == divPoint(1));
end
x = (0:M-1);
b1 = x'\diffPoints';
X = [ones(length(x),1) x'];
b2 = X\diffPoints';
b2 = (b2/resX)*plotEnd;
regression = b2(2)*(x)+b2(1);
Ei_cyl = real(Ei_cyl);
figure()
hold on;
scatter((diffPoints/resX)*plotEnd,x,20,'filled')
plot(regression,x,'linewidth',2)
grid on;
title('N vs Divergence Point','Interpreter','latex','FontSize',20)
ylabel('$k\rho$','Interpreter','latex','FontSize',17)
xlabel('N','Interpreter','latex','FontSize',17);
legend('Point of 1% error','Lin. Reg. k\rho='+string(b2(2)) +'*N'+string(b2
    (1)),'location','best','FontSize',12)
saveas(gca, 'Task2_reg', 'epsc');
```

Task 5 main document and functions

```
clear all; close all;
tic;
X = 15; %Length i wavelengths
Y = 15; %Height in Wavelengths
a = 5; % Radius of dielectric in wavelengths
res = 20; % Points per wavelength
N = 150; % Number of functions to evalate fields
Nx = X*res;
Ny = Y*res;
epsilon = [1,2*ones(1,N)];
mu__0 = 1;
E__0 = 1;
eta__0 = 377;
k__0 = (2*pi)/(1);
epsilonc = 5;
mu = 1;
```

```
eta__1 = eta__0*sqrt(mu/epsilonc);
k = k__0*(sqrt(epsilonc*mu));
x = -Nx/2:Nx/2;
y = -Ny/2:Ny/2;
[xx yy] = meshgrid(x,y);
u = zeros(size(xx));
distMat = sqrt((xx/res).^2+(yy/res).^2);
u((xx.^2+yy.^2)<(a*res)^2)=-1;   % radius 100, center at the origin
phiMat = (atan2(flip(y),x'))'; %Create angle matrix

%% Calculate Incident Field
Ei = 0;
for n = 0:N
    Ei = Ei + (-1i).^(n)*E__0*epsilon(n+1).*besselj(n,k__0*distMat).*cos(n*
     phiMat);
end
Ei((xx.^2+yy.^2)<(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,real(Ei),'CDataMapping','scaled')

%% Calculate Field inside Dielectric
Et = 0;


for n = 0:N
    Cn = -E__0*(1j)^(-n)*eta__1*(J(n, k__0*a)*Hp(n, k__0*a) - H(n, k__0*a)*Jp
     (n, k__0*a))/(Jp(n, k*a)*eta__0*H(n, k__0*a) - J(n, k*a)*Hp(n, k__0*a)*
     eta__1);
    Et = Et + Cn.*epsilon(n+1).*besselj(n,k*distMat).*cos(n*phiMat);
end
Et(isnan(Et)) = 0;
Et((xx.^2+yy.^2)>(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,abs(Et))

%% Calculate Scattered Field
Es = 0;
for n = 0:N
    Bn = -(-1j)^(n)*E__0*(J(n, k__0*a)*Jp(n, k*a)*eta__0 - J(n, k*a)*Jp(n,
     k__0*a)*eta__1)/(Jp(n, k*a)*eta__0*H(n, k__0*a) - J(n, k*a)*Hp(n, k__0*a)
     *eta__1);
    Es = Es + Bn.*epsilon(n+1).*H(n,k__0*distMat).*cos(n*phiMat);
end
Es((xx.^2+yy.^2)<(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,abs(Es))

%% Calculate field for entire area
Etot = Ei+Et+Es;
Ei((xx.^2+yy.^2)<(a*res)^2)=0;
figure()
imagesc(x/res,y/res,abs(Etot),'CDataMapping','scaled')
title('|E| for \mu_r = 1, \epsilon_r = 5','fontsize',16);
pbaspect([1.1 1 1])
colorbar;
ab=colorbar;

ylabel(ab,'|E_0|','fontsize',13);
```

```
xlabel('x-coordinate (x/\lambda_0)','fontsize',14)
ylabel('y-coordinate (y/\lambda_0)','fontsize',14)
saveas(gca, 'Etotmu1eps5Amplitude', 'epsc');

figure()
imagesc(x/res,y/res,angle(Etot),'CDataMapping','scaled')
title('Phase of E-field for \mu_r = 1, \epsilon_r = 5','fontsize',16);
pbaspect([1.1 1 1])
colorbar;
ab=colorbar;
ylabel(ab,'arg(E)','fontsize',13);
xlabel('x-coordinate (x/\lambda_0)','fontsize',14)
ylabel('y-coordinate (y/\lambda_0)','fontsize',14)
saveas(gca, 'Etotmu1eps5Phase', 'epsc');

%% For epsilon = 5 and mu = 5
epsilonc = 5;
mu = 5;
eta__1 = eta__0*sqrt(mu/epsilonc);
k = k__0*(sqrt(epsilonc*mu));

%% Calculate Incident Field
Ei = 0;
for n = 0:N
    Ei = Ei + (1i).^(-n)*E__0*epsilon(n+1).*besselj(n,k__0*distMat).*cos(n.*
     phiMat);
end
Ei(X/2:end,(X/2-a):(X/2+a))
Ei((xx.^2+yy.^2)<(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,real(Ei),'CDataMapping','scaled')

%% Calculate Field inside Dielectric
Et = 0;
for n = 0:N
    Cn = -E__0*1j^(-n)*eta__1*(J(n, k__0*a)*Hp(n, k__0*a) - H(n, k__0*a)*Jp(n
     , k__0*a))/(Jp(n, k*a)*eta__0*H(n, k__0*a) - J(n, k*a)*Hp(n, k__0*a)*
     eta__1);
    Et = Et + Cn.*epsilon(n+1).*besselj(n,k*distMat).*cos(n.*phiMat);
end
Et(isnan(Et)) = 0;
Et((xx.^2+yy.^2)>(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,real(Et))

%% Calculate Scattered Field
Es = 0;
for n = 0:N
    Bn = -1j^(-n)*E__0*(J(n, k__0*a)*Jp(n, k*a)*eta__0 - J(n, k*a)*Jp(n, k__0
     *a)*eta__1)/(Jp(n, k*a)*eta__0*H(n, k__0*a) - J(n, k*a)*Hp(n, k__0*a)*
     eta__1);
    Es = Es + Bn.*epsilon(n+1).*H(n,k__0*distMat).*cos(n.*phiMat);
end
Es((xx.^2+yy.^2)<(a*res)^2)=0;
figure()
imagesc((x/res)+X/2,(y/res)+Y/2,real(Es))

%% Calculate field for entire area and Plot
```

```
Etot = Ei+Et+Es;
figure()
imagesc(x/res,y/res,abs(Etot),'CDataMapping','scaled')
title('|E| for \mu_r = 5, \epsilon_r = 5','fontsize',16);
pbaspect([1.1 1 1])
colorbar;
ab=colorbar;
ylabel(ab,'|E_0|','fontsize',13);
xlabel('x-coordinate (x/\lambda_0)','fontsize',14)
ylabel('y-coordinate (y/\lambda_0)','fontsize',14)
saveas(gca, 'Etotmu5eps5Amplitude', 'epsc');

figure()
imagesc(x/res,y/res,real(Etot),'CDataMapping','scaled')
title('\real for \mu_r = 5, \epsilon_r = 5','fontsize',16);
pbaspect([1.1 1 1])
colorbar;
ab=colorbar;
ylabel(ab,'|E_0|','fontsize',13);
xlabel('x-coordinate (x/\lambda_0)','fontsize',14)
ylabel('y-coordinate (y/\lambda_0)','fontsize',14)
saveas(gca, 'Etotmu5eps5Real', 'epsc');

figure()
imagesc(x/res,y/res,angle(Etot),'CDataMapping','scaled')
title('Phase of E-field for \mu_r = 5, \epsilon_r = 5','fontsize',16);
pbaspect([1.1 1 1])
colorbar;
ab=colorbar;
ylabel(ab,'arg(E)','fontsize',13);
xlabel('x-coordinate (x/\lambda_0)','fontsize',14)
ylabel('y-coordinate (y/\lambda_0)','fontsize',14)
saveas(gca, 'Etotmu5eps5Phase', 'epsc');
toc;
```

```
function out = H(nu,Z)
%H2 Summary of this function goes here
%   Detailed explanation goes here
out = besselh(nu,2,Z);
end
```

```
function out = Hp(mu,Z)
%HP Summary of this function goes here
%   Detailed explanation goes here
out = besselh(mu-1,2,Z)-(mu/Z)*besselh(mu,2,Z);
end
```

```
function output = J(mu,Z)
%J Summary of this function goes here
%   Detailed explanation goes here
output = besselj(mu,Z);
end
```

```
function out = Jp(mu,Z)
%JP Summary of this function goes here
%   Detailed explanation goes here
out = besselj(mu-1,Z)-(mu/Z)*besselj(mu,Z);
end
```