Signalbehandling for computer-ingeniører

COMTEK-5, E20

&

Signalbehandling

EIT-5, E20

12. The Discrete Fourier Transform, cont.

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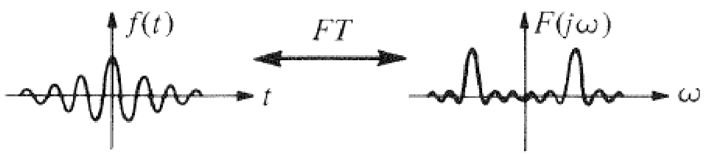
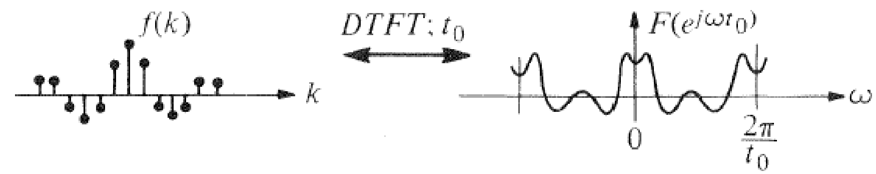
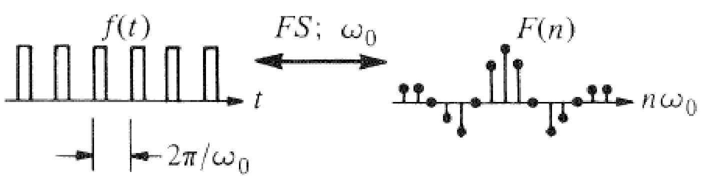
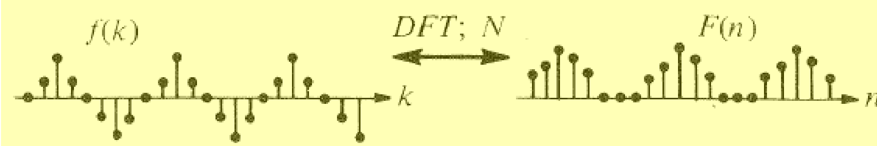
The Discrete Fourier Transform, DFT

In our previous lecture we discussed the basic math associated with the Discrete Fourier Transform, which is the Fourier Transform that we want to use for calculating the spectral content of a finite-length physically observable time-discrete signal, i.e., a signal for which we do not know any closed-form mathematical representation.

Today, first of all we will recap some of the highligths from last week's lecture and then we will elaborated a bit on some of the important DFT properties…

Fourier Transform – a classification

	Continuous in time	Discrete in time – Periodic in frequency
Continuous in frequency	 $f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega$ $F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$ **Fourier transform**	 $f(k) = \frac{t_0}{2\pi}\int_{-\pi/t_0}^{\pi/t_0} F(e^{j\omega t_0})\, e^{jk\omega t_0}\, d\omega$ $F(e^{j\omega t_0}) = \sum_{k=-\infty}^{\infty} f(k)\, e^{-jk\omega t_0}$ **Discrete-time Fourier transform**
Discrete in frequency – Periodic in time	 $f(t) = \sum_{n=-\infty}^{\infty} F(n)\, e^{jn\omega_0 t}$ $F(n) = \frac{\omega_0}{2\pi}\int_{-\pi/\omega_0}^{\pi/\omega_0} f(t)\, e^{-jn\omega_0 t}\, dt$ **Fourier series**	 $f(k) = \frac{1}{N}\sum_{n=0}^{N-1} F(n)\, (e^{j2\pi/N})^{kn}$ $F(n) = \sum_{k=0}^{N-1} f(k)\, (e^{j2\pi/N})^{-kn}$ **Discrete Fourier transform**

Our starting point was the Discrete Fourier Series, DFS

$$\text{Analysis equation:} \quad \tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] W_N^{kn}.$$

$$\text{Synthesis equation:} \quad \tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] W_N^{-kn}.$$

Remember that tilde denotes priodicity – so, $\tilde{x}[n]$ as well as $\tilde{X}[k]$ are periodic sequences…

$W_N = e^{-j(2\pi/N)}$ is the twiddle factor.

Important properties of the DFS

Linearity:

$$\left.\begin{array}{l}\tilde{x}_1[n] \overset{DFS}{\longrightarrow} \tilde{X}_1[k] \\ \tilde{x}_2[n] \overset{DFS}{\longrightarrow} \tilde{X}_2[k]\end{array}\right\} \text{yields}$$

$$a\,\tilde{x}_1[n] + b\,\tilde{x}_2[n] \overset{DFS}{\longrightarrow} a\,\tilde{X}_1[k] + b\,\tilde{X}_2[k]$$

Time-shift:

$$\tilde{x}[n-m] \overset{DFS}{\longrightarrow} W_N^{km}\,\tilde{X}[k]$$

Phase shift

Frequency-shift:

$$\tilde{X}[k-l] \overset{DFS}{\longrightarrow} W_N^{-nl}\,\tilde{x}[n]$$

Heterodyne

Another important property: Periodic Convolution

Let $\tilde{x}_1[n]$ and $\tilde{x}_2[n]$ be two periodic sequences, each with period N and with discrete Fourier series coefficients denoted by $\tilde{X}_1[k]$ and $\tilde{X}_2[k]$, respectively. If we form the product

$$\tilde{X}_3[k] = \tilde{X}_1[k]\tilde{X}_2[k],$$

then the periodic sequence $\tilde{x}_3[n]$ with Fourier series coefficients $\tilde{X}_3[k]$ is

$$\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m].$$

This is an interesting, but not surprising result; Remember that multiplication in frequency results in convolution in time – and the right-hand side looks very much like the well-known convolution sum, as we know from aperiodic discrete convolution.

There are two differences though…

$\tilde{x}_3[n]$ is periodic and the sum is only over *N* samples.

This is know as periodic convolution – see the math on p. 659.

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Periodic Convolution

$$\tilde{X}_3[k] = \tilde{X}_1[k]\tilde{X}_2[k]$$

$$\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$$

$$\sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m] \stackrel{\mathcal{DFS}}{\longleftrightarrow} \tilde{X}_1[k]\tilde{X}_2[k]$$

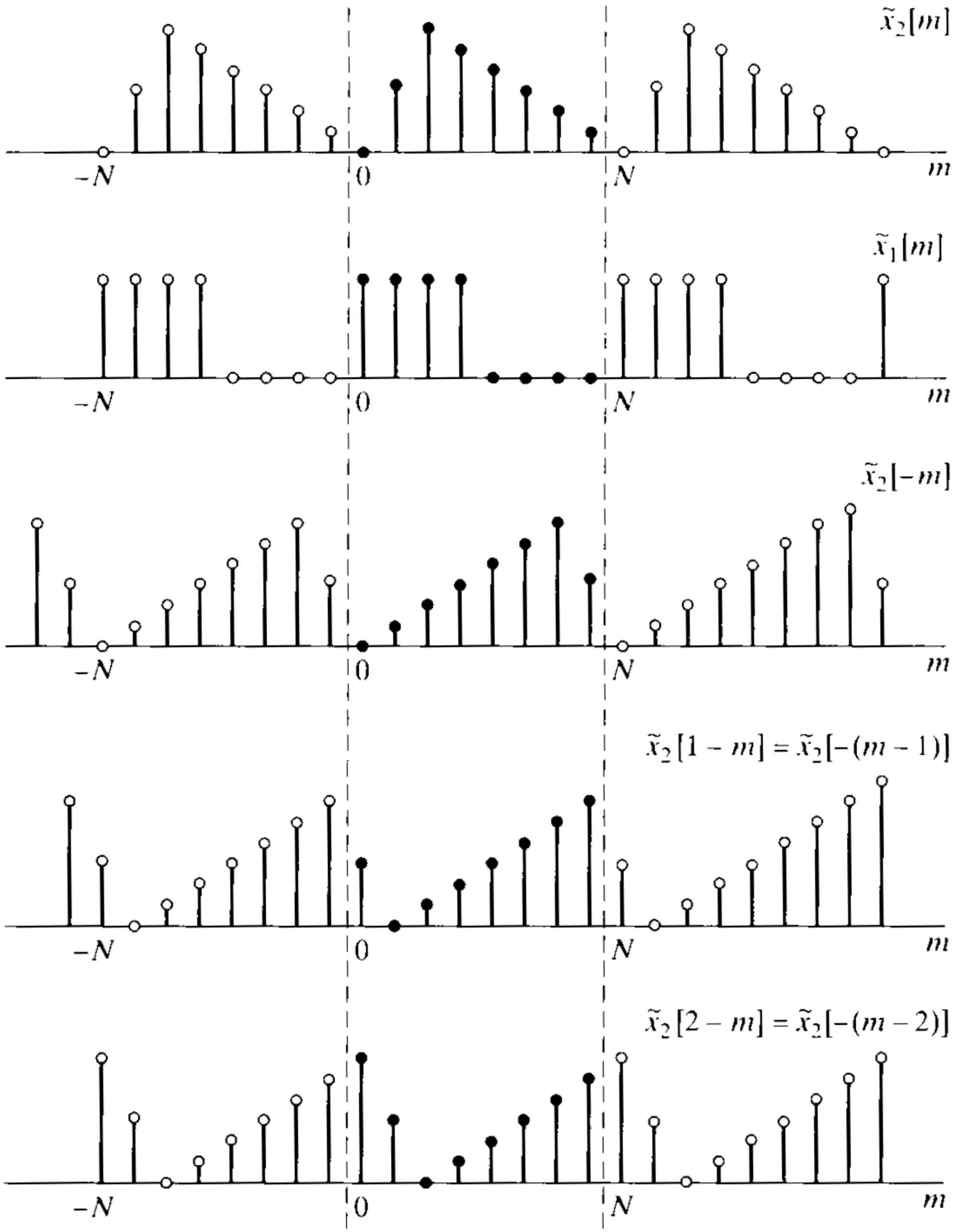
The periodic convolution of periodic sequences thus corresponds to multiplication of the corresponding periodic sequences of Fourier series coefficients.

Let's have a look at $\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$ which

calls for the product of sequences $\tilde{x}_1[m]$ and $\tilde{x}_2[n-m] = \tilde{x}_2[-(m-n)]$ viewed as functions of m with n fixed. This is the same as for an aperiodic convolution, but with the following two major differences:

1. The sum is over the finite interval $0 \leq m \leq N-1$.
2. The values of $\tilde{x}_2[n-m]$ in the interval $0 \leq m \leq N-1$ repeat periodically for m outside of that interval.

Let's see an example…

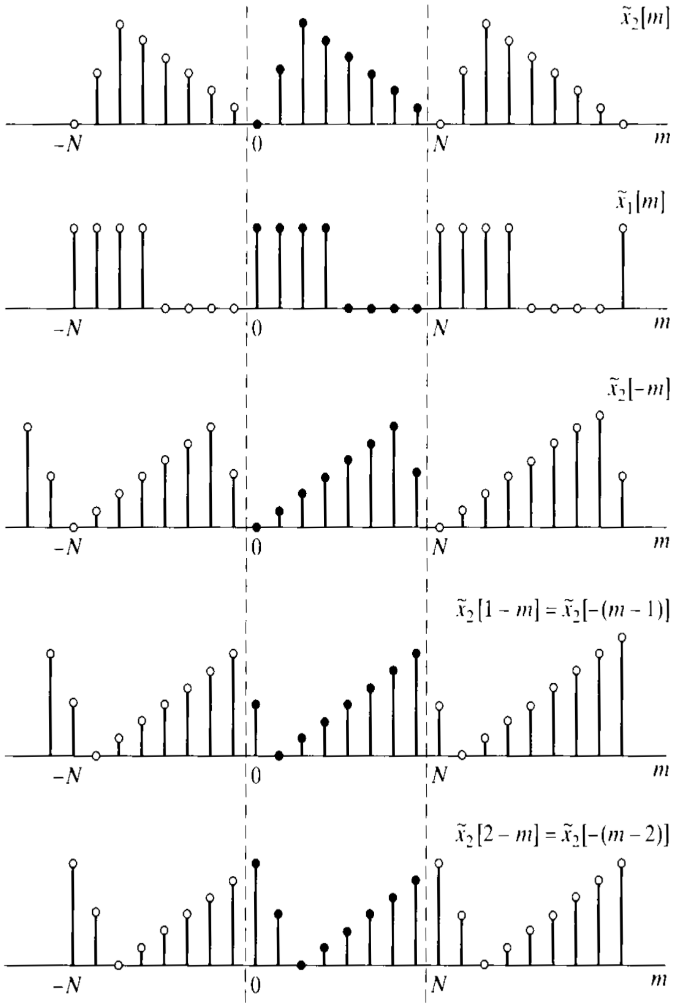


Two periodic sequences $\tilde{x}_2[n]$ and $\tilde{x}_1[n]$

Here $\tilde{x}_2[n]$ is flipped – we may choose any period around which we do the flip since the sequence is infinite

Here $\tilde{x}_2[n]$ is delayed one sample

Here $\tilde{x}_2[n]$ is delayed two samples



Two periodic sequences $\tilde{x}_2[n]$ and $\tilde{x}_1[n]$

Here $\tilde{x}_2[n]$ is flipped – we may choose any period around which we do the flip since the sequence is infinite

Here $\tilde{x}_2[n]$ is delayed one sample

Here $\tilde{x}_2[n]$ is delayed two samples

Note that $\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$ will repeat itself outside *0>n>N-1* and thus $\tilde{x}_3[n]$ is periodic with period *N.*

SUMMARY OF PROPERTIES OF THE DFS

Periodic Sequence (Period N)	DFS Coefficients (Period N)
1. $\tilde{x}[n]$	$\tilde{X}[k]$ periodic with period N
2. $\tilde{x}_1[n], \tilde{x}_2[n]$	$\tilde{X}_1[k], \tilde{X}_2[k]$ periodic with period N
3. $a\tilde{x}_1[n] + b\tilde{x}_2[n]$	$a\tilde{X}_1[k] + b\tilde{X}_2[k]$
4. $\tilde{X}[n]$	$N\tilde{x}[-k]$
5. $\tilde{x}[n-m]$	$W_N^{km}\tilde{X}[k]$
6. $W_N^{-\ell n}\tilde{x}[n]$	$\tilde{X}[k-\ell]$
7. $\sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$ (periodic convolution)	$\tilde{X}_1[k]\tilde{X}_2[k]$
8. $\tilde{x}_1[n]\tilde{x}_2[n]$	$\frac{1}{N}\sum_{\ell=0}^{N-1} \tilde{X}_1[\ell]\tilde{X}_2[k-\ell]$ (periodic convolution)
9. $\tilde{x}^*[n]$	$\tilde{X}^*[-k]$
10. $\tilde{x}^*[-n]$	$\tilde{X}^*[k]$
11. $\mathcal{R}e\{\tilde{x}[n]\}$	$\tilde{X}_e[k] = \frac{1}{2}(\tilde{X}[k] + \tilde{X}^*[-k])$
12. $j\mathcal{J}m\{\tilde{x}[n]\}$	$\tilde{X}_o[k] = \frac{1}{2}(\tilde{X}[k] - \tilde{X}^*[-k])$
13. $\tilde{x}_e[n] = \frac{1}{2}(\tilde{x}[n] + \tilde{x}^*[-n])$	$\mathcal{R}e\{\tilde{X}[k]\}$
14. $\tilde{x}_o[n] = \frac{1}{2}(\tilde{x}[n] - \tilde{x}^*[-n])$	$j\mathcal{J}m\{\tilde{X}[k]\}$
Properties 15–17 apply only when $x[n]$ is real.	
15. Symmetry properties for $\tilde{x}[n]$ real.	$\begin{cases} \tilde{X}[k] = \tilde{X}^*[-k] \\ \mathcal{R}e\{\tilde{X}[k]\} = \mathcal{R}e\{\tilde{X}[-k]\} \\ \mathcal{J}m\{\tilde{X}[k]\} = -\mathcal{J}m\{\tilde{X}[-k]\} \\ \lvert\tilde{X}[k]\rvert = \lvert\tilde{X}[-k]\rvert \\ \sphericalangle\tilde{X}[k] = -\sphericalangle\tilde{X}[-k] \end{cases}$
16. $\tilde{x}_e[n] = \frac{1}{2}(\tilde{x}[n] + \tilde{x}[-n])$	$\mathcal{R}e\{\tilde{X}[k]\}$
17. $\tilde{x}_0[n] = \frac{1}{2}(\tilde{x}[n] - \tilde{x}[-n])$	$j\mathcal{J}m\{\tilde{X}[k]\}$

Last time we also discussed sampling of the Fourier Transform

The idea is that most real-life signals $x[n]$ are not naturally periodic – and that's a major problem as related to Fourier analysis since periodicity was a prerequisite for DFS.

Therefore, if we instead

- find the DTFT of $x[n]$, i.e., $X(e^{j\omega})$, we then have a periodic function with period 2π
- next sample one period of $X(e^{j\omega})$
- and finally let these samples represent the periodic DFS sequence

then we have what we are looking for;

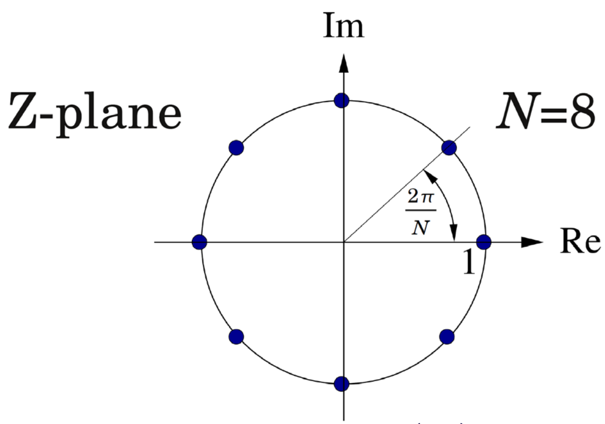
…a periodic sequence of Fourier coefficients.

Remember that we concluded, that this sequence can be viewed as one separate period or it can be seen as an infinite series of consecutive periods – it's the same…

The math behind it…

Remember that the DTFT of a sequence $x[n]$, i.e., $X(e^{j\omega})$, is identically equal to the z-transform $X(z)$ on the unit circle; $z = e^{j\omega}$.

We now consider *N* equally spaces point on the unit circle, in this example *N*=*8*;



We next sample $X(z)$ in the points $\mathrm{z} = e^{j\left(\frac{2\pi}{N}\right)k}$ which leads to a sampling of the Fourier transform;

Tilde denotes ”periodicity” in *k* with period *N*

$$X(z)\big|_{z=e^{j\left(\frac{2\pi}{N}\right)k}} = X\left(e^{j\left(\frac{2\pi}{N}\right)k}\right) = X\left(e^{j\omega_k}\right) = \tilde{X}[k] \qquad 0 \leq k \leq N-1$$

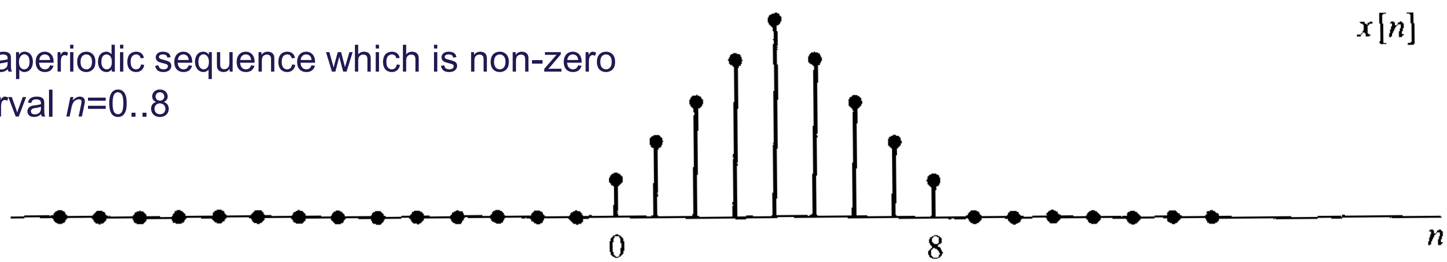
The DFS coefficients

$$\tilde{X}[k] = X(z)\big|_{z=e^{j\left(\frac{2\pi}{N}\right)k}} = X\left(e^{j\left(\frac{2\pi}{N}\right)k}\right) = X\left(e^{j\omega_k}\right) \qquad 0 \le k \le N-1$$

This expression represents an *N*-periodic sequence of samples which could be the sequence of Discrete Fourier Series coefficients of a sequence $\tilde{x}[n]$.

On p. 667 you'll find the math leading to the conclusion that $\tilde{x}[n]$, which corresponds to $\tilde{X}[k]$ obtained by sampling $X(z)$, is formed from $x[n]$ by adding together an infinite number of shifted replicas of $x[n]$. The shifts are all positive and negative integer multiples of *N.*

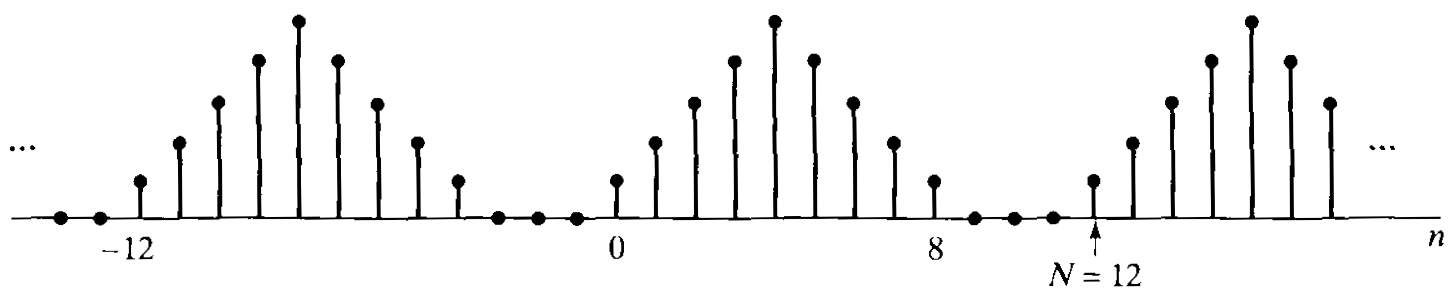
x[*n*] is an aperiodic sequence which is non-zero in the interval *n*=0..8



(a)

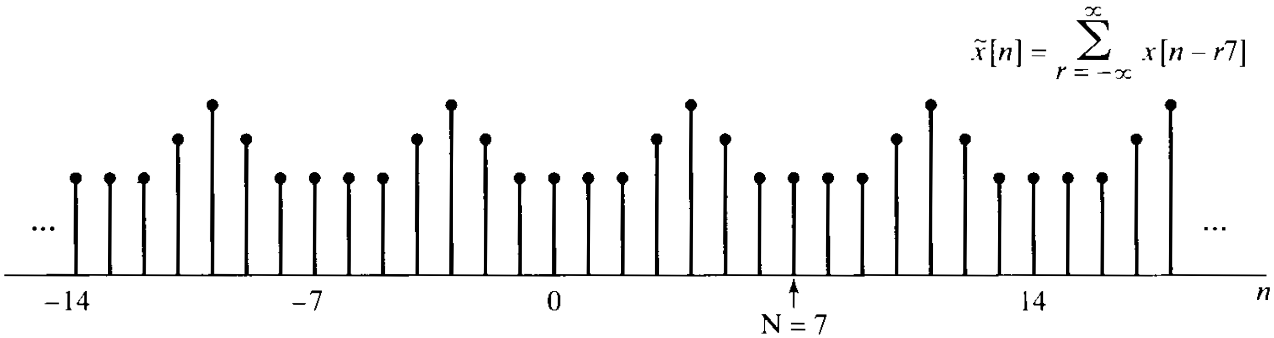
Here we see the periodic sequence derived by aperiodic convolution of *x*[*n*] with a periodic unit-impulse train. The period *N*=12.

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n - r12]$$

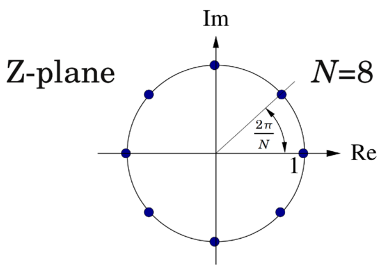


What happens if the period is less than 9 (in the example)..?

Here we have the same sequence $x[n]$, but now the period N=7



Basically what we see here is ”an overlap in the time domain” which can be considered as ”aliasing” – the period N is too small, i.e., there are too few samples in the frequency domain…



Consequently, time domain aliasing can be avoided only if $x[n]$ has finite length, just as frequency domain aliasing can be avoided only for signals that have bandlimited Fourier transforms.

OBS…!!

The Discrete Fourier Transform

We begin by considering a finite-length sequence $x[n]$ of length N samples such that $x[n] = 0$ outside the range $0 \leq n \leq N-1$. In many instances, we will want to assume that a sequence has length N even if its length is $M \leq N$. In such cases, we simply recognize that the last $(N-M)$ samples are zero. To each finite-length sequence of length N, we can always associate a periodic sequence

This is known as "zero padding"

$$\tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n - rN].$$

So, N is the length of the period, and r is number of the period.

$$\tilde{x}[n] = x[(n \text{ modulo } N)]$$

For convenience, we will use the notation $((n))_N$ to denote (n modulo N)

$$\tilde{x}[n] = x[((n))_N]$$

These two sequences are identical only when $x[n]$ has length less than or equal to N

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See slide no. 13.

Now, the sequence of discrete Fourier series coefficients $\tilde{X}[k]$ of the periodic sequence $\tilde{x}[n]$ is itself a periodic sequence with period N. To maintain a duality between the time and frequency domains, we will choose the Fourier coefficients that we associate with a finite-duration sequence to be a finite-duration sequence corresponding to one period of $\tilde{X}[k]$. This finite-duration sequence, $X[k]$, will be referred to as the discrete Fourier transform (DFT). Thus, the DFT, $X[k]$, is related to the DFS coefficients, $\tilde{X}[k]$, by

$$X[k] = \begin{cases} \tilde{X}[k], & 0 \le k \le N-1, \\ 0, & \text{otherwise}, \end{cases}$$

and

$$\tilde{X}[k] = X[(k \text{ modulo } N)] = X[((k))_N].$$

So, the DFS is a periodic sequence, whereas the DFT represents only one period

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The DFS vs. the DFT

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] W_N^{kn},$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] W_N^{-kn}$$

$$X[k] = \begin{cases} \sum_{n=0}^{N-1} x[n] W_N^{kn}, & 0 \le k \le N-1, \\ 0, & \text{otherwise}, \end{cases}$$

$$x[n] = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}, & 0 \le n \le N-1, \\ 0, & \text{otherwise}. \end{cases}$$

Generally, the DFT analysis and synthesis equations are written as follows:

$$\text{Analysis equation:} \quad X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn},$$

$$\text{Synthesis equation:} \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}.$$

That is, the fact that $X[k] = 0$ for k outside the interval $0 \leq k \leq N-1$ and that $x[n] = 0$ for n outside the interval $0 \leq n \leq N-1$ is implied, but not always stated explicitly.

However, since the DFT represents one period of the DFS, the inherent periodicity is always present…!!

It means essentially that the DFT spectrum $X[k]$ is also periodic with period 2π despite that it represents a finite length sequence $x[n]$.

Some Properties of the DFT
- Circular Shift of a Sequence

From DTFT theory we remember;

$$x[n-n_d] \quad (n_d \text{ an integer}) \quad \longleftrightarrow \quad e^{-j\omega n_d}X(e^{j\omega})$$

Now we will consider the operation in the time domain that corresponds to multiplying the DFT coefficients of a finite-length sequence $x[n]$ by the linear phase factor $e^{-j(2\pi k/N)m}$. Specifically, let $x_1[n]$ denote the finite-length sequence for which the DFT is $e^{-j(2\pi k/N)m}X[k]$; i.e., if

$$x[n] \overset{\mathcal{DFT}}{\longleftrightarrow} X[k],$$

then we are interested in $x_1[n]$ such that

$$x_1[n] \overset{\mathcal{DFT}}{\longleftrightarrow} X_1[k] = e^{-j(2\pi k/N)m}X[k].$$

Since the N-point DFT represents a finite-duration sequence of length N, both $x[n]$ and $x_1[n]$ must be zero outside the interval $0 \le n \le N-1$, and consequently, $x_1[n]$ cannot result from a simple time shift of $x[n]$.

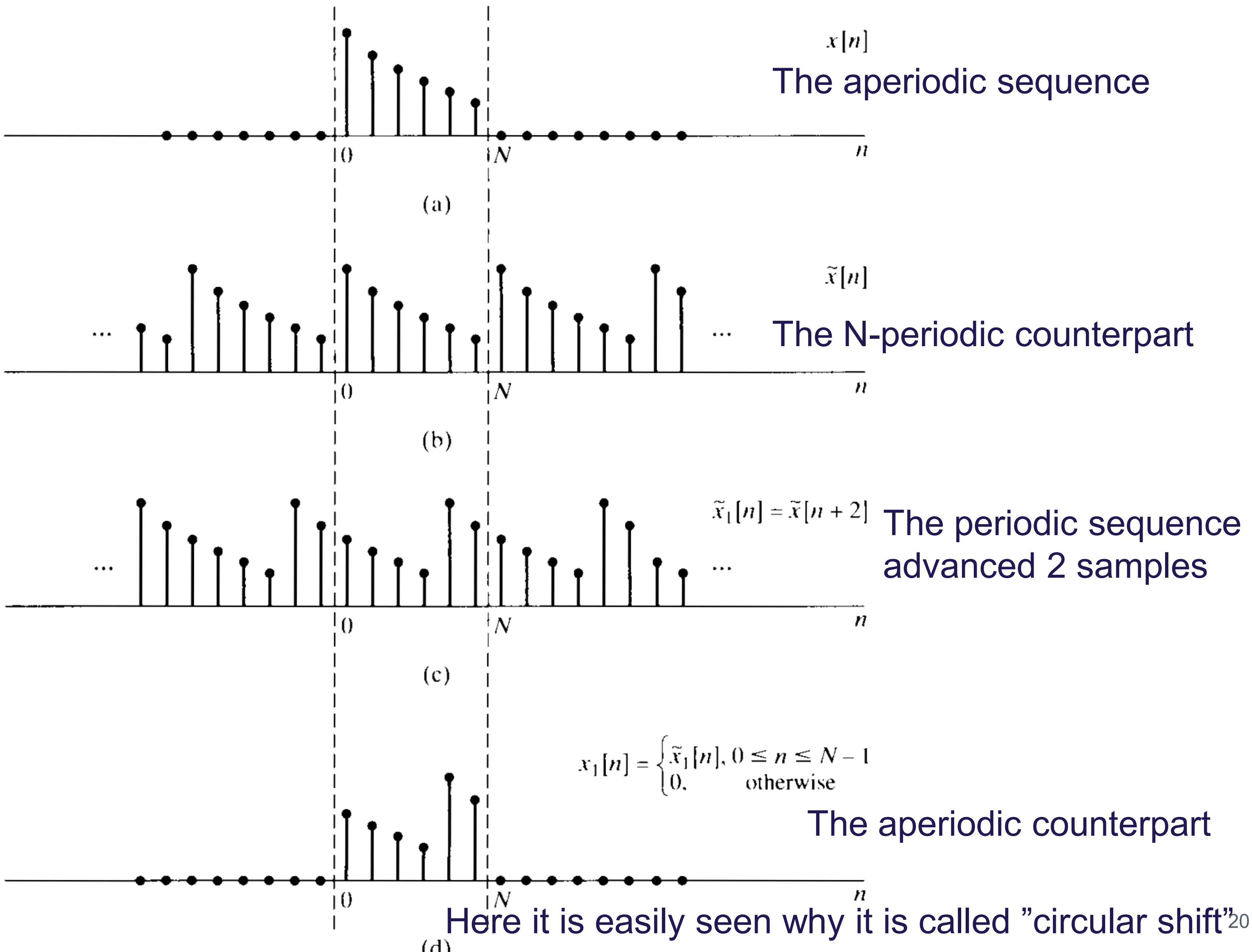
because it needs to be periodic…

See all the math argument on p. 676-677

Thus, the finite-length sequence $x_1[n]$ whose DFT is given by $X_1[k] = e^{-j(2\pi k/N)m}X[k]$ is

$$x_1[n] = \begin{cases} \tilde{x}_1[n] = x[((n-m))_N], & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$$

Circular Shift of a Sequence – An Example



Here it is easily seen why it is called ”circular shift”

Circular Convolution

Here we consider two *finite-duration* sequences $x_1[n]$ and $x_2[n]$, both of length N, with DFTs $X_1[k]$ and $X_2[k]$, respectively, and we wish to determine the sequence $x_3[n]$ for which the DFT is $X_3[k] = X_1[k]X_2[k]$.

Specifically, $x_3[n]$ corresponds to one period of $\tilde{x}_3[n]$, which is given by $\tilde{x}_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m]$

$$x_3[n] = \sum_{m=0}^{N-1} \tilde{x}_1[m]\tilde{x}_2[n-m], \qquad 0 \le n \le N-1,$$

Periodic Convolution
see. p.6

or, equivalently,

$$x_3[n] = \sum_{m=0}^{N-1} x_1[((m))_N]x_2[((n-m))_N], \qquad 0 \le n \le N-1.$$

Since $((m))_N = m$ for $0 \le m \le N-1$, $x_3[n]$ can be written as;

$$x_3[n] = \sum_{m=0}^{N-1} x_1[m]x_2[((n-m))_N], \qquad 0 \le n \le N-1.$$

N-point Circular Convolution $x_3[n] = x_1[n] \circledN x_2[n]$

$$x_3[n] = \sum_{m=0}^{N-1} x_1[m] x_2[((n-m))_N], \qquad 0 \le n \le N-1$$

Looks like a linear convolution as we know it, but it differs in two important respects;

- The sequence x_2 is circularly time reversed respect to x_1.
- The sequence x_2 is circularly shifted with respect to x_1.

”Circularly Time Reversed” – what does that actually mean…????

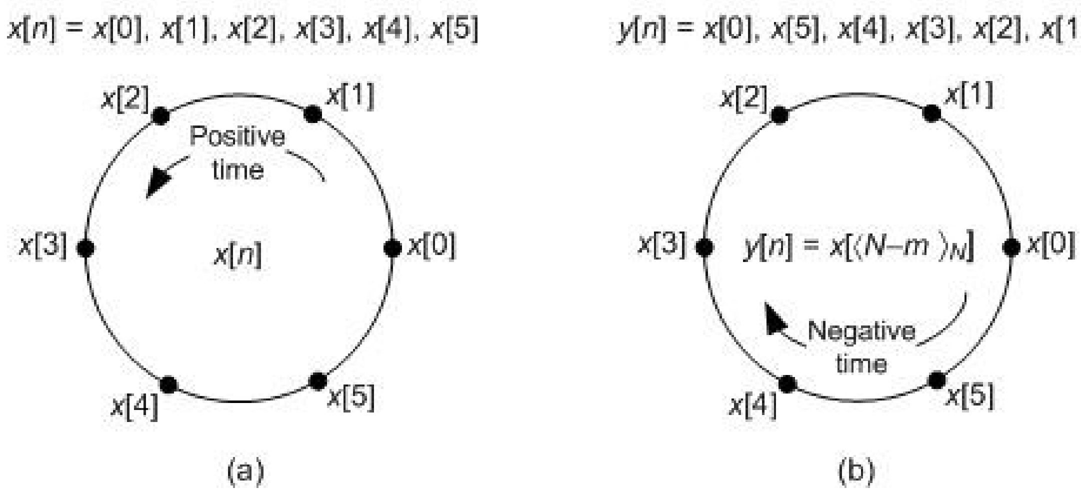


FIGURE A-1. Graphical description of circular time-reversal: (a) an $x[n]$ time sequence; (b) $y[n]$ sequence equal to a circular reversed $x[n]$.

Circular convolution is a commutative operation

Likewise linear convolution, circular convolution is commutative operation.

$$x_3[n] = x_1[n] \circledN x_2[n] \qquad x_3[n] = \sum_{m=0}^{N-1} x_1[m] x_2[((n-m))_N]$$

$$x_3[n] = x_2[n] \circledN x_1[n] \qquad x_3[n] = \sum_{m=0}^{N-1} x_2[m] x_1[((n-m))_N]$$

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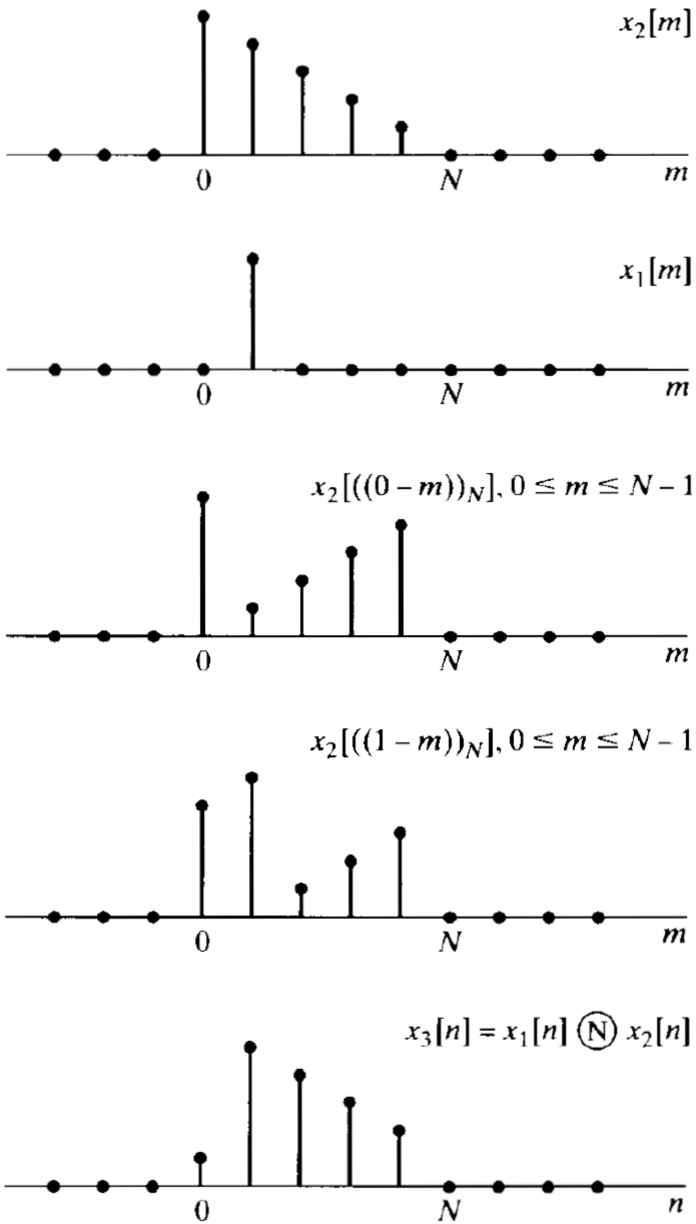
Circular convolution – an example
Convolution with $x_1[n] = \delta[n-1]$



$$x_1[n] = \begin{cases} 0, & 0 \le n < n_0, \\ 1, & n = n_0, \\ 0, & n_0 < n \le N-1. \end{cases}$$

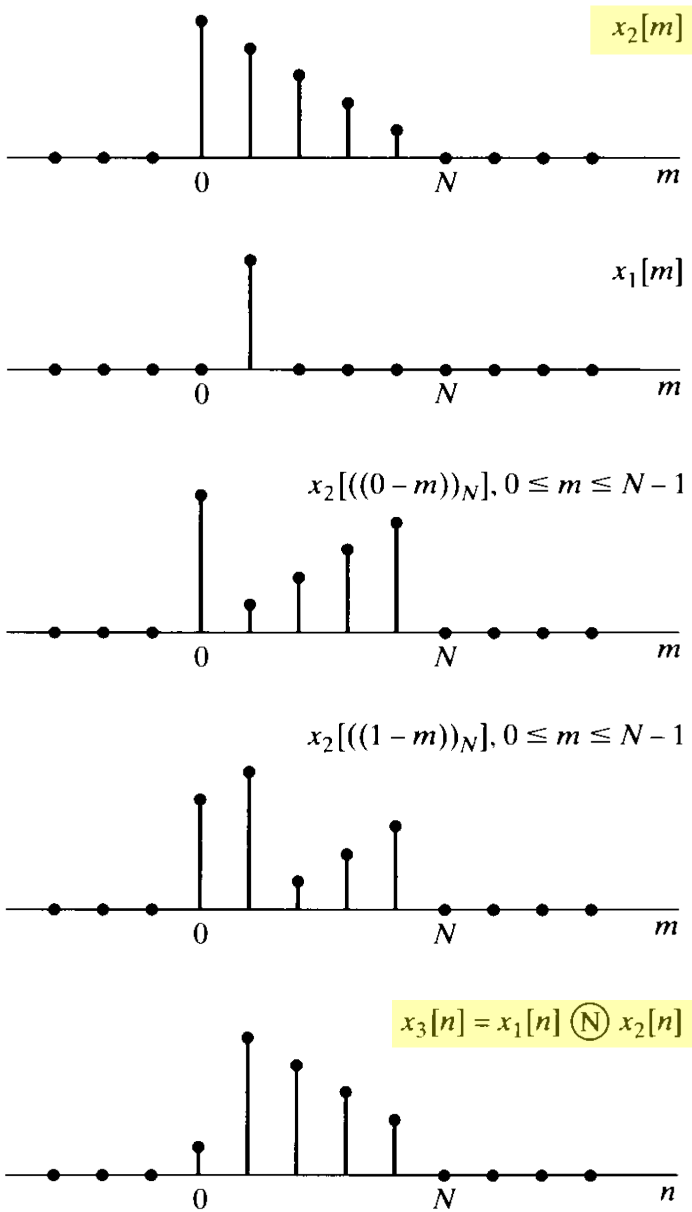
Here x_2 is circularly time reversed and shifted 0

Here x_2 is circularly time reversed and shifted 1

etc.…

Continuing this shifting process for *n=0..N-1* and for every *n* multiply with $x_1[n]$. Finally, add the *N* resulting sequences together to get $x_3[n]$

What else to learn from this example…??



Phase shift…

The DFT of $x_1[n]$ is $X_1[k] = W_N^{kn_0}$

Since convolution in the time domain is equivalent to multiplication in the frequency domain, we have;

$$X_3[k] = W_N^{kn_0} X_2[k]$$

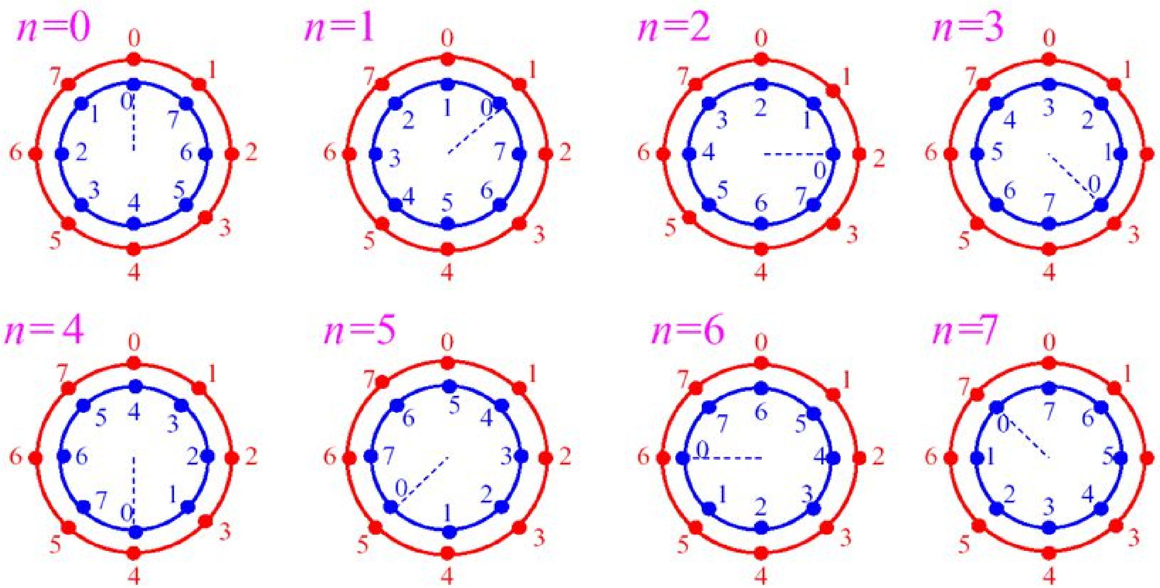
Now, the finite-length sequence $x_3[n]$ which corresponds to $X_3[k]$, is then seen to be $x_2[n]$ shifted right by $n_0 = 1$ sample.

Since things are circular, a ”one sample right shift” is equivalent to a ”one sample right rotation”…

An illustrative explanation of Circular Convolution

The idea is to represent the two finite length sequences on two concentric circles – one linearly (blue) and the other circularly time reversed (red).
Then these two circles are rotated relative to each other, and for every shift the appropriate sample values are multiplied and added.

Illustration of circular convolution for N = 8:



	Finite-Length Sequence (Length N)	N-point DFT (Length N)	
1.	$x[n]$	$X[k]$	
2.	$x_1[n], x_2[n]$	$X_1[k], X_2[k]$	
3.	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$	Linear
4.	$X[n]$	$Nx[((-k))_N]$	Duality
5.	$x[((n-m))_N]$	$W_N^{km} X[k]$	Time shift
6.	$W_N^{-\ell n} x[n]$	$X[((k-\ell))_N]$	Frequency shift
7.	$\sum_{m=0}^{N-1} x_1(m) x_2[((n-m))_N]$	$X_1[k] X_2[k]$	Circular convolution in time
8.	$x_1[n] x_2[n]$	$\frac{1}{N} \sum_{\ell=0}^{N-1} X_1(\ell) X_2[((k-\ell))_N]$	Circular convolution in frequency
9.	$x^*[n]$	$X^*[((-k))_N]$	
10.	$x^*[((-n))_N]$	$X^*[k]$	
11.	$\mathcal{R}e\{x[n]\}$	$X_{ep}[k] = \frac{1}{2}\{X[((k))_N] + X^*[((-k))_N]\}$	
12.	$j\mathcal{J}m\{x[n]\}$	$X_{op}[k] = \frac{1}{2}\{X[((k))_N] - X^*[((-k))_N]\}$	
13.	$x_{ep}[n] = \frac{1}{2}\{x[n] + x^*[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$	
14.	$x_{op}[n] = \frac{1}{2}\{x[n] - x^*[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$	
	Properties 15–17 apply only when $x[n]$ is real.		
15.	Symmetry properties	$\begin{cases} X[k] = X^*[((-k))_N] \\ \mathcal{R}e\{X[k]\} = \mathcal{R}e\{X[((-k))_N]\} \\ \mathcal{J}m\{X[k]\} = -\mathcal{J}m\{X[((-k))_N]\} \\ \lvert X[k]\rvert = \lvert X[((-k))_N]\rvert \\ \sphericalangle\{X[k]\} = -\sphericalangle\{X[((-k))_N]\} \end{cases}$	
16.	$x_{ep}[n] = \frac{1}{2}\{x[n] + x[((-n))_N]\}$	$\mathcal{R}e\{X[k]\}$	
17.	$x_{op}[n] = \frac{1}{2}\{x[n] - x[((-n))_N]\}$	$j\mathcal{J}m\{X[k]\}$	

Finally, some more practical matters concerning the DFT…

Spectral analysis is often based on continuous time signals $s_c(t)$.

This is sampled to $x[n]$ and a window $w[n]$ is multiplied, since a subsequent DFT requires sequences of finite length.

The finite length analysed is then:

$$v[n] = w[n]x[n] \Rightarrow$$

$$V(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})W(e^{j(\omega-\theta)})\,d\theta$$

According to DFT this is sampled and yields:

$$V[k] = V(e^{j\omega})\big|_{\omega=2\pi k/N}$$

The distance between the DFT frequencies is:

$$\frac{2\pi}{N}, \text{ where } \omega = \Omega T$$

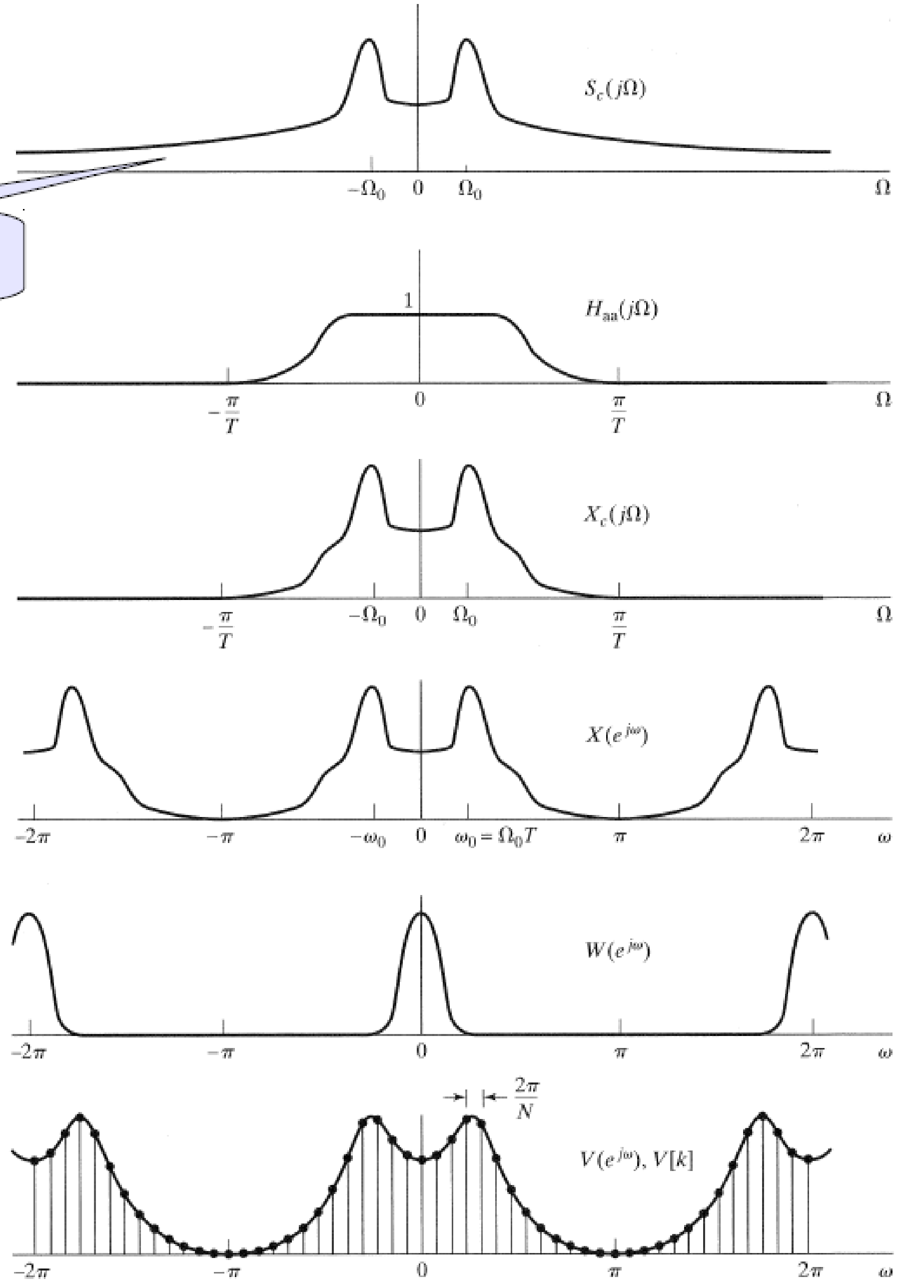
The DFT frequencies then corresponds to the continuous Ω_k-frequencies:

$$\Omega_k = \frac{2\pi k}{NT}$$

So, from this we realize that ”the size” of the DFT (N point) has to have a certain value in order produce a spectrum with a ”sufficient” accuracy…

From continuous time signal to the DFT spectrum

Wide-band



Numerical considerations – an example

Continuous signal limited so it only exists below 2500 Hz, i.e.

$$X_c(j\Omega)=0 \text{ for } |\Omega|\geq 2\pi\cdot 2500$$

The sampling frequency is 5000 Hz, i.e.

$$\frac{1}{T}=f_s=5000 \text{ samples per second.}$$

We want the DFT to give information about X_c for each 10 Hz $(\Delta\Omega=2\pi\cdot 10)$ so the <u>"frequency resolution"</u> is 10 Hz.

How long must the DFT then be?

$$\frac{2\pi}{NT}\leq 2\pi\cdot 10 \Rightarrow \frac{2\pi}{N\cdot\frac{1}{5000}}\leq 20\pi \Rightarrow$$

$$\underline{N\geq 500.}$$