AALBORG UNIVERSITY
DENMARK

Written exam in
Signal processing, 5 ECTS

Thursday, January 8, 2015
9.00 – 13.00

Read carefully :

- Remember to write your **full name on every sheet** you return!
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets**.
- Problems are weighted as listed.
- Results **without sufficient explanations will not give full credits**!
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times.
- **Communication with others is strictly prohibited**.

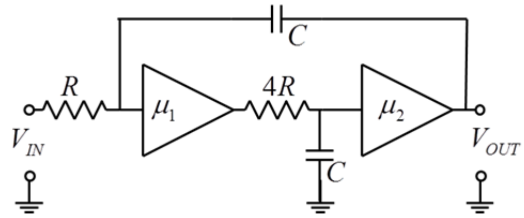
ITC5/EIT5 Signal Processing / Analog Filters 2015-01-02/OKJ

Note: In case your calculator does not have hyperbolic functions, you may use the following equations if needed:

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right) \quad \sinh(x) = \frac{e^x - e^{-x}}{2} \quad \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

A.1 (Weight 11 %)

A second order filter section like the one shown has the transfer function (You do not need to analyse the circuit):



$$H(s) = \frac{\dfrac{\mu_1 \mu_2}{(2CR)^2}}{s^2 + \dfrac{5 - \mu_1 \mu_2}{4CR} s + \dfrac{1}{(2CR)^2}}$$

It is given that:
- $C = 10\ nF$
- $\mu_2 = 2$

And it is required that:
- ω_0 is 2π·1000 rad/s
- Q is 4

a. Explain if it is a low-pass, high-pass or band-pass filter.

b. Find the values of μ_1 *and* R.

c. Find the gain in dB at DC and at ω_0

d. Find the sensitivity of the Q-value with respect to μ_1 , $S_{\mu_1}^{Q}$.

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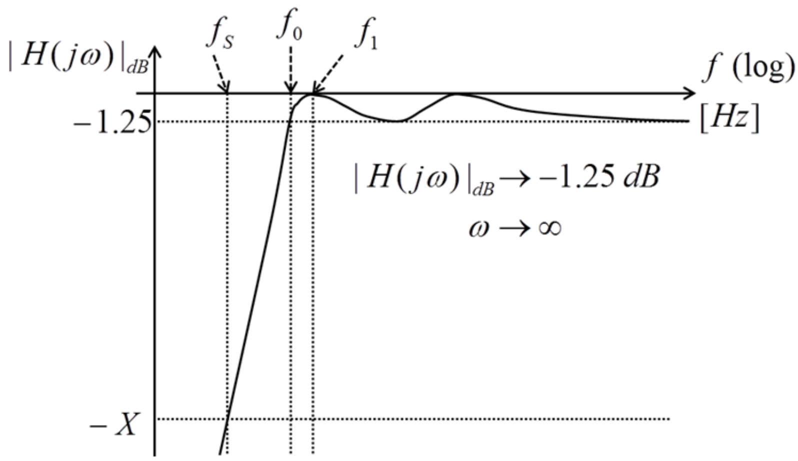
A.2 (Weight 11 %)

The rough sketch shows a response of a high-pass filter.

- $f_S = 50\ Hz$ is the frequency, where X dB of attenuation is obtained.
- $f_0 = 200\ Hz$ is the lowest frequency, where the attenuation is 1.25 dB.
- f_1 is the lowest of two frequencies, where the attenuation is 0 dB



a. Explain which type of filter it is and which order it has. No calculations are needed.

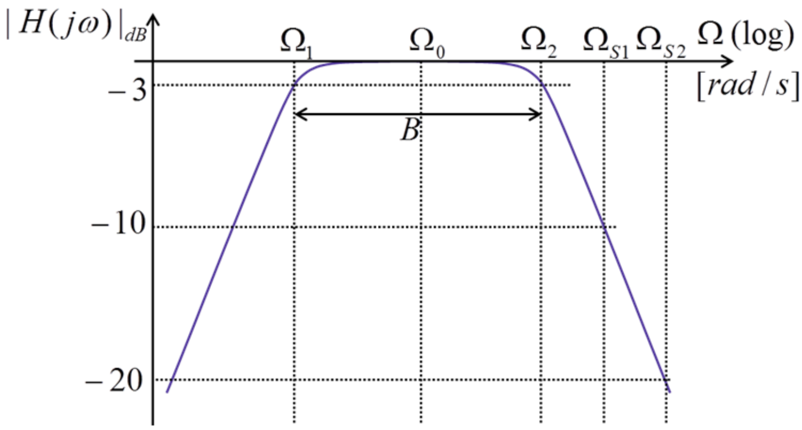
b. Find the attenuation, X in dB at $f_S = 50\ Hz$. Hint: Use the LP←→HP transformation and the normalized filter response.

c. Find the value of f_1.

A.3 (Weight 11 %)

The sketch shows a response of a band-pass filter, which is based on a 2nd order Butterworth prototype (so the BP-filter has 4 poles). The two frequencies, where the attenuation is 10 dB and 20 dB, respectively, are given as:

- $\Omega_{S1} = 3000\ rad/s$ and
- $\Omega_{S2} = 4000\ rad/s$

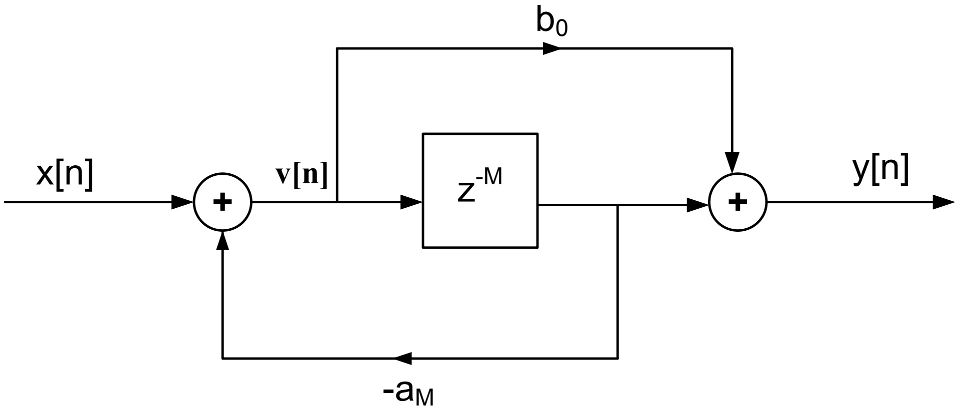


a. For the normalized low-pass prototype, find the two radian frequencies, ω_{S1} and ω_{S2}, where the attenuation is 10 dB and 20 dB, respectively.

b. Use the LP←→BP frequency mapping to find the value of the "centre" radian frequency, Ω_0 and the 3-dB radian bandwidth, B.

Problem 1 (weighted with 10% - Digital filters)

A digital filter is has two coefficients a_M, b_0 and a delay block z^{-M} as illustrated by the flow graph below.



Questions:

1. Determine the transfer function H(z).
   (hint: it may turn helpful to use the v[n] as shown above)
2. Determine the difference equation.
3. Determine an expression for the magnitude response $|H(e^{j\omega})|$.
4. Compute the DC gain (i.e. $|H(e^{j\omega})|$ for $\omega=0$).

Problem 2 (Weighted with 12% - Digital filters)

Transform the 1. order analog Butterworth filter:

$$H_a(s) = \frac{1}{1+\frac{s}{\Omega_c}} \quad , \quad \text{where } \Omega_c \text{ is the cut-off frequency}$$

into a digital filter using the bilinear transformation. The sampling frequency is fs= 1000Hz. The digital filter should have -3 dB cutoff frequency at 200 Hz. The DC gain should be 0 dB.

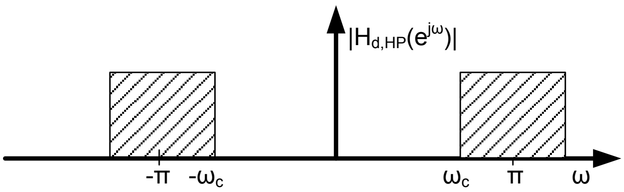
Questions:

2.1 Determine the transfer function H(z).

2.2 Compute the gain in dB at 200 Hz.

Problem 3 (weighted with 12% - Digital filters)

Design a high-pass FIR filter using the window method. The ideal amplitude response is illustrated in the figure below.



$$H_{d,HP}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & \omega_c \le |\omega| \le \pi \\ 0 & 0 \le |\omega| \le \omega_c \end{cases}$$

Use a rectangular window, a sampling frequency of f_s=20 kHz, a cut-off frequency of f_c=5 kHz and a filter order of M=4.

Questions:

3.1 Determine the filter coefficients.
3.2 Determine the transfer function H(z).
3.3 Draw a signal flow graph for the filter.
3.4 Determine the gain in dB at 5 kHz
3.5 Describe the characteristics of the phase response?
3.6 Compute the group delay.

Signal Processing - Spectral Estimation

*Please demonstrate that you can do **all** the computations manually! Simple "Matlab results" won't count!*

In the following we shall investigate the difference between straight convolution in time domain and what we can achieve by multiplication in frequency domain. In principle they should be the same, but...

Exercise 1 (4%)
At a sampling frequency of f_S=44.1 kHz a signal x[n]=[1 1 -1 1] is sent through a filter. The filter has the impulse response h[n]=[2 0 4 0]. What is the output $y_1[n]$ of the filter?

Exercise 2 (4%)
Draw the output signal $y_1[n]$ and put numbers for each mark n on the x-axis. What is the difference Δt in time [μs] between sample $y_1[2]$ and sample $y_1[4]$?

Exercise 3 (9%)
Find the frequency response X[k] of x and the frequency response H[k] of h.

Exercise 4 (4%)
Multiplying in frequency domain corresponds to convolving in time domain, so let's try to multiply X[k] and H[k]. Find Y[k]=X[k] H[k]. What is the frequency difference Δf [Hz] between sample Y[3] and Y[2]?

Exercise 5 (7%)
Final step is to bring Y[k] back to time domain and call the result $y_2[n]$. Find $y_2[n]$, draw it and put numbers for each mark n on the x-axis.

Exercise 6 (5%)
Compare $y_1[n]$ and $y_2[n]$. In which way are they different? And why? How do you think they would compare, if the input signal x[n] was e.g. 100 times longer?