ITC5/EIT5 Signal Processing / Analog Filters

A.1 (Weight 11 %)
A normalized low-pass filter should have:
- 3^{rd} order Chebyshev characteristic
- 0.4 dB pass-band ripple
- A pass-band (ripple) bandwidth of 1 rad/s

a. Find the stop-band attenuation at ω_S = 20 rad/s
b. Find the pole locations of the filter
Note: You may use the following equations:

$$\cosh(x)=\frac{e^x+e^{-x}}{2},\ \cosh^{-1}(x)=\ln\left(x+\sqrt{x^2-1}\right),\ \sinh(x)=\frac{e^x-e^{-x}}{2},\ \sinh^{-1}(x)=\ln\left(x+\sqrt{x^2+1}\right).$$

a. $\varepsilon=\sqrt{10^{Riple,dB/10}-1}=\sqrt{10^{0.04}-1}=0.3106$

$C_3(20)=\cosh(3\cdot\cosh^{-1}(20))=31940$

$-H(j20rad/s)=10\log(1+\varepsilon^2C_3^2(20))=79.9dB$

b.

$$s_k=\sin\frac{(2k-1)\pi}{2n}\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right)+j\cos\frac{(2k-1)\pi}{2n}\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right)\qquad k=n+1,\ldots 2n$$

s_4 = -0.3354 - j1.0428

s_5 = -0.6708 - j0.0000

s_6 = -0.3354 + j1.0428

A.2 (Weight 11 %)
The requirements for a Butterworth high-pass filter are:
- 3 dB attenuation at f = 20 kHz ($\Omega = 2\pi\cdot 20\cdot 10^3$ rad/s)
- Minimum 40 dB stopband attenuation at f = 4 kHz ($\Omega = 2\pi\cdot 4\cdot 10^3$ rad/s)

a. Make a sketch (graph) of the filter requirements and the corresponding low-pass prototype filter. Use the LP-HP frequency mapping to find the frequency in the stop-band of the low-pass prototype that corresponds to the high-pass filter frequency of f = 4 kHz ($\Omega = 2\pi\cdot 4\cdot 10^3$ rad/s) where 40 dB attenuation must be obtained.
b. Find the necessary filter order
c. Find the attenuation of the high-pass filter at f = 2 kHz ($\Omega = 2\pi\cdot 2\cdot 10^3$ rad/s)

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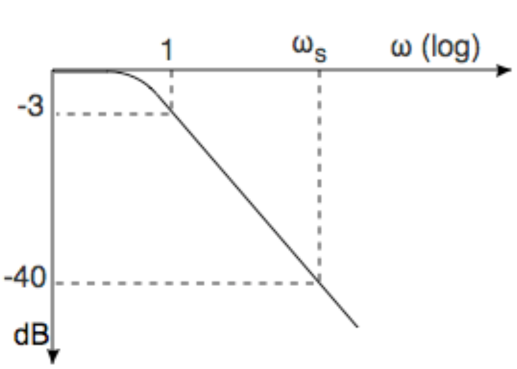
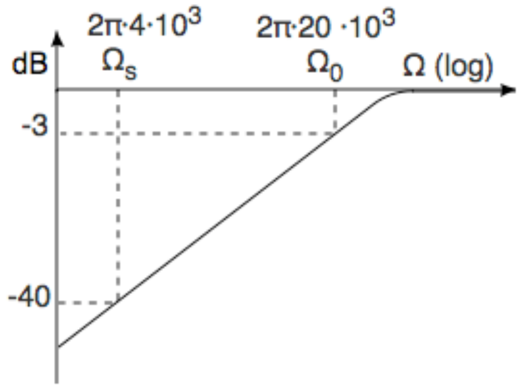
a.





$$s = \frac{\Omega_0}{S} \qquad |\omega| = \frac{\Omega_0}{\Omega}$$

$$|\omega_S| = \frac{\Omega_0}{\Omega_S} = \frac{2\pi \cdot 20 \cdot 10^3}{2\pi \cdot 4 \cdot 10^3} = 5 \quad (rad/s)$$

$$|H_{LPP}(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

b. $10\log(1+\omega_S^{2n}) = 40$

$$\omega_S^{2n} = 10^{40/10} - 1$$

$$n = \frac{\log(10^{40/10}-1)}{2\log(5)} = 2.86 \rightarrow 3$$

c.

$$-10\log\left(\frac{1}{1+\left(\frac{\Omega_0}{\Omega}\right)^{2n}}\right) = 10\log\left(1+\left(\frac{\Omega_0}{\Omega}\right)^{2n}\right) = 10\log\left(1+\left(\frac{2\pi \cdot 20 \cdot 10^3}{2\pi \cdot 2 \cdot 10^3}\right)^{2\cdot 3}\right) = 60 \quad dB$$

A.3 (Weight 11 %)

A normalized low-pass filter with the transfer function:

$$H_{LPP}(s) = \frac{1}{1+s}$$

is to be transformed into a band-pass filter with:

- Centre frequency, Ω_0 = 15 rad/s
- Bandwidth, B = 5 rad/s.

  a. Find the transfer function of the band-pass filter, $H_{BP}(S)$
  b. Find the locations of poles and zero of the band-pass filter

a.

$$\because \qquad s = \frac{S^2 + \Omega_0^2}{B \cdot S}$$

$$and \qquad H_{LPP}(s) = \frac{1}{1+s}$$

$$\therefore$$

$$H_{BP}(S) = \frac{1}{\frac{S^2 + \Omega_0^2}{B \cdot S} + 1}$$

$$H_{BP}(S) = \quad \frac{B \cdot S}{S^2 + B \cdot S + \Omega_0} = \frac{5S}{S^2 + 5S + 225}$$

b.

Zeros:

$B \cdot S = 0 \Rightarrow Zero = 0$

Poles:

$S^2 + 5S + 225 = 0 \quad rad/s$

$\Rightarrow$

$$Poles = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 225}}{2} = \begin{cases} -2.5 + j14.8 \\ -2.5 - j14.8 \end{cases} rad/s$$