Signalbehandling for computer-ingeniører COMTEK-5, E20 & Signalbehandling

10. Realization Structures for Digital Filters, and Finite Word Length Effects

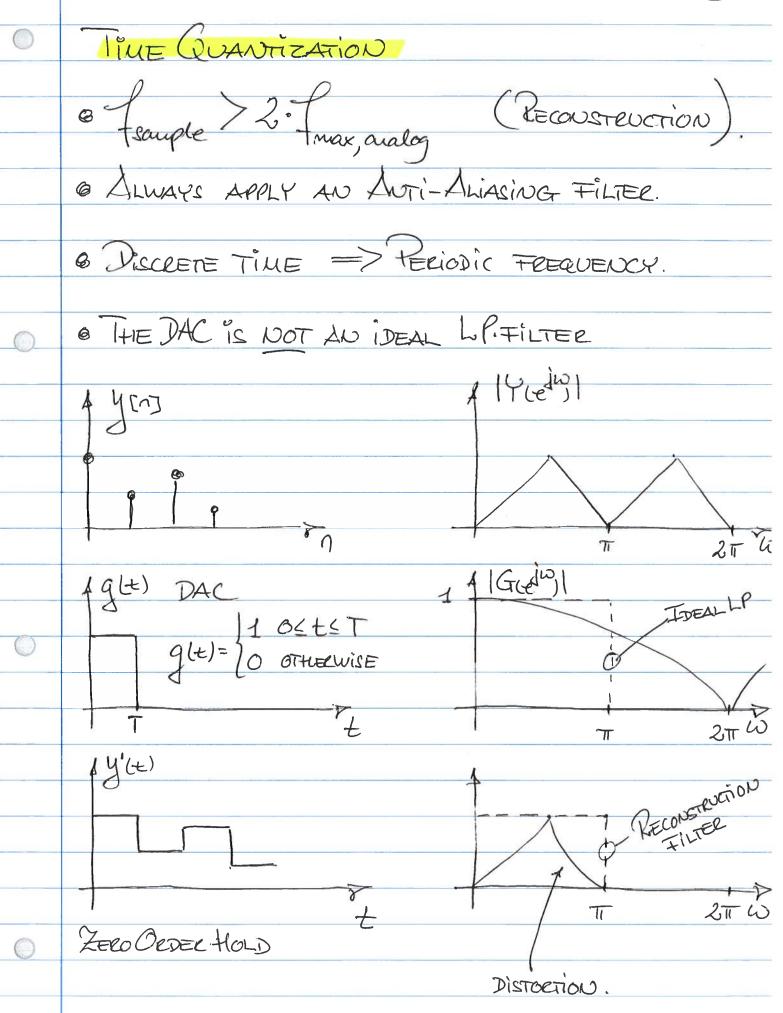
EIT-5, E20

Assoc. Prof. Peter Koch, AAU

REALISATION OF DIGITAL FILTERS



0	
	THE SIGNAL CHAIN ZERO DEDER HOLD
Xte	
	THROUGH THE SIGNAL CHAIN, SEVERAL ERRORS - OR GUANTIZATIONS - ARE INTRODUCED. S/H: TIME QUANTIZATION ADC: VARIABLE QUANTIZATION
0	H(Z) (COUPUTER): VARIABLE + COEFFICIENT QUANT. DAC: TÎME QUANTIZATION.



0	VARIABLE QUANTIZATION
75-2-1	<i>N</i>
	e ADC />
	Courinuous in AMP 9 Discourse in AMP
	Λ)
	N bit ADC => 2 POSSIBLE VALUES FOR X
0	7
	9
	QUAUTIZATION STEP
	$\Delta = 1/2^{N}$
	SO, AT THE ADC OUTPUT WE HAVE A
	FINITE SIGNAL-TO-NOISE PATIO (SNR)
0	SiGNAL (Rus)
	SNR = 20. log SiGNAL (ens) Noise (ens)
	3
	WITHOUT PROOF;
- 100	
	THE SUR ON THE OUTPUT OF THE ADC IS
	INCREASED WITH GOB WHEN WIS
	INCREASED WITH 1.
0	
	SNR = 6.02N + 1.76 DB



a H(z) (COMPUTER)

MULTIPLICATION BETWEEN SIGNAL AND COEF.

M

Y[N] = La.yn-k] + Lbex[n-k]

k=1

l=0

(EN) N

2N

Do X[N]

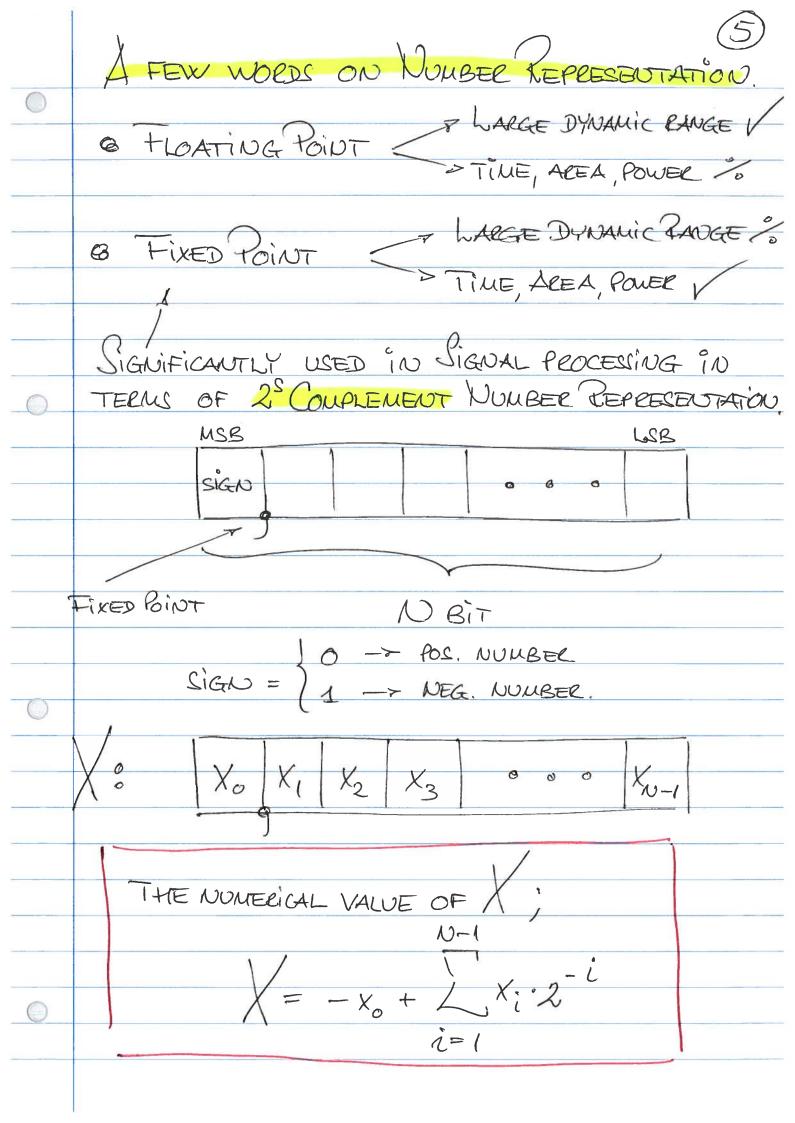
QUANTIZER.

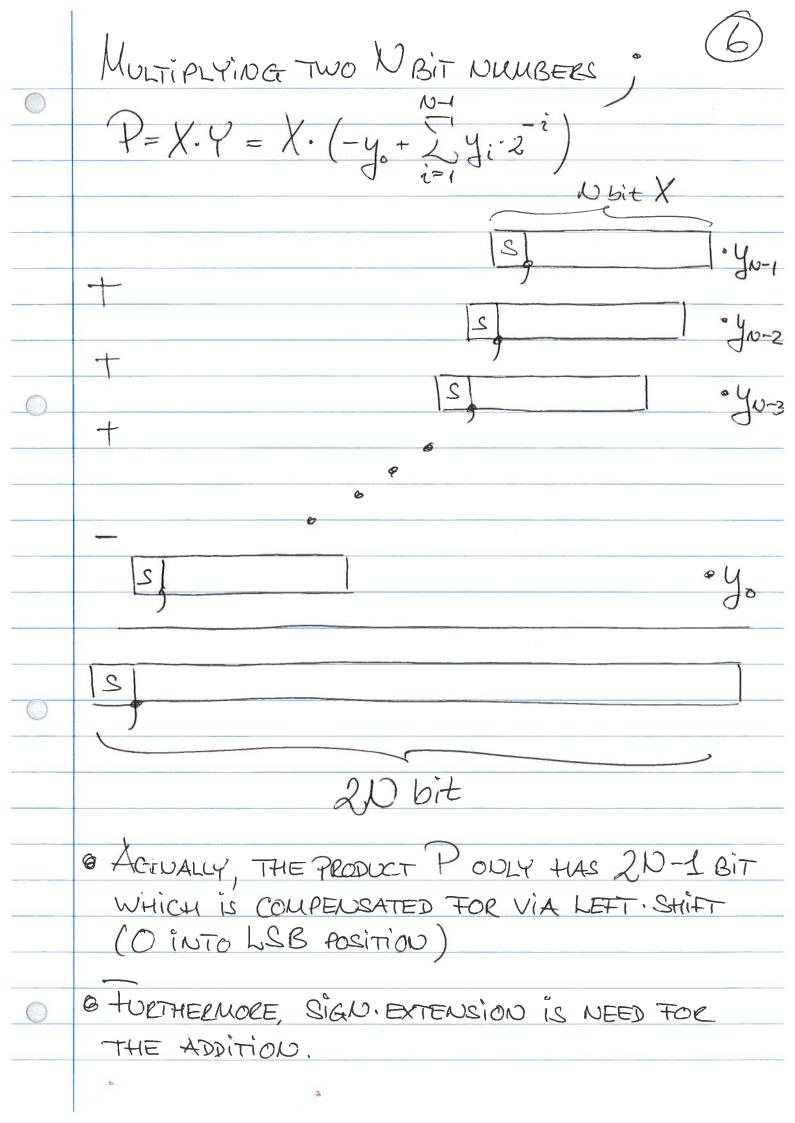
POUND DOWN ROUNDUP

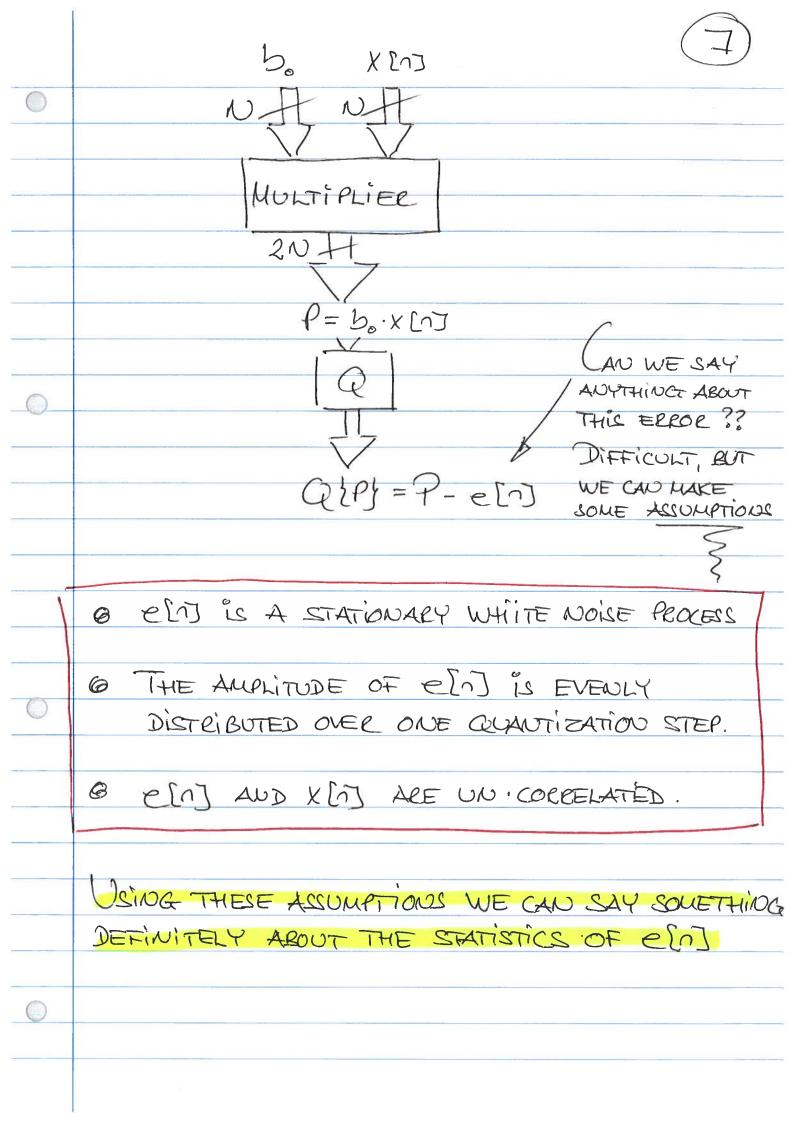
SO, THE QUANTIZER WHICH TAKES US BACK FROM 2N BIT TO N BIT, INTRODUCES AN ELROR;

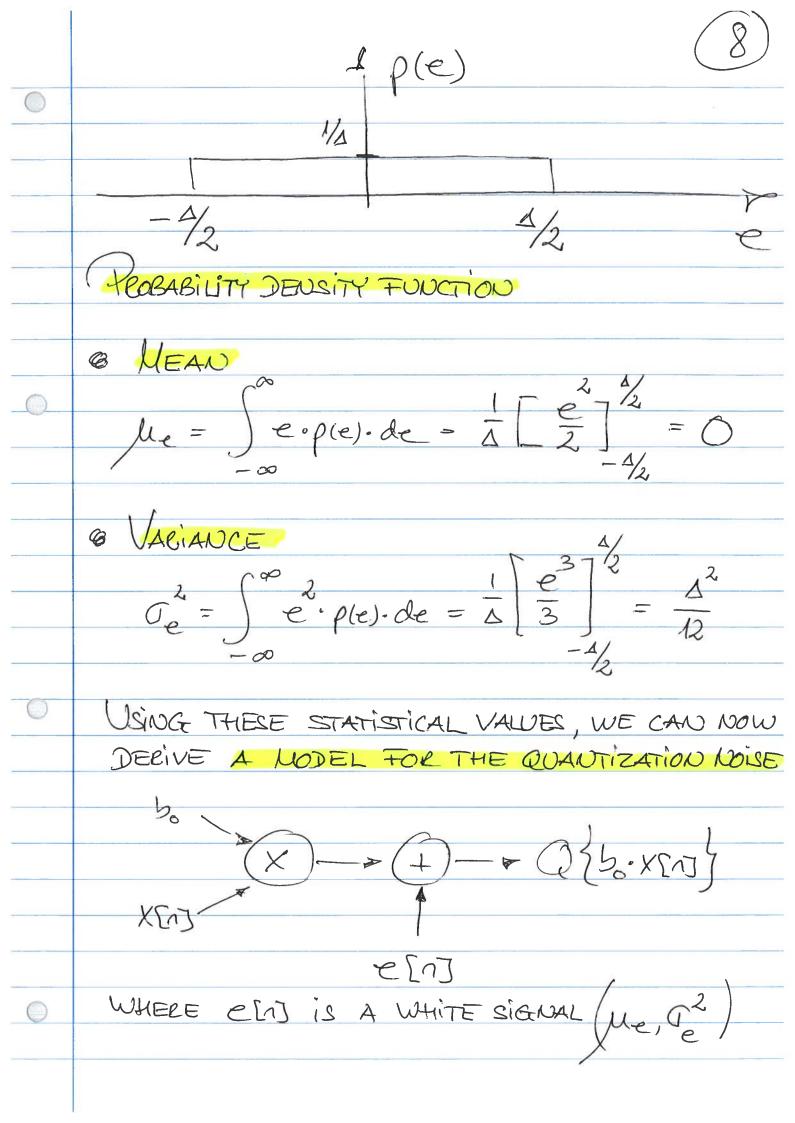
e[n] = b. XEnj - Q(b. XEnj)
2NBIT NBIT

THE COMPUTATION ITSELF MAKES NOISE =









EXAMPLE: 1 ORDER LIRITIER. y[n] = a, y[n-1] + b, x[n] + b, x[n-1] GRAPHICAL REPRESENTATION; e, M SINCE THESE ARE ALL LINEAR OPERATIONS, WE CAN GE-ORGANIZE THE GRAPH enj M3X WHERE e[n] =) e.[n], AND THUS; & THE TOTAL NOISE IMPACTS DIRECTLY THE OUTPUT. @ AT THE OUTPUT THE SNR IS NOT INFINITE



CEFFICIENT QUANTIZATION (in H(Z))

EXACT QUANTIZATION ERROR.

WHERE BO IS THE EXACT COEFFICIENT (CAL-CULATED WITH MANY DIGITS) AND ABO IS THE ELECE INTRODUCED ON THE COEFFICIENT WHEN IT IS REPRESENTED IN N BIT

$$\frac{\lambda}{1} = \frac{\lambda}{2} = \frac{\lambda}{2}$$

$$\frac{\lambda}{1} = \frac{\lambda}{2}$$

SINCE, GENERALLY, by the AND at the WILL SEE SOME CHANGES IN THE POLE/ZERO LOCATIONS.

ONE POSSIBLE METHOD TO INVESTIGATE HOW SEVERE THIS PROBLEM IS, IS TO MAKE AN ANALYSIS OF THE POLE-SENSITIVITY



SENSITIVITY. ANALYSIS (WITHOUT PROOF)
$$A(z) = 1 - \sum_{k=1}^{N} a_k z^{-k} = 11 (1 - d_k z^{-1})$$

WHERE Of ARE THE COORS IN A(Z) (POLES)

BASICALLY WE WANT TO KNOW , THE CHANGE IN THE POLES AS RELATED TO THE CHANGE IN THE FILTER COEFFICIENTS";

$$\frac{\partial di}{\partial a_{k}} = \frac{\partial di}{\partial a_{k}}$$

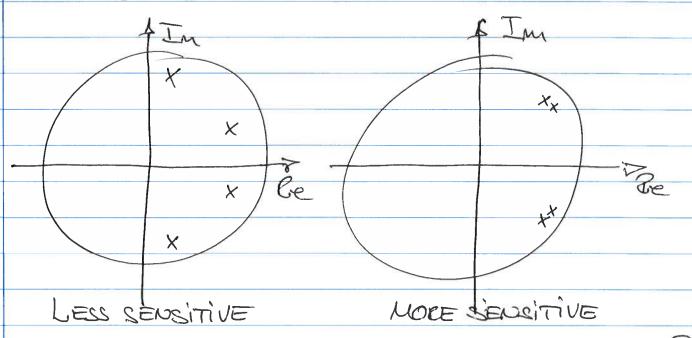
$$\frac{\partial di}{\partial a_{k}} = \frac{\partial di}{\partial a$$

(THIS EQUATION IS NOT SHOW IN THE BOOK)

SO, WHAT WE SEE IS THAT A CHANGE IN THE POLE - LOCATION AS CELATED TO A CHANGE IN THE COEFFICIENTS (DUE TO QUANTIZATION) IS INVERSE PROPORTIONAL TO THE DISTANCE BETWEEN THE POLES



SO, THE CLOSER TOGETHER THE POLES ARE LOCATED, THE MORE SENSITIVE THE POLE WORATION IS DUE TO A CHANGE IN THE FILTER COEFFICIENTS



CAN WE DO SCHETHING ABOUT THIS PROBLEM .

YES WE CAN .

0

WE MAY BE-ORGANIZE HIZ) IN SUCH A
WAY THAT CLOSELY SPACES POLES ARE
DE-COUPLED INTO INDIVIDUAL FILTER SECTIONS

TAKE THE EXAMPLE ABOVE;

$$H(z) = \frac{B(z)}{A(z)} = \frac{B(z)}{1 - \frac{4}{2}az^{2}} = \frac{B(z)}{(1 - \frac{2}{2}az^{2})(1 - \frac{2}{2}az^{2})}$$

$$B(z) = \frac{B(z)}{1 - \frac{4}{2}az^{2}} = \frac{B(z)}{1 - \frac{2}{2}az^{2}}$$

$$B(z) = \frac{B(z)}{1 - \frac{2}{2}az^{2}} = \frac{B(z)}{1 - \frac{2}{2}az^{2}}$$

 $= \overline{A_1(z) \cdot A_2(z)}$

A CASCADE OF 2ND ORDER SECTIONS (EVT.

ALSO ONE 1ST ORDER SECTION).

- [H₂(2)] + (3(2)] - · · · - [H₂(2)]

H(z) = $\frac{1}{11}$ H(z) (FACTORIZATION) i=0 $\frac{1}{2}$ (Respectively) WHERE $\frac{1}{2}$ (Respectively)

Now, QUESTION is , WHICH ZEROS FROM HICZ)

(i.E., ROOTS FROM BCZ) SHOULD BE PAIRED

WITH A: (7) ??

A: (12) ??

July Complex Conjugated

Complex Conjugated

Complex Conjugated

Re

A: (12) ??

Complex Conjugated

The Complex Conjugated

Complex Conjugated

T

PAIRS ARE MERGED INTO ONE SECTION H. (Z)



0	NEXT QUESTION;
	is H(2) H2(2) iDENTICAL TO
	- H(z) - H(z) - 2
	IF WE DO THE COMPUTATION IN INFINITE
	WORDLENGTH (FLOATING POINT), THEN THE
0_	ANSWER IS YES".
1117	
	HOWEVER, FOR FIXED POINT COMPUTATION, THE
	ANSWER MOST LIKELY IS NO.
	HENCE, WHICH ONE TO CHOOSE .
)
	TO ANSWER THIS QUESTION, WE DISTINGUES
	BETWEEN BROAD BAND AND NARROW BAND
Θ	SECTIONS; HOLZ) AND HOLZ)
	1 Hy
	To the true
8	1) Ha(z) FIRST - REDUCTION OF NOISE SPECTEUM
	THROUGH THE CASCADE
	2) HO(2) LAST - THE AMPLITUDE LEVEL IS

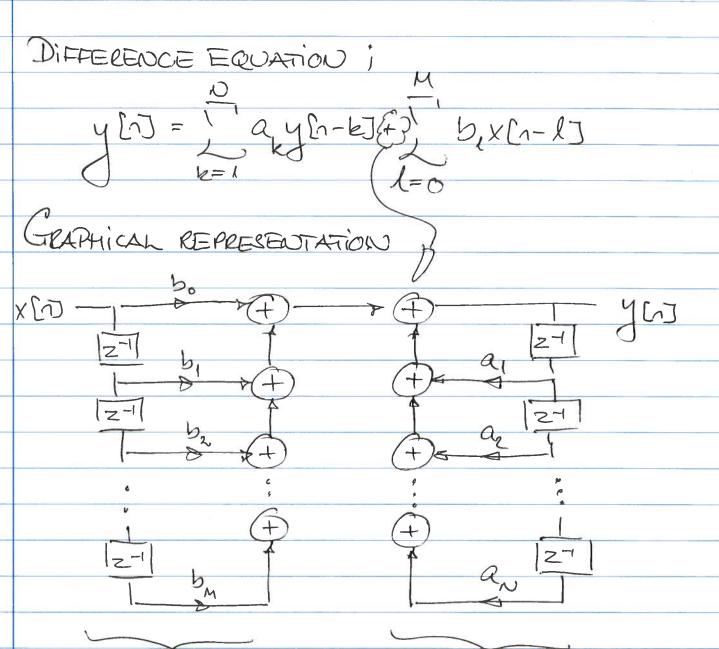
REDUCED ONLY AT THE OUTPUT OF THE FILTER

RECOMMENDED

NEALISATION STRUCTURES

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Itis STEUCTURE IS KNOWN AS THE

DieEct Foeu I

2EPOS

BECAULE IT IS A DILECT LEALISATION OF THE DIFFERBUCE EQUATION



NOW, THE DF-I STEUTURE CAN BE RE-ORGANIZED INTO WHAT IS KNOWN AS DF-II

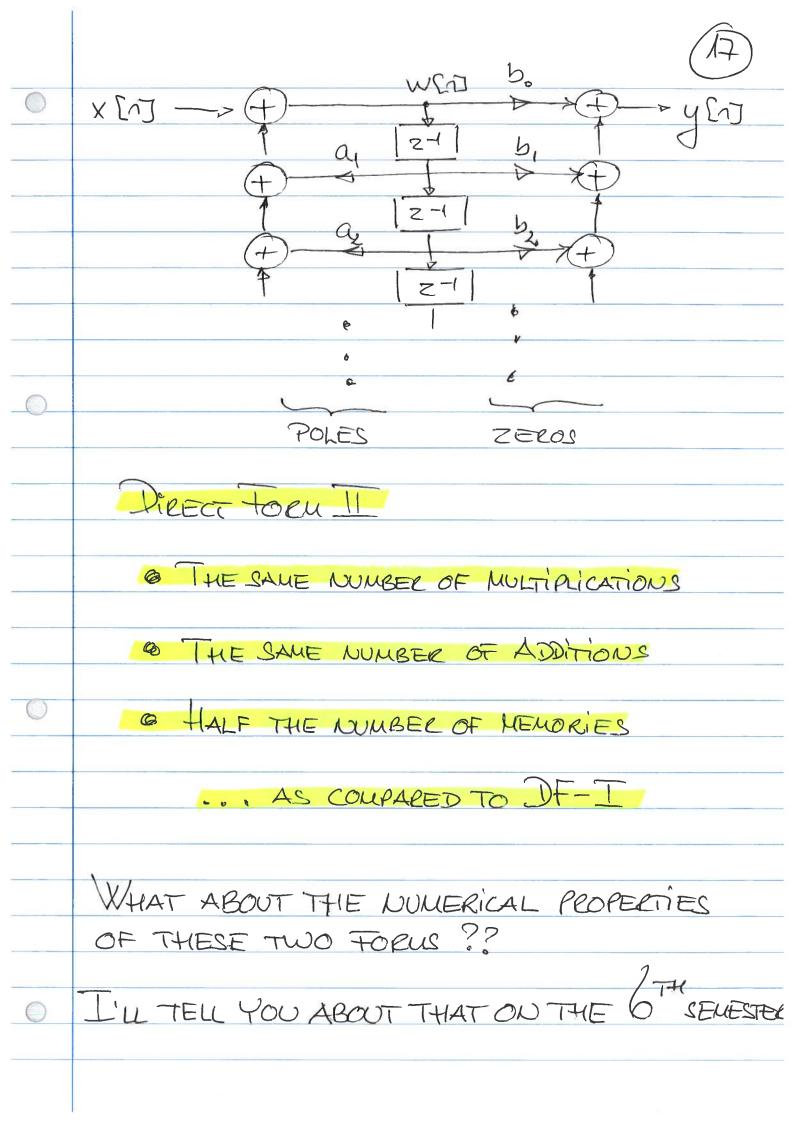
$$H(z) = \begin{cases} \frac{1}{N} & \text{if } \frac{N}{N} \\ \frac{N}{N} &$$

$$\begin{cases} W(z) \cdot (1 - \lambda a_k z^k) = X(z) \\ k = 1 \end{cases}$$

$$W(z)$$
 $b_1 \cdot z^{-1} = 1(z)$

$$Z^{-1} = x [n] + \sum_{k=1}^{N} a_k w [n-k]$$

SO, THE INTERMEDIA VARIABLE WIND IS GENERATED AT THE INPUT AND IS USED TO GENERATE THE CUTPUT

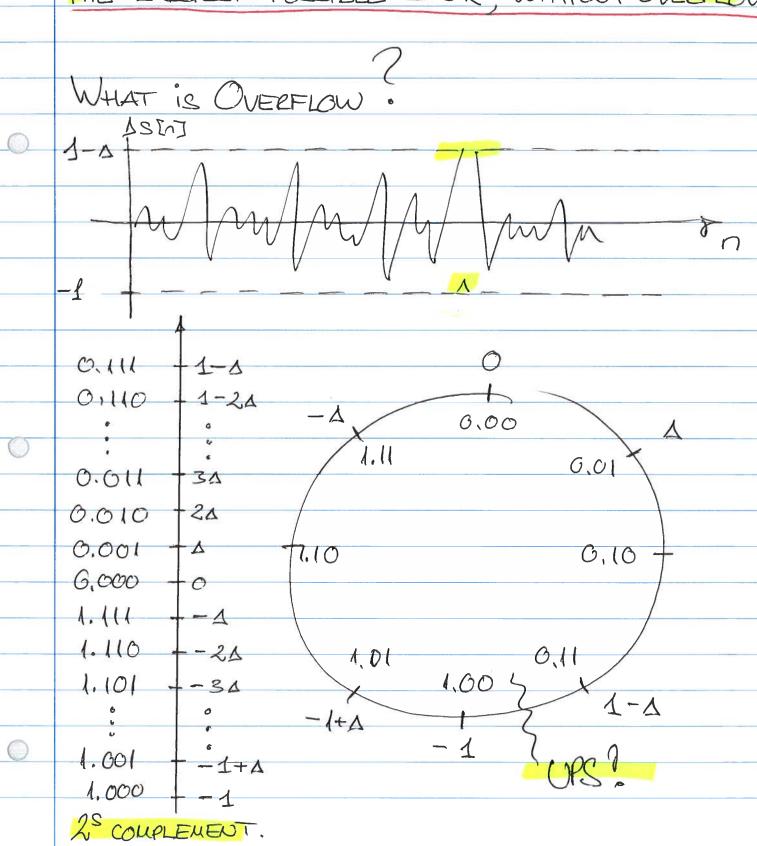


SCALING

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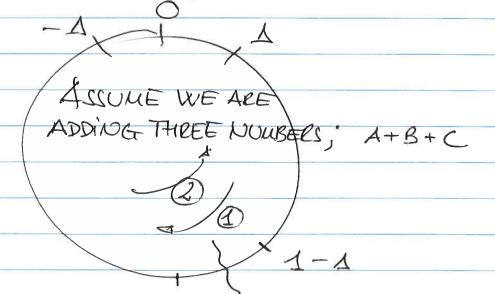


NO MATTER WHETHER WE ARE USING DF-I OR DF-II, WE HAVE TO ENSURE THAT OUR IMPLEMENTATION OF THE FILTER PROVIDES THE LARGEST POSSIBLE SNR, WITHOUT CREEFLOW



OVERFLOW IS POSSIBLE ONLY WHEN ADDINGT

THEREFORE, 2 COMPLEMENT IS VERY USEFUL COMBATING OVERFLOW.



- 1 |A+B| >1 => OVERFLOW
- (2) (A+B)+C <1 => NO OUTEFLOW.

THUS; THERE MAY BE SOME INTERNAL VARIABLES
IN OUR FILTER STRUCTURE WHERE THE

PARTIAL SUMS ARE ALLOWED TO

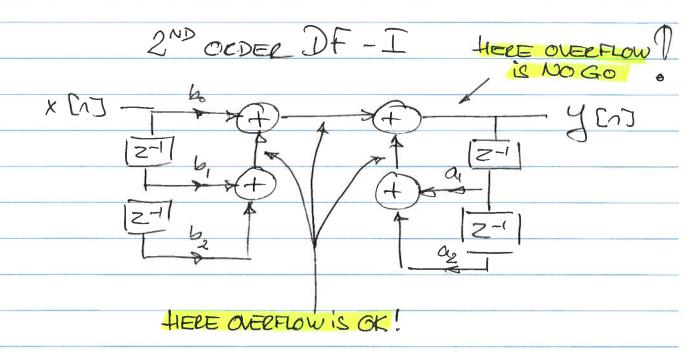
OVERFLOW, AS LONG AS WE EUSURE

THAT THE TOTAL SUM DOES NOT OVERFLOW

AN ALTERNATIVE FORMULATION;



WE WANT TO DESIGN OUR FILTER WITH BEST POSSIBLE UTILIZATION OF THE DYNAMIC PANGE (I.E., BEST SNR) OF THE VARIABLES, AND AT THE SAME TIME REDUCE THE RISC OF OVERFLOW IN THOSE VARIABLES WHERE OVERFLOW IS NOT ALLOWED

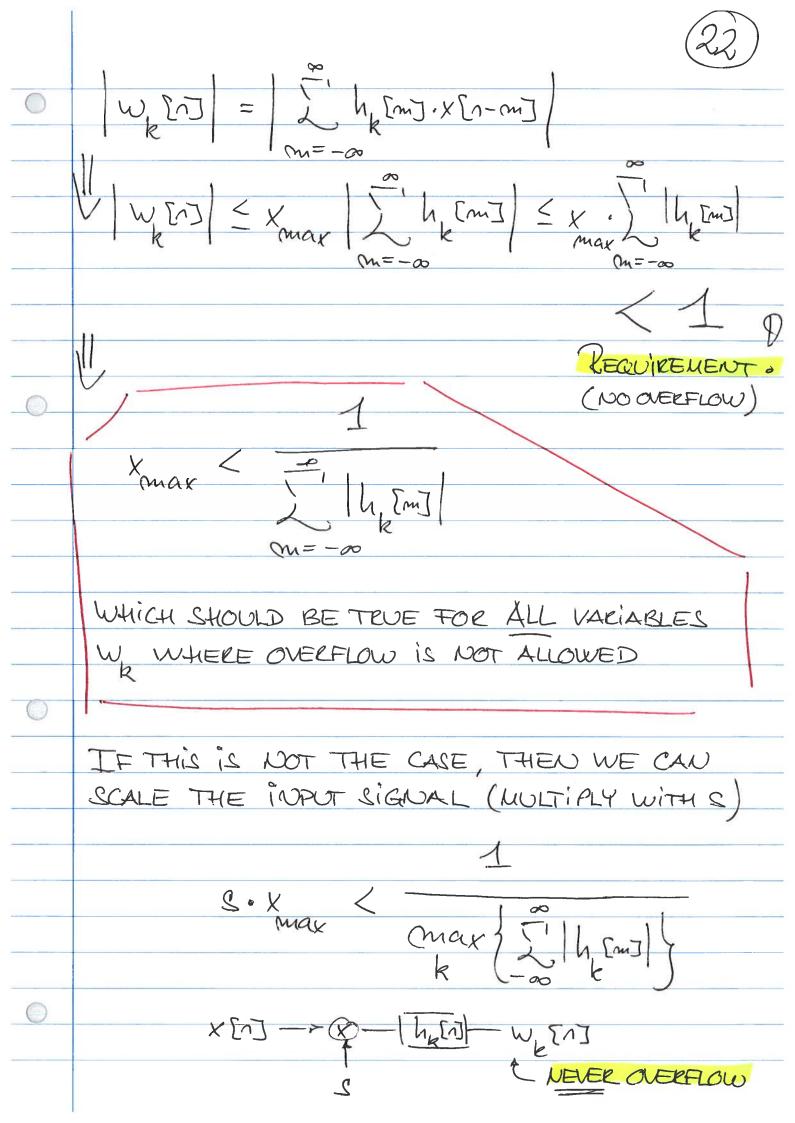


" ... LEDUCE THE EISC OF OVERFLOW ...

OUR PROBLEM IS THAT ON BISC AND A HIGH SNR (FULL UTILIZATION OF THE DYNAMIC BANGE) ARE CONFLICTING REQUIREMENTS

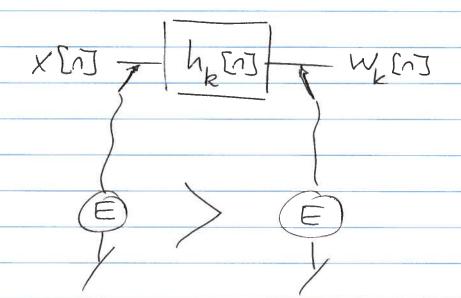


	THREE POSSIBLE SCALING STRATEGIES PP. 463-465
0	
	1) MAX-VALUE SCALING
	THE ÎDEA ÎS TO SCALE THE ÎNPUT SIGNAL
	SUCH THAT OVERFLOW DOES NOT OCCUR IN
	THOSE VACIABLE WHERE OVERFLOW IS NOT
	ALLOWED.
	- SUCH A VARIABLE IS DENOTED W, AND
	- SUCH A VACIABLE IS DENOTED WE AND THE SEQUENCE IN THIS VACIABLE IS WENT;
	XENJ - WENJ
	THE INPUT TO THE VARIABLE W.
	FROM THE INPUT TO THE VARIABLE W.
0	$ w = \int h [m] \times [m]$
	k (M) - / W (M-100)
	m=-00
	DALL ACCUMENTUATION THE LANGUE CALL
	NOW, ASSUME THAT WE KNOW THE NUMERICAL LARGEST VALUE IN X [N] - X MAX
	LAKGESI VALUE IIV X (I)



- AN ISSUE WITH THE MAX-VALUE SCALING STRATEGY IS THAT & MAY BECOME SO SMALL THAT IT COMPROMISES THE SNK OF THE FILTER.
 - 2) SINUSOID SCALING MAINLY USE FOL NARROW BAND SIGNALS — SELF STUDY.
 - 3) VARIANCE SCALING

THE IDEA HERE IS TO ENSURE THAT THE ENERGY IN ALL VARIABLE WHERE OVERFLOW IS NOT ALLOWED, IS LESS THAN THE ENERGY IN THE INPUT SIGNAL.



THE ENERGY (OR VACIANCE) IS THE POWER INTEGRATED OVER TIME.

