Løsningsforslag, 9. forelæsning
Signalbehandling, EIT-5 og COMTEK-5, E20

1. Frekvensanalyse af H(z) (fra 7. mm)

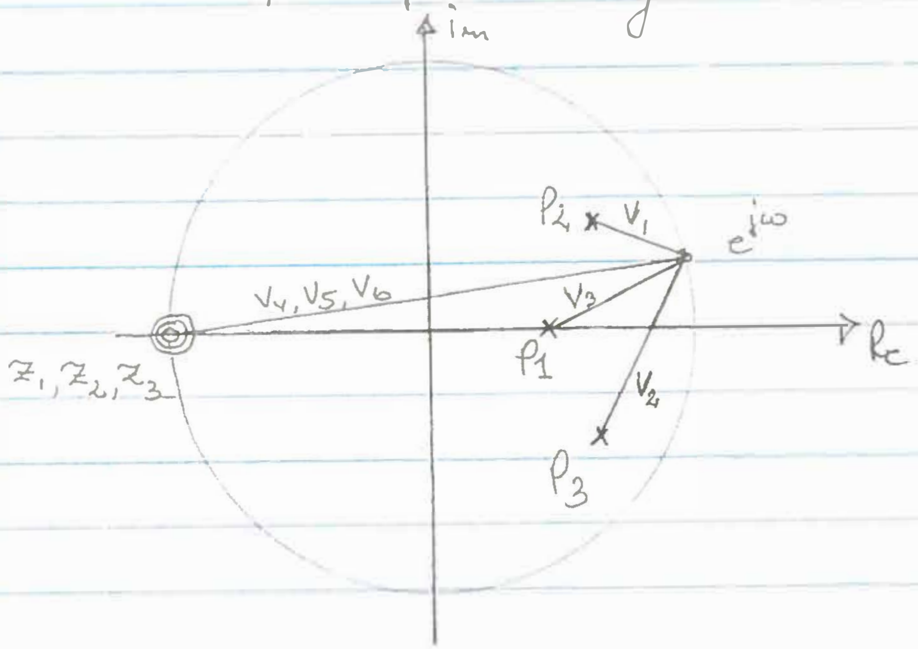
$$H(z) = 0.0317 \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1-1.4590z^{-1}+0.9104z^{-2}-0.1978z^{-3}}$$

Bestem poler/nulpunkter. (evt. vha. Matlab – roots)

- 3 nulpkt. i $z=-1$ (z_1, z_2, z_3)
- Reel pol i $z=0.4142$ (p_1)
- Kompleks konjugeret polpar
  $z = 0.5224 \pm j0.4524$ (p_2, p_3)

$$H(z) = 0.0317 \cdot \frac{(z+1)^3}{(z-0.4142)(z-(0.5224+j0.4524))(z-(0.5224-j0.4524))}$$

Pol/Nulpunkts diagram



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Vi søger nu et udtryk for amplitüderesponsen vha. vektorene $|\vec{V}_i|$, $i = \{1, \cdots, 6\}$

$$|H(e^{j\omega})| = 0.0317 \cdot \frac{|V_4| \cdot |V_5| \cdot |V_6|}{|V_1| \cdot |V_2| \cdot |V_3|} = 0.0317 \frac{|V_4|^3}{|V_1| \cdot |V_2| \cdot |V_3|}$$

Vi ønsker at lave et program, som kan ūdtegne amplitüderesponsen i frekvens-intervallet $0..\pi$. Derfor opstilles først ūdtryk for modulus af de fire vektorer.

$$|V_4| = \sqrt{(\cos\omega + 1)^2 + (\sin\omega)^2}$$

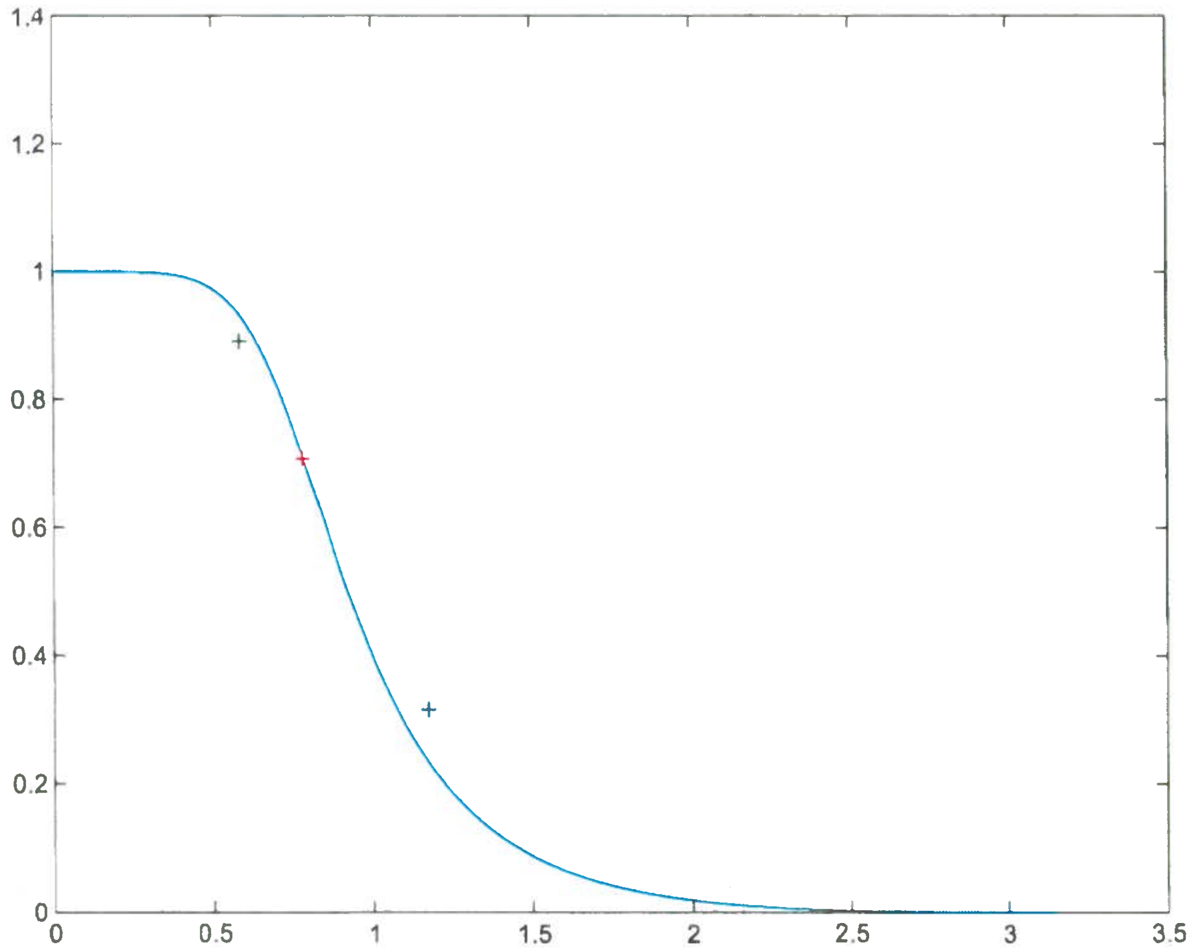
$$|V_1| = \sqrt{(\cos\omega - 0.5224)^2 + (\sin\omega - 0.4524)^2}$$

$$|V_2| = \sqrt{(\cos\omega - 0.5224)^2 + (\sin\omega + 0.4524)^2}$$

$$|V_3| = \sqrt{(\cos\omega - 0.4142)^2 + (\sin\omega)^2}$$

```
FOR i = 0..π STEP π/1000
    |V4| = ... ;
    |V1| = ... ;
    |V2| = ... ;
    |V3| = ... ;
    h(i) = 0.0317 · |V4|^3 / (|V1|·|V2|·|V3|) ;
ENDFOR;
```

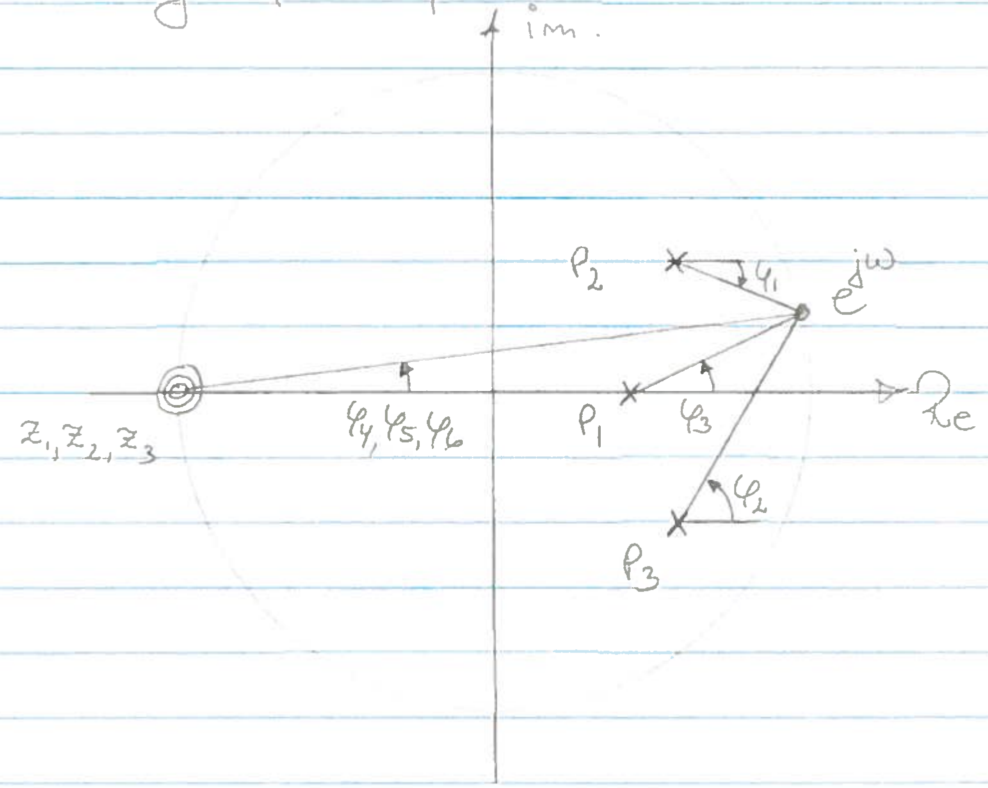
```
% Dette MATLAB-program beregner amplituderesponsen
% af H(z) fremkommet ved bilineær transformation.

clear;

% Frekvens-sweep
for i=0:999,
  omega(i+1) = pi*i/999;
end;

% For hver værdi af omega beregnes amplituden.
for i=1:1000,
  % Først beregnes længden af vektorerne
  v4 = sqrt((cos(omega(i)) + 1)^2 + (sin(omega(i)))^2);
  v1 = sqrt((cos(omega(i)) - 0.5224)^2 + (sin(omega(i)) - 0.4524)^2);
  v2 = sqrt((cos(omega(i)) - 0.5224)^2 + (sin(omega(i)) + 0.4524)^2);
  v3 = sqrt((cos(omega(i)) - 0.4142)^2 + (sin(omega(i)))^2);
  % Herefter bestemmes amplituden
  h(i) = 0.0317 * ((v4)^3)/(v1 * v2 * v3);
end;

% og til slut plottes amplituderesponsen sammen med specifikationerne
plot(omega,h,omega(187),0.8913,'+',omega(250),1/sqrt(2),'+',omega(375),0.3162,'+');
```



1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5

PK

Beregn faserespons:



Vi søger nu et udtryk for fase responsen vha. vinklerne φ_i, $i = \{1..6\}$.

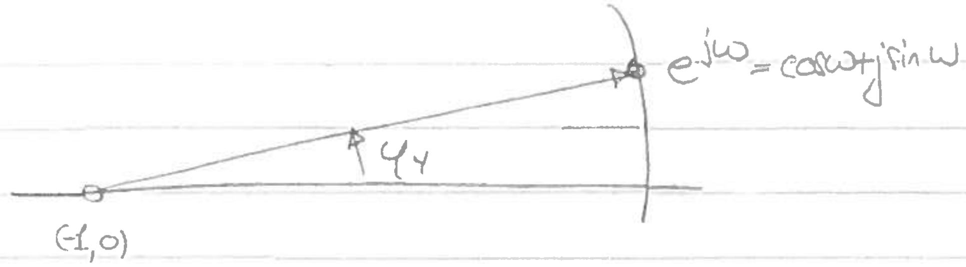
$$\underline{\angle H(e^{j\omega})} = \text{Arg}(0.0317) + \sum_{i=4}^{6} \varphi_i - \sum_{j=1}^{3} \varphi_j = 3 \cdot \varphi_4 - \sum_{j=1}^{3} \varphi_j$$

Argumentet fra nulpunkterne

Argumentet fra polerne

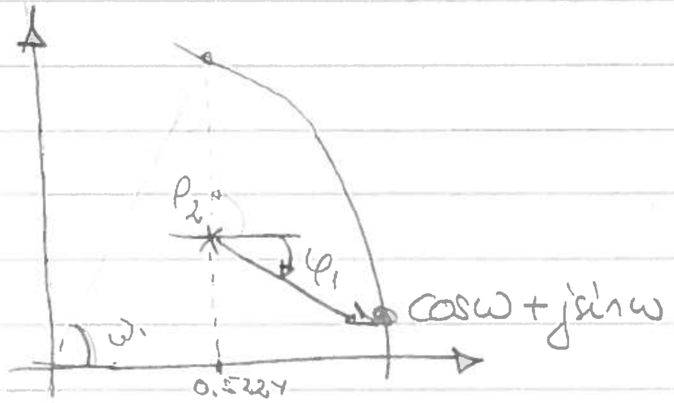
Vi ønsker at lave et program, som kan udtegne faseresponsen i frekvensintervallet $0..\pi$. Derfor opstilles først udtryk for de fire vinkler

φ_4



$$\tan\varphi_4 = \frac{\sin\omega}{\cos\omega + 1} \Rightarrow \varphi_4 = \arctan\left(\frac{\sin\omega}{\cos\omega+1}\right)$$

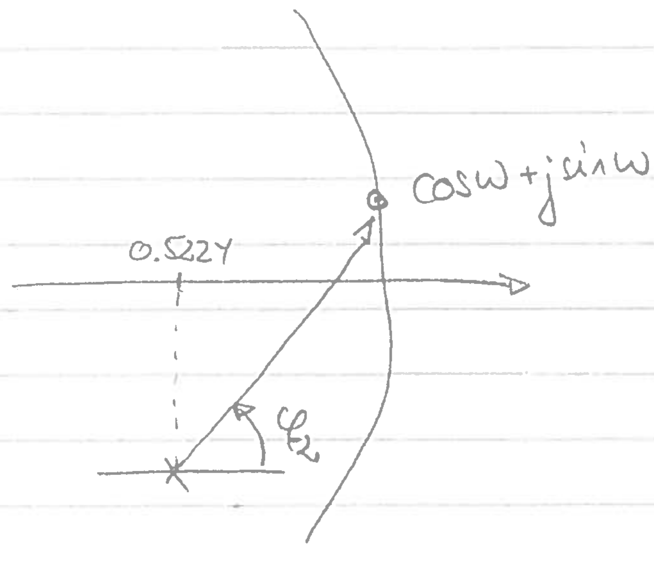
φ_1



$$\tan\varphi_1 = \frac{\sin\omega - 0.4524}{\cos\omega - 0.5224} \Rightarrow \varphi_1 = \arctan\left(\frac{\sin\omega - 0.4524}{\cos\omega - 0.5224}\right)$$

OBS: bemærk at φ_1 „springer" $-\pi$ når ω passerer $\arccos(0.5224)$!

φ_2



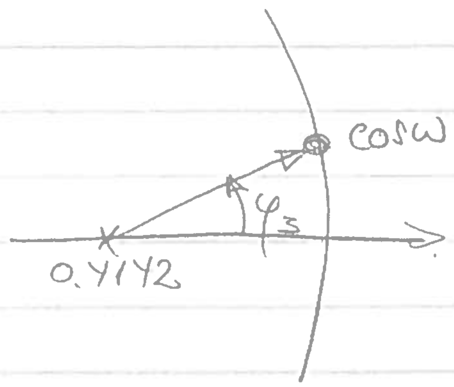
$$\tan\varphi_2 = \frac{\sin\omega + 0.4524}{\cos\omega - 0.5224}$$

$$\Downarrow$$

$$\varphi_2 = \arctan\left(\frac{\sin\omega + 0.4524}{\cos\omega - 0.5224}\right).$$

OBS. φ_2 springer $-\pi$ for $\omega = \arccos(0.5224)$

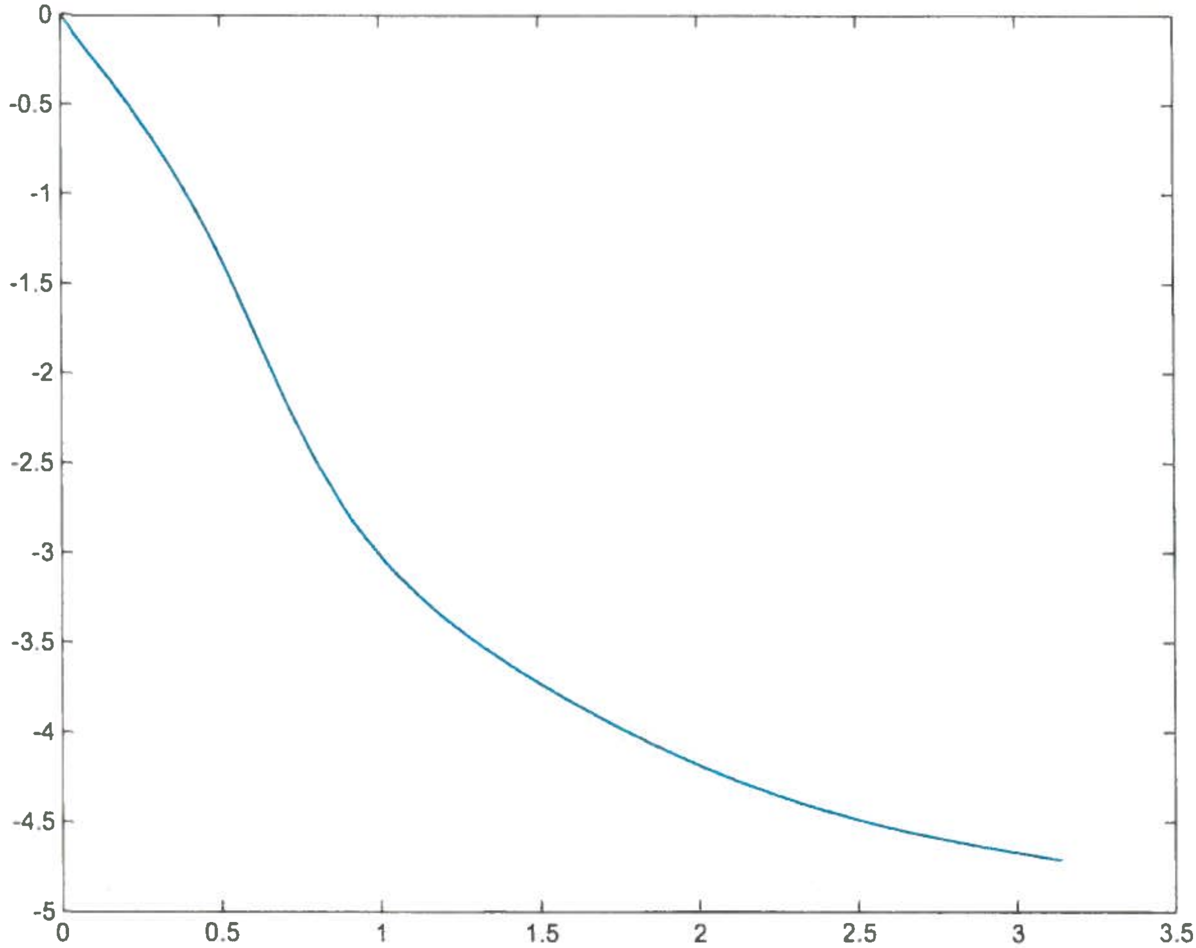
φ_3



$$\tan\varphi_3 = \frac{\sin\omega}{\cos\omega - 0.4142} \Rightarrow \varphi_3 = \arctan\left(\frac{\sin\omega}{\cos\omega - 0.4142}\right)$$

PROGRAMMER i MATLAB!

```
% Dette MATLAB-program beregner amplituderesponsen
% af H(z) fremkommet ved bilineær transformation.

clear;

% Frekvenssweep i intervallet 0..pi, 1000 samples
for i=1:1000;
  w(i) = (pi/1000)*i;
end;

% Beregning af vinklen hidrørende fra nulpunkterne
for i=1:999,
  phi4(i) = atan(sin(w(i))/(cos(w(i))+1));
end;
phi4(1000)=pi/2;

% Beregning af vinklen fra pol i 0.5224+j0.4524
for i=1:1000,
  if w(i) < acos(0.5224), % Der tages højde for spring i Arctan
     phi1(i) = atan(sin(w(i)-0.4524)/(cos(w(i))-0.5224));
  else
     phi1(i) = atan(sin(w(i)-0.4524)/(cos(w(i))-0.5224)) + pi;
  end
end;

% Beregning af vinklen fra pol i 0.5224-j0.4524
for i=1:1000,
  if w(i) < acos(0.5224), % Der tages højde for spring i Arctan
     phi2(i) = atan(sin(w(i)+0.4524)/(cos(w(i))-0.5224));
  else
     phi2(i) = atan(sin(w(i)+0.4524)/(cos(w(i))-0.5224)) + pi;
  end
end;

% Beregning af vinklen fra pol i 0.4142
for i=1:1000,
  if w(i) < acos(0.4142), % Der tages højde for spring i Arctan
     phi3(i) = atan(sin(w(i))/(cos(w(i))-0.4142));
  else
     phi3(i) = atan(sin(w(i))/(cos(w(i))-0.4142)) + pi;
  end
end;

%Beregning af den samlede fasevinkel
for i=1:1000,
  vinkel(i)=3*phi4(i) - (phi1(i)+phi2(i)+phi3(i));
end;

plot(w,vinkel);
```


0
-0.5
-1
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-2
-2.5
-3
-3.5
-4
-4.5
-5
0
0.5
1
1.5
2
2.5
3
3.5

PK.

Opg 2 Frekvenstransformation af LP-filter.

$$H_{LP}(Z) = 0.0317 \frac{b_0 + b_1 Z^{-1} + b_2 Z^{-2} + b_3 Z^{-3}}{1 + a_1 Z^{-1} + a_2 Z^{-2} + a_3 Z^{-3}}$$

Fra tabel 1 side 553 har vi, at

$$H_{HP}(z) = H_{LP}(Z) \Big|_{Z^{-1} = -\frac{z^{-1}+\alpha}{1+\alpha z^{-1}}}$$

og $$\alpha = -\frac{\cos\left(\frac{\Theta_p + \omega_p}{2}\right)}{\cos\left(\frac{\Theta_p - \omega_p}{2}\right)}$$

hvor Θ_p er lavpasfilterets 3 dB-frekvens og ω_p er den ønskede 3 dB-frekvens for højpasfilteret.

$$\Theta_p = \frac{\pi}{4} \quad \text{og} \quad \omega_p = \frac{3\pi}{4}$$

$\Downarrow$

$$\alpha = -\frac{\cos\left(\frac{\frac{\pi}{4}+\frac{3\pi}{4}}{2}\right)}{\cos\left(\frac{\frac{\pi}{4}-\frac{3\pi}{4}}{2}\right)} = -\frac{\cos\frac{\pi}{2}}{\cos(-\frac{\pi}{4})} = 0$$

$\Downarrow$

$$Z^{-1} = -z^{-1}$$

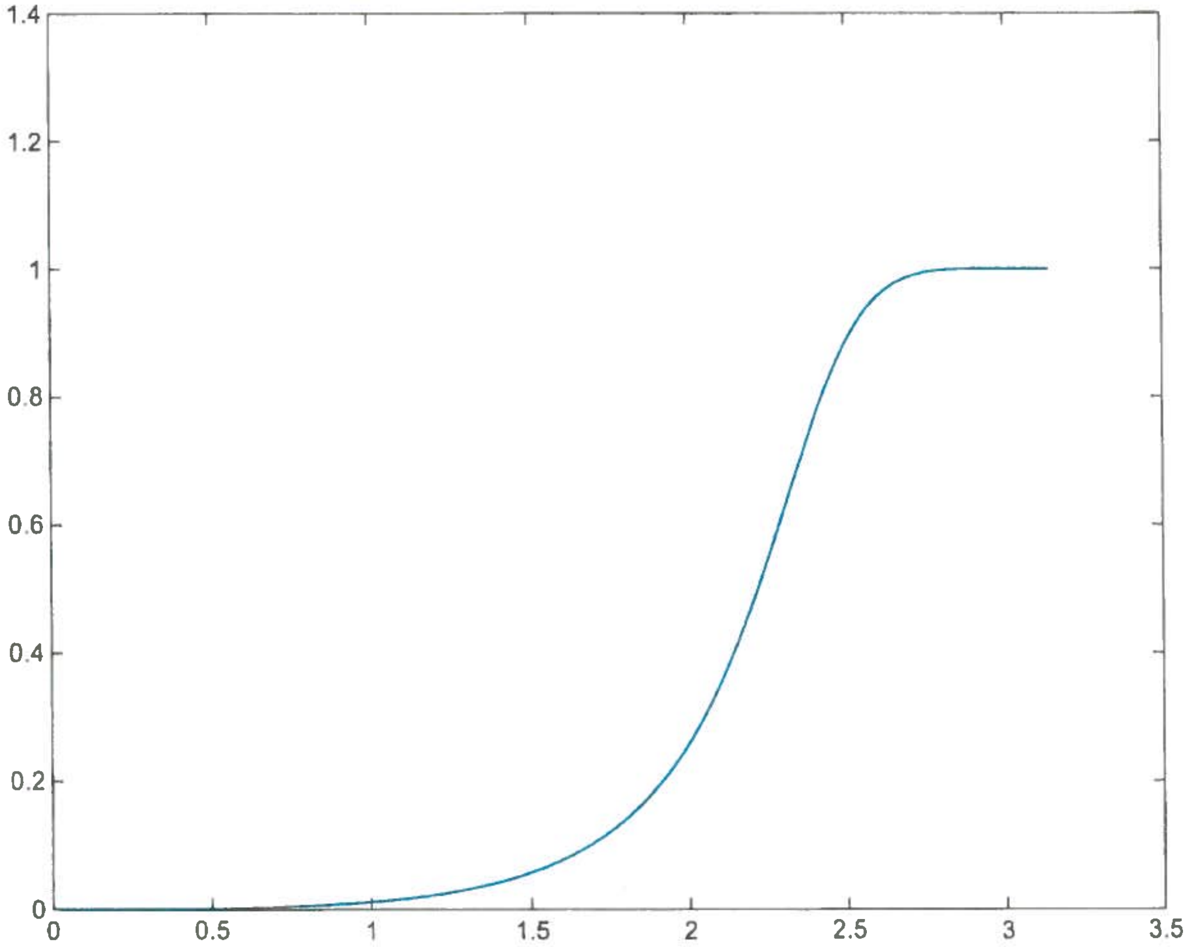
PIC

$$H_{HP}(z) = G \cdot \frac{b_0 - b_1 z^{-1} + b_2 z^{-2} - b_3 z^{-3}}{1 - a_1 z^{-1} + a_2 z^{-2} - a_3 z^{-3}}$$

$\Downarrow$

$$H_{HP}(z) = 0.0317 \cdot \frac{1 - 3z^{-1} + 3z^{-2} - z^{-3}}{1 + 1.4590z^{-1} + 0.9104z^{-2} + 0.1978z^{-3}}$$

Foretag frekvensanalyse vha pol/nulpkt - diagram — eller direkte vha Matlab - funktionen FREQZ.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