2020-09-16/MS

# Signal Processing, #3

**Topics:**

- Frequency transformations: LP → HP, LP → BP and LP → BS
- Sensitivity analysis

**Literature:**

Kendal Su: "Analog Filters", Kluwer Academic Publishers, 2nd ed. 2002, ISBN 1-4020-7033-0
The book is available in electronic form at http://www.en.aub.aau.dk.

| Topic | Pages ({*} ⇔ supplementary lit.) |
|---|---|
| Inverse Chebyshev characteristic | 37-38 |
| Bessel/Thomson characteristic | {66-71} |
| Frequency transformations | 77-88 |
| Sensitivity analysis | 171-184 |

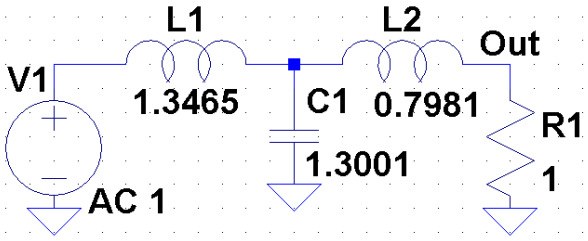
Supplementary: Lecture presentation "slides"

**Exercises:**

**3.1**

A Chebyshev filter must pass frequencies above 1 kHz (not rad/s) with max. 0.5 dB attenuation and must attenuate frequencies below 100 Hz with at least 60 dB.

a. Make a rough sketch of the filter requirements.
b. Find the necessary filter order, n, using the HP<->LP frequency mapping, and the "n = " equation from the slides from lecture 2
c. Find (analytically) the actual attenuation at 100 Hz.
d. Check the result in c. by making a plot in Matlab.
e. A normalized 3rd order Chebyshev LP-filter with 0.5 dB ripple can be made using the circuit shown with:
   $L_1$ = 1.3465 H
   $C_2$ = 1.3001 F
   $L_3$ = 0.7981 H
   Find the component values in a 1 kHz HP-filter.




**3.2**

A BP-filter made from a 4th order LP-prototype has:

- Butterworth characteristic
- Lower passband edge (-3 dB) = 10 kHz
- Upper passband edge (-3 dB) = 15 kHz

a. Find (analytically) the attenuation at 1 kHz and 20 kHz using the frequency transformation and $|H(j\omega)|^2$ for the low-pass prototype.
b. Check the result in a. by making a plot in Matlab.

**3.3**

Use Matlab to plot the step response of 5th order low-pass filters:

- Butterworth ($\omega_{-3dB}$ = 1 rad/s)
- Elliptic with 1 dB passband ripple and 40 dB stopband attenuation. It must be re-normalized to have a 3 dB cut-off radian frequency of 1 rad/s to make a fair comparison.

a. Make a Bode-plot to check the re-normalization.
b. Plot step-responses

**3.4**

A high-pass filter section has the transfer function:

$$H(s) = -\frac{\frac{C_1}{C_2}s^2}{s^2 + \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3}s + \frac{1}{R_1 R_2 C_2 C_3}}$$

The capacitor values are:

- $C_1$ = 10 nF, $C_2$ = 15 nF and $C_3$ = 15 nF

a. Find an expression for Q as a function of the component values.
b. Find the sensitivity of Q with respect to $C_1$, $S_{C_1}^{Q}$

**Results:**

**3.1**
a.
b. *3*
c. *62.8 dB*
d.
e. *118 μF, 122 μH, 199 μF*

**3.2**
a. *118 dB, 31.8 dB*
b.

**3.3**
a.
b.

**3.4**
c.
d. -0.25