AALBORG UNIVERSITY
DENMARK

Written exam in
Signal processing, 5 ECTS

Tuesday January 7, 2020
9.00 – 13.00

Read carefully:

- Remember to write your **full name on every sheet** you return!
- Write **legible** with a pen that allows your answers to be scanned electronically.
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets.**
- Problems are weighted as listed.
- Results **without sufficient explanations will not give full credits**!
- Grading is dependent on the number of correct answers but also on the depth as well as width of the answers. You should demonstrate knowledge in all three main subjects: analog filters, digital filters and spectral estimation
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times.
- **Communication with others is strictly prohibited**.

EIT5/ITC5 Signal Processing / Analog Filters

Note: You may use the following equations:

$$\cosh(x) = \frac{e^x + e^{-x}}{2},\ \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right),\ \sinh(x) = \frac{e^x - e^{-x}}{2},\ \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right).$$

Problem A.1 (Weight 9 % - Analog Filters)

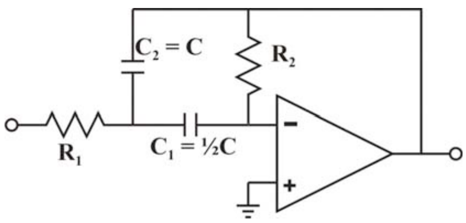
A normalized low-pass filter should have:

- 5th order Chebyshev characteristic
- A ripple-bandwidth of 1 rad/s
- At least 60 dB attenuation at $\omega_S = 5$ rad/s

a. Make a sketch of the attenuation function.
b. Find the pass-band ripple (in dB) that corresponds to these requirements.
c. Find the frequency where the attenuation is 40 dB.

Problem A.2 (Weight 14 % - Analog Filters)

In the (MFB-) bandpass biquad section shown, the two capacitors are chosen to:



- $C_1 = ½C = 1$ nF
- $C_2 = C = 2$ nF

The transfer function is shown in the equation: $$H_{MFB-BP}(s) = -\frac{\frac{1}{R_1 C} s}{s^2 + \frac{3}{CR_2} s + \frac{2}{R_1 R_2 C^2}}$$

The requirements for the section are:

- Centre frequency $f_0 = 8$ kHz ($\omega_0 = 2\pi \cdot 8 \cdot 10^3$ rad/s)
- Q-value = 5

a. Find the values of R_1 and R_2.
b. Find the gain (in dB) at the center frequency, f_0.
c. Find the sensitivity of Q with respect to variations in R_2 ($S^Q_{R_2}$).

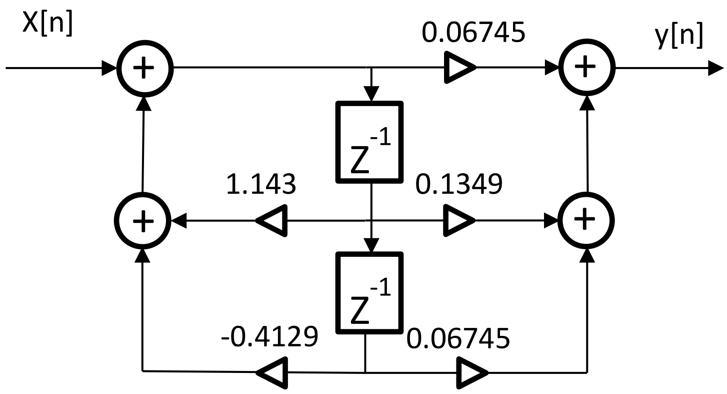
Problem A.3 (Weight 10% - Analog Filters)

A band-pass filter should have:

- Butterworth characteristic
- Center radian frequency (0 dB loss) at $\Omega_0 = 1000$ rad/s
- 3-dB radian bandwidth B = 400 rad/s
- At least 40 dB attenuation at $\Omega_S = 800$ rad/s

a. Make a rough sketch of the filter requirements and calculate the band-edge radian frequencies, Ω_1 and Ω_2 (where the attenuation is 3 dB).
b. Using the normalized low-pass Butterworth function and the LP-BP frequency mapping, find the necessary filter order to obtain the required 40 dB attenuation at $\Omega_S = 800$ rad/s.

Problem B.1 (weighted with 12% - Digital filters)
A discrete time filter is defined by the flow graph below:



The sampling frequency is 8kHz.

Questions:

a) Find the transfer function H(z)
b) Determine the expression for y[n]
c) Draw a pole-zero diagram with all poles and zeros.
d) Make an approximate sketch of the amplitude response using the knowledge of the location of poles and zeros
e) Adjust the amplitude gain of the filter to ensure a DC gain of 3dB

Problem B.2 (weighted with 10% - Digital filters)
Design a <u>low-pass</u> finite-impulse response (FIR) filter using the window method. The specifications for the filter are:

Sampling frequency: 4000 Hz   Cutoff frequency: 250 Hz
Order: 5   Window: Hamming
Passband gain: 0 dB

Questions:

a) Compute the filter coefficients
b) Determine the transfer function H(z)
c) Compute the filter's amplitude response at DC (i.e. 0 Hz)
d) Plot the phase response for the filter

Problem B.3 (weighted with 12% - Digital filters)
An analog filter is defined by:

$$H(s) = \frac{800}{800 + s}$$

Transform the analog filter into a digital filter by applying the bi-linear transformation. The sampling frequency is fs=1000Hz.

Questions:

a) Compute the -3dB frequency for the analog filter in Hz.
b) Use the bi-linear transformation to find the transfer function H(z) (remember to pre-warp the cut off frequency)
c) Determine the amplitude response for H(z) at the frequency corresponding to 800 rad/s.

Problem C.1 (weighted with 8% - Spectral estimation)
Consider the following 8-point signals $x[n], 0 \leq n \leq 7$.

Signal 1: [1,1,1,0,0,0,1,1]
Signal 2: [1,1,0,0,0,0,-1,-1]
Signal 3: [0,1,1,0,0,0,-1,-1]
Signal 4: [0,1,1,0,0,0,1,1]

a) Which of these signals have a real-valued 8-point DFT?
b) Which of these signals have an imaginary-valued 8-point DFT?

Do not use any computer to solve this problem and do not explicitly compute the DFT; instead use the properties of the DFT.

Problem C.2 (weighted with 8% - Spectral estimation)
The first 5 points of the 8-point DFT of a real-valued finite length sequence (of length 8) are

$$\{0.25,\ 0.125 - j0.3018,\ 0,\ 0.125 - j0.0518,\ 0\}$$

Determine the remaining 3 points.

Hint: Use symmetry properties of the Discrete Fourier Transform.

Problem C.3 (weighted with 8% - Spectral estimation)
If a 512-point radix-2 FFT takes about 50 microseconds on a particular computer, how long would you expect a 2048-point radix-2 FFT to take on the same computer?

Problem C.4 (weighted with 9% - Spectral estimation)

A 10 millisecond segment of a signal is sampled at a rate of 10 kHz and the resulting samples are saved. It is desired to compute the spectrum of that segment as 128 equally spaced frequencies covering the range from 2.5 kHz to 5 kHz. We would like to use a Fast Fourier Transform (FFT) algorithm to perform this computation. The algorithm takes as input an N-dimensional vector x of time samples. Its output is an N-dimensional vector X

a) What value of N should we use?
b) How is the algorithm's input vector defined in terms of the time samples we collected?
c) Exactly what DFT indices k and DFT values X[k] corresponds to the 128 spectral values correspond to the 128 spectral values that we wish to compute?