2020-09-16/MS

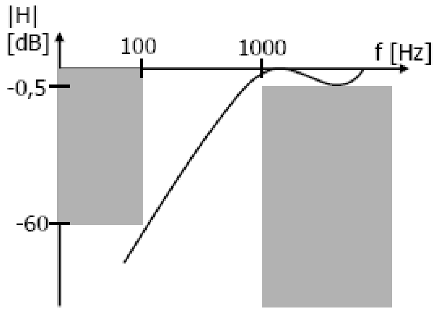
EIT/ITC5 Signal Processing, #3

Suggested solutions to exercises:

3.1

A Chebyshev filter must pass frequencies above 1 kHz (not rad/s) with max. 0.5 dB attenuation and must attenuate frequencies below 100 Hz with at least 60 dB.

a. Make a rough sketch of the filter requirements.



b. Find the necessary filter order, n, using the HP<->LP frequency mapping, and the "n = " equation from the slides from lecture 2

$$|H_{HP}(j\Omega)| = \left|H_{LPP}\left(\frac{-\Omega_0}{\Omega}\right)\right| = \left|H_{LPP}\left(\frac{2\pi f_0}{2\pi f}\right)\right| = \left|H_{LPP}\left(\frac{1000}{100}\right)\right|$$

$$n \geq \frac{1}{\cosh^{-1}(10)} \cosh^{-1}\sqrt{\frac{10^{60/10}-1}{10^{0,5/10}-1}} = 2.89 \rightarrow 3$$

c. Find (analytically) the actual attenuation at 100 Hz.

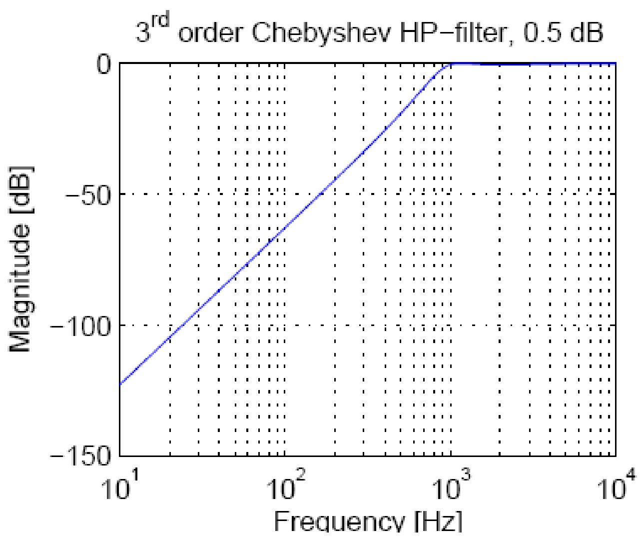
$$\varepsilon = \sqrt{10^{0,5/10}-1} = 0.3439$$

$$C_3(10) = \cosh(3 \cdot \cosh^{-1}(10)) = 3970$$

$$-H(j10)_{dB} = 10 \cdot \log(1+\varepsilon^2 C_3^2(10)) = 62.8\ dB$$

d. Check the result in c. by making a plot in Matlab.

```
[NumPoly DenomPoly] = cheby1(3,0.5,2*pi*1e3,'high','s');
sys = tf(NumPoly,DenomPoly);
bode(sys,{2*pi*10,2*pi*10e3});
grid;  % Right click on axis > Properties > Units > Hz
title('3^r^d order Chebyshev HP-filter, 0.5 dB');

% Alternative:
% freq = logspace(1,4,2000);
% om = 2*pi*freq;
% jom = j*om;
% H = poly-
val(NumPoly,jom)./polyval(DenomPoly,
jom);
% H_dB = 20*log10(abs(H));
%
% semilogx(freq,H_dB);
% grid;
% xlabel('Frequency [Hz]');
% ylabel('Magnitude [dB]');
% title('3^r^d order Chebyshev HP-
filter, 0.5 dB');
% H_dB_100Hz =
H_dB(find(freq>100,1))
```



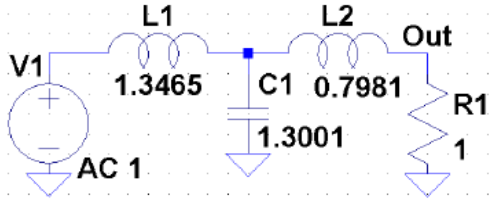
e. A normalized 3rd order Chebyshev LP-filter with 0.5 dB ripple can be made using the circuit shown with:

L_1 = 1.3465 H

C_2 = 1.3001 F

L_3 = 0.7981 H

Find the component values in a 1 kHz HP-filter.

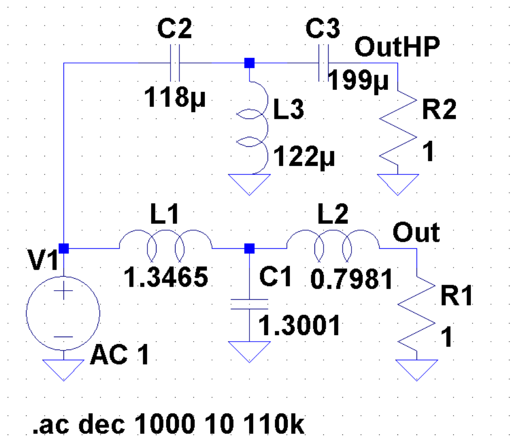


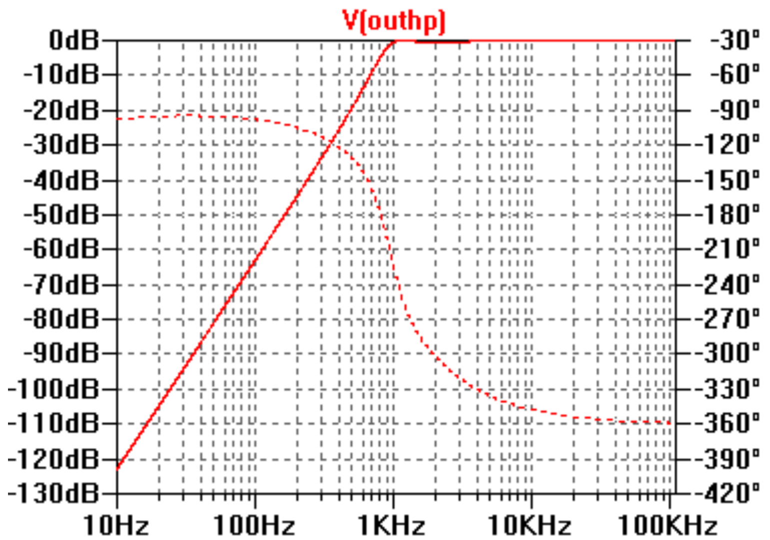
$$C_{HP1} = \frac{1}{\Omega_0 L_1} = \frac{1}{2\pi 10^3 \cdot 1.3465} F = 118\ \mu F$$

$$L_{HP2} = \frac{1}{\Omega_0 C_2} = \frac{1}{2\pi 10^3 \cdot 1.3001} H = 122\ \mu H$$

$$C_{HP3} = \frac{1}{\Omega_0 L_3} = \frac{1}{2\pi 10^3 \cdot 0.7981} F = 199\ \mu F$$

LT-spice simulation (not required):





3.2

A BP-filter made from a 4th order LP-prototype has:

- Butterworth characteristic
- Lower passband edge (-3 dB) = 10 kHz
- Upper passband edge (-3 dB) = 15 kHz

a. Find (analytically) the attenuation at 1 kHz and 20 kHz using the frequency transformation and $|H(j\omega)|^2$ for the low-pass prototype.

$$\Omega_1 = 2\pi 10^4\ rad/s \quad \Omega_2 = 2\pi \cdot 15 \cdot 10^3\ rad/s$$

$$B = \Omega_2 - \Omega_1 = 2\pi \cdot 5 \cdot 10^3\ rad/s$$

$$\Omega_0 = \sqrt{\Omega_1 \Omega_2} = 2\pi \cdot 12.25 \cdot 10^3\ rad/s$$

$$\left|H_{BP}(j2\pi 10^3)\right| = \left|H_{LPP}\left(\frac{-(2\pi 10^3)^2 + (2\pi \cdot 12.25 \cdot 10^3)^2}{j2\pi 10^3 2\pi \cdot 5 \cdot 10^3}\right)\right| = \left|H_{LPP}\left(\frac{-10^6 + (12.25 \cdot 10^3)^2}{j10^3 \cdot 5 \cdot 10^3}\right)\right| = \left|H_{LPP}(-j29.81)\right|$$

$$-H_{LPP,Butter4,dB}(-j29.81) = 10 \cdot \log(1 + 29.81^{2 \bullet 4}) = 118.0\ dB$$

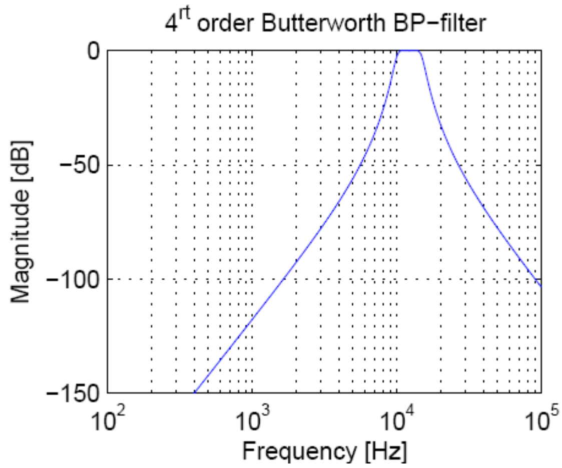
$$\left|H_{BP}(j2\pi 20 \cdot 10^3)\right| = \left|H_{LPP}\left(\frac{-(20 \cdot 10^3)^2 + (12.25 \cdot 10^3)^2}{j20 \cdot 10^3 \cdot 5 \cdot 10^3}\right)\right| = \left|H_{LPP}(j2.50)\right|$$

$$-H_{LPP,Butter4,dB}(j2.50) = 10 \cdot \log(1 + 2.50^{2 \cdot 4}) = 31.8\ dB$$

b. Check the result in a. by making a plot in Matlab.

```
[NumPoly DenomPoly] = butter(4,2*pi*[10 15]*1e3,'bandpass','s');
sys = tf(NumPoly,DenomPoly);
bode(sys,{pi*2e3, pi*2e5});   % Right click on axis > Properties > Units > Hz
title('4^t^h order Butterworth BP-filter');
```

```
grid;
% ALTERNATIVE:
% [NumPoly DenomPoly] = butter(4,2*pi*[10 15]*1e3,'bandpass','s');
% freq = logspace(2,5,20000);
% om = 2*pi*freq;
% jom = j*om;
% H = poly-
val(NumPoly,jom)./polyval(DenomPoly,jom);
% H_dB = 20*log10(abs(H));
%
% semilogx(freq,H_dB);
% grid;
% xlabel('Frequency [Hz]');
% ylabel('Magnitude [dB]');
% title('4^t^h order Butterworth BP-
filter');
% H_dB_1kHz = H_dB(find(freq>1000,1))
% H_dB_20kHz = H_dB(find(freq>20000,1))
% set(gca,'Ylim',[-150 0]);
```



3.3

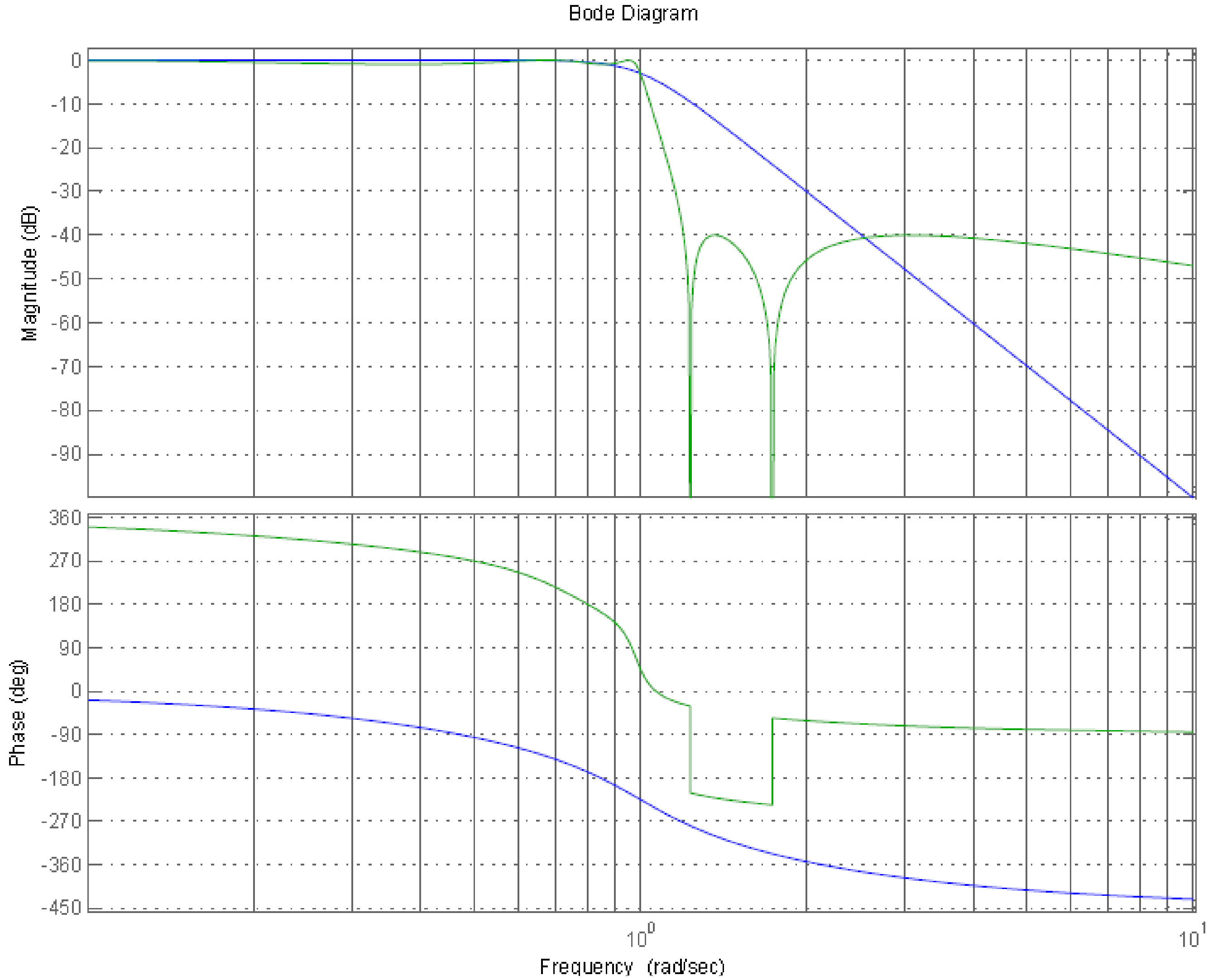
Use Matlab to plot the step response of 5th order low-pass filters:

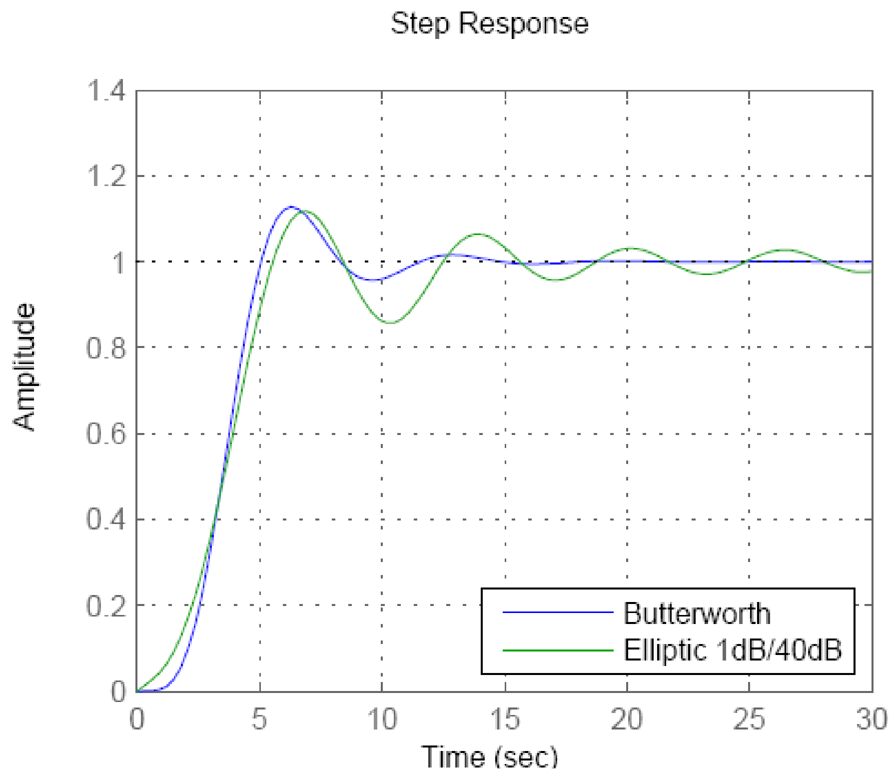
- Butterworth (ω_{-3dB} = 1 rad/s)
- Elliptic with 1 dB passband ripple and 40 dB stopband attenuation. It must be re-normalized to have a 3 dB cut-off radian frequency of 1 rad/s to make a fair comparison.

a. Make a Bode-plot to check the re-normalization.

b. Plot step-responses

```
Order = 5;                   % Filter order
Rip_dB = 1;                  % Passband ripple (not Butterworth)
Stop_dB = 40;                % Stopband attenuation (not Butterworth)

% Butterworth:
[NumPoly DenomPoly] = butter(Order,1,'s');    % Numerator/denominator polyno-
mials
sysBut = tf(NumPoly, DenomPoly);

% First, the elliptic filter is found for a ripple bandwidth of 1 rad/s:
[NumPoly DenomPoly] = ellip(Order,Rip_dB,Stop_dB,1,'s');
sysElli = tf(NumPoly, DenomPoly);
[Mag Phase Omeg] = bode(sysElli,1:0.001:1.2);   % Fine frequency resolution
Mag = squeeze(Mag);                             % Remove excessive dimensions
om3dB = Omeg(find(Mag<1/sqrt(2),1));
% Then find the re-scaled transfer function:
[NumPoly DenomPoly] = ellip(Order,Rip_dB,Stop_dB,1/om3dB,'s');
sysElli = tf(NumPoly, DenomPoly);

% Bode-plots:
figure(1);
bode(sysBut,sysElli,{0.1,10});
grid;

% Step response
figure(2);
step(sysBut,sysElli,30);
grid;
legend('Butterworth','Elliptic 1dB/40dB','Location','SouthEast');
```


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
Phase (deg)
360
270
180
90
0
-90
-180
-270
-360
-450
10^0
10^1
Frequency (rad/sec)


Step Response
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
Time (sec)
Butterworth
Elliptic 1dB/40dB

3.4

A high-pass filter section has the transfer function:

$$H(s) = -\frac{\frac{C_1}{C_2}s^2}{s^2 + \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3}s + \frac{1}{R_1 R_2 C_2 C_3}}$$

The capacitor values are:

- C_1 = 10 nF, C_2 = 15 nF and C_3 = 15 nF

a. Find an expression for Q as a function of the component values.
b. Find the sensitivity of Q with respect to C_1, $S_{C_1}^{Q}$

a. Comparing the transfer function with the standard second order function [K.S. (10.15)] gives:

$$\omega_0^2 = \frac{1}{R_1 R_2 C_2 C_3} \quad \wedge \quad \frac{\omega_0}{Q} = \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3} \quad \Rightarrow$$

$$Q = \frac{\sqrt{R_2 C_2 C_3}}{\sqrt{R_1}(C_1 + C_2 + C_3)}$$

b. Using the rules [K.S. (8.5) & (8.8) or mm.4.slide.15ff]:

$$S_x^{k\cdot y} = S_x^y \quad S_x^{1/y} = -S_x^y$$

You obtain:

$$\underline{\underline{S_{C_1}^{Q}}} = -S_{C_1}^{(C_1+C_2+C_3)} = -\frac{C_1}{(C_1 + C_2 + C_3)} \cdot \frac{d(C_1 + C_2 + C_3)}{dC_1} = \frac{-C_1}{(C_1 + C_2 + C_3)} = \frac{-10}{10 + 15 + 15} = \underline{\underline{-0.25}}$$