AALBORG UNIVERSITY
DENMARK

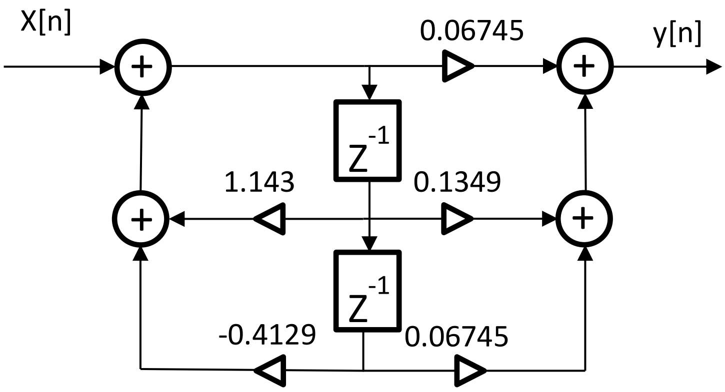
Written exam in
Signal processing, 5 ECTS

Monday, January 7, 2020
9.00 – 13.00

Read carefully:

- Remember to write your **full name on every sheet** you return!
- Write **legible** with a pen that allows your answers to be scanned electronically.
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets.**
- Problems are weighted as listed.
- Results **without sufficient explanations will not give full credits**!
- Grading is dependent on the number of correct answers but also on the depth as well as width of the answers. You should demonstrate knowledge in all three main subjects: analog filters, digital filters and spectral estimation
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times.
- **Communication with others is strictly prohibited**.

Problem B.1 (weighted with 12% - Digital filters)
A discrete time filter is defined by the flow graph below:



The sampling frequency is 8kHz.

Questions:

a) Determine the transfer function H(z)

- We introduce a variable v[n] which is located between x[n] and y[n]:
    - $y[n] = 0.06745v[n] + 0.1349v[n-1] + 0.06745v[n-2]$
    - $v[n] = x[n] + 1.143v[n-1] - 0.4129[n-2]$

$$\frac{Y[z]}{V[z]} = 0.06745 + 0.1349z^{-1} + 0.06745z^{-2}$$

$$\frac{V[z]}{X[z]} = \frac{1}{1 - 1.143z^{-1} + 0.4129z^{-2}}$$

$$H[z] = \frac{Y[z]}{V[z]}\frac{V[z]}{X[z]} = \frac{0.06745 + 0.1349z^{-1} + 0.06745z^{-2}}{1 - 1.143z^{-1} + 0.4129z^{-2}}$$

b) Determine the expression for y[n]

- y[n]= 0.06745x[n]+0.1349x[n-1]+0.06745x[n-2]+ 1.143y[n-1]-0.4129y[n-2]

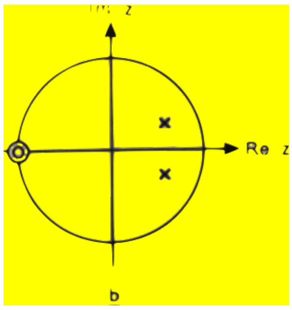
c) Draw a pole-zero diagram with all poles and zeros.

The poles and zeros are determined using Matlab's function. First we multiply H(z) with $z^{2/}z^{2}$ and get:

$$H[z] = \frac{0.06745z^2 + 0.1349z^1 + 0.06745}{z^2 - 1.143z^1 + 0.4129}$$

$$= \frac{0.06745(z+1)(z+1)}{(z-(0.5715+j0.2937))(z-(0.5715-j0.2937))}$$

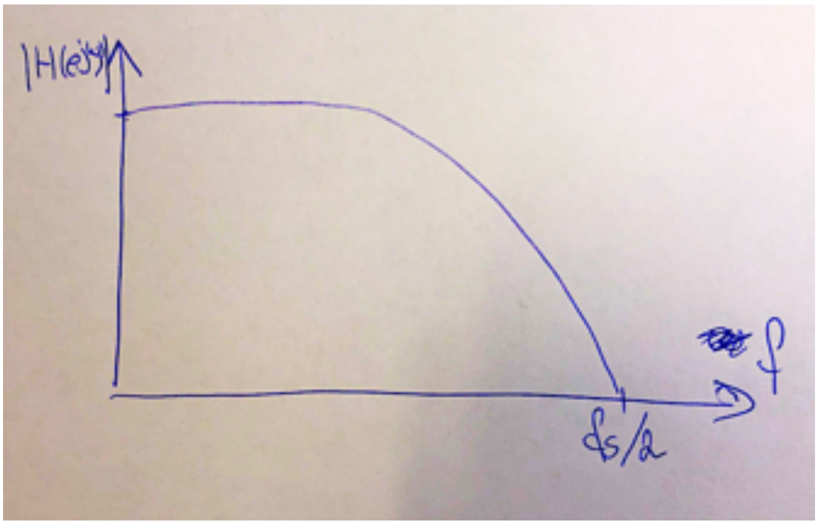
Zero: -1 (double zero)
Poles: 0.5715+/- j0.2937



d) Make an approximate sketch of the amplitude response using the knowledge of the location of poles and zeros

We look at the length of the vectors from poles/zeros to a specific points along the unit circle according to:

$$|H(z)| = \frac{|V_1||V_2|}{|V_3||V_4|}$$



e) Scale the filter to ensure a DC gain of 3dB

$$H(z) = \frac{0.06745 + 0.1349z^{-1} + 0.06745z^{-2}}{1 - 1.143z^{-1} + 0.4129z^{-2}}$$

$$H(e^{j\omega}) = \frac{0.06745 + 0.1349e^{-j\omega} + 0.06745e^{-2j\omega}}{1 - 1.143e^{-j\omega} + 0.4129e^{-2j\omega}}$$

$$H(e^{j\omega})\big|_{\omega=0} = \frac{0.06745 + 0.1349 + 0.06745}{1 - 1.143 + 0.4129} = \frac{0.2698}{0.2699} \approx 1$$

Hence the input x[n] should be multiplied with:

G=$\sqrt{2}$

Problem B.2 (weighted with 10% - Digital filters)
Design a low-pass finite-impulse response (FIR) filter using the window method. The specifications for the filter are:

Sampling frequency: 4000 Hz
Cutoff frequency: 250 Hz
Order: 5
Window: rectangular
Passband gain: 0 dB

Questions:

a) Compute the filter coefficients

$$\omega_c = \frac{2\pi f_c}{f_s} \Rightarrow \omega_c = \frac{2\pi 250}{4000} = \frac{\pi}{8}$$

$$h_d[n] = \frac{1}{2\pi}\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} e^{-\frac{j\omega M}{2}} e^{j\omega n}\, d\omega$$

$$h_d[n] = \frac{\sin\left(\frac{\pi}{8}\left(n-\frac{M}{2}\right)\right)}{\pi\left(n-\frac{M}{2}\right)}$$

For M=5

$$h_d[0] = h_d[5] = 0.1059$$

$$h_d[1] = h_d[4] = 0.1179$$

$$h_d[2] = h_d[3] = 0.1242$$

Hamming window:

$$w[n] = 0.5 - 0.46\cos\left(\frac{2\pi n}{M}\right) \qquad 0 \le n \le M$$

$$w[0] = w[5] = 0.0800$$
$$w[1] = w[4] = 0.3979$$
$$w[2] = w[3] = 0.9121$$

$$h = h_d * w$$

$$h[0] = h[5] = 0.00850$$
$$h[1] = h[4] = 0.0469$$
$$h[2] = h[3] = 0.1133$$

b) Determine the transfer function H(z)

$$H(z) = 0.0085 + 0.0469z^{-1} + 0.1133z^{-2} + +0.1133z^{-3} + 0.0469z^{-4} + 0.0085z^{-5}$$

c) Compute the filter's amplitude response at DC (i.e. 0 Hz)

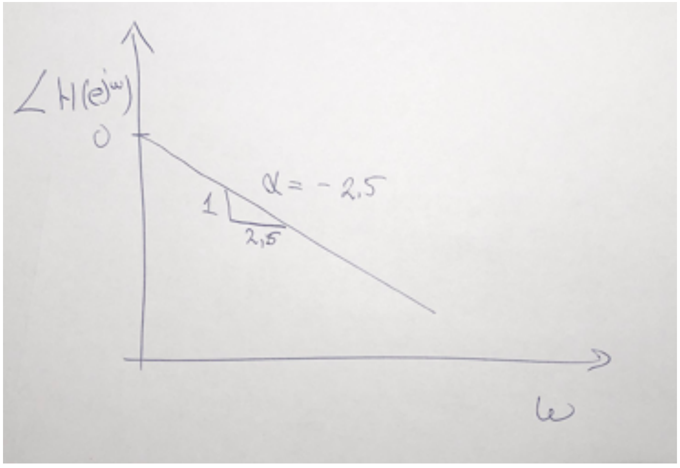
$$H(e^{j\omega}) = 0.0085 + 0.0469e^{-j\omega} + 0.1133e^{-2j\omega} + +0.1133e^{-3j\omega} + 0.0469e^{-4j\omega} + 0.0085e^{-5j\omega}$$

$$H(e^{j\omega})\big|_{\omega=0} = 0.0085 + 0.0469 + 0.1133\ + +0.1133 + 0.0469 + 0.0085$$
$$= 0.3373 = -9.4388dB$$

d) Plot the phase response for the filter

Phase response is given by:

$$\angle H(e^{j\omega}) = -\frac{M}{2} \Rightarrow \angle H(e^{j\omega}) = -2.5$$



Problem B.3 (weighted with 12% - Digital filters)

An analog filter is defined by:

$$H(s) = \frac{800}{800 + s}$$

Transform the analog filter into a digital filter by applying the bi-linear transformation. The sampling frequency is f_s=1000Hz.

- Compute the gain at 800 rad/s.

$$|H(s)|_{\Omega=800} = \frac{800}{800 + j800} = \frac{1}{\sqrt{2}} = -3dB$$

- Compute the -3dB frequency for the analog filter in Hz.

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow f = 127.3Hz$$

- Use the bi-linear transformation to find the transfer function H(z). (remember to pre-warp the cut off frequency)

$$\Omega_{pre} = \frac{2}{T_d}\tan\frac{\omega_c}{2} \Rightarrow \Omega_{pre} = 845\ rad/sec$$

$$H(z) = \frac{\Omega_{pre}}{\Omega_{pre} + \frac{2}{T_d}\frac{z-1}{z+1}}$$

$$H(z) = \frac{845(z+1)}{845(z+1) + 2000(z-1)}$$

$$H(z) = \frac{845(z+1)}{845(z+1) + 2000(z-1)}$$

$$H(z) = \frac{845(z+1)}{2845z - 1155} = \frac{0.2970(1+z^{-1})}{(1-0.4024z^{-1})}$$

- Determine the gain the frequency corresponding to 800 rad/s.

$$|H\left(e^{j\omega}\right)| = |\frac{0.2970\left(1+e^{-j\omega}\right)}{1-0.4024e^{-j\omega}}|$$

$$= |\frac{0.2970(1+cos\omega - jsin\omega)}{1-0.4024(cos\omega - jsin\omega)}|$$

$$= \frac{\sqrt{(0.2970 + 0.2970cos\omega)^2 + (0.2970sin\omega)^2}}{\sqrt{(1-0.4024cos\omega)^2 + (0.4024sin\omega)^2}}$$

$$\omega_c = 2\pi\frac{f_c}{f_s} = 2\pi\frac{127.3}{1000} = 0.7998 \quad indsættes$$

$$\left|H\left(e^{j\omega}\right)\right| = 0.70 = -3.0dB$$