2018-09-05/ MS/OKJ

Signal Processing, lecture 1

Suggested solutions to exercises:

1.1

A Butterworth low-pass filter is wanted, with the two (standard) requirements:

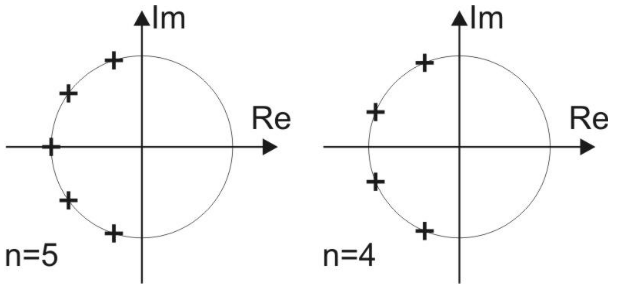
- 20·log|H(j·1 rad/s)| = -3 dB
- 20·log|H(j0)| = 0 dB

a. Calculate and plot the pole locations for filters of order n = 4 and n = 5.

$$p_k = e^{j\left(\frac{2k-1}{2n}\pi + \frac{\pi}{2}\right)} \quad k = 1,2,3,\ldots n$$

$$n = 4: \quad p_k = e^{j\left(\frac{2k-1}{8}\pi + \frac{\pi}{2}\right)} \quad \{k = 1,2,3,4\} \quad \begin{cases} e^{j\frac{5}{8}\pi} = -0.383 + j0.924 \\ e^{j\frac{7}{8}\pi} = -0.924 + j0.383 \\ e^{j\frac{9}{8}\pi} = -0.924 - j0.383 \\ e^{j\frac{11}{8}\pi} = -0.383 - j0.924 \end{cases}$$

$$n = 5: \quad p_k = e^{j\left(\frac{2k-1}{10}\pi + \frac{\pi}{2}\right)} \quad \{k = 1,2,3,4,5\} \quad \begin{cases} e^{j\frac{6}{10}\pi} = -0.309 + j0.951 \\ e^{j\frac{8}{10}\pi} = -0.809 + j0.588 \\ e^{j\frac{10}{10}\pi} = -1 \\ e^{j\frac{12}{10}\pi} = -0.809 - j0.588 \\ e^{j\frac{14}{10}\pi} = -0.309 - j0.951 \end{cases}$$



b. Find an expression for the transfer function from the pole locations for n = 4.

$$H(s) = \frac{1}{(s - e^{j\frac{5}{8}\pi})(s - e^{j\frac{11}{8}\pi})(s - e^{j\frac{7}{8}\pi})(s - e^{j\frac{9}{8}\pi})}$$

$$H(s) = \frac{1}{(s^2 + 2\cdot 0.383s + 1)(s^2 + 2\cdot 0.924s + 1)}$$

$$H(s) = \frac{1}{s^4 + 2.614s^3 + 3.416s^2 + 2.614s + 1}$$

c. Check the results from a and b using Matlab

[dummy pole4 A0] = buttap(4); *% Returns the poles (and A0 = 1)*

```
pole4
[dummy pole5 A0] = buttap(5);
pole5
denom_coeff4 = poly(pole4)              % Returns the a_n's
pole4 =
  -0.3827 + 0.9239i
  -0.3827 - 0.9239i
  -0.9239 + 0.3827i
  -0.9239 - 0.3827i
pole5 =
  -0.3090 + 0.9511i
  -0.3090 - 0.9511i
  -0.8090 + 0.5878i
  -0.8090 - 0.5878i
  -1.0000
denom_coeff4 =
    1.0000    2.6131    3.4142    2.6131    1.0000
```

1.2

A 2nd order Butterworth low-pass filter is to be used in a class-D audio amplifier to pass the audio signal and attenuate the signal at the switching frequency.

- The attenuation at 20 kHz shall be 0.5 dB
- The attenuation at the switching frequency shall be 30 dB

a. For a normalized (ω_{3dB} = 1 rad/s) 2nd order Butterworth filter, find the radian frequency, where the attenuation is 0.5 dB.

$$\left|H_{norm}(j\omega_{0.5dB})\right|^2 = \frac{1}{1+(\omega_{0.5dB})^{2n}} = 10^{-0.5/10}$$

$$(\omega_{0.5dB})^{2\cdot 2} = 10^{0.5/10} - 1$$

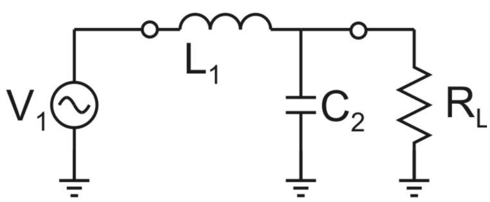
$$\omega_{0.5dB} = \left(10^{0.5/10} - 1\right)^{1/4} = 0.591\, rad/s$$

b. For the normalized filter, find the radian frequency, where the attenuation is 30 dB, and find the transition band ratio, $\omega_{30dB}/\omega_{0.5dB}$.

$$\omega_{30dB} = \left(10^{30/10} - 1\right)^{1/4} = 5.62\, rad/s$$

$$\omega_{30dB}/\omega_{0.5dB} = 9.51$$

- A 2nd order Butterworth filter with ω_{3dB} = 1 rad/s, can be made using the circuit shown with L_1 = 1.4142 H and C_2 = 0.7071 F and R_L = 1 Ω.

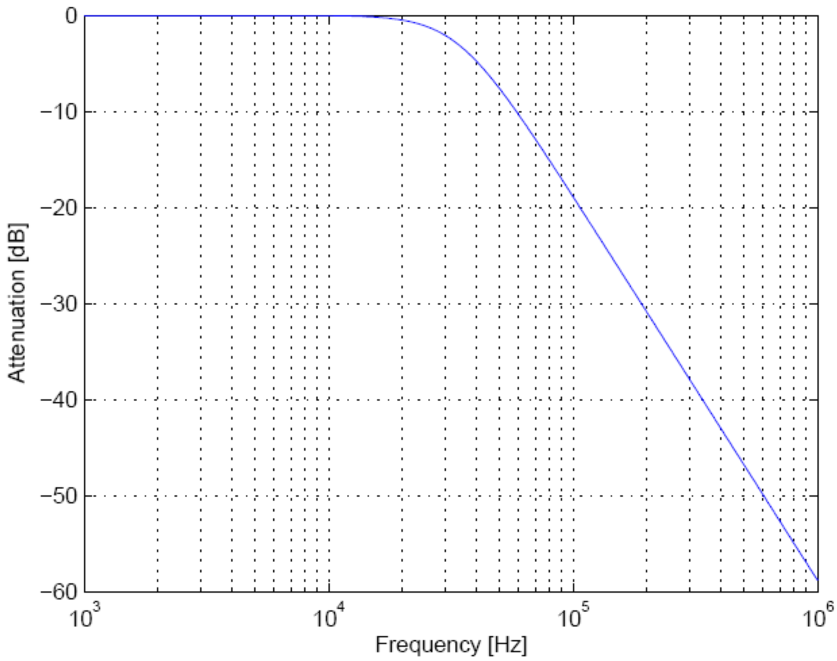


c. Make a frequency scaling so that the scaled filter has an attenuation of 0.5 dB at 20 kHz. What is the needed frequency scaling factor?
Freq. scaling factor, k_f = (2π20e3)/0.591 = 212.6e3
$L_1 = L_{1norm}/k_f$ = 6.65 μH
$C_2 = C_{2norm}/k_f$ = 3.33 μF

d. Find the frequency, where the scaled filter has an attenuation of 30 dB.
$f_{30dB} = \omega_{30dBnorm} \cdot k_f/2/\pi$ = 190 kHz

e. Plot the magnitude of the transfer function (dB) in Matlab

```
n = 2;                          % Filter order
kf = 212.6e3;                   % Freq scaling factor
freq = logspace(3,6,1000);      % Real frequency
om_norm = 2*pi*freq/kf;         % Normalized radian frequency
HdB = -10*log10(1+om_norm.^(2*n));
semilogx(freq,HdB);
grid;
xlabel('Frequency [Hz]');
ylabel('Attenuation [dB]');
f_sw = freq(find(HdB+30<0,1,'first'))
```



f. Make an impedance scaling to R_L = 4 Ω and find the new values, $L_{1,scaled}$ and $C_{2,scaled}$.
   Imp scaling factor, $k_z = 4/1 = 4$.
   $L_{1,scaled} = L_1 \cdot k_z = 26.6\ \mu H$
   $C_{2,scaled} = C_2/k_z = 831\ nF$

1.3

The following requirements are set for a Butterworth low-pass filter:

- The attenuation at ≥ 30 kHz shall be ≥ 20 dB
- The attenuation at ≤ 10 kHz shall be ≤ 1 dB

a. Find the necessary filter order
b. Find the 3-dB bandwidth, when the attenuation at 10 kHz is chosen to be 1 dB

$$n \geq \frac{1}{2\log\frac{\omega_S}{\omega_P}} \log\frac{10^{\alpha_{S,dB}/10}-1}{10^{\alpha_{P,dB}/10}-1} = \frac{1}{2\log\frac{30}{10}} \log\frac{10^2-1}{10^{0.1}-1} = 2.7 \sim \underline{\underline{3}}$$

$$\left|H_{Norm}(j\omega)\right|^2 = \frac{1}{1+\omega^{2n}} \sim -1\ dB \Leftrightarrow \omega_{1dB} = \sqrt[2\cdot 3]{10^{0.1}-1} = 0.798$$

$$k_f = \frac{2\pi 10e3}{0.798} = 78702$$

$$f_{3dB,scaled} = \frac{k_f}{2\pi} 1\ rad/s = \underline{\underline{12.53\ kHz}}$$