AALBORG UNIVERSITY
DENMARK

Written exam in
Signal processing, 5 ECTS

Monday, January 9, 2017
9.00 – 13.00

Read carefully:

- Remember to write your **full name and study number on every sheet** you return!
- Write **legible** with a pen that allows your answers to be scanned electronically.
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets**.
- Problems are weighted as listed.
- Results **without sufficient explanations will not give full credits**!
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times.
- **Communication with others is strictly prohibited**.

EIT5/ITC5 Signal Processing / Analog Filters
Written examination Jan. 09 2017

Note: You may use the following equations:

$$\cosh(x)=\frac{e^{x}+e^{-x}}{2},\ \cosh^{-1}(x)=\ln\left(x+\sqrt{x^{2}-1}\right),\ \sinh(x)=\frac{e^{x}-e^{-x}}{2},\ \sinh^{-1}(x)=\ln\left(x+\sqrt{x^{2}+1}\right).$$

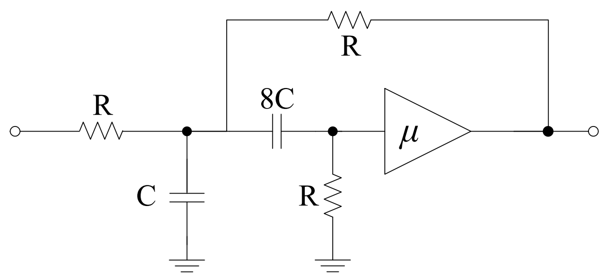
A.1 (Weight 12%)
A band-pass filter has to fulfill the requirements:

- Chebyshev characteristic with 1.0 dB ripple.
- Max. 1.0 dB attenuation at frequencies between 900 and 1000 Hz.
- Min. 40 dB attenuation at 800 Hz.
- Min. 30 dB attenuation at 1100 Hz.

a. Make a sketch (graph) of the band-pass filter.
b. Find the necessary order of the filter using the LP←→BP frequency transformation.
c. Find the actual attenuation at 800 Hz.
d. Find the actual attenuation at 950 Hz. (Note: The Chebyshev Cn-function expressions are different in the passband and stopband)

A.2 (Weight 9 %)
The band-pass filter shown has the transfer function:

$$H(s)=\frac{s\dfrac{\mu}{RC}}{s^{2}+s\dfrac{25-8\mu}{8RC}+\dfrac{1}{4R^{2}C^{2}}}$$



The component values are:

- R = 5 k Ω
- C = 2 nF
- Gain of the gain block, μ = 3.

a. Find the centre radian frequency, ω0, and the Q-value.
b. Find the gain at the centre radian frequency.

A.3 (Weight 12 %)
A band-pass circuit has the transfer function:

$$H(s)=\frac{s\dfrac{\mu}{4RC}}{s^{2}+s\dfrac{12-2\mu}{6RC}+\dfrac{1}{4R^{2}C^{2}}}$$

a. Find an expression for the sensitivity of the Q-value with respect to changes in the gain bloc gain μ, S_{μ}^{Q}, and its value for Q = 5.
b. Find a simple expression of the transfer function, H(**jw**$_0$), at the centre radian frequency.
c. Find an expression for the sensitivity of the transfer function at the centre radian frequency with respect to changes in the gain block gain μ, $S_{\mu}^{H(j\omega_0)}$, and its value for Q = 5.

Problem B.1 (weighted with 10% - Digital filters)
Design a high-pass finite impulse response (FIR) filter with linear phase using the window method. Use a sampling frequency f_s=10kHz, a -3dB cutoff frequency of f_c=2.5kHz, a rectangular window and an order of M=5.

Questions:

a) Determine the filter coefficients.
b) What is the gain in dB at 0 Hz (DC)?
c) Draw a signal flow graph for the filter showing the actual coefficients and delays
d) Draw a transposed version of the signal flow graph
e) Determine the initial 6 values of the step response for the transposed filter

Problem B.2 (weighted with 14% - Digital filters)
A digital filter is characterized by:

- Two complex conjugate poles at: $p_1,p_2 = 0.1848 \pm j0.4021$
- Two real zeros at: $z_1,z_2=1$

The sampling frequency is: f_s=10kHz

Questions:

a) Make the pole-zero plot for the filter
b) Sketch the amplitude response by using vector considerations. Describe how you arrive at the sketch.
c) Determine the transfer function H(z)
d) Scale the filter such that the gain at 5kHz (half of the sampling frequency) is 0 dB
e) Draw a signal flow graph for the filter as a single second order section. Include all coefficients in the figure.
f) Determine the initial 5 samples of the impulse response

Problem B.3 (weighted with 10% - Digital filters)
An analog second order Butterworth low pass filter with a -3dB cut-off frequency of 2kHz is described by the transfer function:

$$H_a(s) = \frac{\Omega_c^2}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2}$$

Use the bilinear transformation to determine a digital filter with the same cut-off frequency. The sampling rate is f_s=10kHz.

Questions:

a) Determine H(z)
b) Compute the DC gain
c) Verify that the -3dB cut-off frequency for H(z) is 2 kHz

Problem C.1 (weighted with 4% - Spectral estimation)
A digital filter H is implemented at the sampling frequency f_S=10 kHz and has the impulse response h[n] with the length N=4. By using a DFT the following frequency response is achieved: H[k]=[0   1+j   0   1-j].

Find the sequence h[n].

Problem C.2 (weighted with 3% - Spectral estimation)

Draw the discrete sequence H[k] for each k and put frequency values in Hz on the x-axis.

Problem C.3 (weighted with 4% - Spectral estimation)
We now consider a continuous sinusoid signal with frequency f=1 kHz, s(t)=cos(Ω t), where Ω=2π f and t is the time. The signal is sampled at f_S in order to make a sequence x[n].

Find x[n] using a rectangular window of length L=4.

Problem C.4 (weighted with 4% - Spectral estimation)
The sampled sine signal x[n] is now sent through the filter H so that an output sequence y[n] is obtained.

Find y[n].

Problem C.5 (weighted with 5% - Spectral estimation)
On your drawing above (see C.2) 1 kHz is not represented, so one might think that the filter doesn't let through this frequency. However, y[n] is clearly not a zero-sequence.

Explain why y[n] is not just a sequence of zeros.

Problem C.6 (weighted with 4% - Spectral estimation)

Calculate the amplification (gain) of the filter H at 1 kHz.

Problem C.7 (weighted with 4% - Spectral estimation)

Sketch the continuous frequency response |H(ω)| and put frequency (in Hz) marks on the x-axis and gain on the y-axis.

Problem C.8 (weighted with 5% - Spectral estimation)
The continuous signal s(t) is now sampled again at f_S but this time utilizing a Blackman window with the length L=256. Hereafter the signal is analyzed with a 256-points FFT.

What is the "effective frequency resolution" Δf using the analysis above, i.e. its ability to separate tones (frequencies) in close proximity to each other?