EIT/ITC5 Signal Processing, 4

Suggested solutions to exercises:

4.1

The filter in Kendall Su: "Analog filters" Figure 8.4 has the transfer function given in (8.22). Component values are chosen as: $C_1 = C_2 = C$, $R_1 = R_2 = R$ and $R_3 = 2R$. In this case the transfer function can be reduced to:

$$H(s) = \frac{\frac{\mu}{CR}s}{s^2 + \frac{3-\mu}{CR}s + \frac{1}{(CR)^2}}$$

a. Find expressions for ω_0 and Q

$$\underline{\underline{\omega_0 = \frac{1}{CR}}} \qquad \frac{3-\mu}{CR} = \frac{\omega_0}{Q} = \frac{1}{CRQ} \quad \Rightarrow \quad \underline{\underline{Q = \frac{1}{3-\mu}}}$$

b. Find an expression for S_μ^Q and its value for Q = 5

$$S_\mu^Q = -S_\mu^{(3-\mu)} = \frac{-\mu}{3-\mu}\cdot\frac{\partial(3-\mu)}{\partial\mu} = \frac{\mu}{3-\mu}$$

$$Q = \frac{1}{3-\mu} \Leftrightarrow 3Q - \mu Q = 1 \Leftrightarrow \mu = 3 - \frac{1}{Q}$$

$$\underline{\underline{S_\mu^Q}} = Q\left(3 - \frac{1}{Q}\right) = \underline{\underline{3Q-1}}$$

$$Q = 5 \Rightarrow \underline{\underline{S_\mu^Q = 14}}$$

This is similar to the value found in the slides

c. Find a simple expression for $H(j\omega_0)$ and for $S_\mu^{H(j\omega_0)}$ and their values for Q = 5

$$\underline{\underline{H(j\omega_0)}} = H(s)\big|_{s=j\omega_0} = \left.\frac{\frac{\mu}{CR}s}{s^2 + \frac{3-\mu}{CR}s + \frac{1}{(CR)^2}}\right|_{s=j\omega_0} = \frac{\frac{\mu}{CR}}{\frac{3-\mu}{CR}} = \underline{\underline{\frac{\mu}{3-\mu} = \mu Q}}$$

$$\underline{\underline{S_\mu^{H(j\omega_0)}}} = \frac{\mu}{\frac{\mu}{3-\mu}}\cdot\frac{\partial\frac{\mu}{3-\mu}}{\partial\mu} = (3-\mu)\left(\frac{1}{3-\mu} + \frac{\mu}{(3-\mu)^2}\right) = 1 + \frac{\mu}{3-\mu} = \frac{3}{3-\mu} = \underline{\underline{3Q}}$$

$$\left(an\ easier\ way: \quad S_\mu^{\mu Q} = S_\mu^\mu + S_\mu^Q = 1 + S_\mu^Q = 1 + (3Q-1) = 3Q\right)$$

$$Q = 5 \Rightarrow \mu = 3 - \frac{1}{Q} = 2.8 \quad \Rightarrow \quad \underline{\underline{H(j\omega_0) = 14}} \qquad \underline{\underline{S_\mu^{H(j\omega_0)} = 15}}$$

4.2

A 2nd order Butterworth normalized low-pass prototype filter is transformed to a BP-filter (4 poles) having:

- Lower passband edge (-3 dB) = 10 kHz
- Upper passband edge (-3 dB) = 15 kHz

The filter is made using 2 biquad filter sections of the type analyzed in exercise 4.1

a. Find the pole locations of the BP-filter. Hint: Use Matlab:

```
OrderLPP = 2;
Om = [10 15]*pi*2e3;
[Numpoly DenomPoly] = butter(OrderLPP,Om,'s'); % Bandpass since Om is a vector
Poles = roots(DenomPoly)
  Poles =
    1.0e+004 *
    -1.2710 + 8.8077i
    -1.2710 - 8.8077i
    -0.9504 + 6.5862i
    -0.9504 - 6.5862i
```

b. Find ω_0 and Q for each of the two sections. Hint:

```
 omeg_0_biquad = abs(Poles)
 Q_biquad = -2\abs(Poles)./real(Poles) % some function of Poles
  omeg_0_biquad =
    1.0e+004 *
      8.8989
      8.8989
      6.6545
      6.6545
  Q_biquad =
      3.5007
      3.5007
      3.5007
      3.5007
```

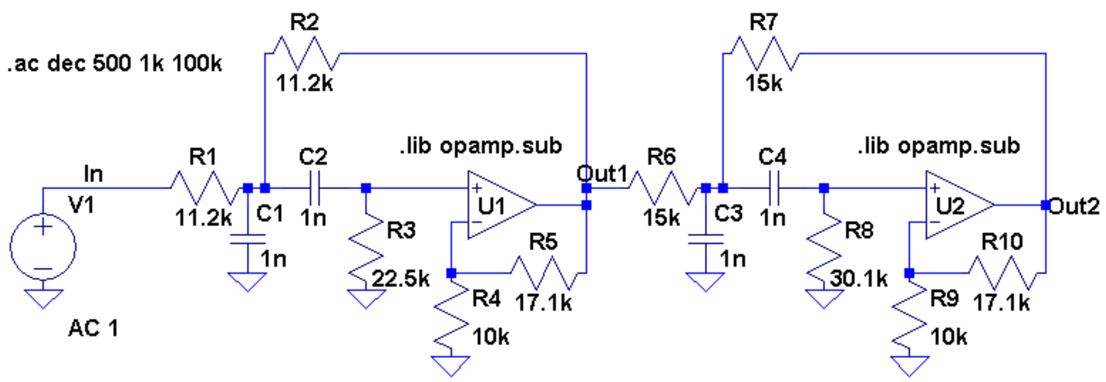
c. Using C = 1 nF, find the resistor values and μ in each section.

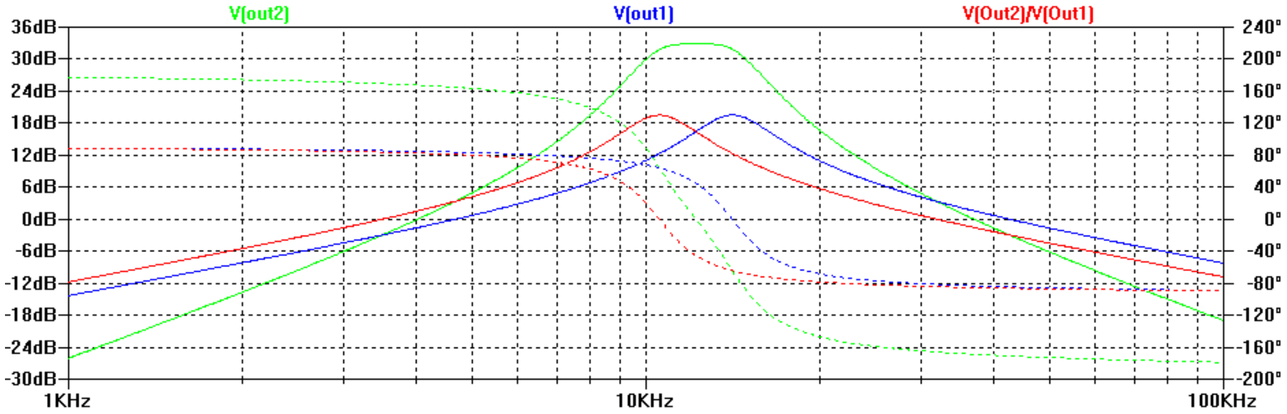
$$\mu = 3 - \frac{1}{Q} \qquad R = \frac{1}{\omega_0 C}$$

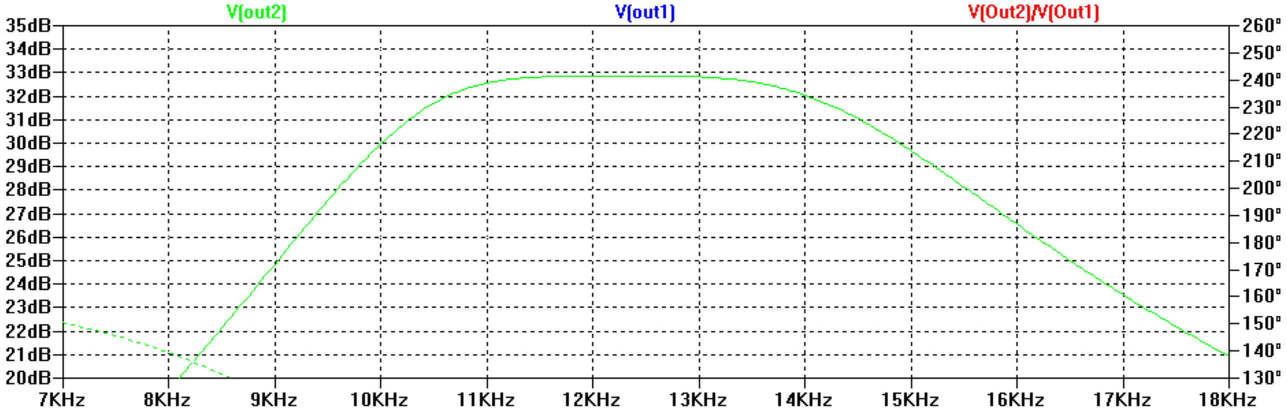
Section	*ω_0 [10^4 rad/s]*	*Q*	*$R = R_1 = R_2$*	*$R_3 = 2R$*	*μ*
1	*8.90*	*3.5*	*11.2 kΩ*	*22.5 kΩ*	*2.71*
2	*6.65*	*3.5*	*15.0 kΩ*	*30.1 kΩ*	*2.71*

d. Optional Spice simulation:

*Check: 20*log10(μQ) = 19.5 dB, 2 sections < 39 dB since the peaks are not coincident.*



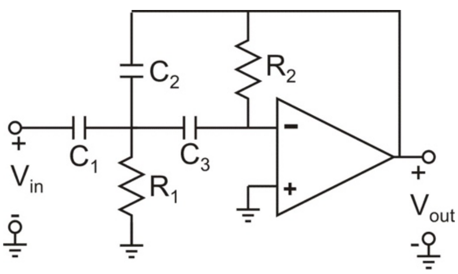




4.3

The 2nd order MFB high-pass section shown has the transfer function:

$$H(s) = -\frac{\frac{C_1}{C_2}s^2}{s^2 + \frac{C_1 + C_2 + C_3}{R_2 C_2 C_3}s + \frac{1}{R_1 R_2 C_2 C_3}}$$



The capacitor values are chosen as:

- $C_1 = C_2 = 10$ nF
- $C_3 = 2C_1 = 2C_2 = 20$ nF

and it is required that:

- $\omega_0 = 2\pi \cdot 10^4$ rad/s
- $Q = 5$

a. Find the values of R_1 and R_2
b. Find the filter gain, $|H(j\omega_0)|$, at ω_0.
c. Find the filter gain, $|H(j\omega)|$, for $\omega \rightarrow \infty$

Suggested solution:

a. Comparing the transfer function with the standard second order function [K.S. (10.15) or mm.5.slide19/33) gives:

$$\frac{\omega_0}{Q}=\frac{C_1+C_2+C_3}{R_2C_2C_3}=\frac{2}{R_2C_1}\Leftrightarrow Q=\frac{\omega_0}{2}R_2C_1$$

$$\omega_0=\sqrt{\frac{1}{R_1R_2C_2C_3}}=\frac{1}{C_1\sqrt{2R_1R_2}}$$

$$Q=\sqrt{\frac{R_2}{R_1}}\cdot\frac{1}{2\sqrt{2}}=5\Leftrightarrow\frac{R_2}{R_1}=(2\sqrt{2}\cdot 5)^2=200$$

$$\omega_0=\frac{1}{\sqrt{400}\cdot C_1R_1}\Leftrightarrow$$

$$\underline{\underline{R_1}}=\frac{1}{20\omega_0C_1}=\frac{1}{20\cdot 2\pi\cdot 10^4\cdot 10\cdot 10^{-9}}=\underline{\underline{79.58\,\Omega}}$$

$$\underline{\underline{R_2}}=200R_1=\underline{\underline{15.92\,k\Omega}}$$

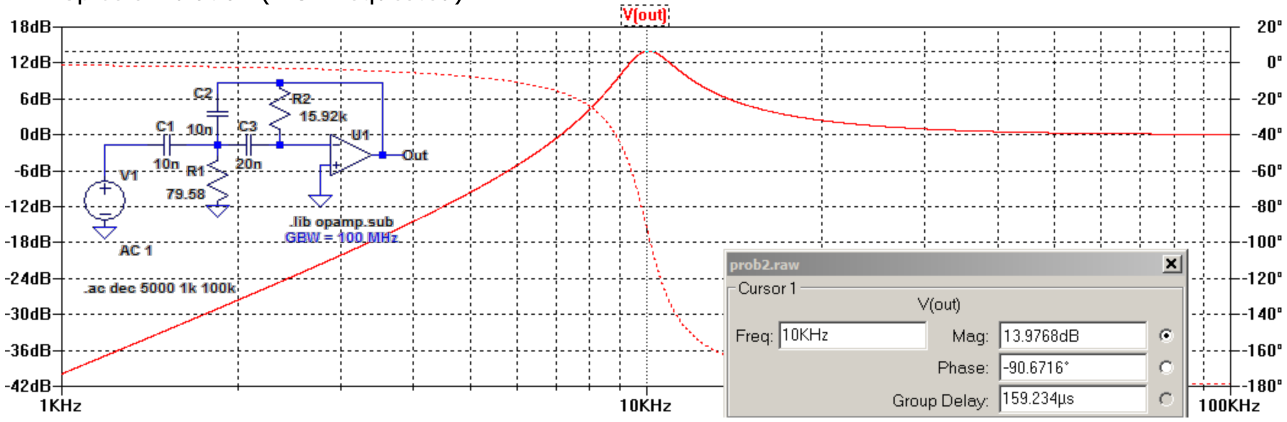
b. For $C_1 = C_2$ you get:

$$\underline{\underline{|H(j\omega_0)|}}=\left|-\frac{\frac{C_1}{C_2}s^2}{s^2+\frac{C_1+C_2+C_3}{R_2C_2C_3}s+\frac{1}{R_1R_2C_2C_3}}\Bigg|_{s=j\omega_0}\right|=\left|-\frac{s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}\Bigg|_{s=j\omega_0}\right|=\underline{\underline{Q=5\sim 14\;dB}}$$

c.

$$\lim_{\omega\to\infty}|H(j\omega)|=\frac{C_1}{C_2}=1\sim 0\;dB$$

FYI: Spice simulation (**NOT** requested):



4.4

A 2nd order Butterworth normalized low-pass prototype filter is transformed to a BP-filter (4 poles) having:

- Lower passband edge (-3 dB) = 10 kHz
- Upper passband edge (-3 dB) = 15 kHz

The filter is made using 2 biquad filter sections:

- An MFB low-pass section, for the poles with the largest ω_0, with $R_1 = R_2 = R_3 = R = 2\ k\Omega$.
- An SK high-pass section, for the poles with the smallest ω_0, with $C_1 = C_2 = C = 10$ nF and μ = 1.

a. Find the pole locations of the BP-filter using Matlab

```
OrderLPP = 2;
  Om = [10 15]*pi*2e3;
    [Numpoly DenomPoly] = butter(OrderLPP,Om,'s'); % Bandpass since Om
    is a vector
  Poles = roots(DenomPoly)
    Poles =
      1.0e+004 *
      -1.2710 + 8.8077i
      -1.2710 - 8.8077i
      -0.9504 + 6.5862i
      -0.9504 - 6.5862i
```

b. Find ω_0 and Q for each of the two sections from the pole locations

```
omeg_0_biquad = abs(Poles)
Q_biquad = -2\abs(Poles)./real(Poles) % some function of Poles
 omeg_0_biquad =
   1.0e+004 *
     8.8989
     8.8989
     6.6545
     6.6545
 Q_biquad =
     3.5007
     3.5007
     3.5007
     3.5007
```

Section	ω_0 [10^4 rad/s]	Q
1 MFB-LP	8.90	3.5
2 SK-HP	6.65	3.5

c. Find the component values of the MFB-LP section

$$Q = \sqrt{\frac{C_1}{C_2}} \cdot \frac{1}{\frac{\sqrt{R_2 R_3}}{R_1} + \sqrt{\frac{R_3}{R_2}} + \sqrt{\frac{R_2}{R_3}}} = \frac{1}{3}\sqrt{\frac{C_1}{C_2}} \quad \Rightarrow C_1 = 9Q^2 C_2$$

$$\omega_0 = \frac{1}{\sqrt{R_2 R_3 C_1 C_2}} = \frac{1}{3QRC_2} \quad \Rightarrow \underline{\underline{C_2}} = \frac{1}{3QR\omega_0} = \underline{\underline{535\ pF}}$$

$$\underline{\underline{C_1}} = 9Q^2 C_2 = \underline{\underline{59.0\ nF}}$$

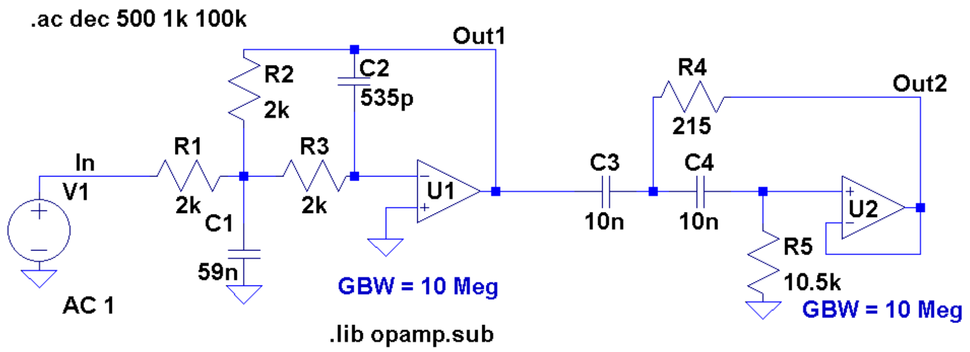
d. Find the component values of the SK-HP section.

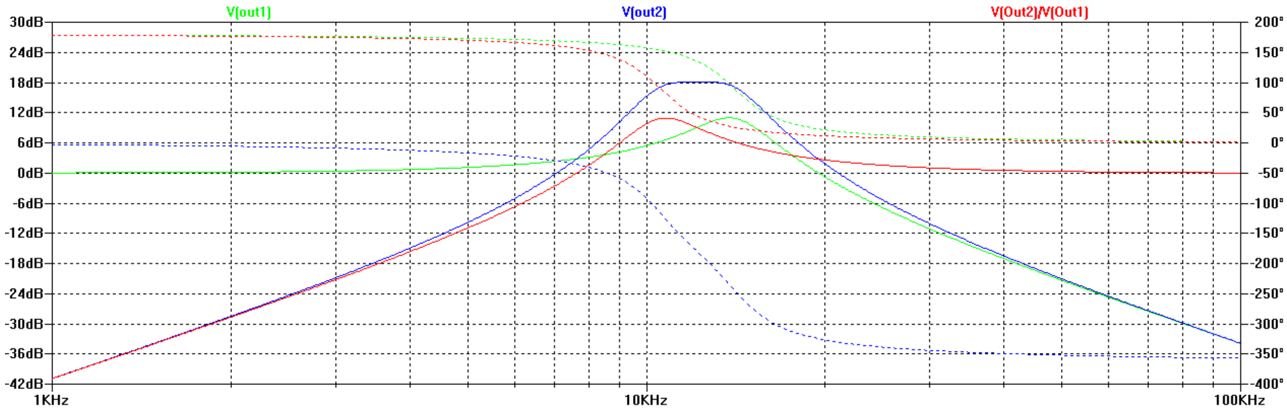
$$Q = \frac{\frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}{\frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} + \frac{1-\mu}{R_1 C_1}} = ½\sqrt{\frac{R_2}{R_1}} \quad \Rightarrow R_2 = 4Q^2 R_1$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2QR_1 C} \quad \Rightarrow \underline{\underline{R_1}} = \frac{1}{2Q\omega_0 C} = \underline{\underline{215\ \Omega}}$$

$$\underline{\underline{R_2}} = 4Q^2 R_1 = \underline{\underline{10.5\ k\Omega}}$$

e. Option: Make a Spice-simulation to check the results


.ac dec 500 1k 100k
Out1
R2
2k
C2
535p
R4
215
Out2
In
V1
R1
2k
R3
2k
C1
59n
U1
C3
10n
C4
10n
U2
R5
10.5k
AC 1
GBW = 10 Meg
.lib opamp.sub
GBW = 10 Meg


V(out1)
V(out2)
V(Out2)/V(Out1)
30dB
24dB
18dB
12dB
6dB
0dB
-6dB
-12dB
-18dB
-24dB
-30dB
-36dB
-42dB
200°
150°
100°
50°
0°
-50°
-100°
-150°
-200°
-250°
-300°
-350°
-400°
1KHz
10KHz
100KHz