ITC5/EIT5 Signal Processing / Analog Filters

Written examination Jan. 08th 2016

A.1 (Weight 11 %)

A normalized low-pass filter should have:

- 3rd order Chebyshev characteristic
- 0.4 dB pass-band ripple
- A pass-band (ripple) bandwidth of 1 rad/s

a. Find the stop-band attenuation at $\omega_S = 20$ rad/s

b. Find the pole locations of the filter
Note: You may use the following equations:

$$\cosh(x) = \frac{e^x + e^{-x}}{2},\ \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right),\ \sinh(x) = \frac{e^x - e^{-x}}{2},\ \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right).$$

a. $\varepsilon = \sqrt{10^{Riple,dB/10} - 1} = \sqrt{10^{0.04} - 1} = 0.3106$

$C_3(20) = \cosh(3 \cdot \cosh^{-1}(20)) = 31940$

$-H(j20rad/s) = 10\log(1 + \varepsilon^2 C_3^2(20)) = 79.9dB$

b.

$$s_k = \sin\frac{(2k-1)\pi}{2n}\sinh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right) + j\cos\frac{(2k-1)\pi}{2n}\cosh\left(\frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}\right) \qquad k = n+1, \ldots 2n$$

$s_4 = \text{-}0.3354 - \text{j}1.0428$

$s_5 = \text{-}0.6708 - \text{j}0.0000$

$s_6 = \text{-}0.3354 + \text{j}1.0428$

A.2 (Weight 11 %)

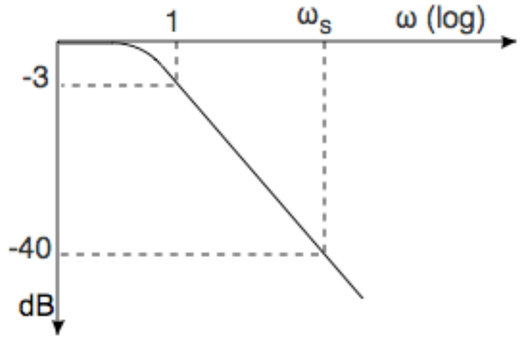
The requirements for a Butterworth high-pass filter are:

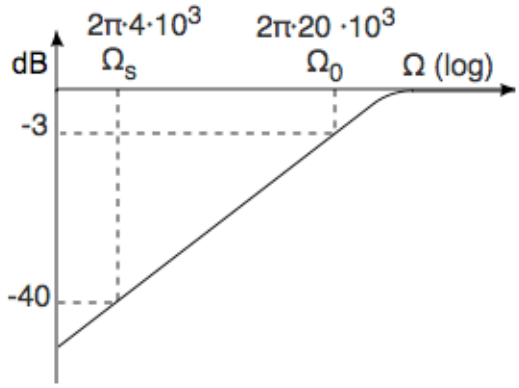
- 3 dB attenuation at f = 20 kHz ($\Omega = 2\pi \cdot 20 \cdot 10^3$ rad/s)
- Minimum 40 dB stopband attenuation at f = 4 kHz ($\Omega = 2\pi \cdot 4 \cdot 10^3$ rad/s)

a. Make a sketch (graph) of the filter requirements and the corresponding low-pass prototype filter. Use the LP-HP frequency mapping to find the frequency in the stop-band of the low-pass prototype that corresponds to the high-pass filter frequency of f = 4 kHz ($\Omega = 2\pi \cdot 4 \cdot 10^3$ rad/s) where 40 dB attenuation must be obtained.

b. Find the necessary filter order

c. Find the attenuation of the high-pass filter at f = 2 kHz ($\Omega = 2\pi \cdot 2 \cdot 10^3$ rad/s) a.





$$s = \frac{\Omega_0}{S} \qquad |\omega| = \frac{\Omega_0}{\Omega}$$

$$|\omega_S| = \frac{\Omega_0}{\Omega_S} = \frac{2\pi \cdot 20 \cdot 10^3}{2\pi \cdot 4 \cdot 10^3} = 5 \quad (rad/s)$$

$$|H_{LPP}(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

b. $10\log(1+\omega_S^{2n}) = 40$

$$\omega_S^{2n} = 10^{40/10} - 1$$

$$n = \frac{\log(10^{40/10}-1)}{2\log(5)} = 2.86 \rightarrow 3$$

c.

$$-10\log\left(\frac{1}{1+\left(\frac{\Omega_0}{\Omega}\right)^{2n}}\right) = 10\log\left(1+\left(\frac{\Omega_0}{\Omega}\right)^{2n}\right) = 10\log\left(1+\left(\frac{2\pi \cdot 20 \cdot 10^3}{2\pi \cdot 2 \cdot 10^3}\right)^{2\cdot 3}\right) = 60 \quad dB$$

A.3 (Weight 11 %)

A normalized low-pass filter with the transfer function:

$$H_{LPP}(s) = \frac{1}{1+s}$$

is to be transformed into a band-pass filter with:

- Centre frequency, Ω_0 = 15 rad/s
- Bandwidth, B = 5 rad/s.

  a. Find the transfer function of the band-pass filter, $H_{BP}(S)$
  b. Find the locations of poles and zero of the band-pass filter

a.

$$\because \qquad s = \frac{S^2 + \Omega_0^2}{B \cdot S}$$

$$and \qquad H_{LPP}(s) = \frac{1}{1+s}$$

$$\therefore$$

$$H_{BP}(S) = \frac{1}{\frac{S^2 + \Omega_0^2}{B \cdot S} + 1}$$

$$H_{BP}(S) = \quad \frac{B \cdot S}{S^2 + B \cdot S + \Omega_0} = \frac{4S}{S^2 + 5S + 225}$$

b.

Zeros:

$$B \cdot S = 0 \Rightarrow Zero = 0$$

Poles:

$$S^2 + 5S + 225 = 0 \quad rad/s$$

$$\Rightarrow$$

$$Poles = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 225}}{2} = \begin{cases} -2.5 + j14.8 \\ -2.5 - j14.8 \end{cases} \; rad/s$$