AALBORG UNIVERSITY
DENMARK

Written exam in
Signal processing,
5th semester

Friday, January 8, 2016
9.00 – 13.00

Read carefully before you continue:

- Remember to write your **full name on every sheet** you return! Sheets without names might **not** be evaluated.
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets**.
- Write **legible** with a clear dark pen/pencil. If it can't be read, no credits will be given.
- Problems are weighted as listed. Prioritize your time accordingly.
- Results **without sufficient explanations will not give full credits**!
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times. It's your responsibility! If you don't know how to turn it off, don't bring it!
- **Communication with others is strictly prohibited**.

ITC5/EIT5 Signal Processing / Analog Filters

Written examination Jan. 08th 2016

A.1 (Weight 11 %)

A normalized low-pass filter should have:

- 3rd order Chebyshev characteristic
- 0.4 dB pass-band ripple
- A pass-band (ripple) bandwidth of 1 rad/s

a. Find the stop-band attenuation at ω_S = 20 rad/s

b. Find the pole locations of the filter

Note: You may use the following equations:

$$\cosh(x) = \frac{e^x + e^{-x}}{2},\ \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right),\ \sinh(x) = \frac{e^x - e^{-x}}{2},\ \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2 + 1}\right).$$

A.2 (Weight 11 %)

The requirements for a Butterworth high-pass filter are:

- 3 dB attenuation at f = 20 kHz (Ω _= $2\pi \cdot 20 \cdot 10^3$ rad/s)
- Minimum 40 dB stopband attenuation at f = 4 kHz (Ω = $2\pi \cdot 4 \cdot 10^3$ rad/s)

a. Make a sketch (graph) of the filter requirements and the corresponding low-pass prototype filter. Use the LP-HP frequency mapping to find the frequency in the stop-band of the low-pass prototype that corresponds to the high-pass filter frequency of f = 4 kHz (Ω = $2\pi \cdot 4 \cdot 10^3$ rad/s) where 40 dB attenuation must be obtained.

b. Find the necessary filter order

c. Find the attenuation of the high-pass filter at f = 2 kHz (Ω = $2\pi \cdot 2 \cdot 10^3$ rad/s)

A.3 (Weight 11 %)

A normalized low-pass filter with the transfer function:

$$H_{LPP}(s) = \frac{1}{1+s}$$

is to be transformed into a band-pass filter with:

- Centre frequency, Ω_0 = 15 rad/s
- Bandwidth, B = 5 rad/s.

a. Find the transfer function of the band-pass filter, $H_{BP}(S)$

b. Find the locations of poles and zero of the band-pass filter

Problem B.1 (weighted with 10% - Digital filters)
A filter, with input x[n] and output y[n], is defined by the difference equation
$$y[n] = \alpha y[n-1] + (1-\alpha)x[n]$$
where $0 < \alpha < 1$ is a real constant.

Questions:
a) Determine the transfer function H(z)
b) Determine the impulse response, h[n], for n = -1, 0, 1, 2
c) State the values for z at which H(z) has a pole or a zero
d) Determine the frequency at which the filter has a gain of -3 dB
(Hint: $\left|H(e^{j\omega})\right|^2 = H(e^{j\omega})H^*(e^{j\omega})$, where * denotes the complex conjugate)

Problem B.2 (weighted with 6% - Digital filters)
A filter is defined by the difference equation:
$$y[n] = x[n] + 0.5\,x[n-1] - 0.5\,x[n-2] - 0.5\,y[n-1]$$
Questions:
a) Draw the direct form II signal-flow-graph (with all coefficients shown)
b) Draw the transposed signal-flow-graph
c) Determine the gain in dB at 0 Hz

Problem B.3 (weighted with 8% - Digital filters)
Use the windowing method with a rectangular window to design a fourth order "low-pass" FIR digital filter whose cut-off frequency is fc=2.5 kHz and linear phase response. The sampling frequency is fs=30 kHz.

Questions:
a) Determine the digital filter's transfer function.
b) Explain why having 'linear phase' is a desirable property for filters.
c) What would happen to the gain and phase response if the order is increased
d) What would happen to the gain and phase response if a Hamming window is used

Problem B.4 (weighted with 3% - Digital filters)
Question:
a) For a given transfer function H(z) explain how the poles and zeros affect the stability and the gain-response of the system.

Problem B.5 (weighted with 7% - Digital filters)
A filter is described by the following difference equation:
$$y[n] = x[n] + 1.21\,x[n-2] - 0.8\,y[n-1]$$
Questions:
a) Plot its poles and zeros on the z-plane
b) Determine whether it is causal and stable
c) Sketch its gain-response.

Signal Processing - Spectral Estimation - January 8th 2016

A digital filter H is given by its discrete impulse response $h[n]$=[-3 0 -3 0] and a frequency response $H[k]$.

C.1
What is the easiest way to compute the gain of H at DC (= 0 Hz)?

C.2
Compute a sampling frequency f_S where the filter can remove noise at 80 Hz completely, i.e. the frequency response at that frequency must be zero.

A continuous input signal defined by $x(t)=\cos(40\pi\, t)$ is now sampled, also using a sampling frequency of f_S and sent through the filter H.

C.3.
What is the amplification (gain) of that signal? (dB is fine but not needed)

C.4
Sketch the continuous amplitude response $|H(\omega)|$ from 0 Hz to f_S and put the frequency in Hertz [Hz] on the x-axis and the gain on the y-axis. (Again, dB is fine but not needed).

We shall now check whether the filter H really is capable of damping or removing noise at 80 Hz. Hence we sample a 80 Hz tone/sinusoidal signal and use the first 10 samples which then is sent through the filter H.

C.5
What is the output $y[n]$ of the filter?

C.6
How long time in milliseconds [ms] does it take until the output of the filter has stabilized at 0?

C.7
Explain why the first sample or samples in y, which are **different** from zero, are not important when we study the steady-state response of the filter H.

A digital filter with a real frequency response H_2=[-1 1] is exposed to the input signal $x_2[n]$. The output from the fitering turns out to be y_2=[0 -4].

C.8
Find the input signal (sequence) $x_2[n]$ by using an inverse DFT.