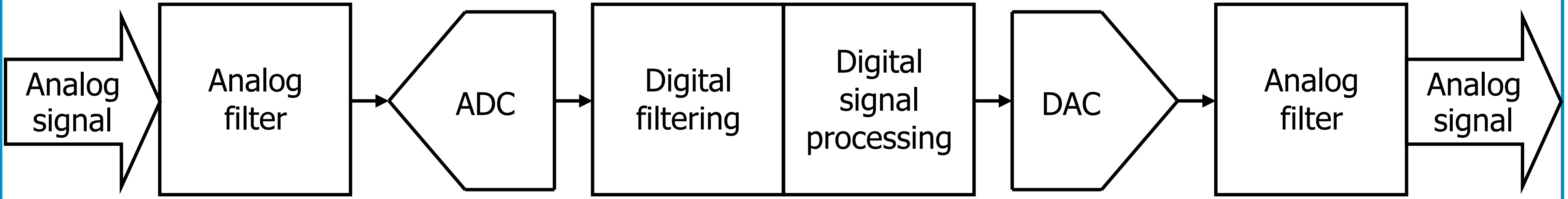
Course overview

Signal processing: (1st Sept. 2017 – 31st Jan. 2018)

rev. 2017-09-05/MS

- Analog filters (Ming Shen) 5 lectures 1 lab exercise
  - Types of filters (Butterworth, Chebyshev etc.)
  - Filter characteristics, e.g. group delay
  - Frequency- and impedance-scaling
  - LP-HP and LP-BP transformation
  - Realisation of active LP, HP and BP-filters
- Digital filtering (Ove Andersen)
  - Synthesis of transfer functions (FIR, IIR)
  - Analysis (frequency response, signal graphs)
  - Realisation/implementation-aspects
- Spectral estimation (Søren Krarup Olesen)
  - Discrete Fourier Transform (DFT)
  - Time- and frequency-sampling, multiplication in the time- and frequency domains
  - Window functions, zero padding, resolution
  - Effective algorithms (FFT)



Examination:

- o Written examination in January (4 hours)
- o You are allowed to use books, slides, your notes etc.
- o Matlab etc. may be used as a calculator, but no examination assignment requires the use of Matlab
- o Grading according to the 7 point scale
- o Formal rules to be announced at: http://www.sict.aau.dk/studienaevn-for-elektronik-og-it/proevedatoer/

Info on Moodle:

- Some course materials (my part) from last year are available FYI - it will be updated – check the dates on the front pages.
- An updated agenda will be available before 12^{00} the day before the lecture.
- Presentation ”slides” and suggested solutions to exercises is supplemental material and may be modified at any time.

Prerequisites

Required qualifications	Courses providing the required skill	
	EIT	ITC
Analog Filters:		
Ability to do calculations with complex numbers and make Laplace- and inverse Laplace-transformations.	Calculus; 2nd sem.	
Application of circuit theory including dynamic circuits: • Impedance and transfer function calculations on circuits with resistors, capacitors, inductors and ideal operational amplifiers • Impulse- and step-response calculations • Ability to make Bode plots • Knowledge of the significance of locations of poles and zeroes of a transfer function	Circuit Theory and Dynamic Systems, 2nd sem.	Linear Circuits, 3rd sem.

Preliminary plan (analog filters)

1. **{1-2}, 3-7, {7-16}, 25-30, 49-57, (app. A)**
   - **Course overview**
   - **Analog filters: Applications**
   - **Ideal and real filters**
   - **The Butterworth approximation**
   - **Design procedure, frequency and impedance scaling**

2. 7-20 (partly repetition), 30-36, 58-62, {App. A}
   - Briefly: Passive filter realisation (ladder structure)
   - The Chebyshev approximation
   - Impact of group delay variations

3. 37-38, {67-71}, 77-88
   - Briefly: Other filter types
   - Frequency transformations, LP-HP, LP-BP & LP-BS

4. 171-184, 187-189, {190-196}, 197-208
   - Sensitivity analysis
     - How sensitive is a given circuit to component variations?
     - Used as a tool to evaluate filter circuits
   - OpAmps applied as building blocks in active RC-filters

6. 217-238, 253-260, 263-264
   - 2nd order Sallen-Key
   - 2nd order multiple feed-back
   - Higher order filters

7. Design/lab. exercise

Kendall Su: ”Analog Filters”,
Kluwer Academic Publishers,
2nd ed. 2002, ISBN 1-4020-7033-0
(Springer: ISBN 978-1-4020-7033-4)

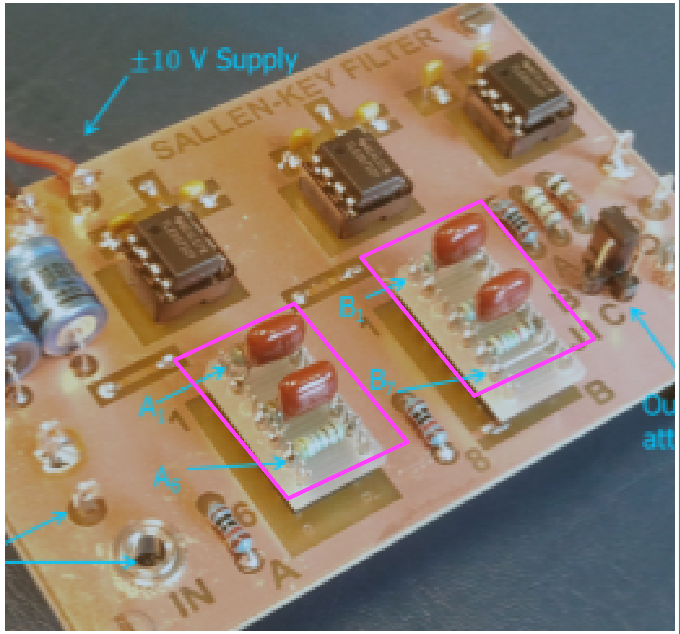
Available in electronic form at
http://site.ebrary.com/lib/aalborguniv
where you can read the book and
print a few pages.

Pages from textbook on Moodle

Preliminary plan - continued

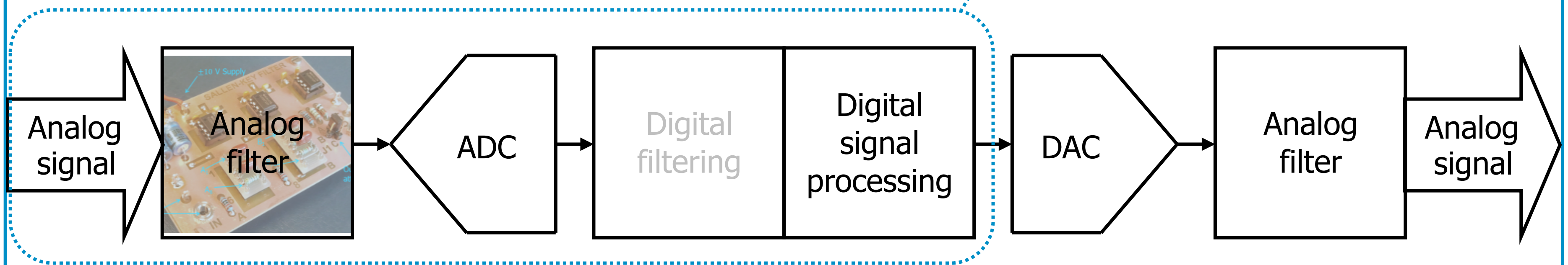
Design and lab. exercise:

9. (Full day – schedule to be revised)
   - Active (analog) filter design
   - Measurement of filter frequency response
   - Sampling of a signal with noise and interference through the filter
   - Analysis of the sampled signal using DFT-techniques (the theory will be given later in the course)
   - Later in the course: Analysis of the signal using digital filtering



Design and lab. exercise

Typical processing chain:



Analog filters: When and why?

- Analog filters
  - History
    - Simple filters (1880s, telegraph)
    - Image filters (1920s, telephone)
    - Network synthesis filters (1930s, WWII)
  - The theory makes a basis for some digital filters
  - Interface to analog surroundings
  - High power, high frequency

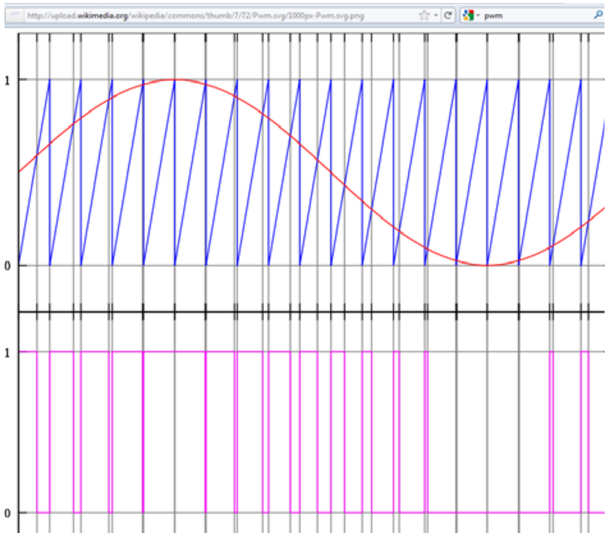
ZZZZ….2 min

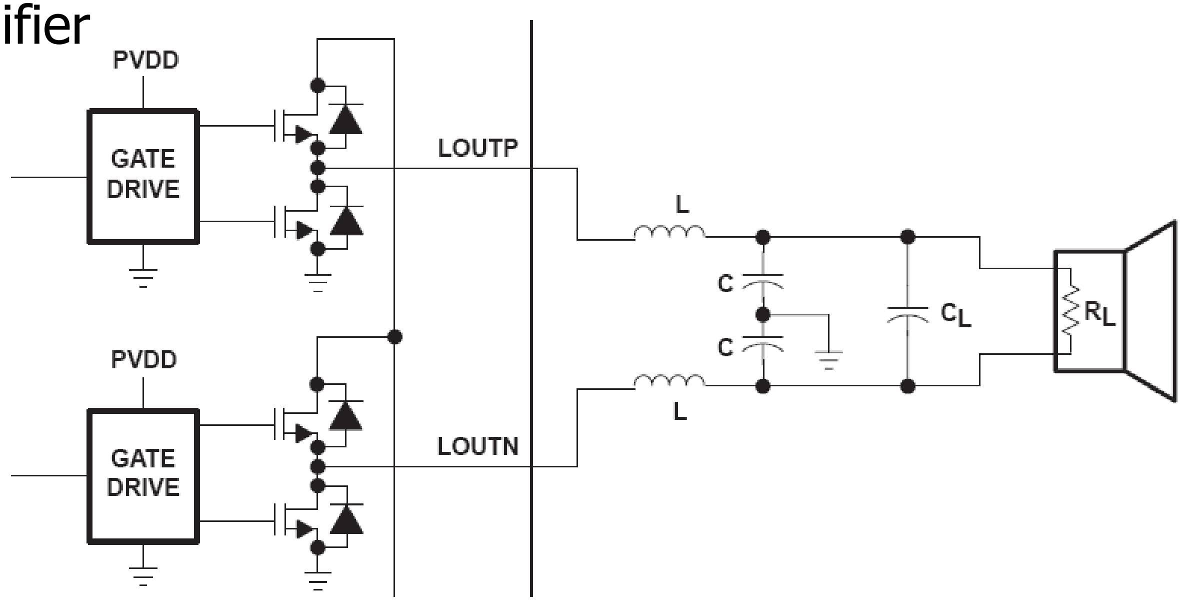
Hear some noise?

Analog filters: Examples

Class D (PWM) audio amplifier

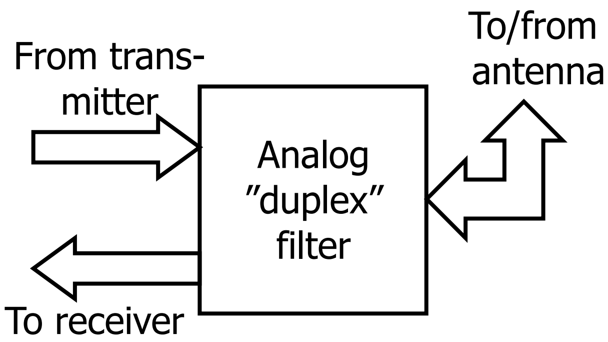
[http://focus.ti.com/lit/an/sloa031/sloa031.pdf]

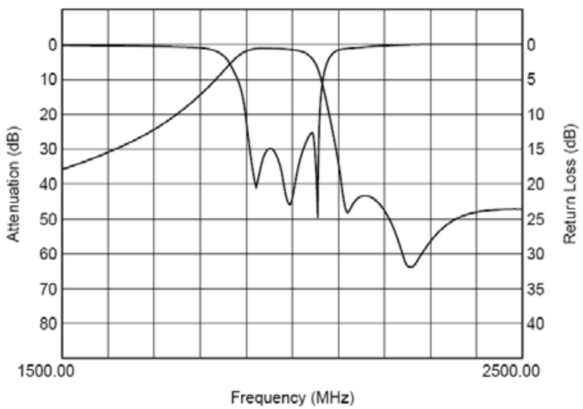




3G mobile phone ~ 2 GHz

[http://murata.com/catalog/o81e.pdf]





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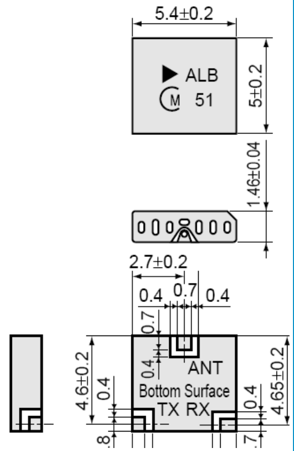
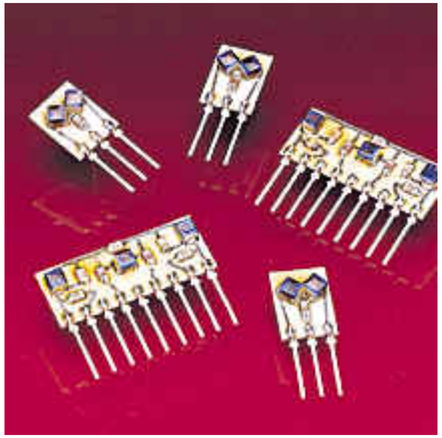
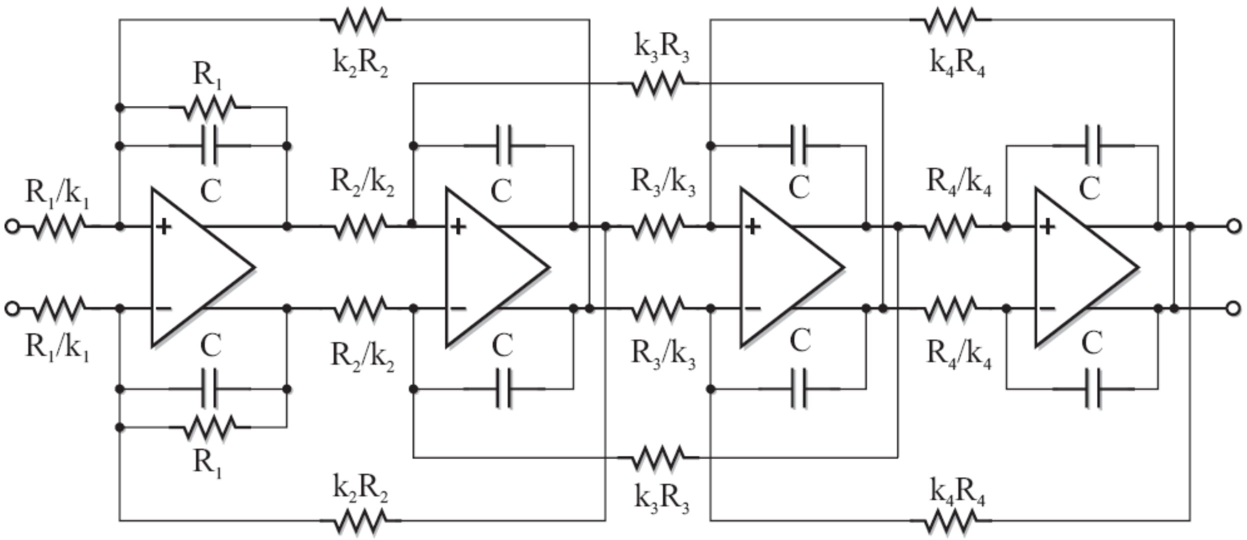


Analog filters: Examples



Coilcraft LC low-pass filters ”xx-xxx” MHz

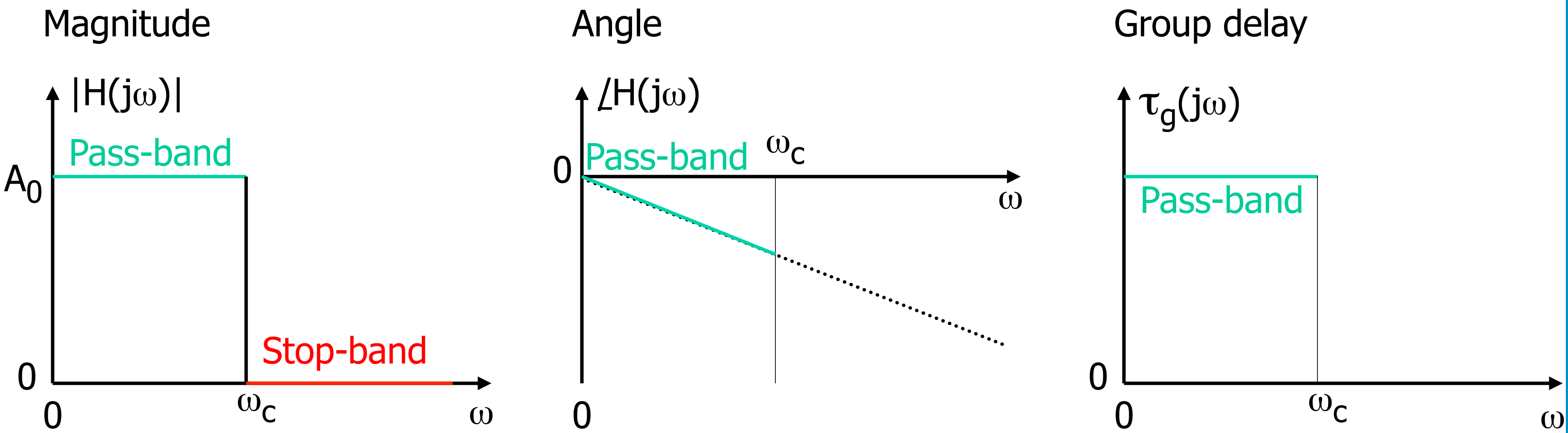


3G mobile phone. Integrated adjustable low-pass filter (2 MHz)
[Jan H. Mikkelsen]

Note: Not all design methods required for design of these filters are covered in this course

Ideal filters

Low-pass filter:



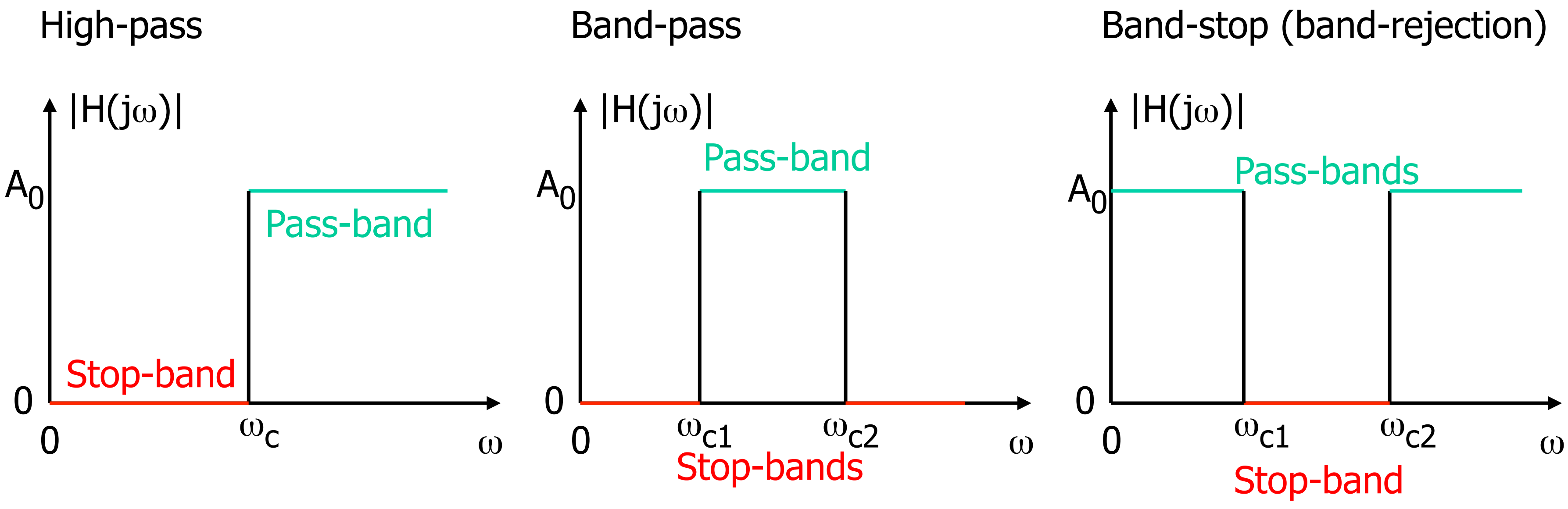
$$\mathcal{L}\left\{f(t)\right\} = F(s)$$

$$\mathcal{L}\left\{f(t-\tau)\cdot u(t-\tau)\right\} = F(s)\cdot e^{-s\tau}$$

$$H(j\omega) = e^{-j\omega\tau} \quad \Leftrightarrow \quad delay$$

$$\tau_g(j\omega) = -\frac{d\angle H(j\omega)}{d\omega}$$

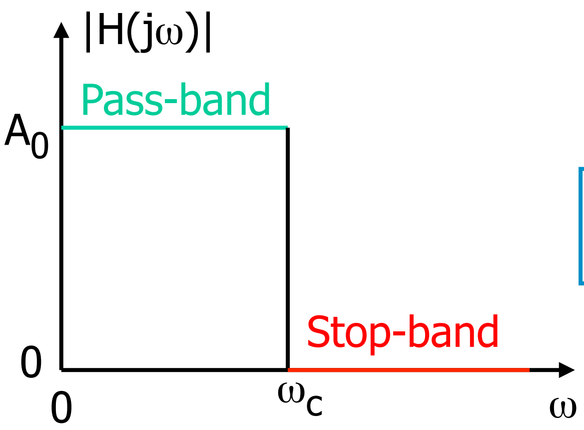
Ideal filters



Ideal filters:

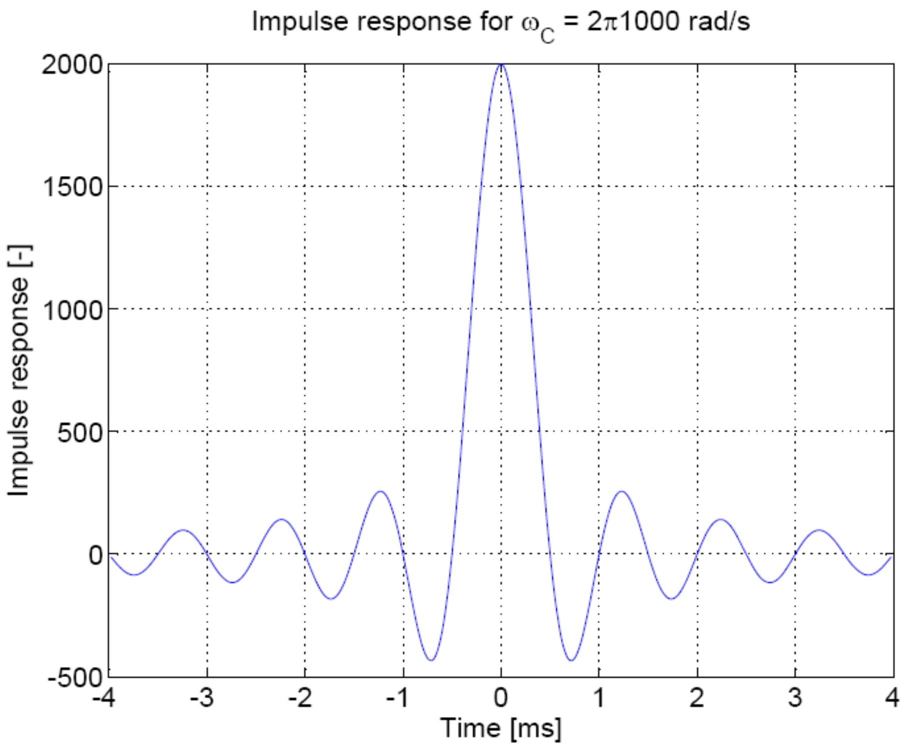
- o Zero transmission in stop-bands
- o Constant transmission magnitude in pass bands
- o Linear transmission phase ⇔ constant group delay in the transmission bands
- o No distance between pass- and stop-bands
- o Impossible to make ☹

Ideal LP-filter



The impulse response may be found as:

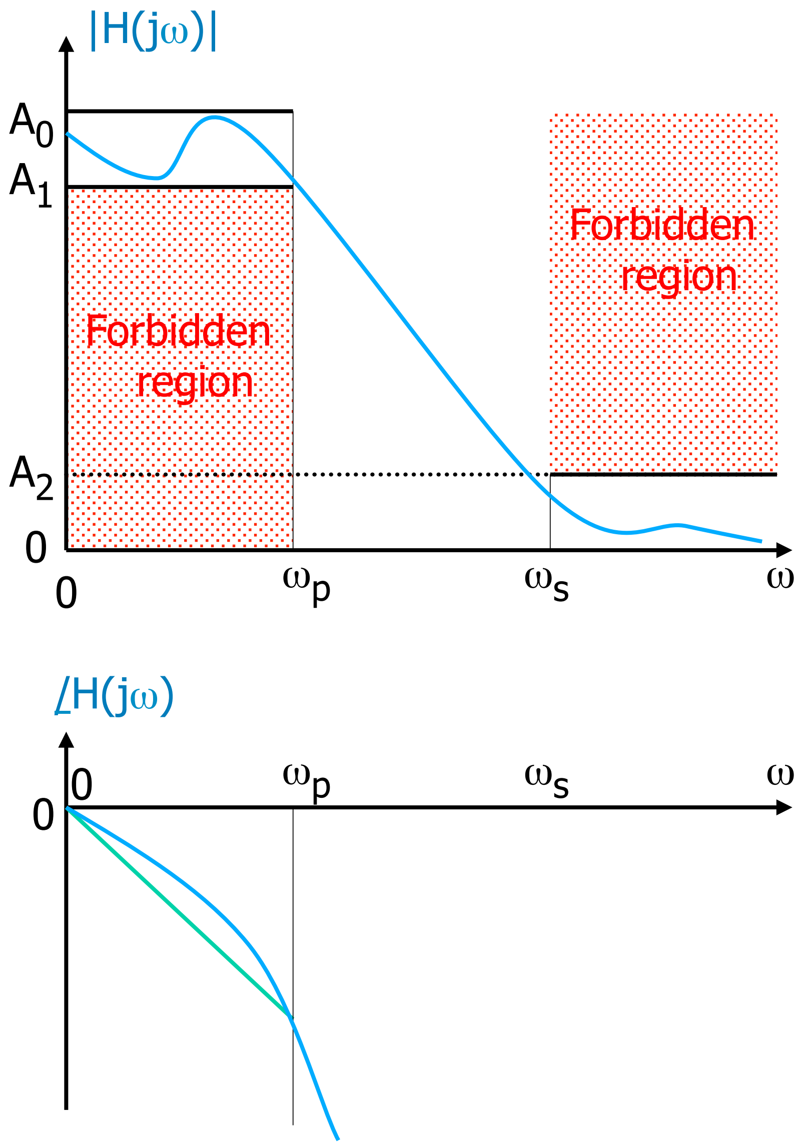
$$h(t) = \frac{\sin(\omega_C t)}{\pi t}$$



h(t) :

- o Has infinite duration
- o Is not causal (output starts before input)

Real filters



A real filters transfer function is not perfect – limits must be specified

Low-pass filter requirements:

- o Maximum attenuation at and below the pass-band edge, ω_p.
- o Minimum attenuation at and beyond the stop-band edge, ω_s.

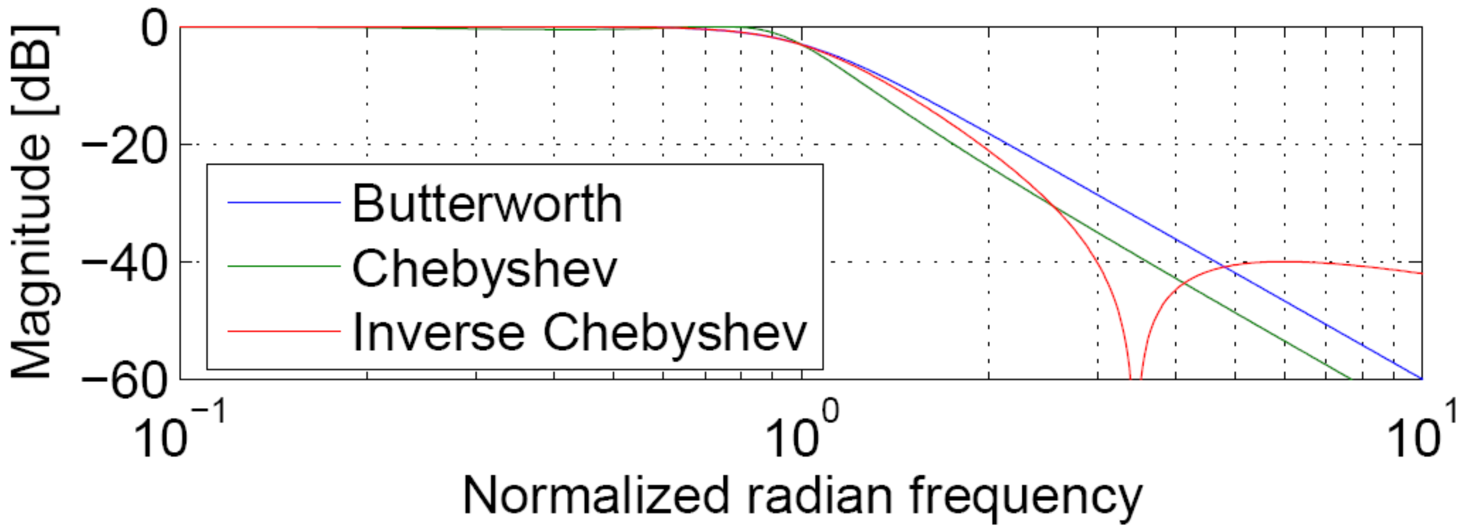
Conflicting requirements:

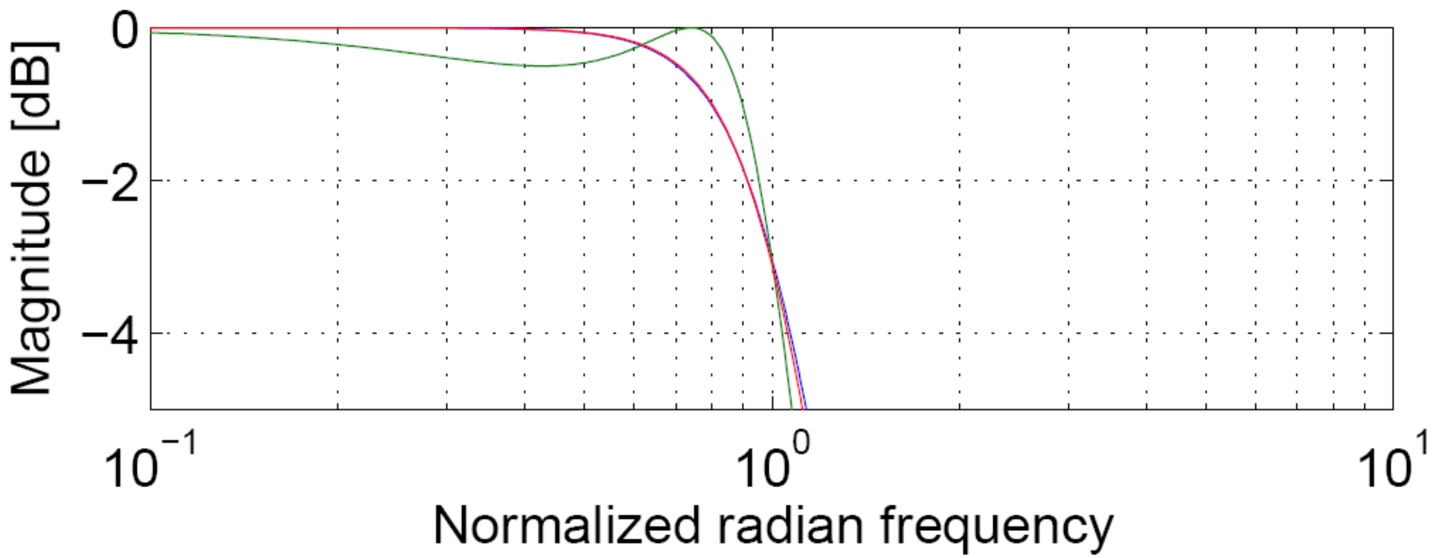
- o Low passband variation, A_0/A_1.
- o High stopband attenuation, A_0/A_2.
- o Low transition band ratio = shape factor, ω_s/ω_p .
- o Simple circuit

Maybe also requirements for:

- o Phase nonlinearity ~ group delay variation in the pass-band

Examples of filter approximations (3rd order)





Filter prototypes

Filter designs are usually based on a library of ”prototypes” describing:

- o Transfer function polynomials for active realisation
- o Component values for a passive realisation

$$H(s) = \frac{K}{a_n s^n + a_{n-1} s^{n-1} \ldots + a_1 s + 1}$$

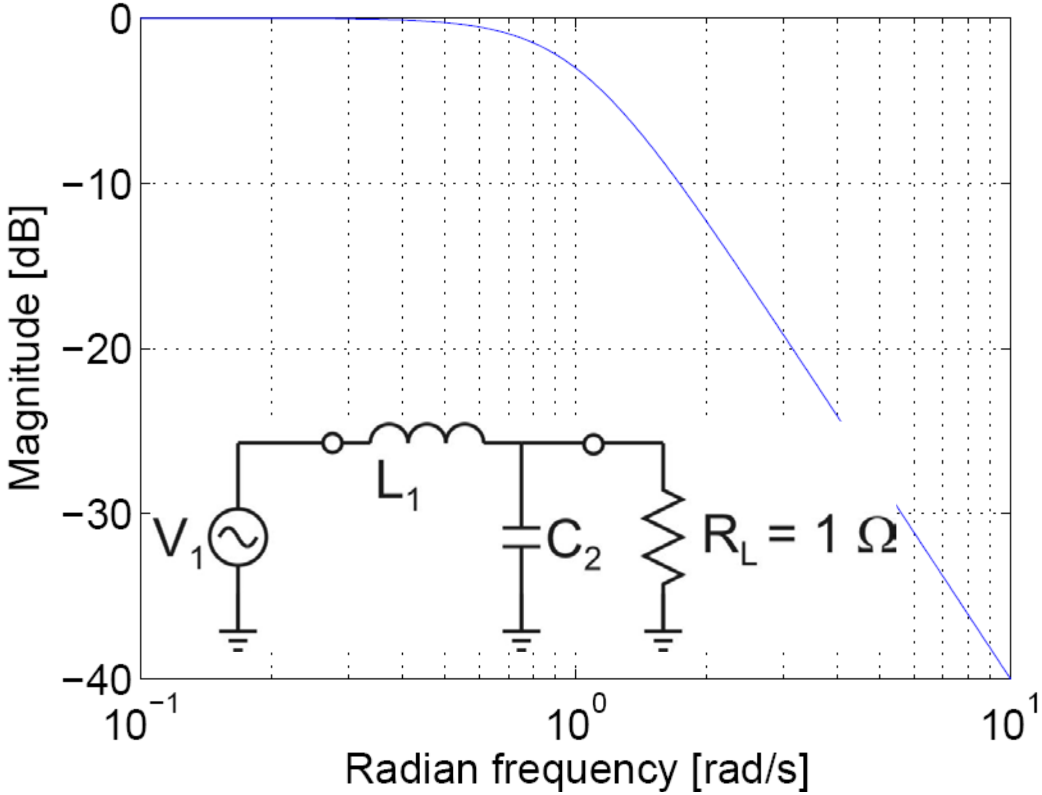
Prototypes are normalized:

- o Passband-edge radian frequency $\omega_{P,Norm}$ = 1 rad/s (~ f_P = 0.159 Hz) (Not necessarily -3 dB)
- o 1 Ω load resistor (passive)

Example (2nd order Butterworth):

$$H(s) = \frac{1}{s^2 L_1 C_2 + s L_1 / R_L + 1}$$

$$C = \frac{1}{\sqrt{2}}\ F \qquad L = \sqrt{2}\ H$$



Disclaimer

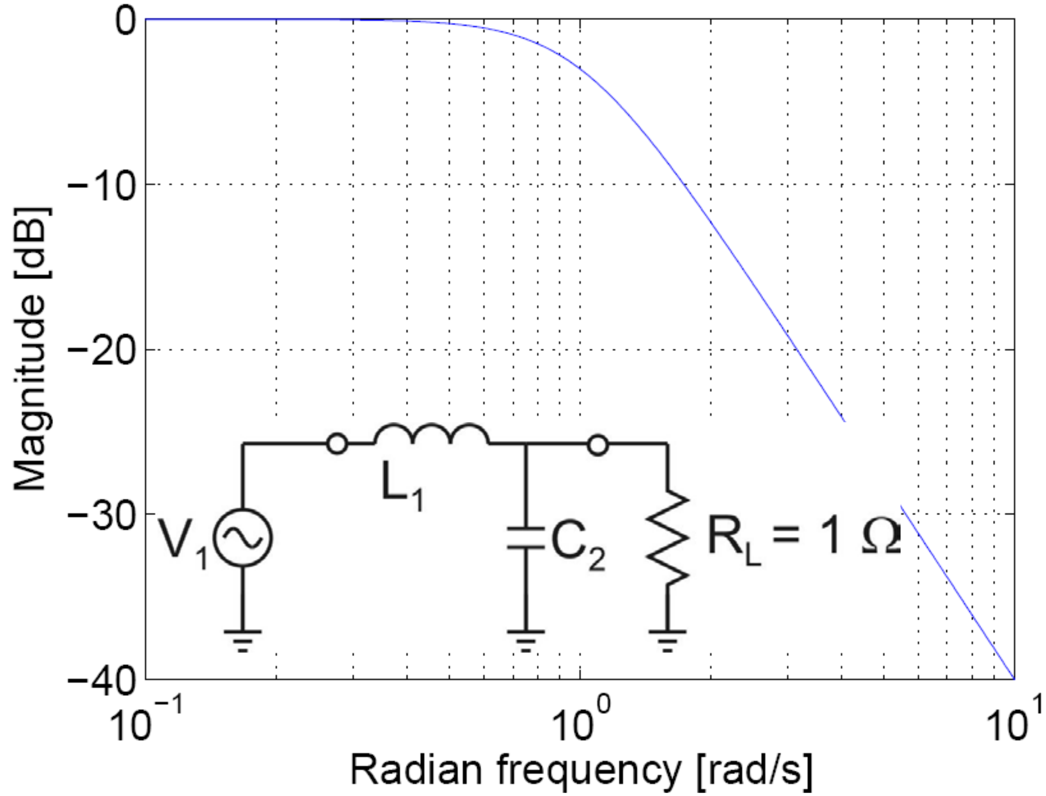
$$H(s) = \frac{1}{s^2 L_1 C_2 + sL_1 / R_L + 1}$$

$$C = \frac{1}{\sqrt{2}}\ F \qquad L = \sqrt{2}\ H$$

$$H(s) = \frac{1}{s^2 + s\sqrt{2} + 1}$$

Units omitted

Mathematically incorrect!!!



In filter literature it is common practice:

- o To use normalized prototypes (1 rad/s, 1 Ω load resistor (passive))
- o Disregard units

This will also be done in this course

zzzz….2 min

Butterworth approximation

Steps in the derivation:

1. The magnitude of the transfer function, $|H(j\omega)|$, is defined
2. Find the poles of the transfer function
3. Find the transfer function, H(s)
4. Find a circuit realizing H(s)
   - Passive
   - Active

$|H(j\omega)|^2$ is more convenient to work with than $|H(j\omega)|$

A transfer function magnitude of the form:

?

$$|H(j\omega)|^2 = \frac{A_0}{1+F(\omega^2)} \quad where$$

$$0 < F(\omega^2) << 1 \quad for \quad \omega < \omega_p$$

$$F(\omega^2) >> 1 \quad for \quad \omega > \omega_s$$

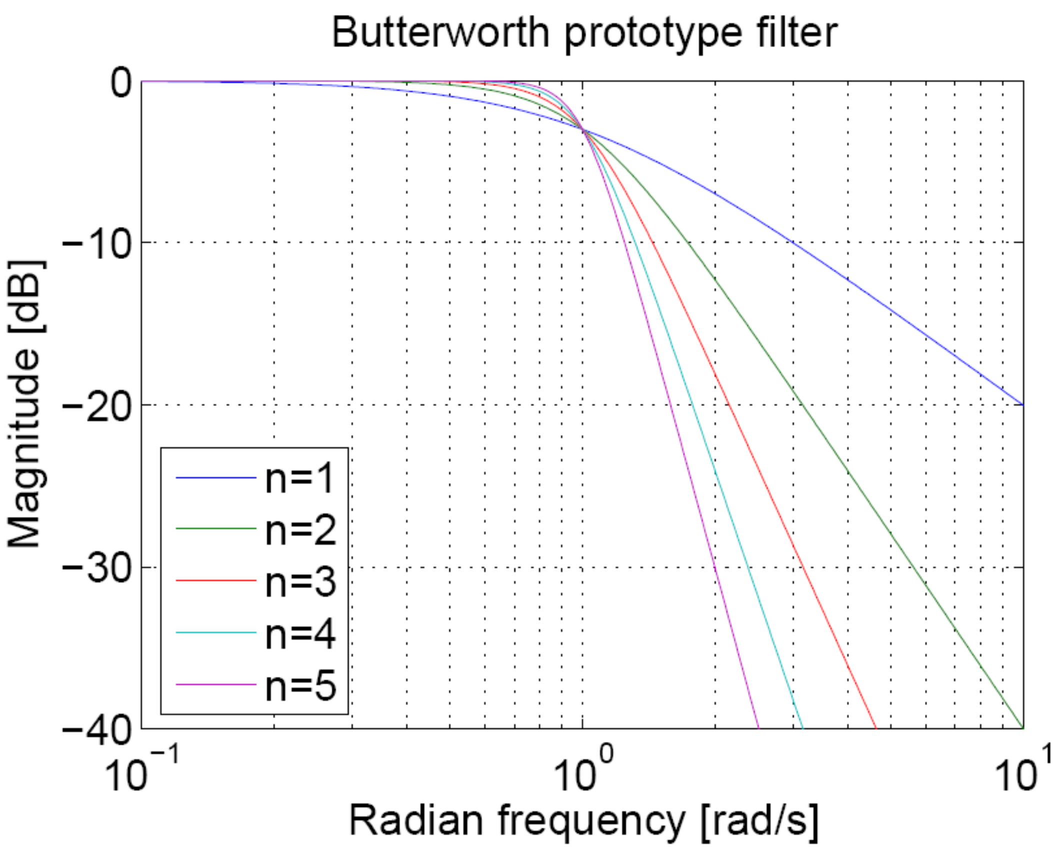
will make a low-pass function

Butterworth approximation/definition (ω_{-3dB} = 1 rad/s)

An n[th] order (n = number of poles) normalized Butterworth filter is defined by:

$$F(\omega^2) = \omega^{2n}$$

$$\boxed{|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}} \quad (A_0 = 1)$$



Properties (independent of order):

$$|H(j\omega)|_{\max} = |H(j0)| = 1$$

$$|H(j1\,rad/s)| = \frac{1}{\sqrt{2}} \sim -3\,dB$$

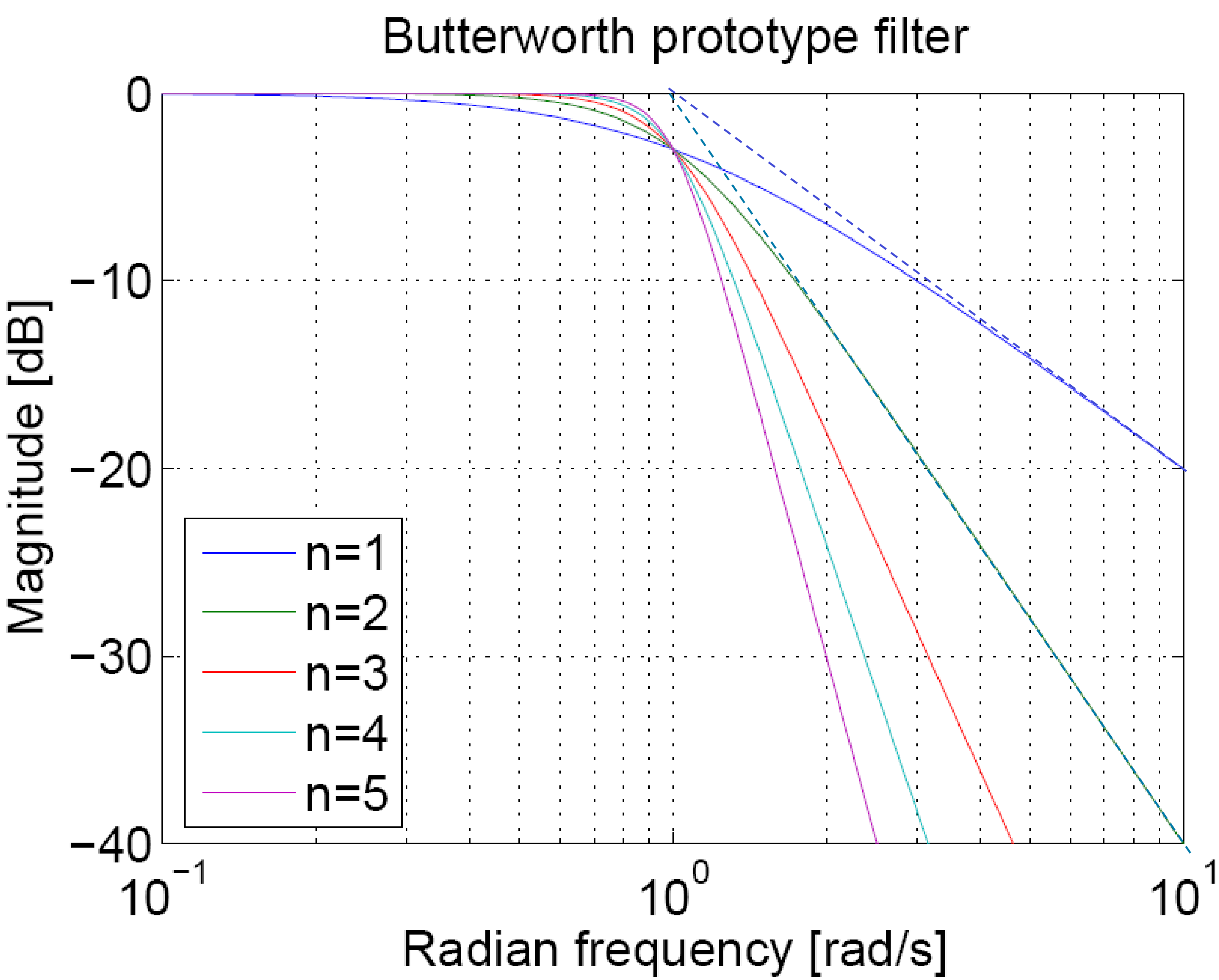
Butterworth approximation

It can be shown that the derivatives:

$$\left.\frac{d^k|H(j\omega)|}{d\omega^k}\right|_{\omega=0} = 0$$

$$for\ k = 1,2,\ldots.2n-1$$

"maximally flat"



Stopband:

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

$$|H(j\omega)|^2 \rightarrow \omega^{-2n}$$

$$\omega \rightarrow \infty$$

$$\sim -n \cdot 20\ dB/dec$$

Butterworth transfer function (ω_{-3dB} = 1 rad/s)

Given:

$$\left|H(j\omega)\right|^2 = \frac{1}{1+\omega^{2n}}$$

Find H(s)

Sorry, it requires a few equations ☹:

A transfer function of the form:

$$H(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_2 s^2 + a_1 s + 1} \quad \textit{where } a_i\textit{'s are real}$$

has the property:

$$H(-j\omega) = H^*(j\omega)$$

so:

$$\left|H(j\omega)\right|^2 = H(j\omega)\cdot H^*(j\omega) = H(s)\cdot H(-s)\Big|_{j\omega=s}$$

$$H(s)\cdot H(-s) = \frac{1}{1+\left(-s^2\right)^n}$$

Butterworth transfer function (ω_{-3dB} = 1 rad/s)

$$H(s)\cdot H(-s) = \frac{1}{1+\left(-s^2\right)^n}$$

Find poles ⇔ roots of:

$$1+\left(-s^2\right)^n = 0$$

$$\left(-s^2\right)^n = -1 = e^{j(-\pi+2k\pi)}$$

$$-s^2 = e^{j\frac{2k-1}{n}\pi}$$

$$s^2 = e^{j\left(\frac{2k-1}{n}+1\right)\pi}$$

(Another choice of signs in the book)

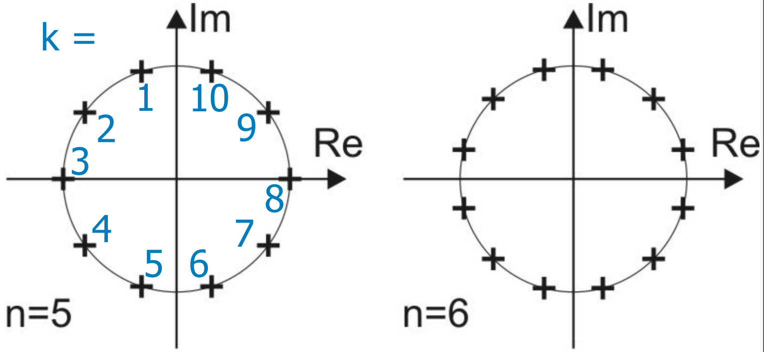
$$s = e^{j\left(\frac{2k-1}{2n}\pi+\frac{\pi}{2}\right)} \qquad k = 1,2,\ldots 2n$$

Butterworth transfer function (ω_{-3dB} = 1 rad/s)

Poles of H(s)·H(-s):

$$p_k = e^{j\left(\frac{2k-1}{2n}\pi + \frac{\pi}{2}\right)} \qquad k = 1,2,\ldots 2n$$

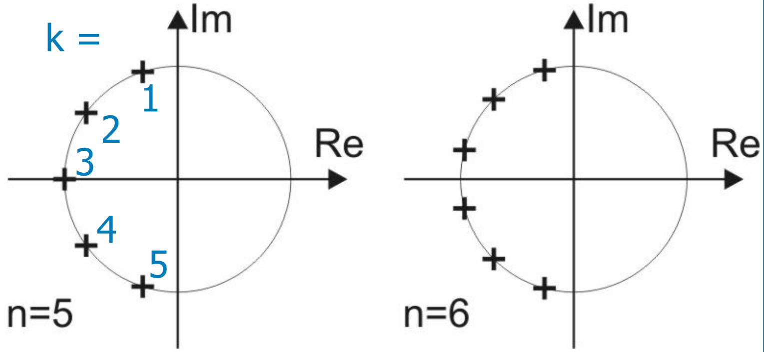
$$\text{PS : Kendall Su : } p_k = e^{j\left(\frac{2k+1}{2n}\pi - \frac{\pi}{2}\right)}$$



Since H(s) must be a stable function, poles in the left half plane are assigned to H(s) (and right hand poles to H(-s))

Poles of H(s) :

$$\boxed{p_k = e^{j\left(\frac{2k-1}{2n}\pi + \frac{\pi}{2}\right)} \qquad k = 1,2\ldots n}$$

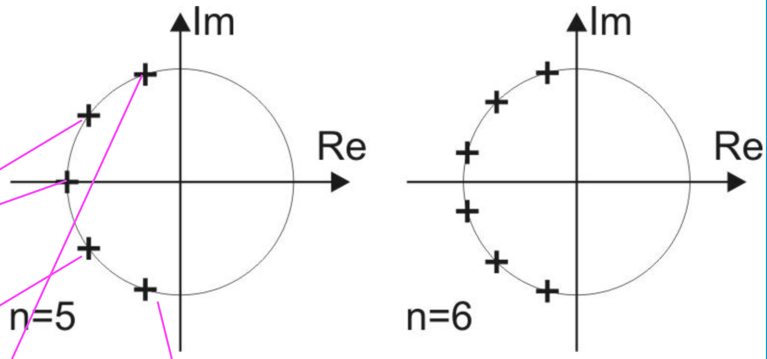


Note that:

- o Only complex conjugated pole pairs when n is even
- o Complex conjugated pole pairs and one real pole when n is odd

Butterworth transfer function (ω_{-3dB} = 1 rad/s)

Poles of H(s) :



Transfer function, n odd:

(p_r ⇔ real pole, p_c ⇔ complex pole)

$$H(s)=\frac{K}{(s-p_r)(s-p_{c1})(s-p_{c1}^*)(s-p_{c2})(s-p_{c2}^*)\ldots\ldots}$$

$$H(s)=\frac{K}{(s-p_r)(s^2-2\,\mathrm{Re}\{p_{c1}\}s+|p_{c1}|^2)(s^2-2\,\mathrm{Re}\{p_{c2}\}s+|p_{c2}|^2)\ldots}$$

$$H(s)=\frac{1}{(s+1)(s^2-2\,\mathrm{Re}\{p_{c1}\}s+1)(s^2-2\,\mathrm{Re}\{p_{c2}\}s+1)\ldots}$$

n even:

$$H(s)=\frac{1}{(s^2-2\,\mathrm{Re}\{p_{c1}\}s+1)(s^2-2\,\mathrm{Re}\{p_{c2}\}s+1)\ldots}$$

Butterworth transfer function (example: 3rd order)

Poles:

$$p_k = e^{j\left(\frac{2k-1}{2\cdot 3}\pi + \frac{\pi}{2}\right)} \quad k = 1,2,3 \quad p_k = \begin{cases} e^{j2\pi/3} \\ -1 \\ e^{j4\pi/3} = e^{-j2\pi/3} \end{cases}$$

Transfer function:

$$H(s) = \frac{1}{(s+1)(s-e^{j2\pi/3})(s-e^{j4\pi/3})}$$

$$= \frac{1}{(s+1)(s^2+s(-e^{j2\pi/3}-e^{j4\pi/3})+1)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{1}{s^3+2s^2+2s+1}$$

10 min. break

Break over

Butterworth transfer function (ω_{-3dB} = 1 rad/s)

The Butterworth transfer functions may be found using the equations on the previous slides.

Alternative 1: Kendall Su: ”Analog Filters”, tables A.1 and A.2
where Butterworth polynomials = denominator of H(s) are given.

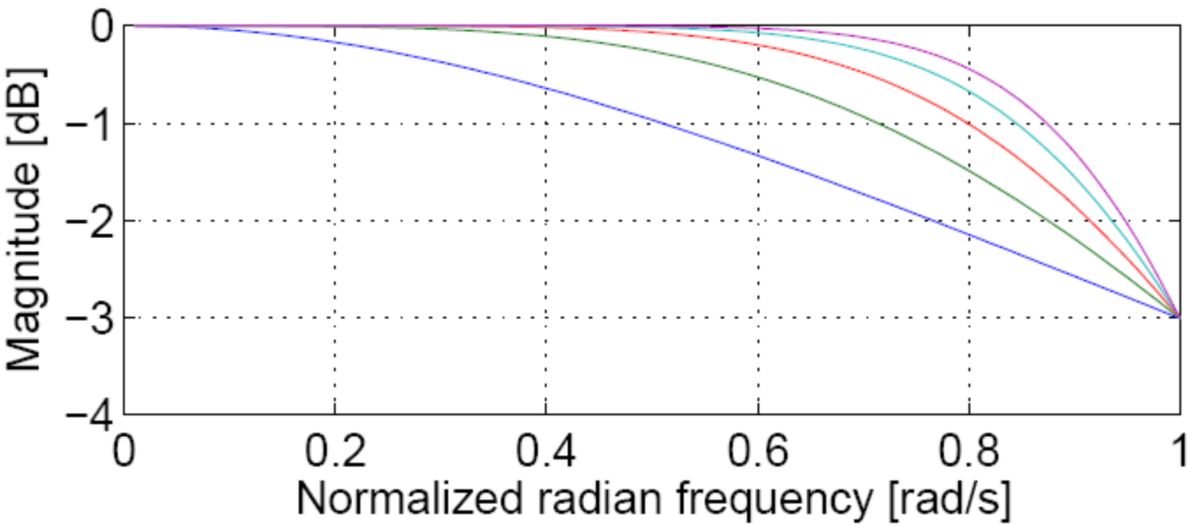
Alternative 2: Matlab – last slides.

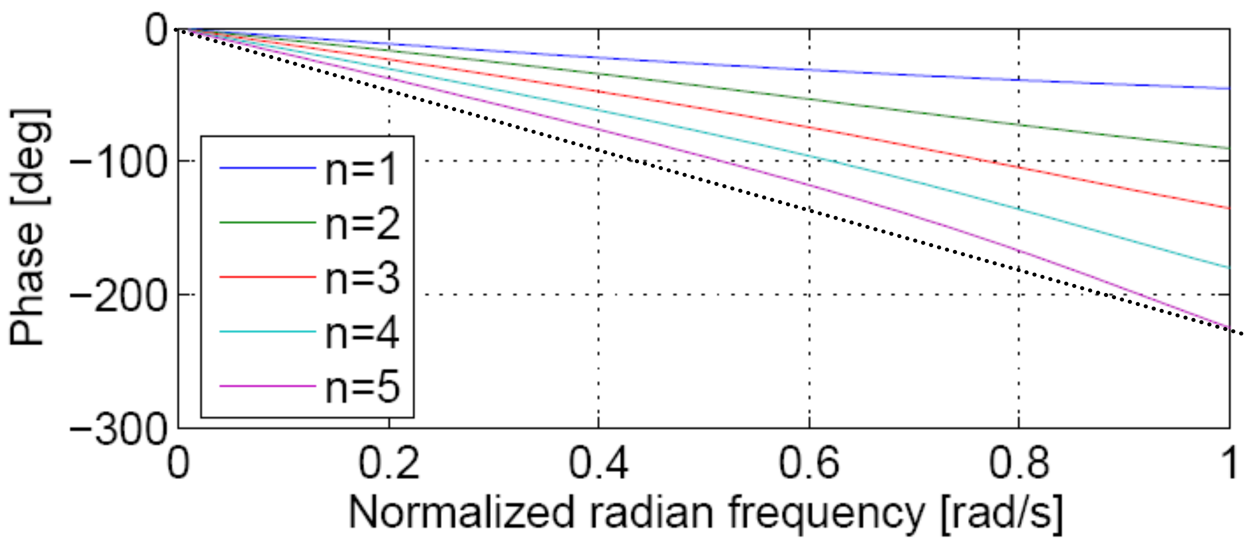
Butterworth phase response

The transfer function is created without conside-ring the phase.

Result: Some deviation from linearity
☺ ?
☹ ?

This will be investigated a bit further in lecture 2

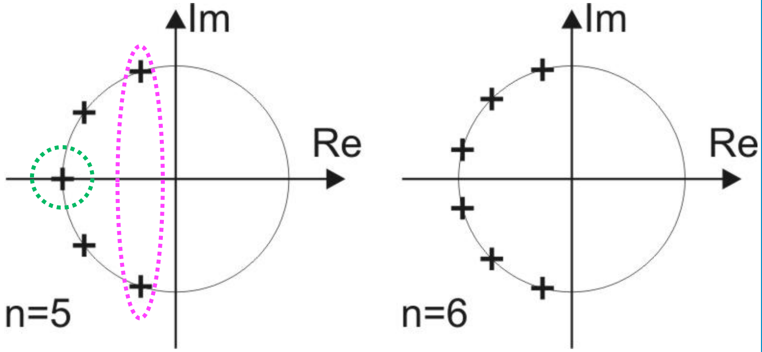




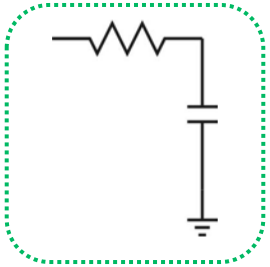
Active realisation (more on this in lecture 4 & 5)

$$H_{odd}(s) = \frac{1}{(s+1)(s^2 - 2\,\mathrm{Re}\{p_{c1}\}s + 1)(s^2 - 2\,\mathrm{Re}\{p_{c2}\}s + 1)\ldots}$$

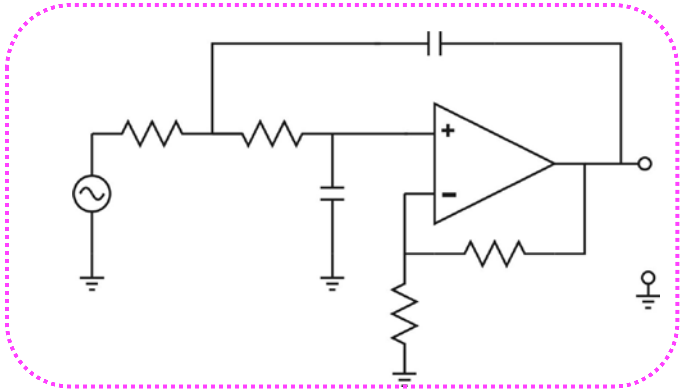
$$H_{even}(s) = \frac{1}{(s^2 - 2\,\mathrm{Re}\{p_{c1}\}s + 1)(s^2 - 2\,\mathrm{Re}\{p_{c2}\}s + 1)\ldots}$$



A real pole is realized by an RC-circuit



A complex pole pair is realized by one of many op-amp circuits, e.g.:



Passive "ladder" realisation (more on this in lecture 2)

The transfer function may be written in the form (a$_n$ = 1 for Butterworth):

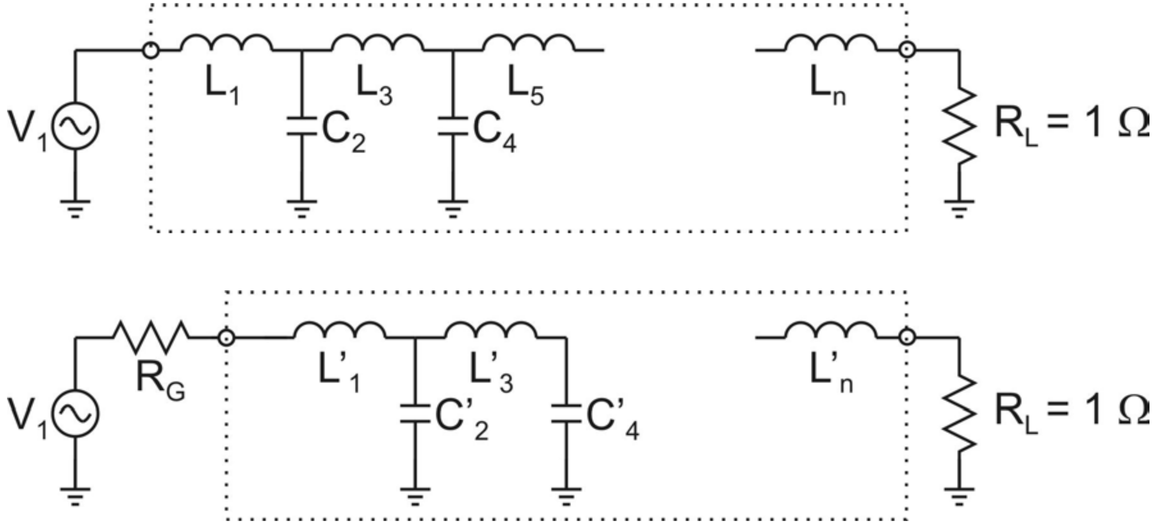
$$H(s) = \frac{1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_2 s^2 + a_1 s + 1} \quad \textit{where } a_i\textit{'s are real}$$

This can be realized with a ladder circuit:

- o Singly terminated, $R_G = 0$
- o Doubly terminated

- o Component values can be found analytically
  Ref.: Kendall Su: "Analog Filters", Ch. 5, 6 & 7.

- o Tables of component values can be found in reference books



n reactive components gives an nth order transfer function

Prototypes are normalized: 1 rad/s, 1 Ω load resistor

Frequency scaling

The prototype has a normalized transfer function

It's wanted to have some other bandedge frequency

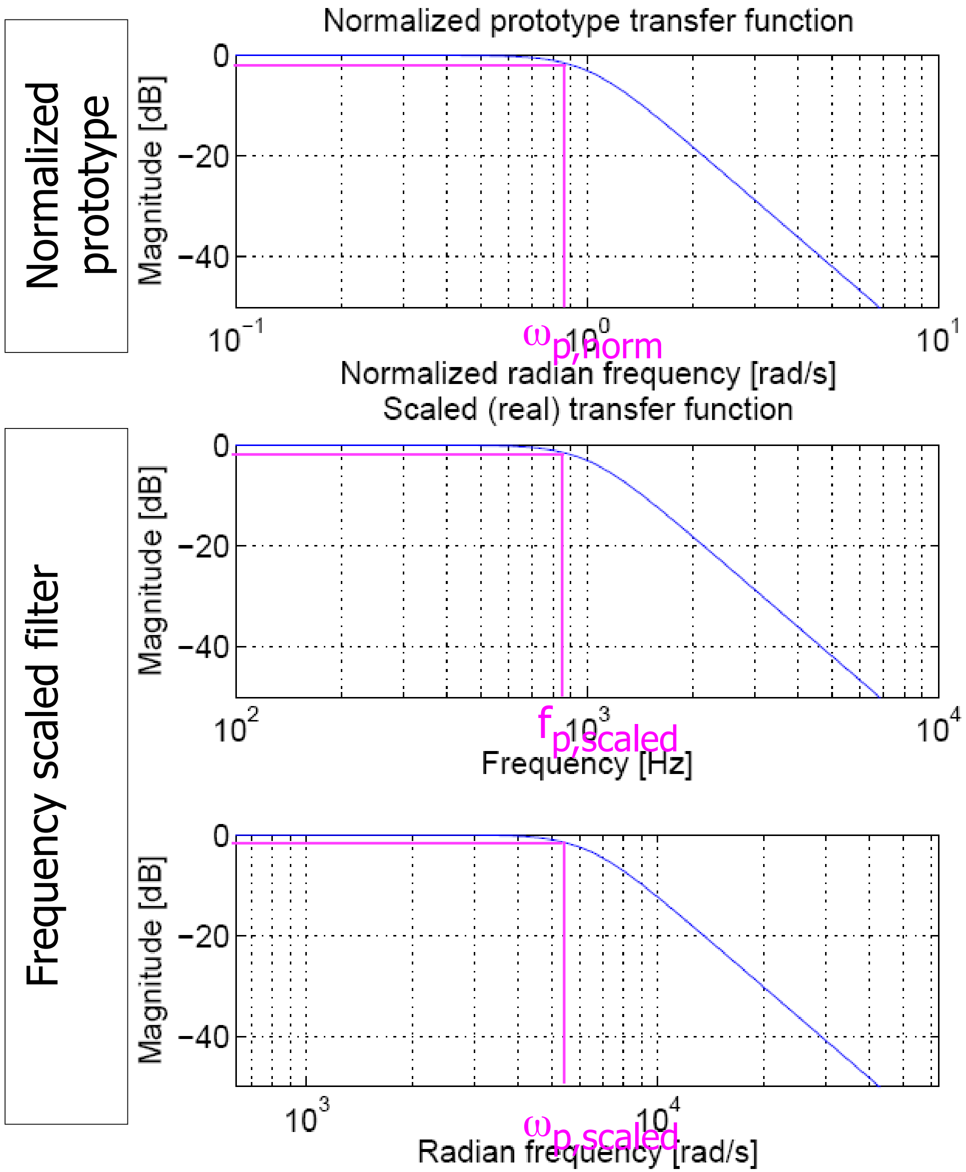
The frequency scaling factor is:

$$k_f = \frac{\omega_{p,scaled}}{\omega_{p,norm}} = \frac{2\pi \cdot f_{p,scaled}}{\omega_{p,norm}}$$

And then the transfer functions are related by:

$$\boxed{H_{scaled}(j\omega) = H_{norm}\left(\frac{j\omega}{k_f}\right)}$$

$$\left|H_{scaled}(j\omega)\right|^2 = \frac{1}{1+\left(\omega / k_f\right)^{2n}}$$

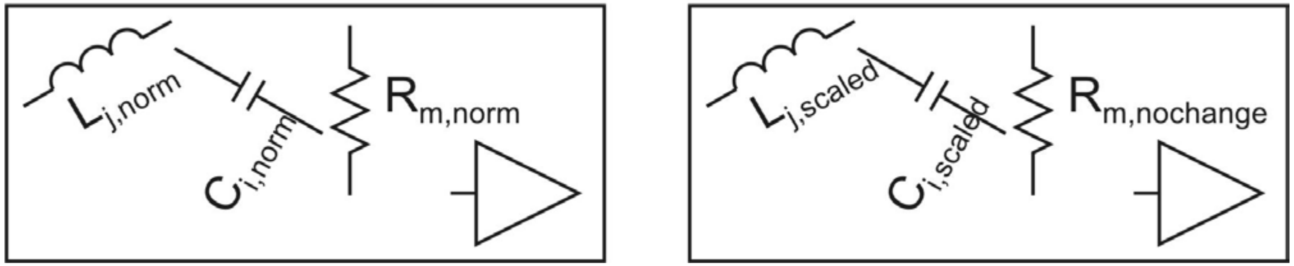


Frequency scaling

The frequency response is obtained by scaling the reactive circuit components:

Normalized prototype

Frequency scaled filter



The same value of transfer function requires the same set of impedances:

$$j\omega_{p,norm}L_{j,norm} = j\omega_{p,scaled}L_{j,scaled}$$

$$L_{j,scaled} = \frac{\omega_{p,norm}}{\omega_{p,scaled}} L_{j,norm}$$

$$L_{j,scaled} = \frac{L_{j,norm}}{k_f}$$

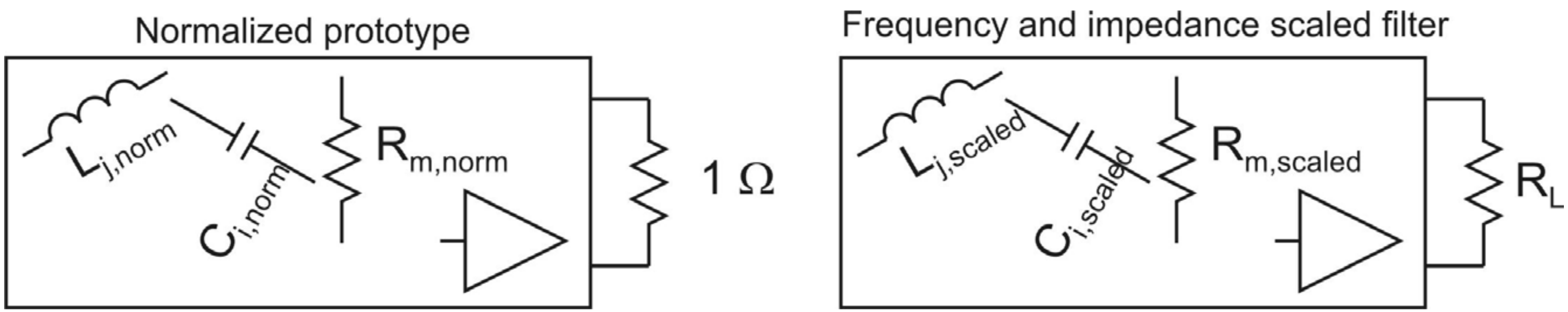
Likewise:

$$C_{i,scaled} = \frac{C_{i,norm}}{k_f}$$

Impedance scaling

Likewise, the impedance level is scaled:

- o Passive: Scaling with load impedance
- o Active: Scaling to an ”appropriate” impedance level



The same transfer function requires the same set of impedance ratios:

Inductor values increase with increasing impedance level

Capacitor values decrease with increasing impedance level

$$k_z = \frac{R_L}{1\,\Omega}$$

$$k_f = \frac{\omega_{p,scaled}}{\omega_{p,norm}}$$

$$L_{j,scaled} = \frac{k_z}{k_f} L_{j,norm}$$

$$C_{i,scaled} = \frac{1}{k_f k_z} C_{i,norm}$$

$$R_{m,scaled} = k_z R_{m,norm}$$

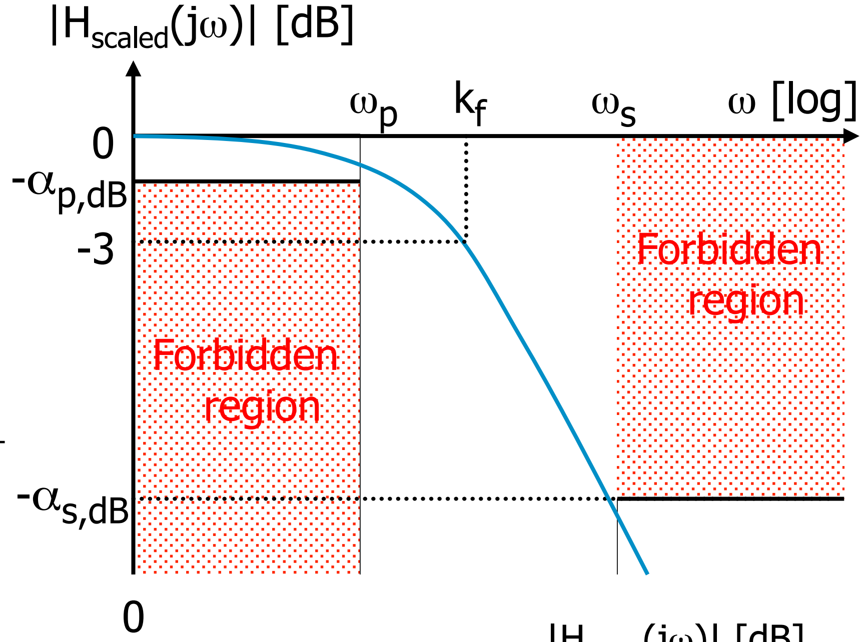
Butterworth, necessary order

Required:

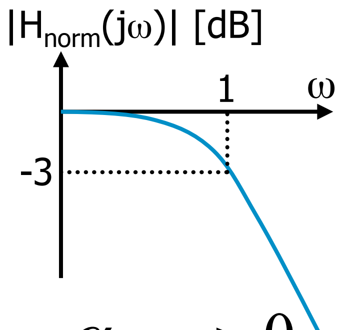
- o Max. $\alpha_{p,dB}$ attenuation at ω_p
- o Min. $\alpha_{s,dB}$ attenuation at ω_s

Given: General form of the Butterworth magnitude function:

$$\left|H_{scaled}(j\omega)\right|^2 = \left|H_{norm}\left(\frac{j\omega}{k_f}\right)\right|^2 = \frac{1}{1+\left(\frac{\omega}{k_f}\right)^{2n}}$$



$$\alpha_{s,dB} = 10\cdot\log\left(1+\left(\frac{\omega_s}{k_f}\right)^{2n}\right) \qquad \alpha_{p,dB} = 10\cdot\log\left(1+\left(\frac{\omega_p}{k_f}\right)^{2n}\right)$$



$$\left(\frac{\omega_s}{\omega_p}\right)^{2n} = \frac{10^{\alpha_{s,dB}/10}-1}{10^{\alpha_{p,dB}/10}-1}$$

$$\boxed{n \ge \frac{1}{2\cdot\log\frac{\omega_s}{\omega_p}}\log\frac{10^{\alpha_{s,dB}/10}-1}{10^{\alpha_{p,dB}/10}-1}}$$

$$\alpha_{p,dB} > 0$$
$$\alpha_{s,dB} > 0$$

Butterworth, scaling factor

The scaling factor is required for design:

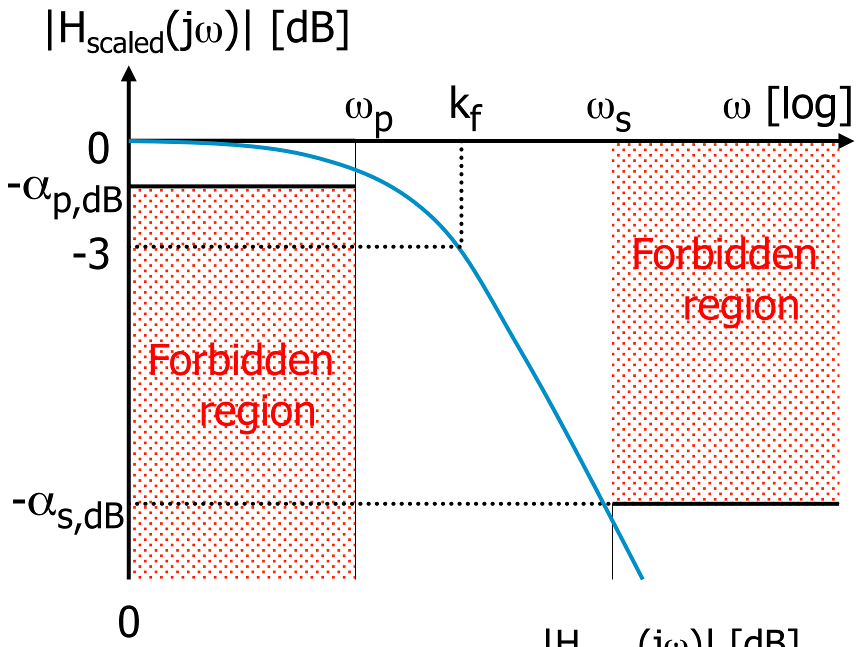
- 3 dB attenuation @ $\omega = k_f$:
- setting $\omega_S = k_f$ (just reuse the equation):

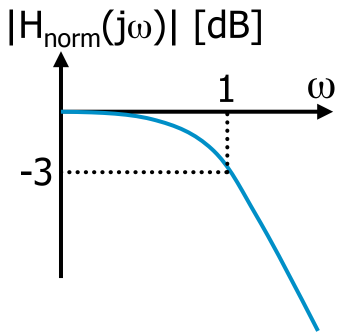
$$\left(\frac{k_f}{\omega_p}\right)^{2n} = \frac{10^{3/10} - 1}{10^{\alpha_{p,dB}/10} - 1}$$

$\Updownarrow$

$$k_f = \frac{\omega_p}{\sqrt[2n]{10^{\alpha_{p,dB}/10} - 1}}$$

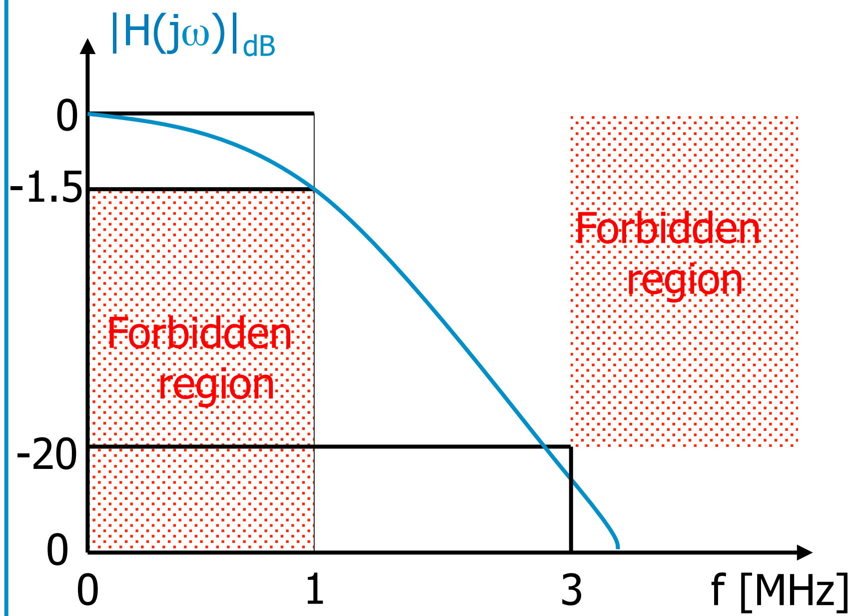
(If $\alpha_{p,dB}$ = 3 dB, then $\omega_p = k_f$)





zzzz....2 min

Butterworth, example



Requirements:

- 1.5 dB attenuation at 1 MHz
- At least 20 dB att. at 3 MHz
- Passive filter, Load resistance = 100 Ω

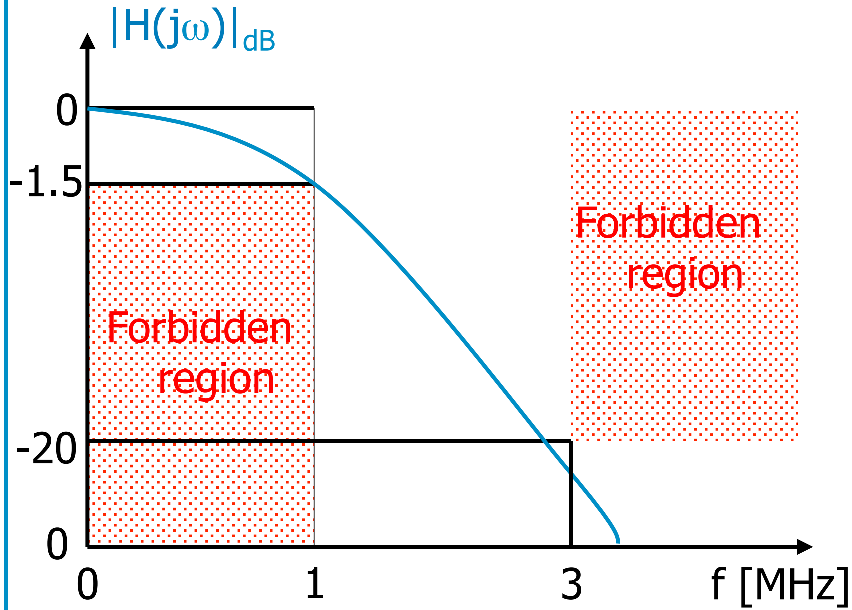
Necessary order:

$$n \geq \frac{1}{2 \cdot \log \frac{\omega_s}{\omega_p}} \log \frac{10^{\alpha_{s,dB}/10} - 1}{10^{\alpha_{p,dB}/10} - 1} = \frac{1}{2 \cdot \log 3} \log \frac{10^{20/10} - 1}{10^{1.5/10} - 1} = 2.49 \rightarrow 3$$

Frequency scaling factor:

$$k_f = \frac{\omega_p}{\sqrt[2n]{10^{\alpha_{p,dB}/10} - 1}} = \frac{2\pi \cdot 10^6}{\sqrt[6]{10^{1.5/10} - 1}} = 7.28 \cdot 10^6$$

Butterworth, example



Actual attenuation @ 3 MHz:

$$\alpha_{s,dB} = 10 \cdot \log\left(1 + \left(\frac{\omega_s}{k_f}\right)^{2n}\right) =$$

$$10 \cdot \log\left(1 + \left(\frac{2\pi \cdot 3 \cdot 10^6}{7.28 \cdot 10^6}\right)^{6}\right) = 24.8dB$$

Alternative:

$$H_{norm}(s) =$$

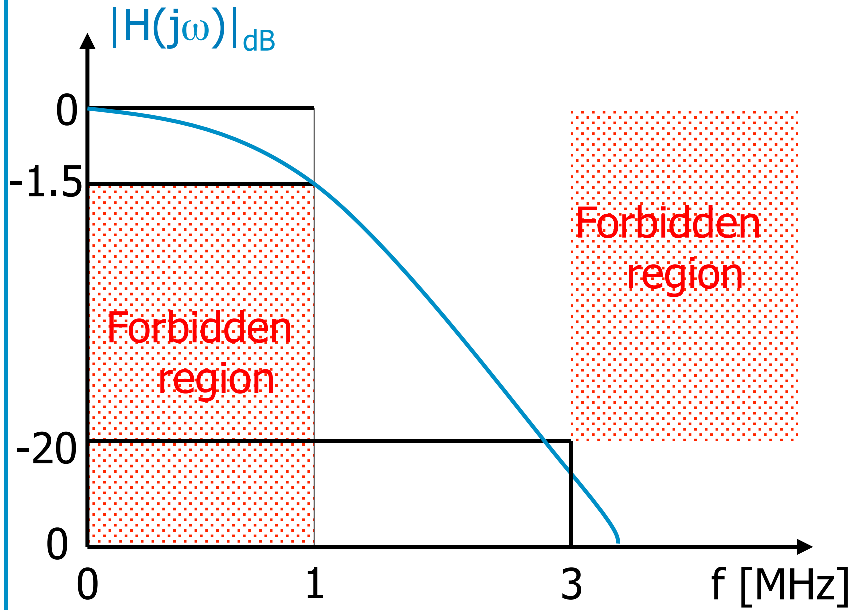
$$\frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$H_{scaled}(s) = \frac{1}{\left(\frac{s}{k_f}\right)^3 + 2\left(\frac{s}{k_f}\right)^2 + 2\frac{s}{k_f} + 1}$$

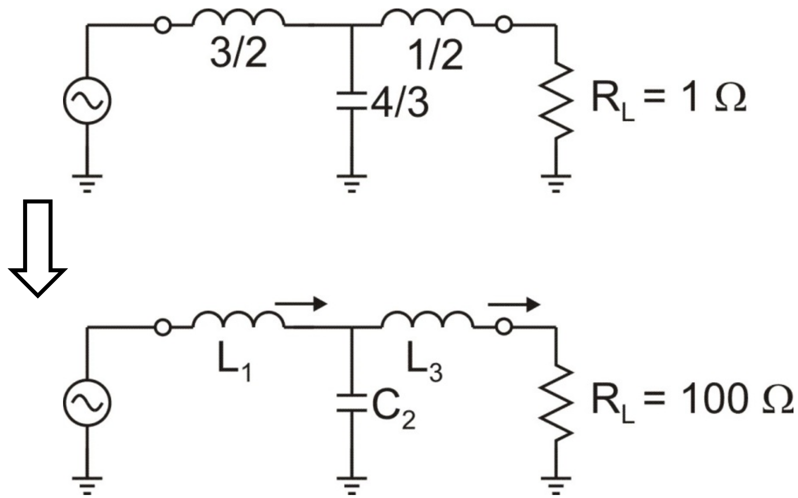
$$H_{scaled}(j2\pi \cdot 3e6) = \frac{1}{\left(\frac{j2\pi \cdot 3e6}{7.28e6}\right)^3 + 2\left(\frac{j2\pi \cdot 3e6}{7.28e6}\right)^2 + 2\frac{j2\pi \cdot 3e6}{7.28e6} + 1}$$

$$H_{scaled}(j2\pi \cdot 3e6) = \frac{1}{(j2.59)^3 + 2(j2.59)^2 + 2 \cdot j2.59 + 1} = 0.0575\angle 136^\circ$$

Butterworth, example



Design based on a prototype circuit:



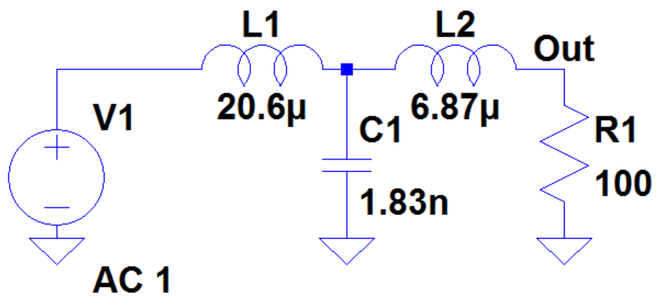
$$L_1 = \frac{3k_z}{2k_f} = \frac{3 \cdot 100}{2 \cdot 7.28e6} = 20.6\ \mu H \qquad C_2 = \frac{4}{3k_f k_z} = \frac{4}{3 \cdot 7.28e6 \cdot 100} = 1.83\ nF$$

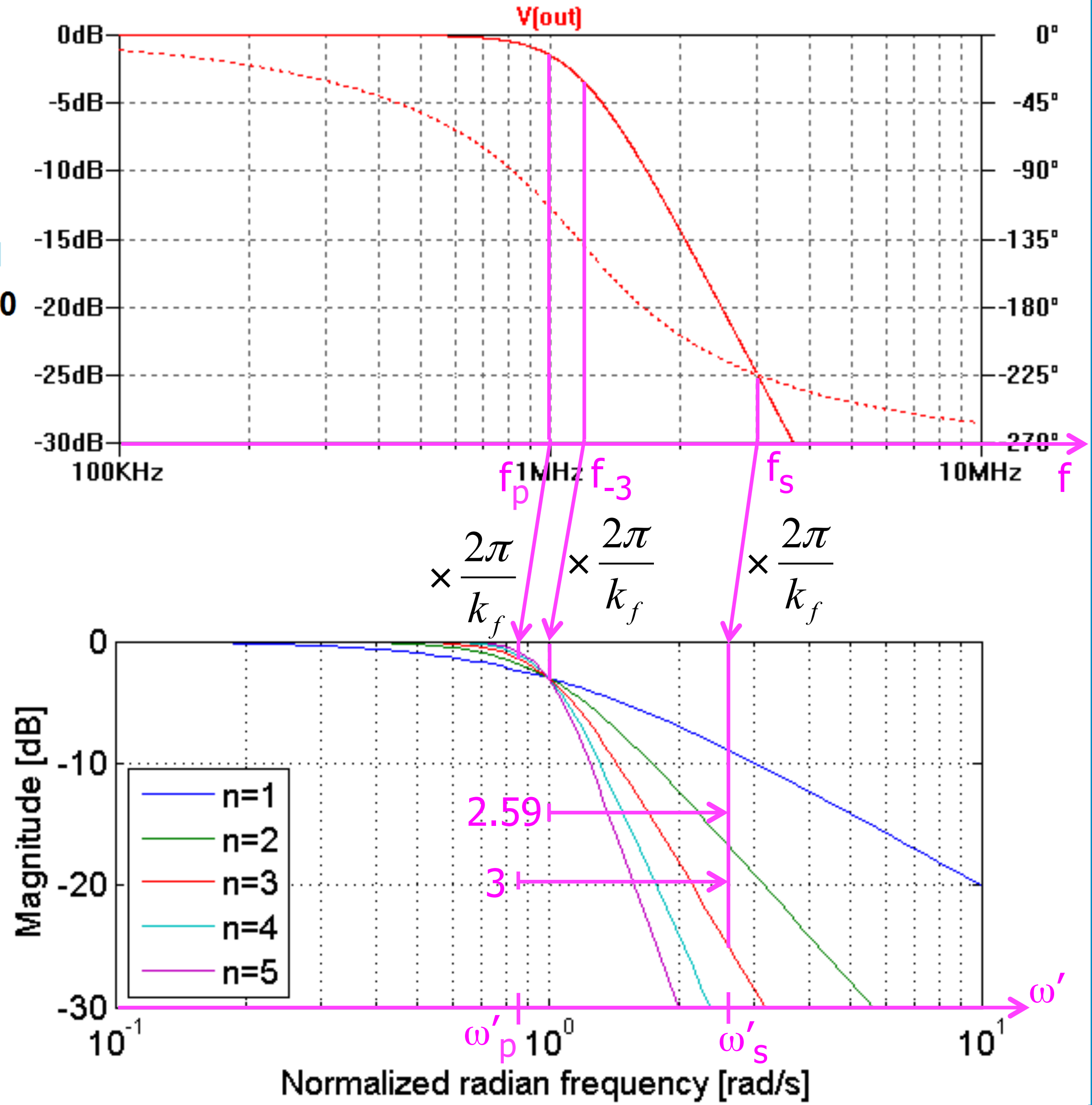
$$L_3 = \frac{1k_z}{2k_f} = \frac{100}{2 \cdot 7.28e6} = 6.87\ \mu H$$

Butterworth, example

Check:

.ac dec 1000 100k 10meg





Frequency mapping to low-pass prototype:

(Often the allowed passband attenuation is 3 dB, so $f_p = f_{-3}$)

Matlab example: Butterworth transfer function

Matlab:

```
>> % Find the Butterworth polynomial the basic way:
n = 3;
k = 1:1:n;
poles = exp(j*pi*((2*k-1)/2/n+0.5));
poles =

  -0.5000 + 0.8660i  -1.0000 + 0.0000i  -0.5000 - 0.8660i
```

$$H(s) = \frac{1}{(s+1)(s-e^{j2\pi/3})(s-e^{j4\pi/3})}$$

```
denom_coeff = poly(poles)      % or denom_coeff = conv([1 -poles(1)],conv([1 -poles(2)],[1 -poles(3)]))
denom_coeff =
1.0000 + 0.0000i     2.0000 + 0.0000i       2.0000 + 0.0000i       1.0000 + 0.0000i

% CAUTION: Note the numbering:
```

$$H(s) = \frac{1}{a_1 s^n + a_2 s^{n-1} + \ldots + a_{n-1}s^2 + a_n s + a_{n+1}}$$

$$(a_1 = 1)$$

```
>> % Find the polynomials in an easier way:
n = 3;
[dummy poles b0] = buttap(n);    % Returns the poles (and b0 = 1)
```

Matlab example: Butterworth transfer function

Matlab (toolbox):

```
>> % Find the Butterworth polynomial the easiest way:
n = 3;                          % Order of the filter
omeg3 = 1;                      % Normalized -3 dB frequency
[b a] = butter(n,omeg3,'s')     % 's' indicates analog filter
b =
        0        0        0    1.0000
a =
    1.0000    2.0000    2.0000    1.0000
```

% CAUTION: Note the numbering:

$$H(s) = \frac{b_1 s^n + b_2 s^{n-1} + \ldots + b_{n-1} s^2 + b_n s + b_{n+1}}{a_1 s^n + a_2 s^{n-1} + \ldots + a_{n-1} s^2 + a_n s + a_{n+1}}$$

$$(a_1 = 1)$$

```
>> % Find the Butterworth polynomial the easiest way:
n = 3;
omeg3 = 2*pi*10;                % 10 Hz bandedge frequency
[b a] = butter(n,omeg3,'s')     % 's' indicates analog filter
b =
  1.0e+005 *
        0        0        0    2.4805
a =
  1.0e+005 *
    0.0000    0.0013    0.0790    2.4805          % CAUTION: a(1) = 1  !!!!
```

Matlab plots: Butterworth transfer function

Matlab (toolbox):

```
% Find the Butterworth polynomial the easiest way:
n = 3;
omeg3 = 1;                      % Normalized frequency
[b a] = butter(n,omeg3,'s');    % 's' indicates analog filter

% Easy plot
ButSys = tf(b,a);               % Creates transfer function
bode(ButSys,{0.1,10});          % Plots from ω=0.1 to ω=10 rad/s. R-Click on axes to set frequencies [Hz]
grid;

>> ButSys = tf(b,a)
Transfer function:
        1
---------------------
s^3 + 2 s^2 + 2 s + 1


% Plot the basic way as function of "real" frequency
freq = logspace(-2,1,400);      % 400 points from 10^(-2) to 10^1 Hz
om = 2*pi*freq;
HdB = -10*log10(1+om.^(2*n));
semilogx(freq,HdB);
xlabel('Frequency [Hz]');
ylabel('Magnitude [dB]');
grid;
```

Matlab roadmap, FYI

buttap
cheb1ap
etc.

Zeros & Poles:

$$K\frac{(s-z_1)(s-z_2)\ldots..}{(s-p_1)(s-p_2)\ldots.}$$

residue

Partial fraction exp.

$$\frac{r_1}{(s-p_1)}+\frac{r_2}{(s-p_2)}+\ldots..k$$

poly

roots

```
ht = r.'*exp(p*time);
```

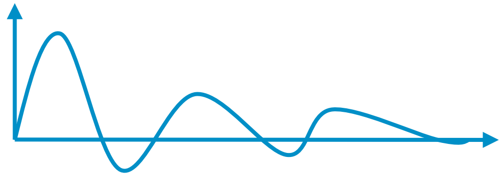
butter
cheby1
etc.

Polynomials:

$$\frac{b_1s^n+\ldots+b_ns+b_{n+1}}{a_1s^n+\ldots+a_ns+a_{n+1}}$$

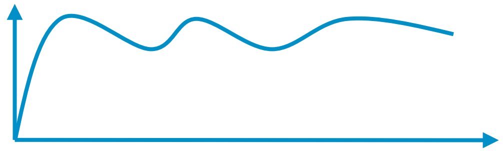
tf
impulse

Impulse response:



tf
step

Step response:

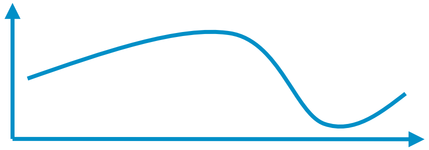


freqs

tf
bode

polyval

Frequency response:



In toolbox'es

Questions & Comments

Group rooms:

ITC:

- 550: B1-215?
- 551: B1-215?

Group rooms:

EIT:

- 510: B2-211
- 511: B2-209
- 512: B2-207
- 513: B2-205
- 514: B2-203
- 515: B2-103
- 516: B2-105
- 517: C*-***
- 518: C*-***
- 519: C*-***
- 520: C*-***
- Any other rooms?

- Please use the black(/white) board for exercises
- In general: There may be more exercises than you have time to do
- Suggested solutions to the exercises will be released ~ 16:30

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