AALBORG UNIVERSITY
DENMARK

Written exam in
Signal processing, 5 ECTS

Tuesday, February 21, 2017
9.00 – 13.00

Read carefully:

- Remember to write your **full name and study number on every sheet** you return!
- Write **legible** with a pen that allows your answers to be scanned electronically.
- The answers concerning analog filters, digital filters and spectral estimation have to be written on **separate sheets**.
- Problems are weighted as listed.
- Results **without sufficient explanations will not give full credits**!
- All ordinary tools may be used e.g. books, notes, programmable calculators and laptops.
- All electronic communication devices **must be turned off** at all times.
- **Communication with others is strictly prohibited**.

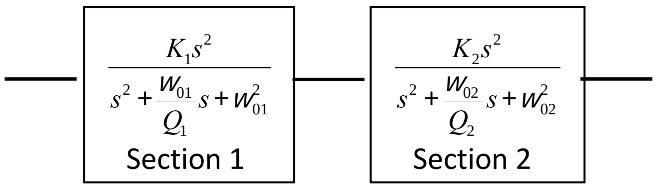
EIT5/ITC5 Signal Processing / Analog Filters
Written examination Feb. 21 2017

Note: You may use the following equations:

$$\cosh(x)=\frac{e^{x}+e^{-x}}{2},\ \cosh^{-1}(x)=\ln\left(x+\sqrt{x^{2}-1}\right),\ \sinh(x)=\frac{e^{x}-e^{-x}}{2},\ \sinh^{-1}(x)=\ln\left(x+\sqrt{x^{2}+1}\right).$$

A.1 (Weight 10%)

A 4th order Butterworth high-pass filter with a cut-off radian frequency of Ω_0 = 800 rad/s is made using a cascade of 2 biquad sections:

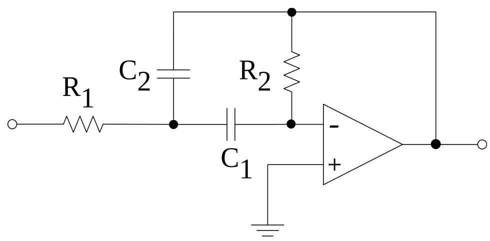


a. Find ω_{01}, Q_1 for section 1.
b. Find ω_{02}, Q_2 for section 2.
Hint: Consider the pole locations.

A.2 (Weight 8 %)

An MFB bandpass section (like the one shown) has the transfer function:

$$H(s)=\frac{s\frac{1}{R_1C_1}}{s^2+s\left[\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



It is required that:

- $\omega_0 = 2\pi\cdot10^3$ rad/s
- Q = 5

and it is chosen that:

- R1 = 1 kΩ
- R2 = 225 kΩ

a. Find the values of C1 and C2 (C2 > C1)

A.3 (Weight 15 %)

The requirements for a high-pass filter are:

- Max. 1.0 dB attenuation at Ω ≥ 1500 rad/s
- Min. 50 dB attenuation at Ω ≤ 500 rad/s

a. Make two sketches of the high-pass filter for both using Chebyshev and Butterworth filters.
b. Find the necessary filter order when a Chebyshev filter is used
c. Find the actual attenuation at Ω = 500 rad/s
d. Find the necessary filter order when a Butterworth filter is used

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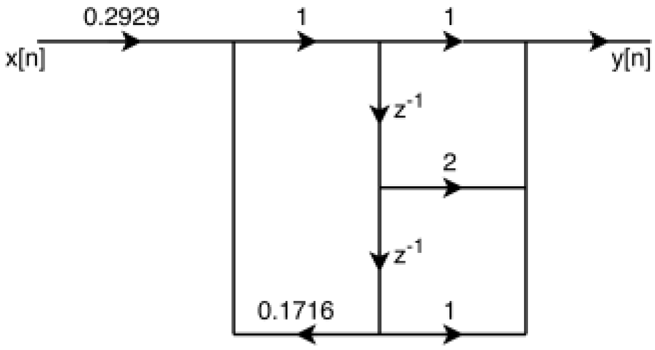
Problem B.1 (weighted with 12% - Digital filters)
A digital filter is illustrated in the figure below:



The sampling frequency is 48kHz.

Questions:

a) How is the length of the impulse response characterized (infinite/finite)?
b) Can this type of filter become unstable (how)?
c) Determine the transfer function H(z)
d) Draw a pole-zero diagram
e) What are the characteristics of the amplitude response (high pass, low pass, band stop, etc).
f) By using the knowledge about location of poles and zeros draw an approximate amplitude response.
g) What is maximum gain of the filter.
h) Perform a variance scaling of the filter.

Problem B.2 (weighted with 12% - Digital filters)
Design a low pass linear phase filter using the window method. The specifications are:

- The sampling frequency: f_s=12kHz
- The order of the filter should be M=6
- The -3dB cut-off frequency: f_c=3kHz
- A rectangular window is to be used

Questions:

a) Determine the filter coefficients
b) Draw a flow graph for the filter.
c) Modify the filter such that it has a 10 dB DC gain.
d) Determine the initial 5 values of the step response
e) Plot the phase response. Remember of include units on the horizontal and vertical axes.
f) What can be said about the stability of the filter?
g) Describe how the filter will change if a Hamming window is used (instead of the rectangular window)?

Problem B.3 (weighted with 10% - Digital filters)
Explain two methods from the course for transforming an analog filter into a digital filter.

The description must include:

- The most important formulas
- Possible limitations.
- Pro and cons of the methods

Two time-domain sequences are given by

$x_1[n] = \delta[n] + 2\,\delta[n-1] + \delta[n-2] - \delta[n-3]$ and

$x_2[n] = 3\,\delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

Problem C.1 (weighted with 5% - Spectral Estimation)

Make a linear convolution of x_1 and x_2 i.e. $x_1 * x_2$, and a circular one i.e. $x_1 \; ④ \; x_2$.

Problem C.2 (weighted with 5% - Spectral Estimation)

Transform both sequences x_1 and x_2 into frequency domain by using a Discrete Fourier Transform (DFT) of length N=4 and call the results X_1 and X_2 accordingly.

If we assume that X_2 is a filter and we run x_1 through that filter X_2 we will of course obtain the same as the linear convolution found above. A linear convolution however, normally takes a long time to compute, so we would like to do the filtering in frequency domain instead and not in time domain.

Problem C.3 (weighted with 5% - Spectral Estimation)

Please filter the signal x_1 by the filter X_2 and do the calculations in frequency domain. Call the result Y[k].

Problem C.4 (weighted with 4% - Spectral Estimation)

Compare the result Y found in C.3 to the circular convolution found in C.1. How are they connected?

Problem C.5 (weighted with 5% - Spectral Estimation)

How can we improve the computation of Y so that its inverse DTF corresponds to the linear convolution?

We now set the sampling frequency to f_S=44.1 kHz.

Problem C.6 (weighted with 3% - Spectral Estimation)

What is the distance in frequency [kHz] between the k's in Y[k]? How long in time [ms] is the sequence x_1?

Problem C.7 (weighted with 6% - Spectral Estimation)

What is the gain of the filter X_2 at 777 Hz?