

Christina Imdahl



Schedule of Second Part

Monday	Tuesday	Wednesday	Thursday	Friday
Oct 4	Oct 5	Oct 6	Oct 7	Oct 8
	Introduction to RL			Case Study Descriptives and Orientation
Oct 11	Oct 12	Oct 13	Oct 14	Oct 15
	Reinforcement Learning – Key Concept			(Homework: Implement RL) Momentum
Oct 18	Oct 19	Oct 20	Oct 21	Oct 22
	Inventory		No Availability	
	Management - Heuristics			Case Study Benchmark
Oct 25	Oct 26	Oct 27	Oct 28	Oct 29
	Wrap-up / Case Study			Case Study
Nov 1	Nov 2	Nov 3	Nov 4	Nov 5
	Submission Case			Case Presentation



Objectives of Today

- Differentiate different ML techniques
- Learn about the basics of Reinforcement Learning
- Understand the interaction between model formulation and learning
- INTUITION on reinforcement learning

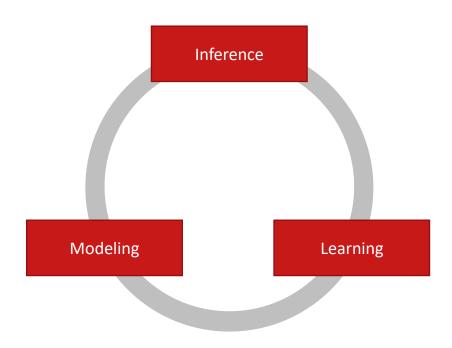


Agenda

ı.	Overview of Machine Learning	
II.	Basic Concepts of Reinforcement Learning	20
III.	The Importance of Modeling	29
	Changing the Reward	36
	Changing the state space	39
IV.	The Case Study	42



Paradigms



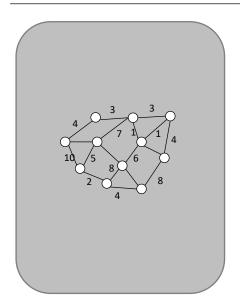


Modelling

Real World Problem



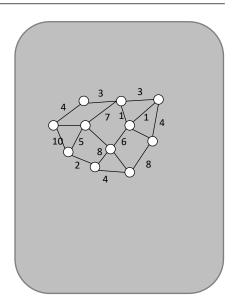
Mathematical Model



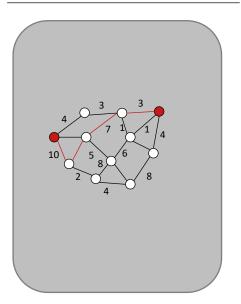


Inference

Full Model



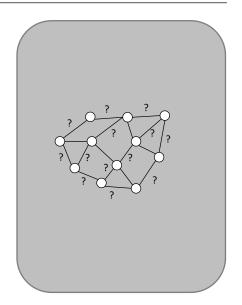




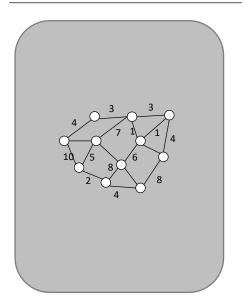


Learning

Model



Learned Model





Overview Machine Learning

Supervised Learning



Predict a value

Input: Labeled Training Data Output: Prediction Model

Type: Regression/Classification

Unsupervised Learning



Identify patterns

Input: Unlabeled Training Data Output: Classes/Associations Type: Clustering/Associations

Reinforcement Learning



Find policy to optimize rewards

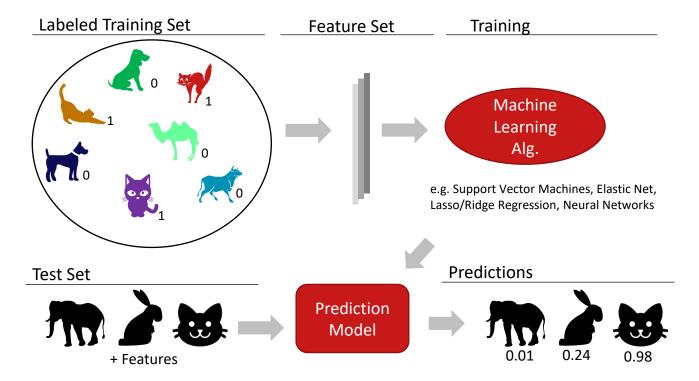
Input: No Predefined Data

Output: Policy

Type: Reward-based



Supervised Learning



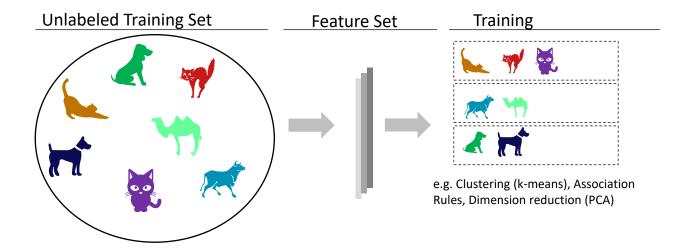


OM Applications

- Predicting Demand & Sales
- Predicting Machine Failures
- Predicting Warehouse Operations
- Predicting Decision-Maker Behavior
- Predicting Preferences



Unsupervised Learning



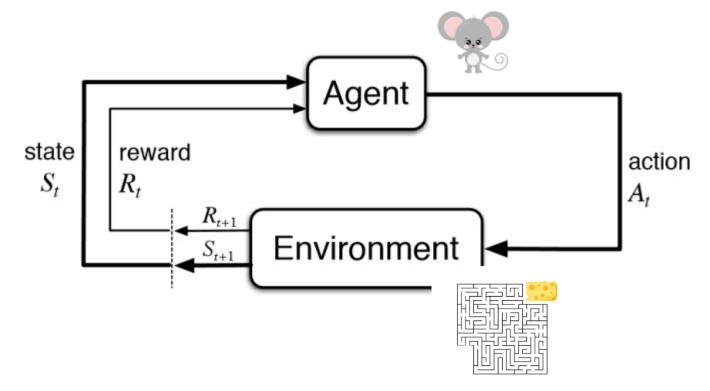


OM Applications

- Market Segementation/Targeted Advertisement
- Fraud Detection
- Basket Analysis
- Recommender Engines (Amazon "Other People Bought", Netflix Movie Recommendation)



Reinforcement Learning





Characteristics of RL

- There is no supervisor, only reward
- Feedback is delayed
- Time matters (sequentiell decision making)
- Actions influence the data the agent receives



RL breaktrough





"I thought AlphaGo was based on probability calculation and that it was merely a machine. But when I saw this move, I changed my mind. Surely, AlphaGo is creative."

-- Lee Sedol



OM Applications











Retail

- Dynamic Pricing
- Item Descriptions
- Anticipatory Shipping
- Fraud Detection

Robotics

- Picking in Warehouses
- Routing Robots
- Manufacturing

Speech Analysis

- Sentiment Analysis
- Chat robots
- Smart Home Tools

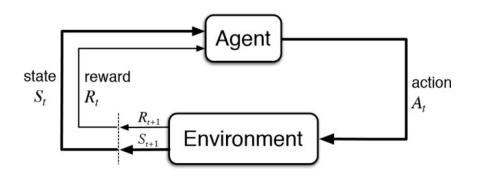


Agenda

l.	Overview of Machine Learning		4
II.		Basic Concepts of Reinforcement Learning	20
III.		The Importance of Modeling	29
	1.	Changing the Reward	36
	2.	Changing the state space	39
IV.		The Case Study	42



Again: Markov Decision Processes



Agent observes state in t: S_t Agent takes action in t: A_t Agent receives reward R in t+1: R_{t+1} Agent moves to the next state: S_{t+1}



RL Terminology

Policy: The agent's behavior given a state a

State-Value Function: how good is a state (s)?

Action-Value Function: how good is a state-action pair (s,a)?

Model: agent's representation of the environment.



Policy

- Determines the agent's behavior given a state
- The policy maps from state to actions
- Deterministe policy: $a = \pi(s)$
- Stochastic policy: $\pi(a|s) = P(A_t = a|S_t = s)$



State-Value function / Value function

- Value of a state under π
- Used to evaluate how good/bad a certain state is
- Used to select between actions
- E.g.:

$$v_{\pi}(s) = \mathbb{E}_{\pi}(\sum_{k=0}^{\infty} \gamma^{k} R_{t+k+1}(s_{t+k}, \pi(a|s_{t+k}), s'_{t+k}))$$

Disc. reward of following policy π



Action-Value function / Q-Function

- Value of an action under π
- Used to evaluate how good/bad a certain stateaction pair is
- Used to select between actions
- E.g.:

$$q_{\pi}(a,s) = \mathbb{E}_{\pi}(R_{t+1}(s_{t+k},a,s'_{t+k}) + \sum_{k=1}^{\infty} \gamma^k R_{t+k+1}(s_{t+k},\pi(a|s_{t+k}),s'_{t+k}))$$

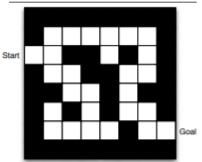
Immediate reward taking a

Disc. reward of following policy π

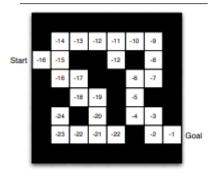


Example: MAZE

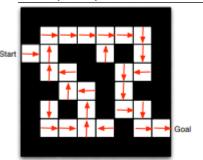
The game



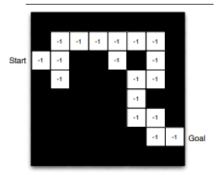
The value function



The policy



The model





Key goal of RL

Determine the policy that maximizes cumulative reward

- Determine the policy directly (policy-based methods)
 - Policy gradient methods, e.g. REINFORCE
- Determine the value function (value-based methods)
 - E.g. Q-Learning, SARSA



Dynamic Programming

- Two methods of dynamic programming can be used to solve MDPs
 - Value Iteration and Policy Iteration

Let's revisit them...



Value Iteration

Update of values based on Bellman Eq.

$$v_{k+1}(s) = \max_{a} \sum_{s' \in S} P(s'|s,a) \underbrace{(R(s,a,s') + \gamma v_{\pi_i,k}(s'))}_{\text{Immediate}} + \underbrace{p_{isc. Value of Sucessor State}}_{\text{Sucessor State}}$$

- 1. Randomly initiate value function
- 2. Update values by above equation until convergence
- 3. Optimal policy is the action ending in the state with the maximum value



Policy Iteration

- Randomly initiate policy
- Policy evaluation: evaluate value function for current policy until convergence

$$v_{\pi_i,k+1}(s) = \sum_{s' \in S} P(s'|s,\pi_i(s))(R(s,\pi_i(s),s') + \gamma v_{\pi_i,k}(s'))$$

Policy improvement: improve policy by

$$\pi_{i+1}(s) = \operatorname{argmax}_a \sum_{s' \in S} P(s'|s, a) (R(s, \pi_i(s), s') + \gamma v_{\pi_i}(s'))$$



Dynamic Programming Differences and Similarities

- Key methods from dynamic programing can be used to solve MDPs
 - Value iteration and policy iteration
- They assume full knowledge of the underlying MDP (transition probabilities, rewards)
- The agent often does not have these (we as programmers may have it)
- The agent shall be able of entering new situations that we don't know



Next Lecture

How do we learn when transition probabilities and values are unknown to the agent.



Main idea: Try & Fail & Try....

Mom: If your friends jumped off a bridge, would you jump too?

Machine learning algorithm:



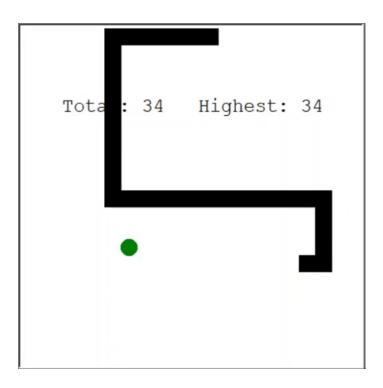


Agenda

	Overview of Machine Learning	4
	Basic Concepts of Reinforcement Learning	20
III.	The Importance of Modeling	29
	Changing the Reward	36
	Changing the state space	39
IV.	The Case Study	42



Snake



Discuss with your neighbour:

Actions

Rewards

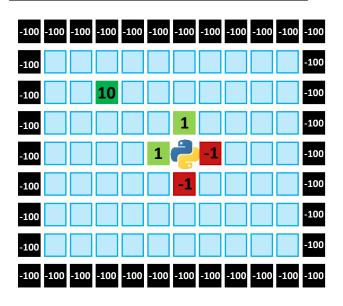


State space



Snake

Rewards



Actions

Snake moves up Snake moves right Snake moves down Snake moves left

State

Apple is above the snake
Apple is on the right of the snake
Apple is below the snake
Apple is on the left of the snake
Obstacle directly above the snake
Obstacle directly on the right
Obstacle directly below the snake
Obstacle directly on the left
Snake direction == up
Snake direction == right
Snake direction == down
Snake direction == left



Snake - Learning

Game 1-4

```
Total: 0 Highest: 0
```



Snake - Learning

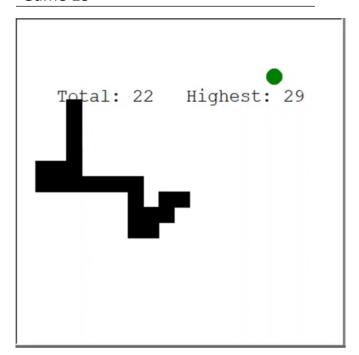
Game 7 – First Apple

Total: 0 Highest: 0



Snake - Learning

Game 13





Snake - Learning

Game 30

```
Total: 5 Highest: 5
```



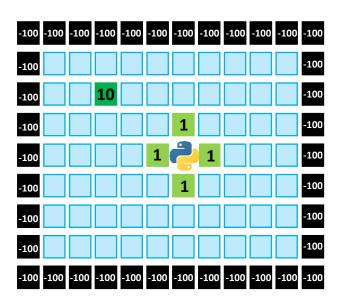
Agenda

l.		Overview of Machine Learning	4
II.		Basic Concepts of Reinforcement Learning	20
III.		The Importance of Modeling	29
	1.	Changing the Reward	36
	2.	Changing the state space	39

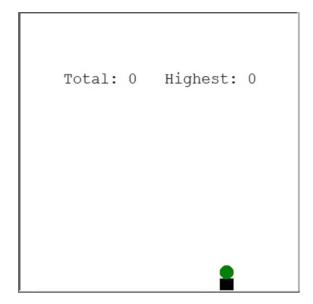


Reward 1 – MOVE!

Rewards



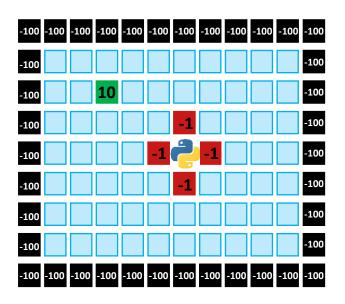
Result



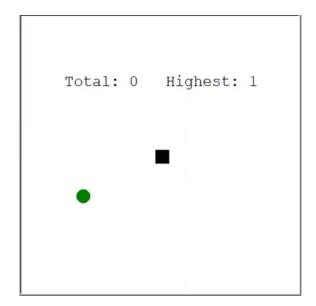


Reward 2 - RUN!

Rewards



Result





Agenda

l.		Overview of Machine Learning	4
II.		Basic Concepts of Reinforcement Learning	20
III.		The Importance of Modeling	29
	1.	Changing the Reward	36
	2.	Changing the state space	39
IV.		The Case Study	42



Different State Space Definitions

1 State (Obstactles = Body/Wall)

Apple is above the snake
Apple is on the right of the snake
Apple is below the snake
Apple is on the left of the snake
Obstacle directly above the snake
Obstacle directly on the right
Obstacle directly below the snake
Obstacle directly on the left
Snake direction == up
Snake direction == right
Snake direction == down
Snake direction == left

State - only walls (Obstacles = Wall)

Same as 1 but body location not included

3 State – coordinates

(x,y) Apple (x,y) Snake

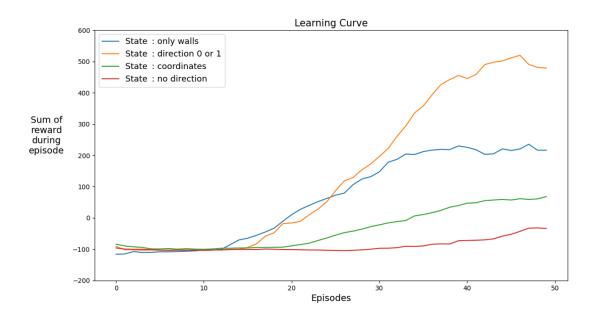
Obstacle directly above the snake
Obstacle directly on the right
Obstacle directly below the snake
Obstacle directly on the left
Snake direction == up
Snake direction == right
Snake direction == down
Snake direction == left

State – no direction

Apple is above the snake
Apple is on the right of the snake
Apple is below the snake
Apple is on the left of the snake
Obstacle directly above the snake
Obstacle directly on the right
Obstacle directly below the snake
Obstacle directly on the left



Learning Curve





Agenda

	Overview of Machine Learning	4
	Basic Concepts of Reinforcement Learning	20
	The Importance of Modeling	29
	Changing the Reward	36
	Changing the state space	39
IV.	The Case Study	42



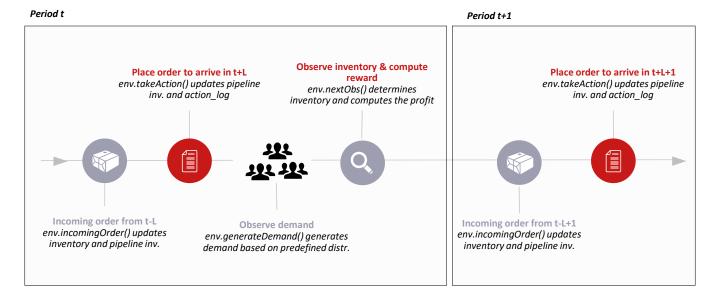
Set-up

- Provider of bike spare parts (brakes, tires ...)
- You start managing the inventory of one item based on part demands
- There is a penalty if you have to much inventory
- There is a penalty if you have to little inventory
- Your objective is to maximize your sales taking into account the costs.





Sequence of events





Two systems to study

Backorder System

- Demand not served at the end of the period can be served the next period
- Optimal policy is a base-stock policy with $S = F^{-1}(\frac{h}{h+h})$
- Can you be equally good?
- How do different parameter/model choices affect your learning?

Lost Sales System

- Demand not served at the end of the period is lost
- Optimal policy is unknown, approximative policies exist
- What is a suitable benchmark?
- Can RL outperform your benchmark?



Acknowledgements

Snake: https://towardsdatascience.com/snake-played-by-a-deep-reinforcement-learning-agent-53f2c4331d36

Snake Code: https://github.com/henniedeharder/snake

RL course by David Silver:

https://www.youtube.com/watch?v=2pWv7GOvuf0&list

=PLqYmG7hTraZDM-

OYHWgPebj2MfCFzFObQ&index=1&t=74s



Back-Up Newsvendor

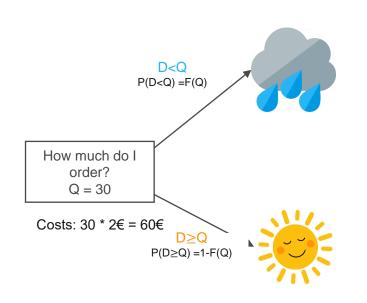


EXAMPLE OF TODAY THE NEWSSTAND

- YOU are a running a small kiosk and need to decide on the optimal amount of papers at the beginning of the day
- You buy newspaper at a costs of 2€
- You sell newspaper at a price of 10€
- At the end of a day all newspapers are shredded
- You expect 30 customers (with a standard deviation of 10, normally distributed).



NEWSVENDOR MISMATCH COSTS



Demand = 10

Revenue: 10 * 10€ = 100€Profit: 100€ - 60€ = 40€

Overstock quantity: 30-10 = 20

Costs of overstocking c_o : 20 * 2€ = 40€

$\underline{\mathsf{Demand} = 50}$

Revenue: 30 * 10€ = 300€Profit: 300€ - 60€ = 240€

Understock quantity: 50-30 = 20

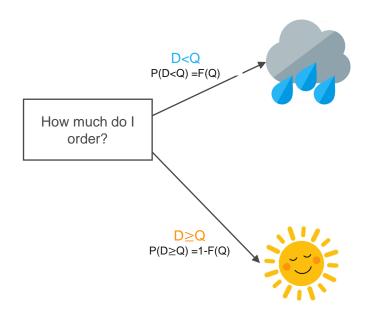
Costs of understocking c_u : 20 * 8€ = 160€

$$C(Q; D) = c_0 \max(0, Q - D) + c_u \max(0, D - Q)$$

Overstock quantity

Understock quantity

NEWSVENDOR MISMATCH COSTS



Overstocking

We need to pay c ($2 \in$) for every newspaper bought, but not sold.

$$c_o = c$$

Marginal expected costs of one additional unit:

$$c_o \cdot F(Q)$$

Understocking

We are losing the margin p-c ($10 \in -2 \in$) for every newspaper that we did not sell, because we had too little.

$$c_u = p - c$$

Marginal expected costs of one additional unit:

$$c_u \cdot (1 - F(Q))$$

MARGINAL ANALYSIS: IS IT BENEFICIAL TO ORDER ONE MORE UNIT?



The optimal order quantity is the point where the expected marginal benefit through an order quantity equals the expected marginal loss through an additional order quantity.

$$c_0 \cdot F(Q) = c_u \cdot (1 - F(Q))$$

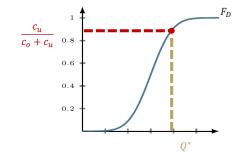
NEWSVENDOR MISMATCH COSTS OPTIMAL ORDER QUANTITY

$$C(Q; D) = c_o \max(0, Q - D) + c_u \max(0, D - Q)$$
Overstock quantity
Understock quantity

Minimizing the expected mismatch costs via optimization yields a simple formula for the optimal order quantity:

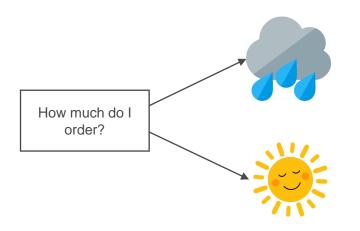
$$F(Q^*) = \frac{c_u}{c_o + c_u}$$

Critical Ratio
= Service level



THE KLU | Christina Imdahl

NEWSVENDOR MISMATCH COSTS



Overstocking

We need to pay c ($2 \in$) for every newspaper bought, but not sold.

$$c_o = c$$

Understocking

We are losing the margin p-c ($10 \in -2 \in$) for every newspaper that we did not sell, because we had too little.

$$c_u = p - c$$

$$F(Q^*) = \frac{c_u}{c_u + c_o} = \frac{p - c}{p - c + c} = \frac{p - c}{p}$$
$$= \frac{10 - 2}{10} = 80\%$$