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Case Study

Bike-Sharing System

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1. Introduction

1.1 Problem Context

Bike-sharing systems have evolved significantly since their inception, with Amsterdam's "Witte Fietsen" program in 1965 marking one of the first attempts to offer public bicycles without regulation. Although this initial effort faced challenges with theft and vandalism, it laid the groundwork for future bike-sharing initiatives. In the late 2000s, self-service bike-sharing programs gained traction globally, with Citi Bike emerging as a prominent example. Launched in New York City in 2013, Citi Bike has grown to encompass over 700 stations and 12,000 bikes, becoming the largest bike-sharing program in the United States.

These systems aim to reduce traffic emissions, alleviate congestion, and address the "last-mile problem" in urban transportation. By providing an accessible alternative to public transit, bike-sharing encourages exercise and promotes public health. However, challenges arise when demand for bikes varies across stations, leading to disparities that necessitate rebalancing efforts. This is accomplished through vehicle routing, optimizing the redistribution of bikes between high- and low-demand stations.

Our objective in this case study is to develop an efficient strategy for managing bike-sharing systems using the framework of the Stochastic Dynamic Inventory Routing Problem. Our goal is to dynamically relocate bikes between stations in response to fluctuating demand while minimizing operational costs and penalties associated with unmet demand or exceeding station capacities. By leveraging real-time data on bike availability and demand patterns, we aim to ensure that customers can easily rent and return bikes without encountering shortages.

By modelling our bike-sharing system as an Stochastic Dynamic Inventory Routing Problem, we can systematically analyse and develop strategies that not only improve the availability of bikes but also enhance the overall efficiency of operations. This includes implementing dynamic decision-making processes that adapt to real-time information about bike usage, station capacities, and travel times. Ultimately, our objective is to create a robust solution that minimizes the total costs.

This report presents a dynamic solution to the rebalancing problem for Citi Bike in New York City. By leveraging ride data from June 2014, we develop a demand distribution model to predict bike and dock needs at each station

1.2 Introduction to SDIRP

We will examine a specific optimization problem for bike-sharing systems: the Stochastic Dynamic Inventory Routing Problem (SDIRP). The SDIRP is a complex optimization problem that arises in various logistics and supply chain contexts. It involves managing inventories in a network where items (such as products or vehicles) are dynamically relocated to meet fluctuating demand. The term stochastic refers to the uncertainty inherent in both demand and travel times, making the problem particularly challenging. In an SDIRP, the goal is to develop strategies that efficiently balance inventory levels across multiple locations while minimizing costs and ensuring service levels.

1.3 Bridging the gap to SSDP

The SDIRP can be understood as a specific instance of a broader class of problems known as Stochastic Sequential Decision Problems (SSDPs). SSDPs encompass all situations where decisions must be made sequentially over time in an uncertain environment. This means that decisions made today can significantly impact future outcomes, especially in the face of uncertainty. In our case, we face the challenge of determining how many bikes to relocate at any given moment, all while considering the future demand and availability of these bikes.

This uncertainty necessitates a strategic approach, wherein we must make informed decisions that account for potential variations in demand and supply. While the SDIRP focuses specifically on inventory management and vehicle routing, it operates within the larger framework of SSDPs. As such, it highlights the importance of sequential decision-making under uncertainty and emphasizes the need for adaptive strategies that respond to evolving conditions in real-time. Understanding our problem as an SSDP allows us to apply a more comprehensive set of tools and methodologies, ultimately leading to more effective solutions for managing the complexities inherent in bike-sharing systems.

1.4 Four Fundamental Policy Approaches in an SSDP

As we delve deeper into the framework of Stochastic Sequential Decision Problems (SSDPs), we can distinguish four fundamental policy approaches that guide decision-making in such contexts, particularly in the case of a Stochastic Dynamic Inventory Routing Problem (SDIRP). These approaches serve as essential tools for selecting appropriate actions in various states of the system.

1. **Policy Function Approximations (PFAs):** PFAs are simple rules that determine actions based on the current state of the system. For example, a PFA might state, "If a station has fewer than five bikes, dispatch a vehicle to deliver bikes." These policies are fast and easy to implement, making them attractive for operational simplicity. However, PFAs often lack precision, as they do not take future uncertainties or changes in demand into account. This can lead to inefficiencies, especially in dynamic environments like bike-sharing systems where demand fluctuates throughout the day.
2. **Cost Function Approximations (CFAs):** In a CFA, decisions are based on the immediate costs associated with a particular action, aiming to minimize short-term expenses. The idea is to select the action that incurs the lowest immediate cost without considering long-term consequences. For instance, suppose we know the cost of transporting bikes between stations and the penalty cost for unmet demand. A CFA might decide to move bikes from one station to another if the transportation cost is lower than the penalty for a shortage. However, this approach does not account for future bike availability at other stations, potentially leading to imbalances down the road. CFAs are useful for minimizing costs in the short term but can result in suboptimal solutions when future conditions are not properly considered.
3. **Value Function Approximations (VFAs):** VFAs extend beyond CFAs by estimating the future rewards or costs associated with the current state. This allows for a more comprehensive decision-making process, taking into account the long-term impact of actions. The goal is to make decisions that are not only cost-effective now but also optimal over an extended horizon. For example, a VFA might assign a value to each station's inventory, representing both its current status and the expected future demand. In practice, this could mean prioritizing the movement of bikes to a station that frequently experiences shortages during peak hours, even if there is no immediate demand. This long-term perspective helps to avoid penalties over time by balancing supply across multiple future periods.
4. **Direct Lookahead Approximations (DLAs):** DLAs are a more advanced approach that simulates potential future scenarios before making a decision. Instead of acting immediately, a DLA projects what might happen over a defined horizon and bases its decisions on these anticipated outcomes. This is especially valuable in environments with highly variable demand, such as bike-sharing systems, where planning ahead for future peaks is crucial. For example, a DLA could simulate bike demand across stations for the next 12 hours and determine the best redistribution strategy based on predicted high-demand periods. If a lookahead suggests a surge in demand at Station A during rush hour, the system would proactively move bikes there well in advance to ensure availability when needed. By anticipating future needs, DLAs can help optimize resource allocation and reduce the likelihood of shortages or overflows.

To effectively manage decision-making in this bike-sharing system, it is crucial to adopt the appropriate policy approach that balances immediate actions with future outcomes. Among the various strategies discussed, Direct Lookahead Approximations (DLAs) play a particularly important role in environments with high demand variability, such as ours. DLAs allow us to project future scenarios and make decisions that account for predicted fluctuations in demand, such as the morning and evening rush hours. This forward-looking capability provides a significant advantage in managing uncertainty over time.

Therefore, in this specific case of the bike-sharing system, a Direct Lookahead Approximation (DLA) is employed to better anticipate future demand. The DLA considers potential future events by simulating bike demand over a particular time horizon, and decisions are made based on these projections. One of the strengths of this approach is its ability to adjust the planning horizon dynamically, depending on the time of day and varying demand patterns.For example, during peak times, such as in the morning when many people rent bikes to commute to work, the DLA will aim to move more bikes to stations with expected high demand. Conversely, during quieter periods, it will reduce the frequency of bike redistributions, conserving resources. This adaptive planning approach ensures that the bike-sharing system is well-prepared for both immediate and future demand changes.

2. Modelling an SSDP

2.1 Core Components of SSDPs

In order to model and solve a Stochastic Sequential Decision Problem (SSDP), it is necessary to define several key components that characterize the system and its decision-making process. These components include global system parameters, state variables, decision variables, exogenous information, the transition function, and the reward function. Each of these elements plays a vital role in capturing the dynamics and behavior of the system over time. Below, we provide an overview of each component:

State Variables

The state variable, denoted S_t, encapsulates the relevant information about the system at time t. This variable includes everything needed to describe the current status of the system, such as the state of resources, time, and other key attributes. The state variable evolves over time as decisions are made and external information is revealed, and it is crucial for determining the available decisions at each period. Formally, we write the state at time t as S_t, which may be a vector comprising multiple attributes relevant to the system.

Decision Variables

At each time period t, a decision x_t is made, based on the current state S_t. The decision variable x_t represents the action taken in response to the observed state and is often drawn from a feasible action space $X(S_t)$. The decision rule $x_t(S_t)$ maps the current state to a particular action, and the notation $x_t \in X_{S_t}$ reflects that the decision depends on the state at time t. The objective of the SSDP is often to optimize the sequence of decisions over time.

Exogenous Information

After each decision x_t is made at time t, new information is revealed that affects the system's evolution. This external information, denoted by W_{t+1}, is typically stochastic and represents factors that are beyond the control of the decision-maker. The exogenous information influences the system's transition to the next state and is a key component in stochastic decision models. The random variable W_{t+1} is revealed after the decision is made, impacting the future state S_{t+1}.

Transition Function

The transition function determines how the system evolves from one state to the next, based on the current state S_t, the decision x_t, and the exogenous information W_{t+1}. This function is typically represented as:

$$S_{t+1} = S^M(S_t, x_t, W_{t+1}),$$

where S^M represents the model that governs the system's dynamics. The transition function captures how the system moves from state S_t to the subsequent state S_{t+1} after a decision is made and external information is observed.

Cost Function

The cost function $R(S_t, x_t)$ quantifies the immediate cost associated with taking a decision x_t in state S_t. The goal in an SSDP is typically to minimize the total expected costs over time. The reward function is often state-dependent, meaning the cost may vary based on the current state and the decision taken. Mathematically, the objective is to minimize the cumulative costs over the entire decision-making horizon:

$$\min_{\pi \in \Pi} \mathbb{E}\left[\sum_{t \in \mathcal{T}} R(S_t, x_t) \,\middle|\, S_0\right],$$

where $\pi \in \Pi$ represents the policy governing decision-making, and the expectation accounts for the stochastic nature of the exogenous information.

2.2 Objective of SSPSs

In Stochastic Sequential Decision Problems (SSDPs), the objective is to minimize the expected total cost over time by making optimal decisions at each step. The objective function is given by a decision policy $\pi \in \Pi$ and a decision rule $X^\pi : S_t \to X(S_t)$. Thus, we write $x_t = X^\pi(S_t)$ as the decision x_t under decision policy π. The objective is to minimize the expected total costs, expressed mathematically as:

$$\min_{\pi \in \Pi} \mathbb{E}\left[\sum_{t \in \mathcal{T}} C(S_t, X^\pi(S_t)) \,\middle|\, S_0\right],$$

where transition function S_{t+1} captures the system's transition to the next state, influenced by the current state S_t, decision $X^\pi(S_t)$, and exogenous information W_{t+1}. The initial state S_0 sets the starting conditions for the system. The objective, therefore, is to optimize decision-making over time while accounting for the inherent uncertainties, minimizing the cumulative expected cost.

To effectively model and solve a Stochastic Sequential Decision Problem (SSDP), it is essential to define the key components that govern the dynamics and decision-making within the system. The objective function, as outlined above, relies on several fundamental elements that together describe the evolution of the system, the available actions, and the resulting rewards or costs. In the next section, we will delve into the core components required to structure and model an SSDP.

3. Problem Formulation

Building upon the foundational concepts outlined in the previous chapter, we now turn our attention to the specific application of Stochastic Sequential Decision Problems (SSDPs) within the context of a bike-sharing system. In this section, we will define the core components of our model tailored to the unique dynamics and operational characteristics of this system.

By establishing precise definitions for the global system parameters, state variables, decision variables, exogenous information, transition function, and reward function, we can effectively address the challenges inherent in managing a bike-sharing system. The following sections will provide a detailed formulation of each component as it pertains to our specific case, guiding the analysis and solution of the underlying decision-making problem.

Global System Parameters

First, we define the global parameters of the system, which form the basis for the dynamics of the bike-sharing system:

- **Bike Stations:** Each station s has a soft maximum capacity C_s, where exceeding C_s incurs a penalty p. The station can still accommodate more bikes than its capacity, but with additional costs.
- **Vehicle:** The vehicle v used for redistribution has a hard capacity C_v. This means that C_v must never be exceeded when transporting bikes between stations.
- **Travel Time:** The travel time T_{ij} between station i and station j is modeled based on the mean travel time of bikers between the stations. If no historical data is available regarding travel times, it is assumed that the vehicle travels at a constant speed of 20 km/h. Thus, the travel time can be approximated as follows:

  $$T_{ij} = \frac{d_{ij}}{20}$$

  where d_{ij} is the distance between station i and station j, measured in kilometers. The travel times T_{ij} are normally distributed with a minimum travel time of 60 seconds.
- **Reward System:** A penalty $p_{missed} = 1$ is incurred for each customer who cannot rent a bike because a station has no available bikes. When a vehicle delivers more bikes than the station can accommodate (C_s), a penalty of $p_{overload} = 10$ is incurred.

State Variables

We define $S_t, t \in T$ as the state observed at period t. A state comprises the hour of the day t_h, the simulation time in seconds t_s, the arrival time of the vehicle t_a, the vector containing the current capacities of all stations C_t^s, the capacity available in the vehicle C_t^v at time t, and the location of the vehicle l_t at time t. Therefore, we write the state variable using the following notation:

$$S_t = (t_h, t_s, t_a, C_t^s, C_t^v, l_t)$$

The time of the state is important as some variables, such as customer arrival rates and vehicle driving speed, are dependent on the hour of the day. Capacities of other stations will play a large role in the decision-making process, as explained in Section 3.2. The vehicle capacity is crucial to register at each state, as capacity constraints can never be violated. Finally, the location of the vehicle l_t will also be an important attribute of the state. This attribute was added to the model, enabling location-based decision-making.

Decision Variables

The moments in time on which a decision is made are denoted by $t \in T$. Therefore, the set of parameters S_t denotes the state on which a decision $x \in X_{S_t}$ is made. Every decision $x_t(S_t)$ is described by a vector of three variables that indicate where the vehicle will pick up bikes (denoted as x_t^p), where the vehicle will drop off the bikes (denoted as x_t^d), and how many bikes will be transported (denoted as x_t^n). Therefore, for the decision variable, we write:

$$x_t = (x_t^p, x_t^d, x_t^n)$$

The first two decision variables are integers ranging from 0 to 327, referring to the index of the station list. The last decision variable is also an integer, ranging from 1 to the maximum vehicle capacity.

Exogenous Information Function

After each action x_t at time t, the system receives new information from the environment that is beyond its control. This is the exogenous information W_{t+1}, relating to customer behavior and other external events.

- **New Customer Data:** The arrival of new customers who want to rent or return bikes is modeled as stochastic processes.
- **Variation in Travel Times:** The travel times $T_{ij}(t)$ between stations i and j are stochastic and follow a normal distribution, with the mean travel time equal to the average observed time of bikers between these stations. The travel time can be expressed as:

$$T_{ij}(t) \sim \mathcal{N}(\mu_{ij}, \sigma_{ij}^2)$$

  where μ_{ij} is the mean travel time derived from historical biker data, and σ_{ij}^2 is the variance representing the variability in travel times. This means that even though the average time is used for planning, actual travel times can deviate due to factors such as traffic conditions and weather.
- **Minimum Travel Time:** Regardless of the stochastic nature of travel times, there is a minimum travel time constraint of 60 seconds, ensuring that the model reflects realistic scenarios.

Transition Function

The transition function describes how we transition towards the state in period $t+1$. It depends on the current state S_t, the decision x_t, and the demand information W_{t+1} that is revealed after x_t is determined.

At time t, the capacities of the stations are known. The decision x_t made at this moment specifies how many bikes are transported to which location. Based on the location of the vehicle and the capacities of the other stations, the function determines from which station to which station the bikes are transported.

Initially, the feasible action x_t is applied to reach a post-decision state S_t^*, after which the exogenous information determines the transition from S_t^* into S_{t+1}. Finally, the transition to S_{t+1} incorporates the new information from W_{t+1}, including demand and travel times.

Cost Function

The cost function $R(S_t, x_t)$ describes the costs arising from taking an action x_t in state S_t. This is minimized by taking actions that limit costs:

We are addressing a state-dependent problem with a cost function that is contingent upon the state S_k, the decision x_k, and the demand information W_{k+1} that becomes available after x_k is determined. The objective is to minimize costs that include:

- **Penalty for unsatisfied demand:** If a customer cannot rent a bike due to an empty station, a penalty of $p_{\text{missed}} = 1$ is incurred.
- **Penalty for overcapacity:** If a vehicle delivers too many bikes at a station, resulting in $b_s(t+1) > C_s$, a penalty of $p_{\text{overload}} = 10$ is incurred.