

# Pension Systems / Demography & Mortality

Lecture notes: Introduction

University of Copenhagen, Autumn 2021

Snorre Jallbjørn & Søren F. Jarner

# Agenda

1. Course structure and contents
  2. Overview of part I: Modelling tools (first four weeks)
  3. Basic demographic concepts
  4. R basics
- Syllabus
    - Chapter 1 of Preston et al. (pp. 1-20)
  - Preparation for Wednesday
    - Chapter 2 & 3 of Preston et al. (pp. 21-70)

# 1. Course structure

- **Co-teaching the first four weeks**
  - We have students from two courses
    - Pension Systems ~26 students
    - Demography & Mortality ~8 students
- **Group assignment**
  - Mandatory!
  - Course week 4
  - Groups of 2 - 4
- **Course-specific teaching**
  - Final three weeks

**Weeks 1-4**

Basic demography and  
mortality modelling  
/ Snorre Jallbjørn &  
Søren F. Jarner

**Weeks 5-7**

Pension Systems  
/ Søren F. Jarner &  
Snorre Jallbjørn

Demography &  
Mortality  
/ Niels R. Hansen &  
friends

# 1. Who are we (1/2)?

## ■ Snorre Jallbjørn

### ■ Education

- 2014: BSc in Actuarial Mathematics, University of Copenhagen
- 2016: MSc in Actuarial Mathematics, University of Copenhagen
- 2019-present: Industrial PhD student, University of Copenhagen

### ■ Employment

- 2014-present: ATP (Danish Labourmarket Supplementary Pension Fund)
  - Student assistant (2014-2016)
  - Actuary (2016-2019)
  - Industrial PhD student (2019-)



# 1. Who are we (2/2)?

## ■ Søren Fiig Jarner

### ■ Education

- 1995 – BSc in Computer Science, University of Copenhagen
- 1996 – BSc in Mathematics, University of Copenhagen
- 1998 – MSc in Mathematical Statistics, University of Copenhagen
- 2002 – PhD in Mathematics (Probability theory), Lancaster University

### ■ Employment

- 2000-2002: Lecturer in Statistics, Lancaster University
- 2002-2021: ATP (Danish Labourmarket Supplementary Pension Fund)
- 2013-today: Affiliated Professor of Insurance Mathematics and Statistics, University of Copenhagen
- 2021-today: Top secret tech startup



# 1. What this course is about

- **Course content - Part I: Modelling tools (4/7)**

- Demographic concepts
- Mortality, fertility, and population projections
- R-programming

- **Course goals - Part I: Modelling tools (4/7)**

- Overview and understanding of basic demographic concepts
- Knowledge of Danish and other demographic data sources
- Understanding of tools to analyze demographic data: Implementation and analysis of demographic models in R
- Specific knowledge on selected mortality models, e.g. the Lee-Carter model

# 1. Lectures, syllabus, exercises and exam

- **Lectures and syllabus**

- Monday: 2-hour lecture module. Wednesday: 2-hour lecture module and 3-hour exercise module
- Syllabus consists of the *Demography* book by Preston et al. and a few additional papers

- **Exercises**

- Simulating, analyzing and comparing different pension systems
- Due to time constraints, you will be given R implementations of demographic models

- **Project**

- Made available on Absalon Friday, September 24
- The deadline for uploading the solution is Sunday, October 3 at 23:59

- **Exam**

- Individual, 3-day exam. More information later!

# 1. Tentative week plan

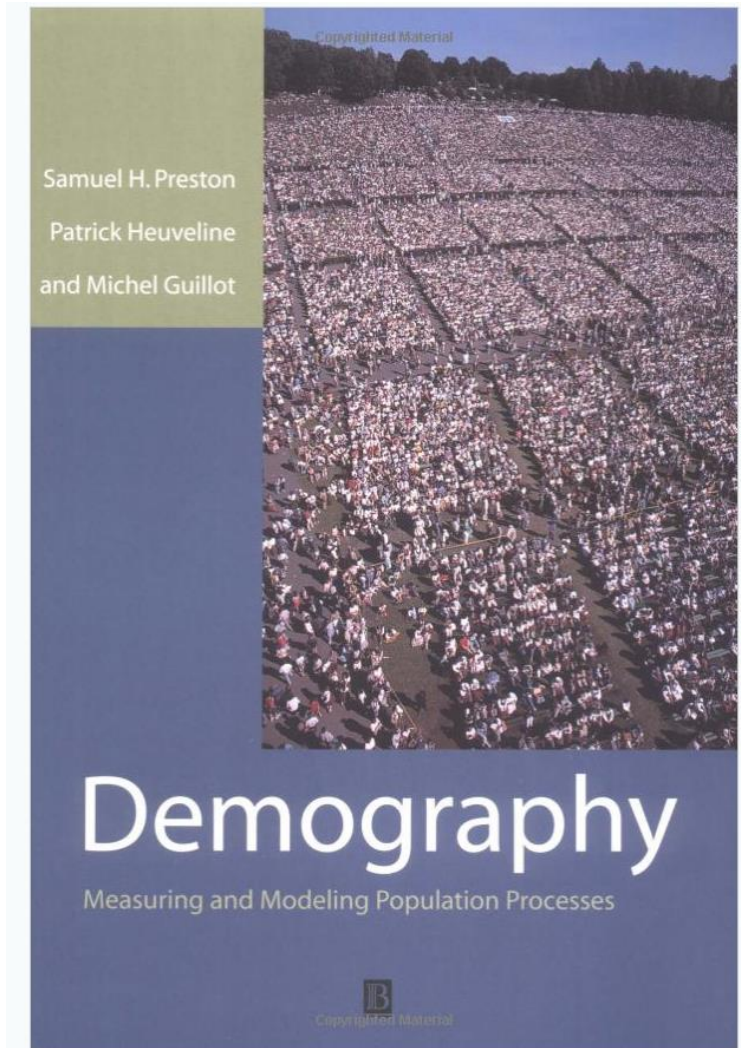
Week	L/E	Topic
36	L	Introduction + R programming
	L	Demography – part 1 Basic concepts
	E	<i>Working with mortality data</i>
37	L	Mortality – part 1 Modelling mortality: The Lee-Carter model
	L	Demography – part 2 Mortality data, fertility, population projections
	E	<i>The Lee-Carter model, life disparity measures, and exposure-to-risk estimation</i>
38	L	Demography – part 3 Stable population theory and momentum
	L	Demography – part 4
	E	<i>Demographic modelling and projection</i>
39	L	Mortality – part 2 Multipopulation mortality forecasting and coherence
	L	Mortality – part 3 Population heterogeneity and selection phenomena
	E	<i>Outsurvival and frailty + <b>Mandatory project</b></i>

Week	L/E	Topic
40	L	
	L	
	E	
41	L	
	L	
	E	
42	Autumn holidays	
43	L	
	L	
	E	
44	EXAM WEEK – 3 day take home exam	



# 1. The book by Preston et al.

- Preston et al. (2001) is the standard textbook
  - *“This will be a bible for demographers in coming years and decades”* – Prof. James Vaupel
- Unlike what you are probably used to, most of the book is set in discrete time ...
  - ... but don't be fooled. Some things turn out surprisingly difficult anyway!



# 1. Many, many data sources

- Demography
  - Eurostat / Statistikbanken
  - UN Population Division
  - Global Burden of Disease (GBD)
    - Risk factors and cause-specific mortality
  - Human Mortality Database (HMD)
    - Global occurrence/exposure data
  - Institute and Faculty of Actuaries
    - Continuous Mortality Investigation (CMI)
  - Ourworldindata
  - Gapminder
  - ...
- Pension systems
  - Eurostat / Statistikbanken
  - OECD
    - Pensions at a Glance 2017
    - Pensions Outlook 2018
  - World Economic Forum
    - Retirement Investment Systems Reform  
part of long-term investing initiative
  - World Bank
  - International Centre for Pension Management (ICPM)
  - ...

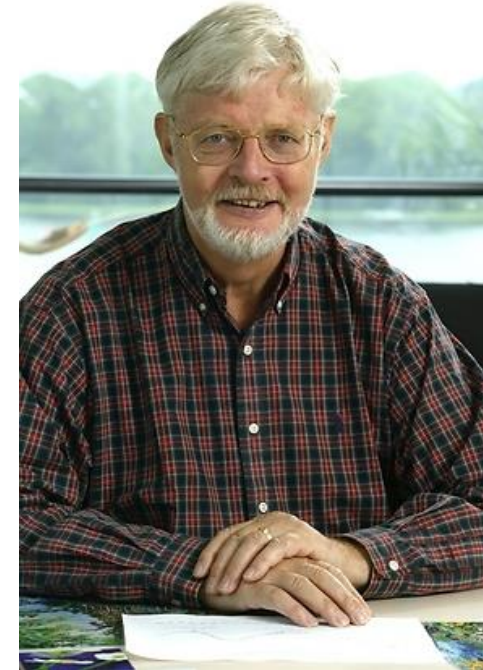
# #2

## Part I: Modelling tools



## 2. Demography

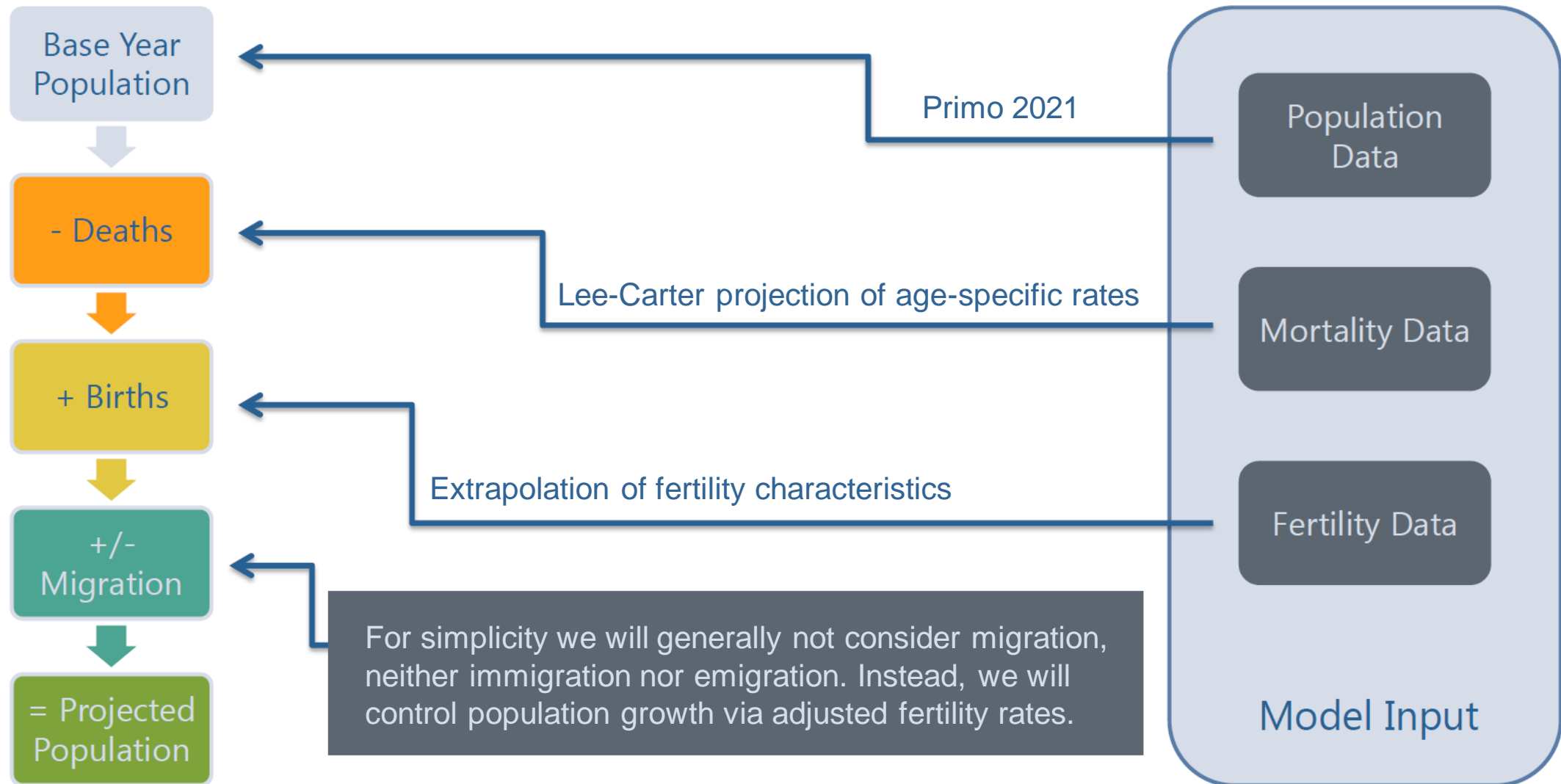
- Demography
  - Study of population processes, e.g., marriage, fertility, mortality etc.
  - Many ingenious indirect estimation methods, e.g. the Brass method
    - Estimation of child mortality based on age of mother and child survivorship
  - In many respects, demography is closely related to actuarial science
- Stable population theory
  - Theoretical study of closed populations with constant vital rates
  - Insights on the age composition over time (and verification tool)
- Momentum of population growth
  - Quantification of the “inevitable” change to population size, i.e. the change that will happen even with replacement level fertility



*Jan M. Hoem (1939 – 2017). Held academic positions as an actuary, statistician and demographer*

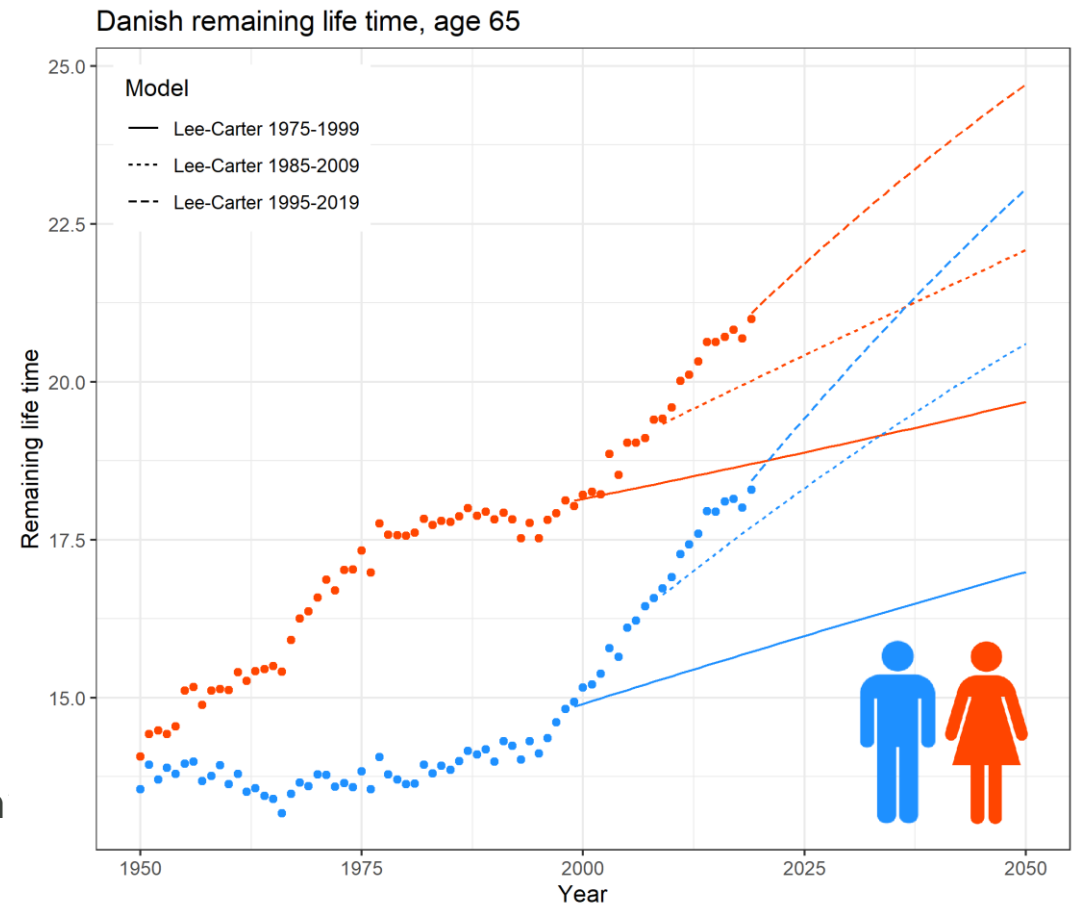
*Professor of Insurance Mathematics at the University of Copenhagen (1974-1981) where he established a state-of-the-art actuarial program*

## 2. Cohort component method for population projection

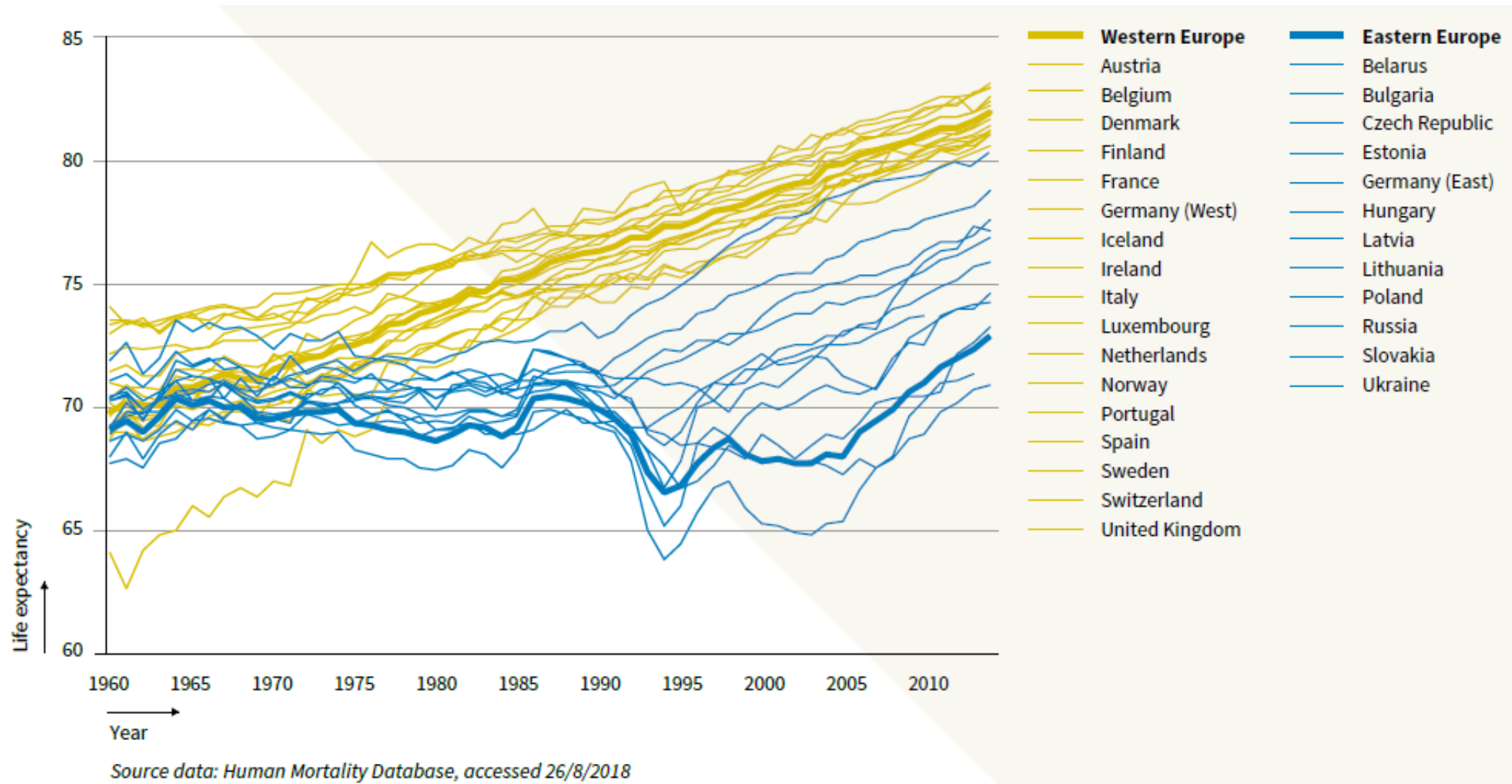


## 2. Mortality

- Life expectancy has increased the last 200 years
  - LE at birth has increased almost 2.5 years pr. decade!
  - Also substantial increases at higher ages (see plot)
- Implications for pensions in Denmark
  - State pension age pegged to life expectancy (2006)
  - Technical provisions must take expected, future improvements into account (2011)
- Huge literature on mortality modelling
  - It is not obvious how to project mortality/life expectancy
  - Can improvements go on forever, or is there a maximum
  - We use a Lee-Carter model
  - Simple, widely used, extrapolative model

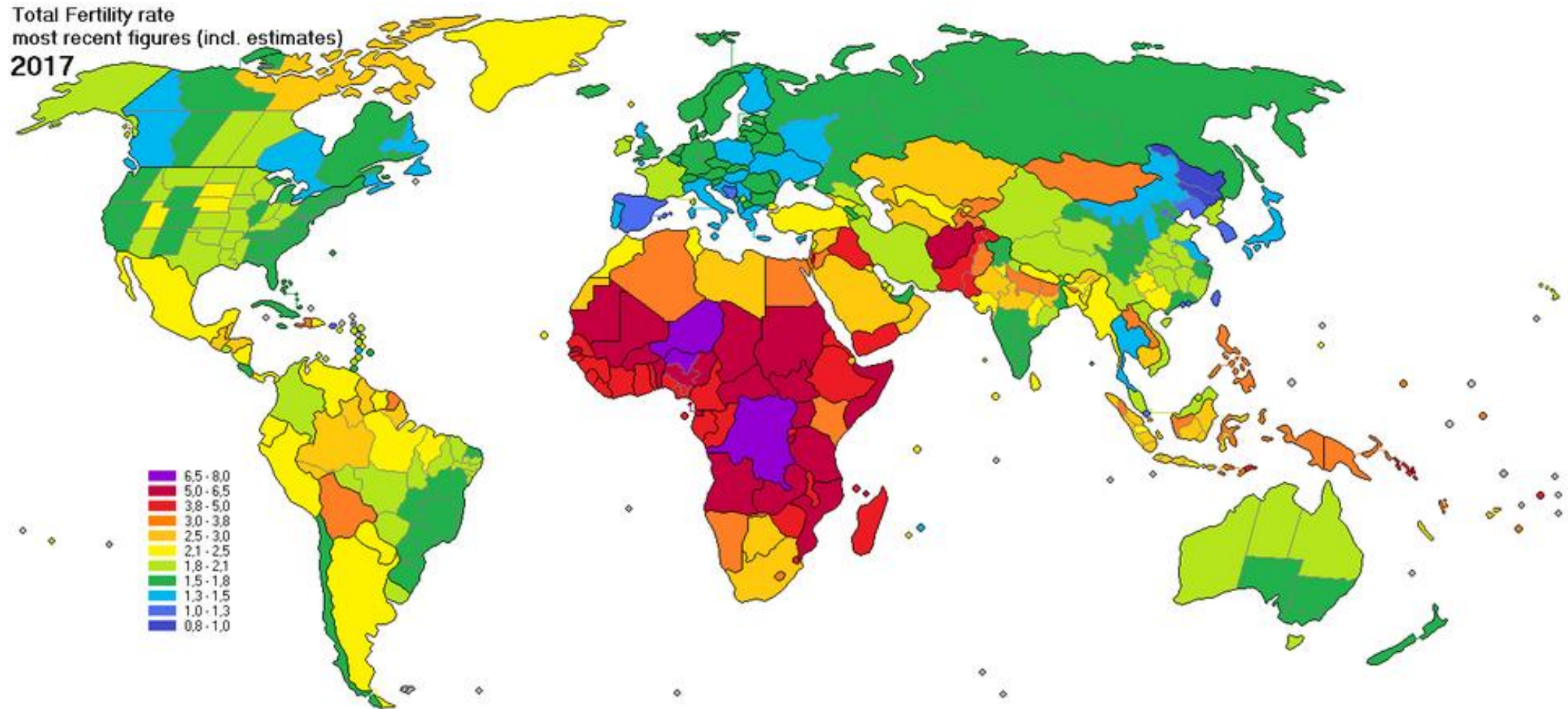


## 2. Mortality in Europe





## 2. Fertility – replacement level is 2.1 children pr. woman





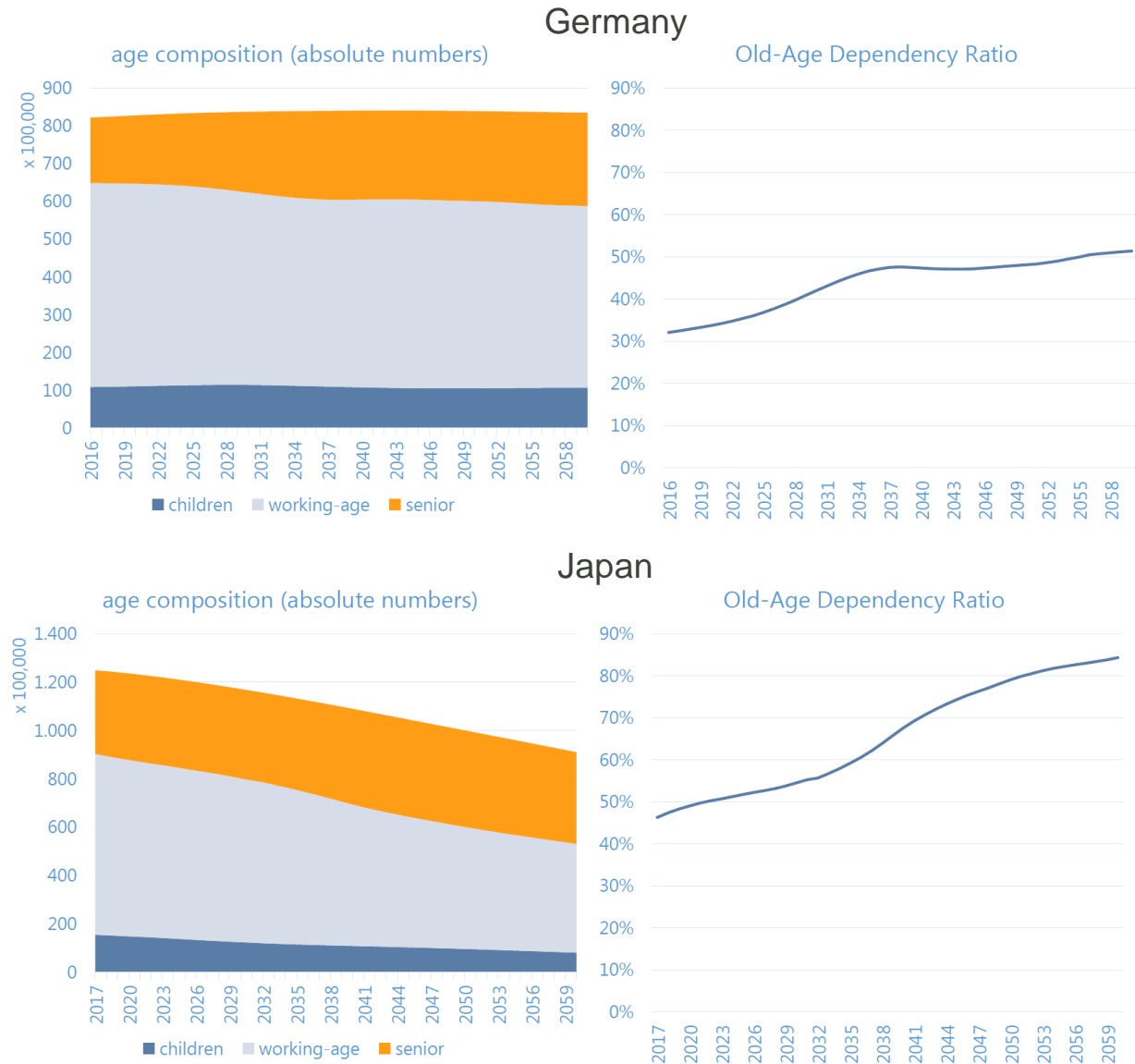
## 2. Main drivers of (un)sustainability of pension systems

- In many developed countries pension systems are affected by a toxic combination of:
  - Demographic changes due to **low fertility** and **increasing life expectancy**
  - Depressed returns due to **low interest rates** and **slow financial growth**

- $\text{Dependency ratio} = \frac{\text{junior pop.} + \text{senior pop.}}{\text{working population}}$

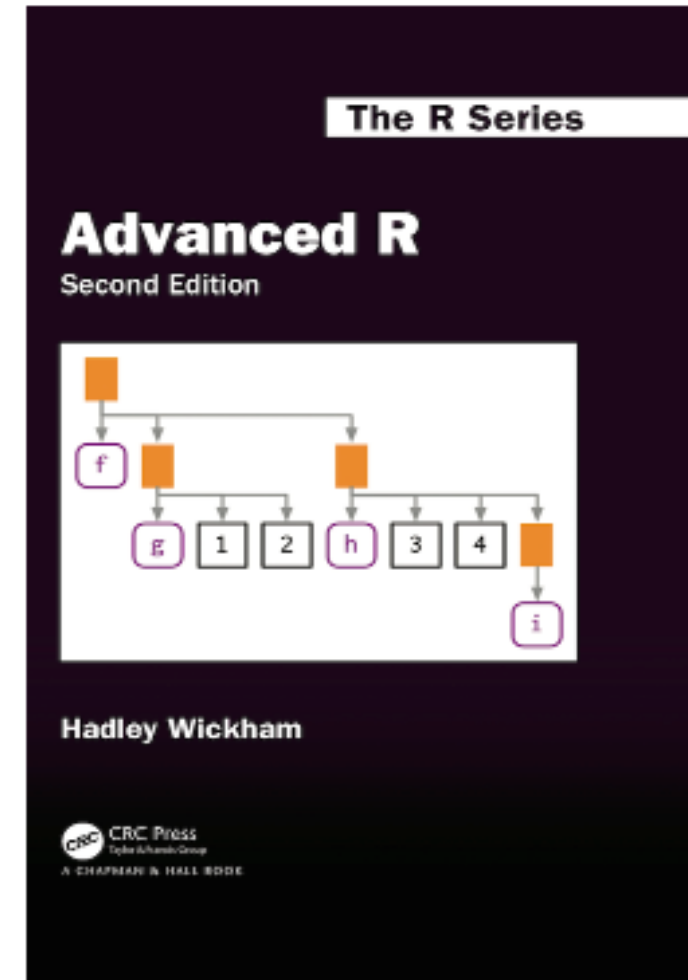
- Junior population (ages 0-14)
- Working population (15-65)
- Senior population (ages 65+)

- $\text{Old-age dependency ratio} = \frac{\text{senior pop.}}{\text{working pop.}}$



## 2. Coding in R

- The programming is not difficult per se, but there is a lot of bookkeeping and many possibilities for error
  - Discipline and structure are essential
- This is not a course on R, but ...
  - Functions and environments
  - Global variables and scoping
  - Loops vs apply
  - Object oriented programming (OOP) with R6 classes
  - Data structures
  - Visualization with ggplot
- Have a look at the website: <https://adv-r.hadley.nz>



# #3

## Basic demographic concepts



# 1. Basic demographic concepts

- Literally translated from the Greek, '*demography*' means '*description of the people*'

- A possible definition:

*“Demography is the study of the size, territorial distribution, and composition of population, changes therein, and the components of such changes, which may be identified as natality, mortality, territorial movement (migration), and social mobility (change of status).”*

(Duncan & Hauser 1972)

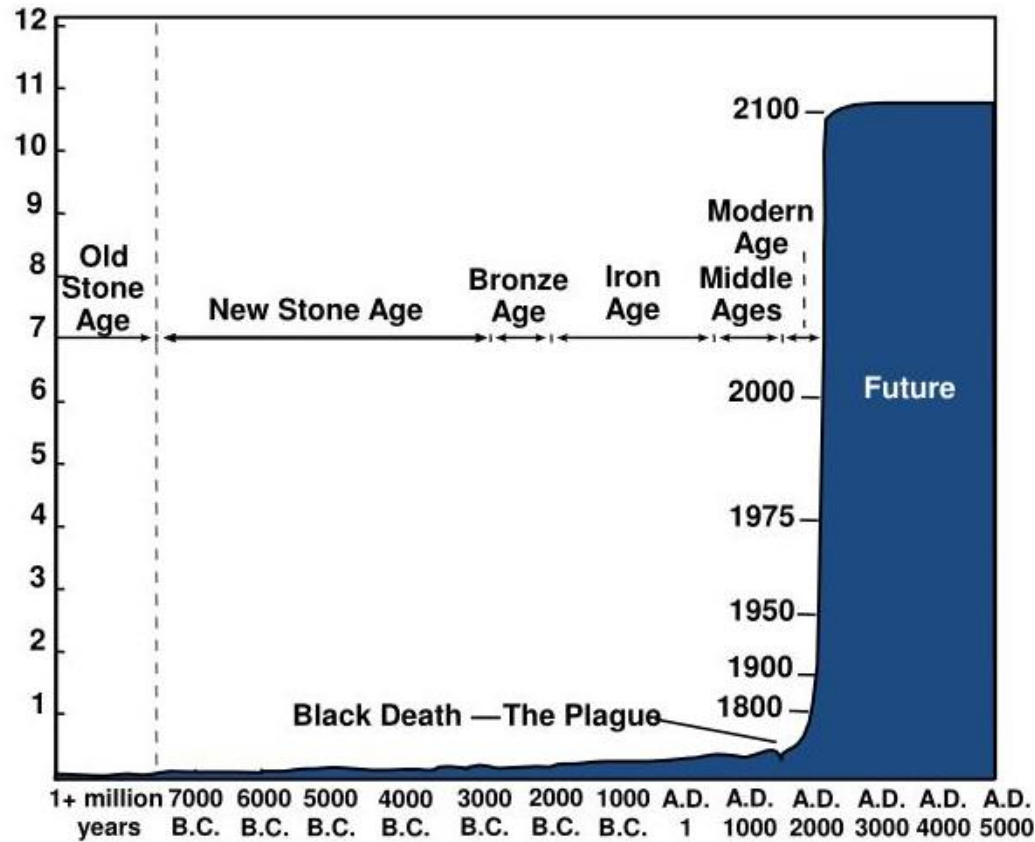
# 1. Meaning of “population”

- Collection of persons alive at a specified point in time who meet certain criteria, e.g.
  - The “population of India on April 1, 1995”
  - The “population of Danish females on January 1, 2019”

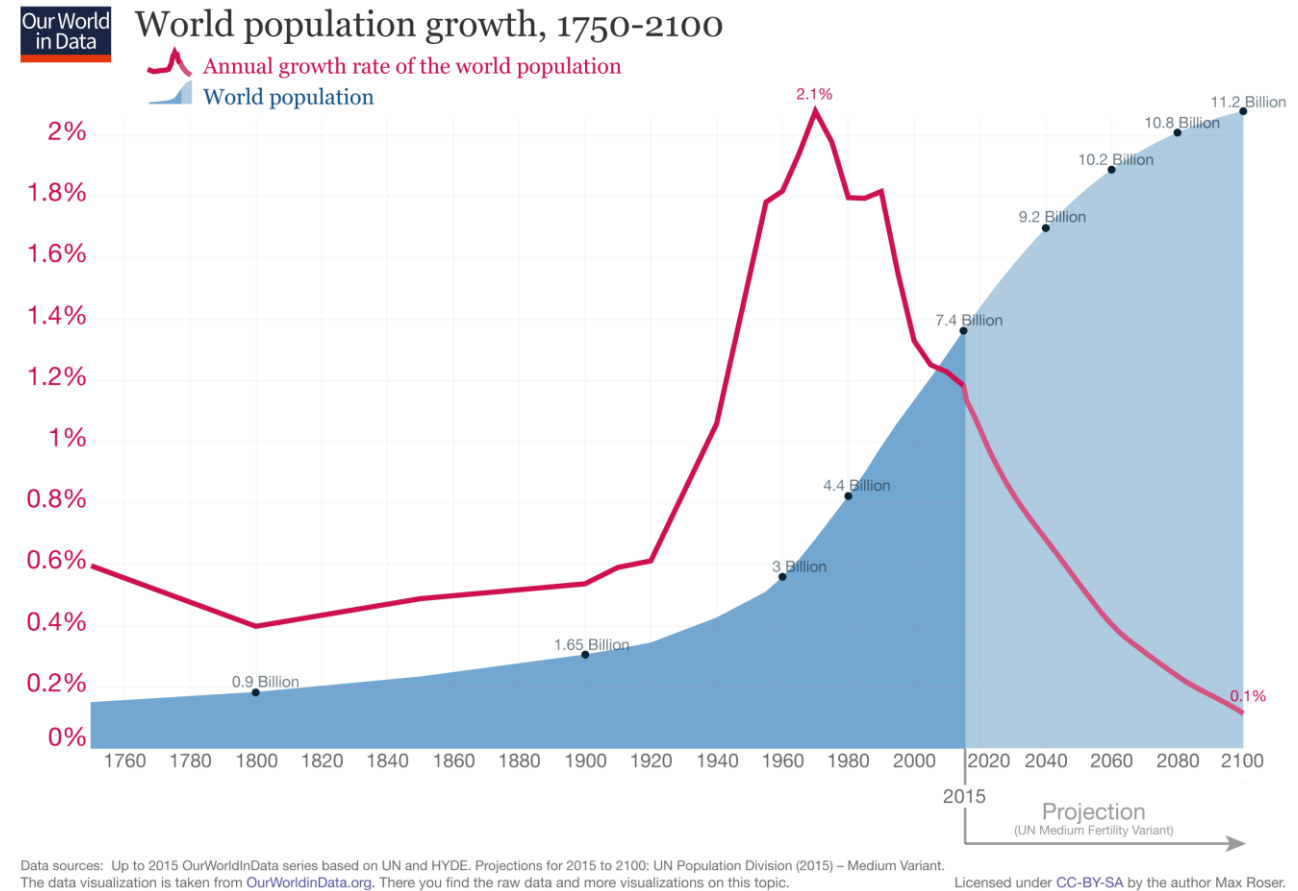
*Kind of collectivity that persists through time even though its members are continuously changing through attrition and accession. Thus, "the population of India" may refer to the aggregate of persons who have ever been alive in the area we define as India and possibly even to those yet to be born there. The collectivity persists even though a virtually complete turnover of its members occurs at least once each century. (Source: Preston et al. (2001))*

- Demographic analysis focuses on this enduring collectivity
  - Studying, e.g., changes in population size, growth rates, vital rates, and composition

# 1. World population growth



Source: Population Reference Bureau; and United Nations, World Population Projections to 2100 (1998)



Currently, global human population is slightly less than 7,7bn and growing with around 82 million per year, or 1,07% p.a.

# 1. The balancing equation

- The “balancing equation” of population change:  $N_1 = N_0 + (B - D) + (I - E)$ 
  - $N_0$  and  $N_1$  are the population sizes at the beginning and end of a period of interest, e.g. a year
  - $B$  is the number of births in the period
  - $D$  is the number of deaths in the period
  - $I$  is the number of immigrations over the period
  - $E$  is the number of emigrations over the period

$B - D$  is the *natural increase*

$I - E$  is the net migration
- Population growth can occur only if
  - Natural increase is positive ( $B > D$ )
  - Net migration is positive ( $I > E$ )
  - Historically, the first effect is most important in understanding population growth

# 1. Period rates and person-years

- In general, a period rate for a population takes the form
  - $Rate[0, T] = \frac{\text{Number of occurrences of interest between time 0 and } T}{\text{Person-years lived in the population between time 0 and } T}$
  - Note, we do not use the population at the beginning of the period in the denominator, but instead a measure of how many have been “at risk” or “exposed” over the period of interest
  - This allows a continuous-time interpretation of the rate
  - In practice, the “person-years lived” is estimated by the population at mid-year (or mid-period)

*Definition of (crude) birth rate from the Population Handbook p. 9*

## Birth Rate

The birth rate (also called the crude birth rate) indicates the number of live births per 1,000 population in a given year. Most annual rates, such as the birth rate, relate demographic events to the population at mid-year (July 1), which is considered to be the average population at risk of the event occurring during the year.

$$\frac{\text{Number of births}}{\text{Total mid-year population}} \times K = \frac{161,042}{7,485,600} \times 1,000 = 21.5$$

There were 22 births per 1,000 population in Israel in 2009.



# 1. Rates of population change

- Rearranging the balancing equation and divide by person-years lived in the period (PY)

- $$\frac{N_1 - N_0}{PY} = \frac{B}{PY} - \frac{D}{PY} + \frac{I}{PY} - \frac{E}{PY}$$

- $$CGR = CBR - CDR + CRIM - CREM = CRNI + CRNM$$

- $CGR$  is the crude growth rate
- $CBR$  is the crude birth rate
- $CDR$  is the crude death rate
- $CRIM$  is the crude rate of immigration
- $CREM$  is the crude rate of emigration
- $CRNI$  is the crude rate of natural increase
- $CRNM$  is the crude rate of net migration

## Growth Rate

The growth rate is the rate at which a population is increasing (or decreasing) in a given year due to natural increase and net migration, expressed as a percentage of the base population.

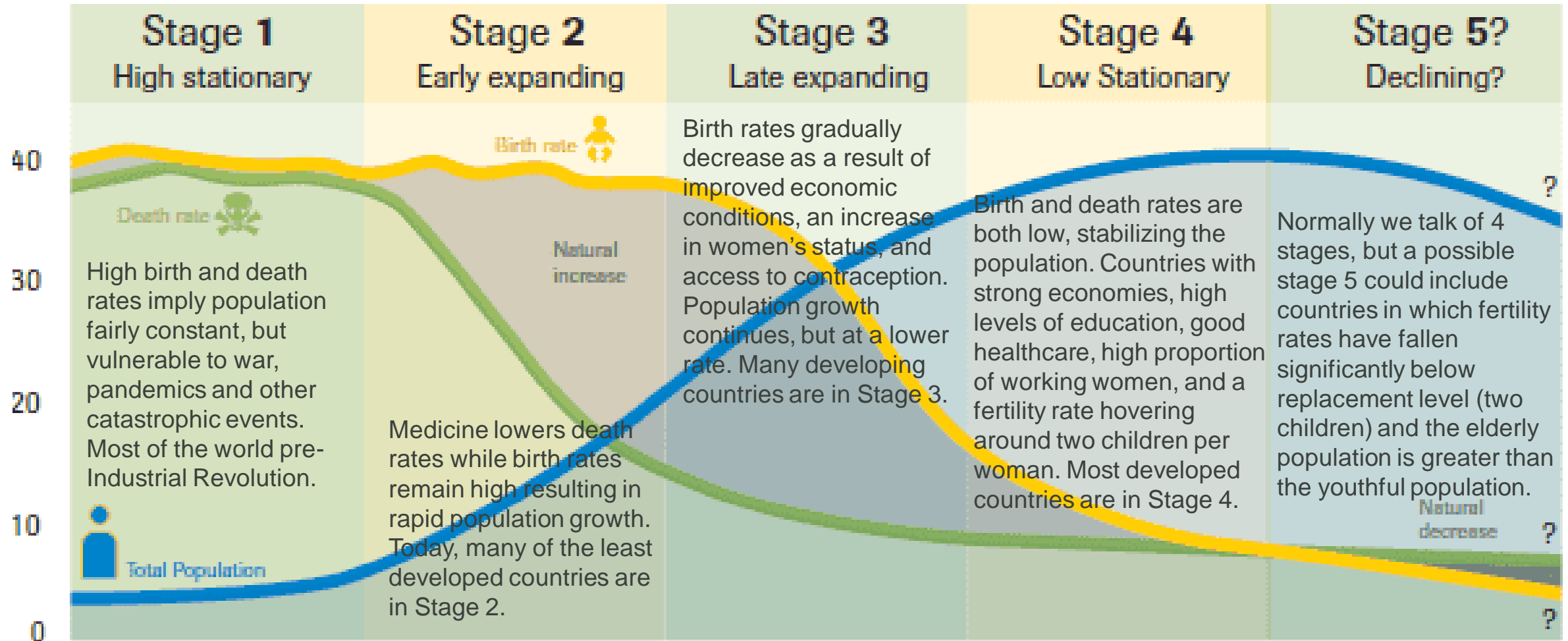
The growth rate takes into account all components of population growth: births, deaths, and migration. It should never be confused with the birth rate, but it sometimes is.

$$\frac{\text{Births 2009} - \text{Deaths 2009} \pm \text{Net migration 2009}}{\text{Total population 2009}} \times K = \frac{111,800 - 90,080 - 63,280}{9,299,000} \times 100 = 0.91$$

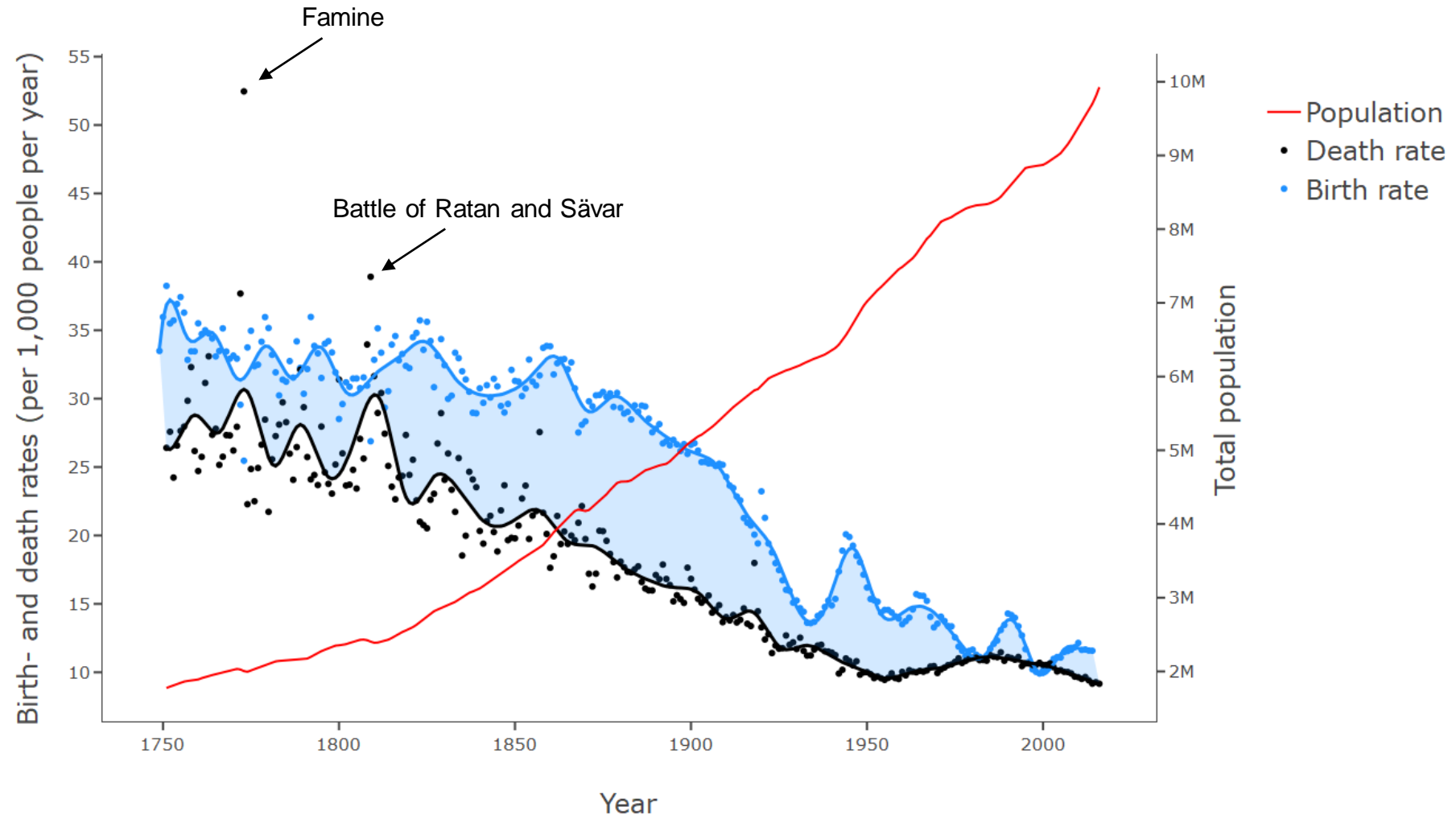
In 2009, the annual growth rate in Sweden was 0.91 percent.

*Definition of (crude) growth rate from the Population Handbook p. 30. For pedagogical reasons, the handbook does not (explicitly) distinguish between crude rates and other types of rates, e.g. age-specific or standardized rates.*

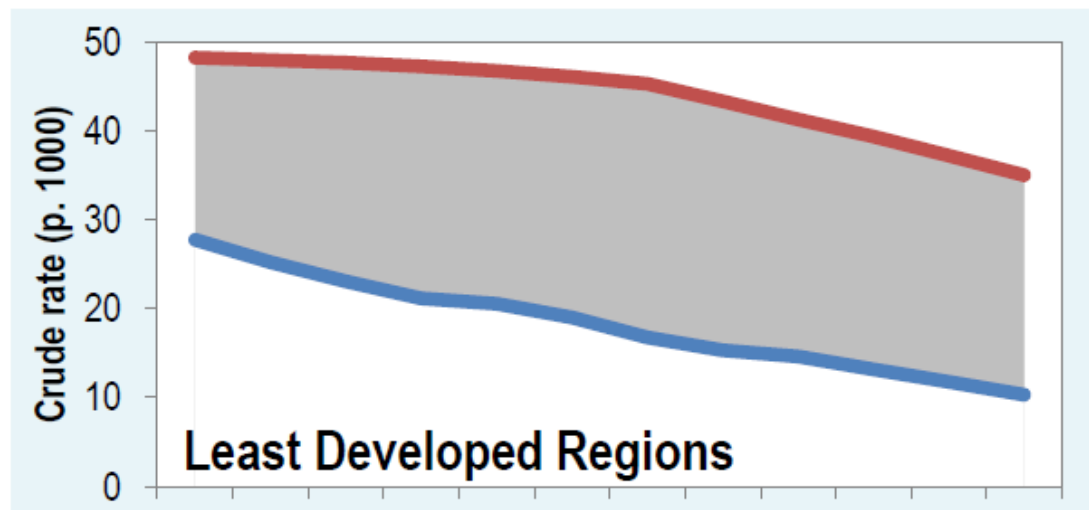
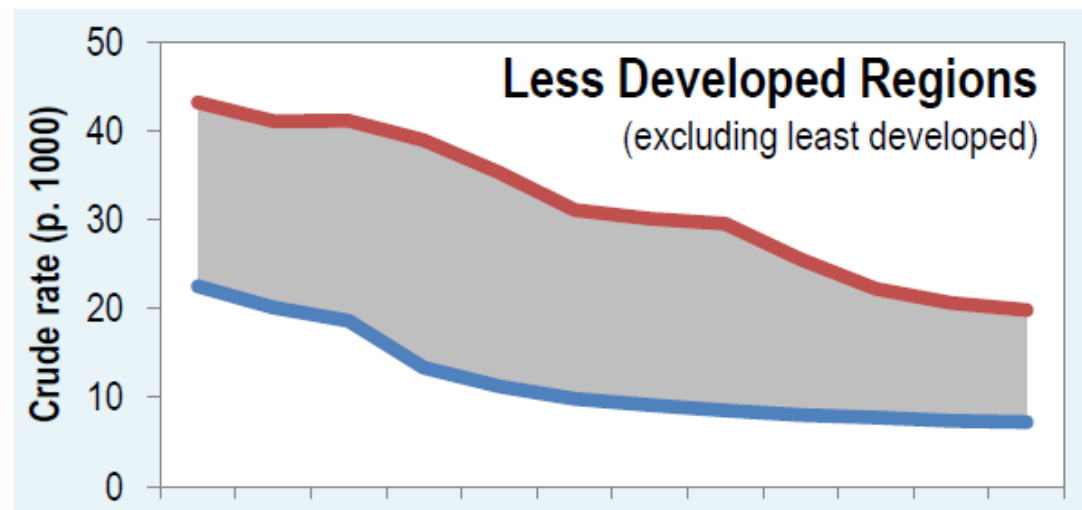
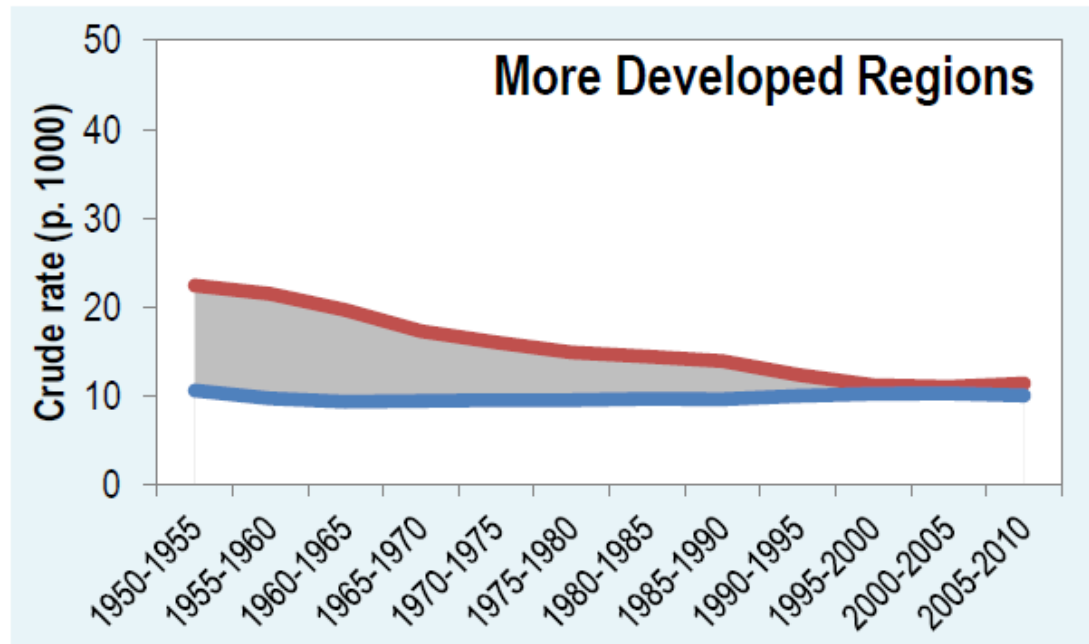
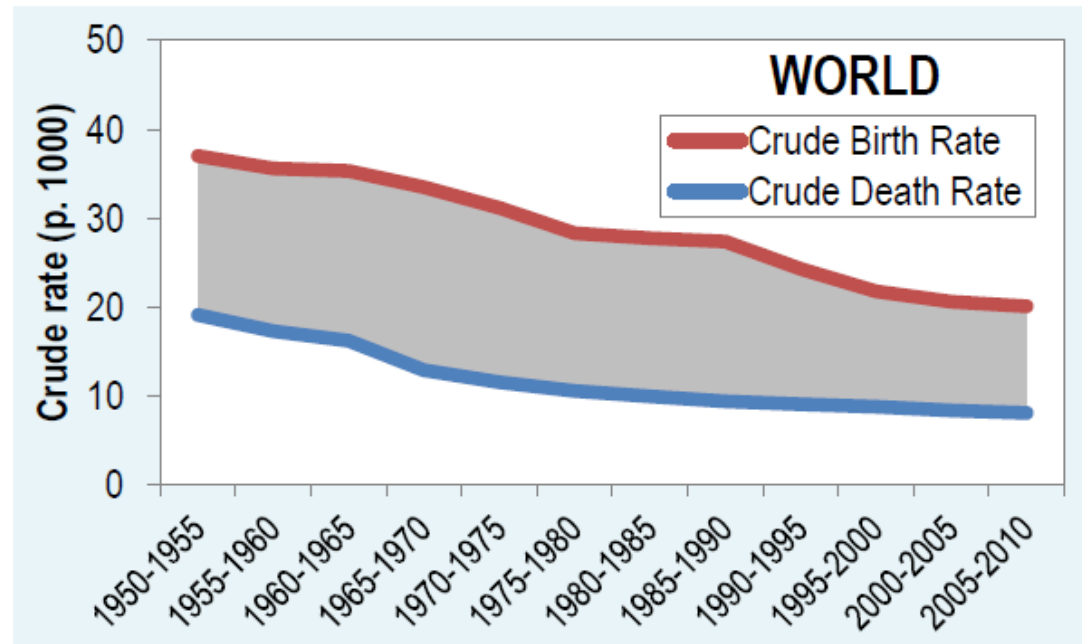
# 1. Demographic transitions – more story than theory



# 1. Demographic transition of Sweden over 250 years

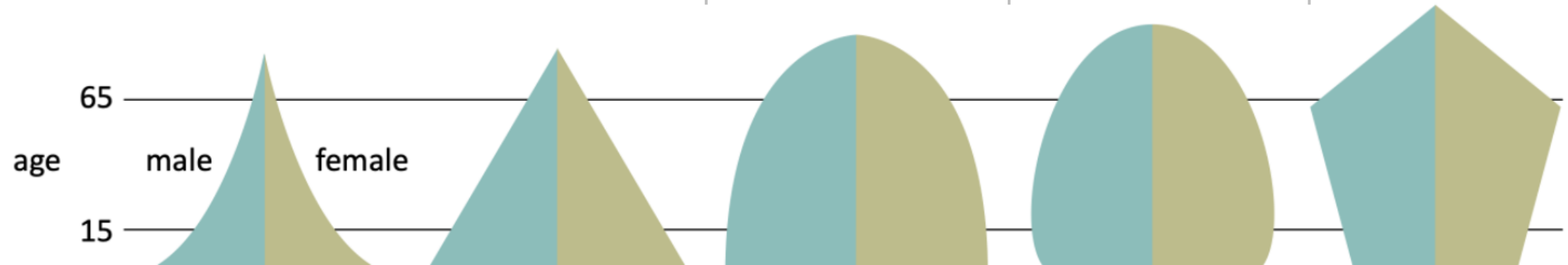


# 1. Demographic transitions around the globe (Source: UNDP 2013)



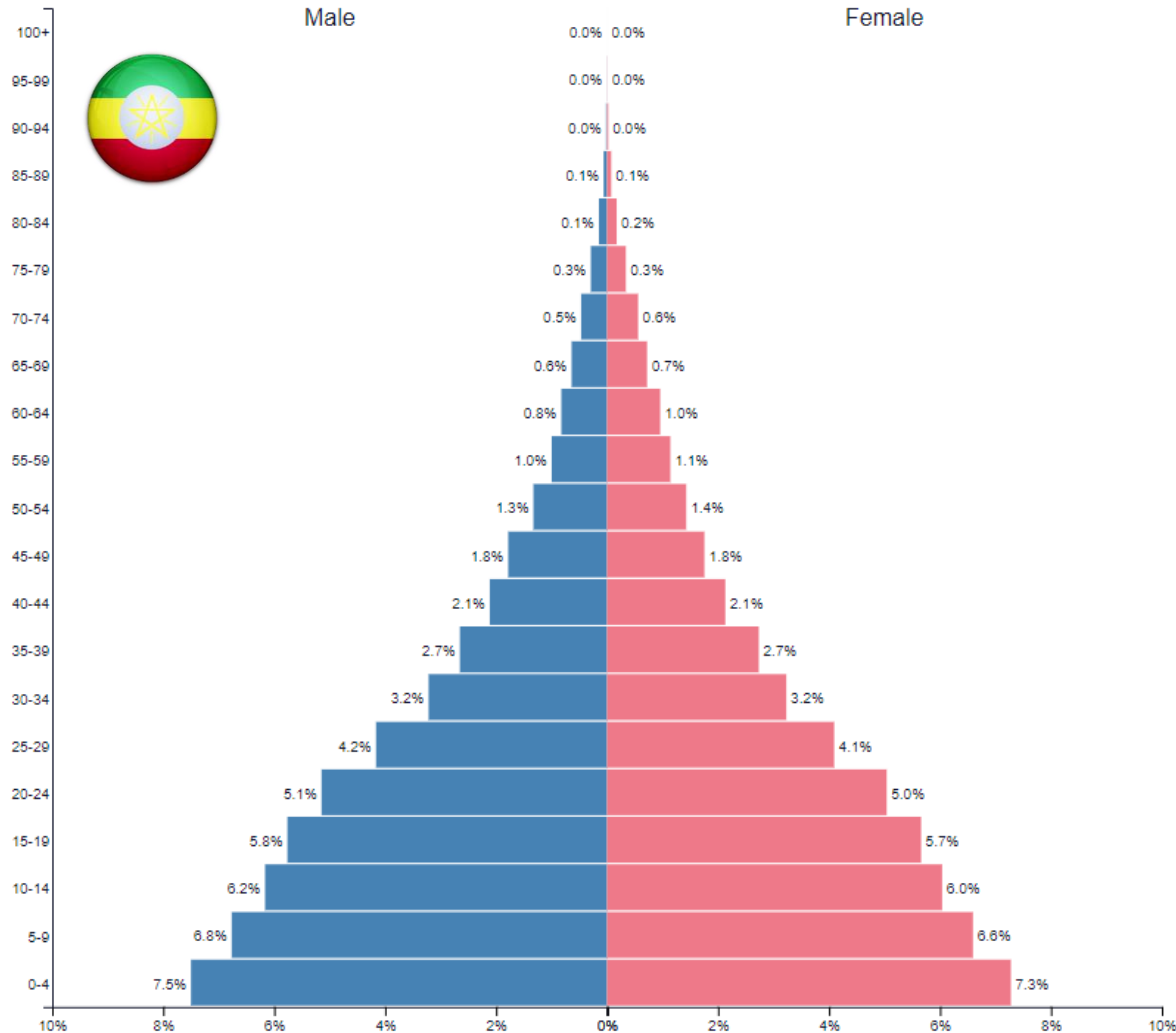
# 1. Changes in age structure through the transitions

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
<b>Birth rate</b>	High	High	Falling	Low	Very low
<b>Death rate</b>	High	Falls rapidly	Falls slowly	Low	Low
<b>Population</b>	Stable / slow increase	Rapid increase	Slow increase	Slow increase / Stable	Stable / slow decrease



# 1. Example: Population pyramids

Ethiopia – Rapid growth



Italy – Decrease

