Pension Systems / Demography & Mortality

Lecture notes: Mortality – part I

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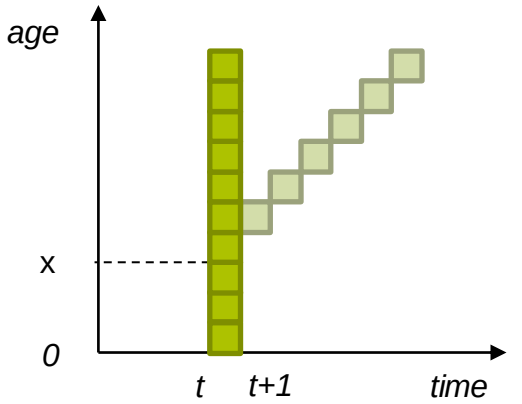
#1

Life expectancies

1. Period and cohort life expectancy

- Period life expectancy
  - Based on **observed** death rates (or smoothed version) in a given year
  - This is a summary statistic characterizing the “level of mortality” at a given point in time; used primarily for illustration of the evolution and comparison
  - Assuming a cell-wise constant mortality surface, we have for integer x
    - $e_x^p(t) = \int_x^{x_{max}} \exp\left(-\int_x^y \mu(z,t)dz\right) dy = \sum_{i=x}^{x_{max}-1} \exp\left(-\sum_{j=x}^{i-1} \mu(j,t)\right) \frac{1-\exp(-\mu(i,t))}{\mu(i,t)}$
- Cohort life expectancy
  - Based on **projected** mortalities for a given cohort
  - The projection takes future, expected mortality improvements into account
  - The cohort life expectancy is the **actual** (remaining) life expectancy
    - $e_x^c(t) = \sum_{i=x}^{x_{max}-1} \exp\left(-\sum_{j=x}^{i-1} \mu(j,t+j-x)\right) \frac{1-\exp(-\mu(i,t+i-x))}{\mu(i,t+i-x)}$



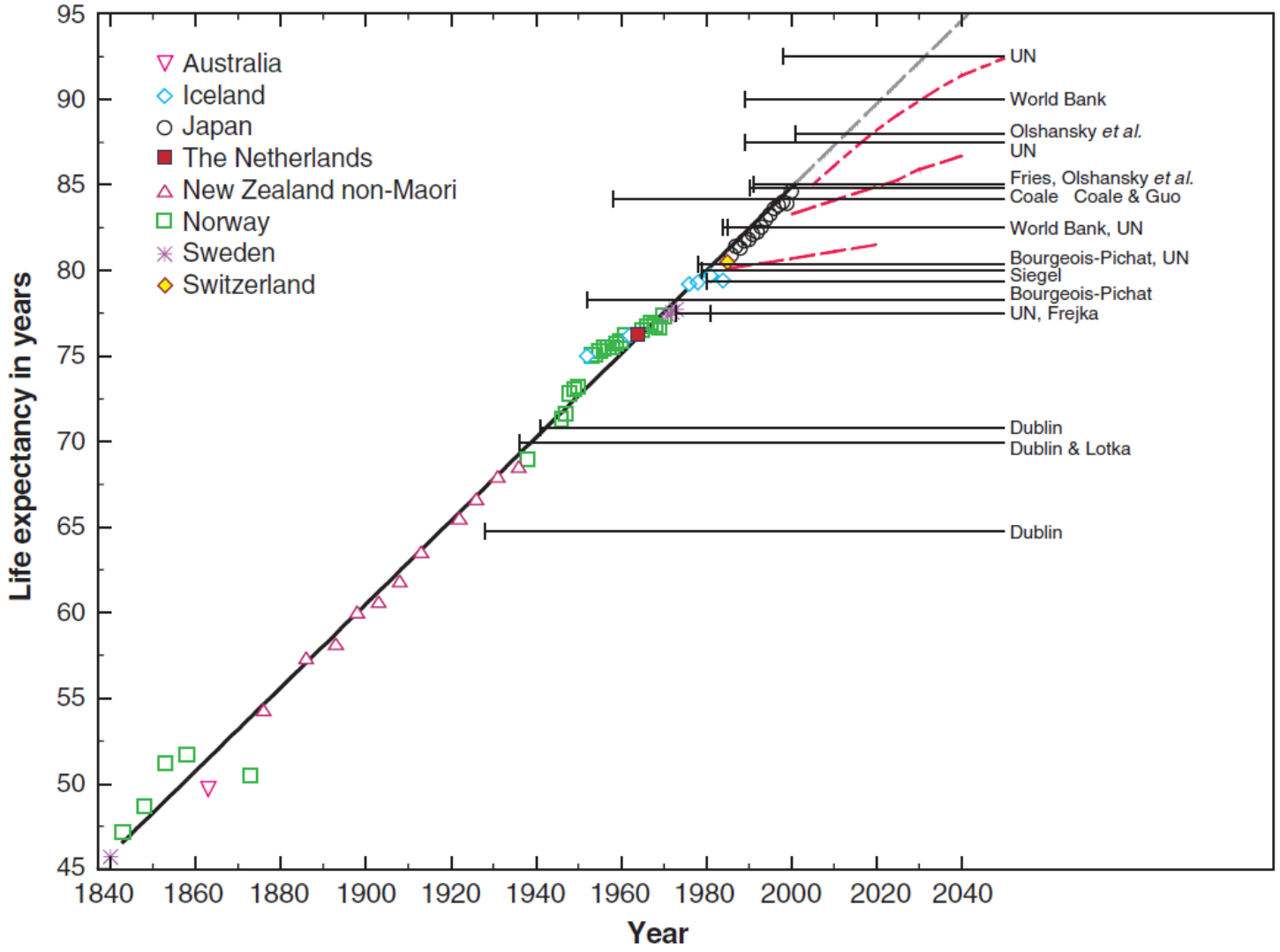
Observed age-specific mortalities in year *t*, *e.g.*, *t=2020*. Used for calculation of period life expectancy

Projected age-specific mortalities for the cohort of age *x* in year *t*. Used for calculation of cohort life expectancy

#2

Mortality modelling basics

The optimal life table: Is there an upper limit to life expectancy?



“In the absence of additional information the best one can do is to extrapolate past trends”

… but what does it mean?

Mortality modelling basics

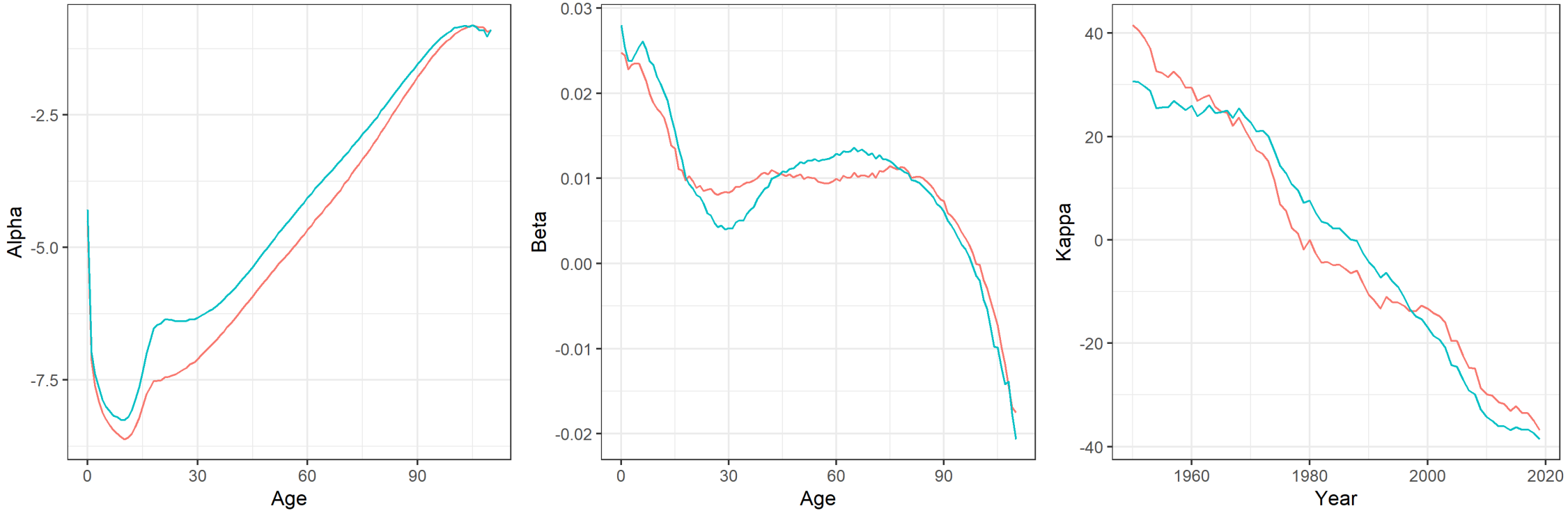
- **Stylized facts**
  - **Death rates increase (roughly) log-linearly with age**
    - … but slower for the oldest-old
    - … and by a different pattern for children and adolescents
  - **Death rates decrease over time for all age groups**
    - Improvement rates decrease with age
    - Old-age improvement rates are increasing over time
    - Young-age improvement rates are decreasing over time
  - **Persistent gender gap**
    - Male mortality higher than female mortality
    - Similar improvements over long horizons
- **Mortality model should achieve these stylized facts**

#3

The Lee-Carter model

… and issues to be aware of

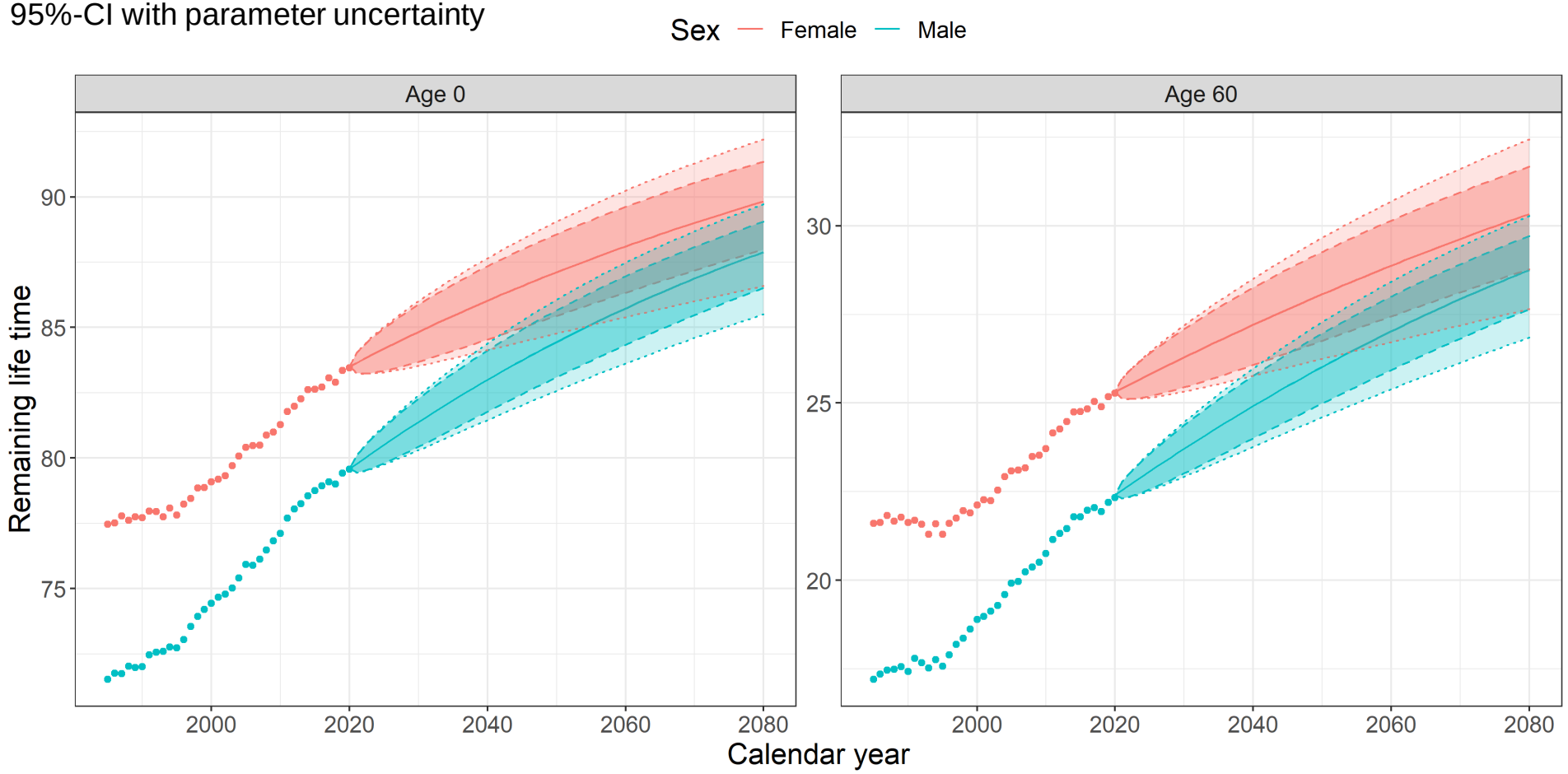
3. Example: Lee-Carter parameters fitted to US data



Tip: You can use the $plotdatEstimates-method (with a mortality-object) to create this plot.

3. LC-model estimated on Danish data

Full line: Median projection
Dashed line: 95%-CI without parameter uncertainty
Dotted line: 95%-CI with parameter uncertainty

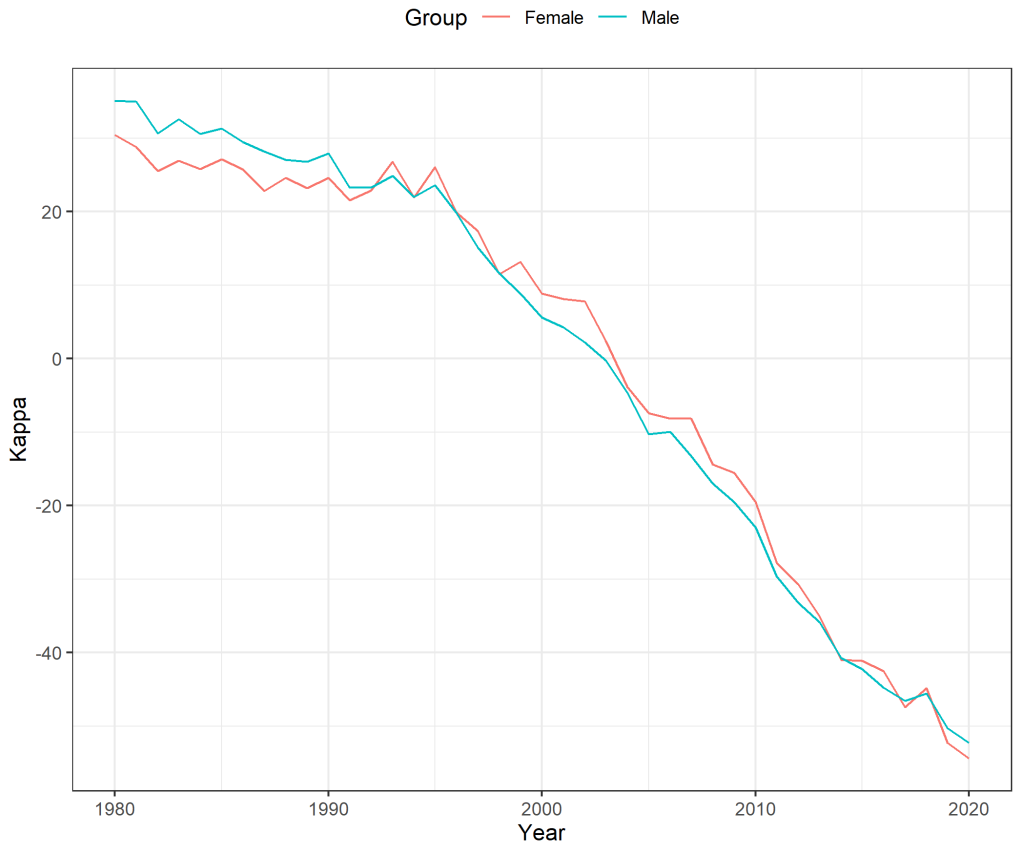


Tip: The geom used to create bands is geom_ribbon.

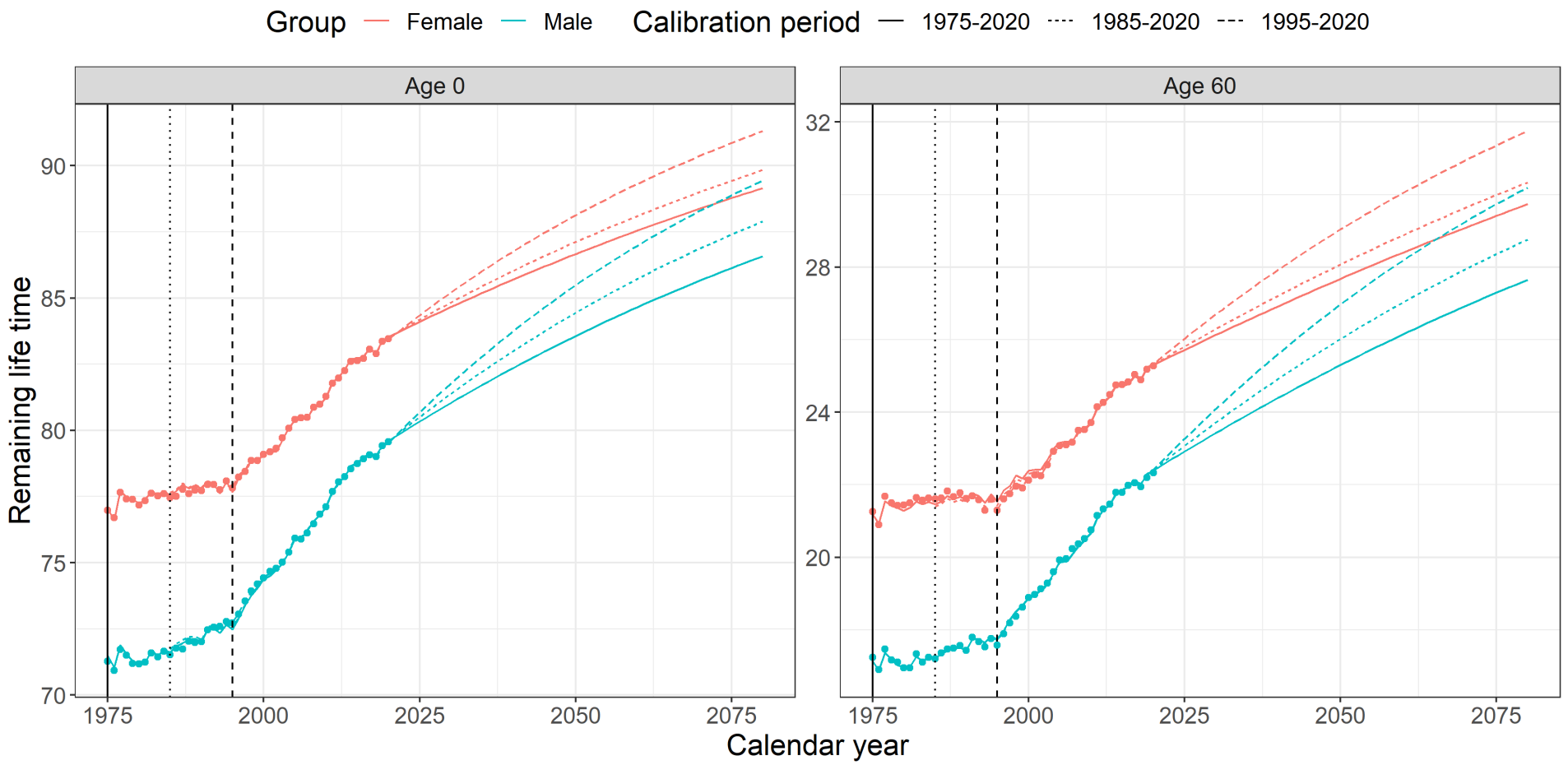
3. Applying the LC-model

- **Model assumption**
  - Age-specific rates of improvement (captured by β) are assumed constant indefinitely (linear projection of κ)
  - This might be appropriate if rates have been constant in the past, but less so when improvements vary over time

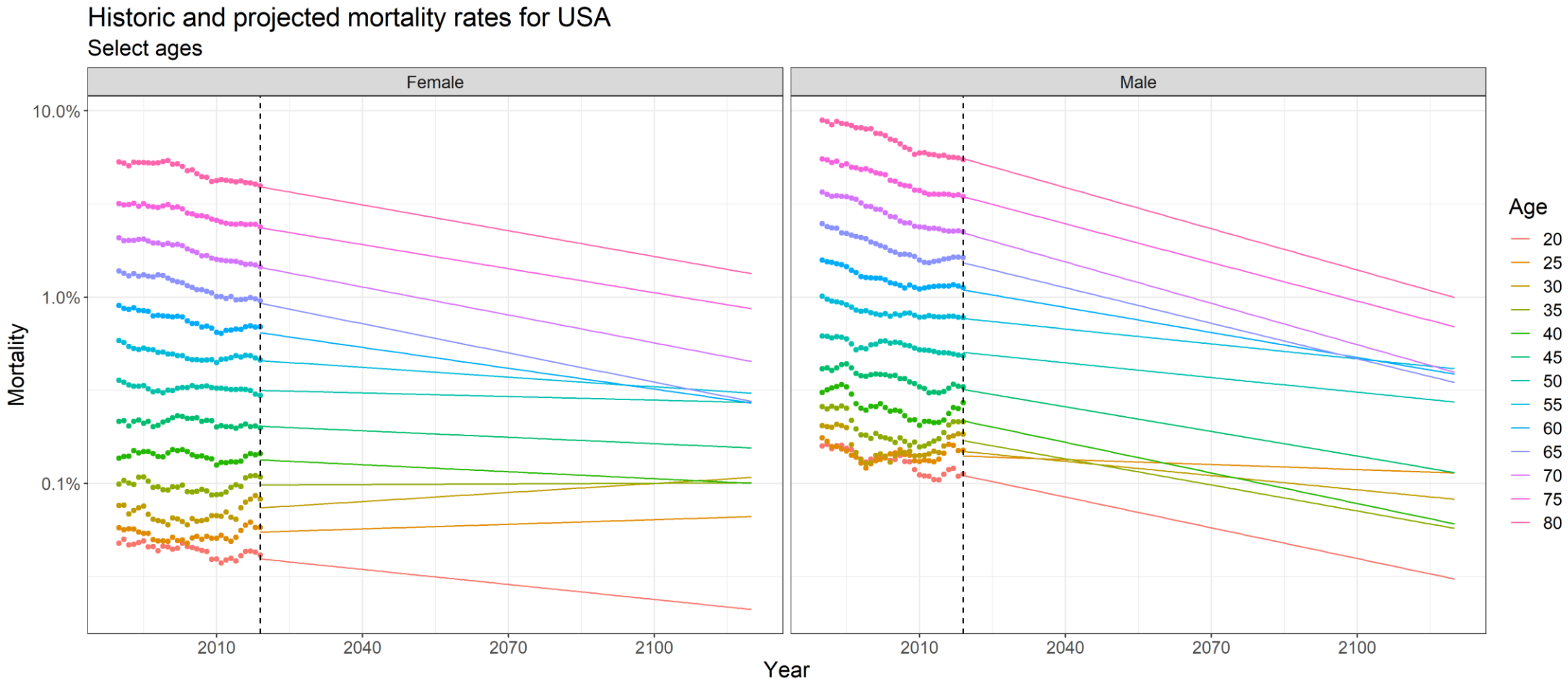
- **Danish mortality**
  - Stagnation in female life expectancy from 1977 to 1995 (primarily due to smoking); steady improvements hereafter
  - Clearly visible as a kink in 1995 in the κ-process
  - The RW-model projects κ using the average drift over the estimation period, which is lower than the current drift
  - The Danish FSA currently uses the period 2000-2019 for its longevity benchmark (but used to have a longer window)



3. Sensitivity to estimation window (Danish data)



3. Inconsistencies may emerge on longer horizons



3. Long-term uncertainty reflects short-term deviations