Pension Systems / Demography & Mortality

Lecture notes: Demography – part IV

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Recap – Stable population

- Definitions:
  - A population with an age distribution which remains constant over time is referred to as **stable**.
  - A stable population of constant size is said to be **stationary**.
- Sharpe and Lotka (1911) showed that
  - Assuming constant age-specific fertility and mortality rates, and zero net migration in all ages
  - *A stable population will arise over time*

- Under general assumptions, it is possible to show that
  - $B(t) \cong B_r e^{rt}$, i.e. the births, and thereby the population itself, grows (asymptotically) exponentially at rate r
  - r is the *intrinsic growth rate* of the population given by the unique solution to $1 = \int_0^\infty e^{-rx} S(x) F(x) dx$
  - B_r is a constant that depends on the initial age-distribution as well as the age-specific mortality and fertility rates

Recap – Equations characterizing a stable population

- Birth process : $B(t) = B_r e^{rt}$
- Persons aged x at time t : $N(x,t) = B(t-x)S(x) = B(t)e^{-rx}S(x)$
- Total population at time t : $N(t) = \int_0^\infty N(x,t)dx = B(t)\int_0^\infty e^{-rx}\,S(x)dx = B(t)e_r$
- Crude birth rate (constant) : $b_r = \frac{B(t)}{N(t)} = \frac{1}{\int_0^\infty e^{-rx}S(x)dx} = \frac{1}{e_r}$
- Age-composition (constant) : $c_r(x) = \frac{N(x,t)}{N(t)} = \frac{e^{-rx}S(x)}{\int_0^\infty e^{-rx}S(x)dx} = b_r e^{-rx}S(x)$

- A stationary population is a stable population with an intrinsic growth rate of $r = 0$
  - In this case, all quantities are time-invariant

- The stable equivalent population of a given population is the population that would eventually arise, if current (or otherwise specified) mortality and fertility rates were to continue indefinitely

Recap – Momentum of population growth

Population momentum can be studied using stable population theory

- Consider a population with survival function S and fertility rates F such that $NRR \equiv \int_0^\infty S(x)F(x)dx = 1$.
  - Such a population will converge to a stationary population of size N_S, say.

- The birth process is asymptotically constant
  - $B(t) \cong B_0 = \frac{\int_0^\infty G(t)dt}{\int_0^\infty xS(x)F(x)dx} = \frac{\int_0^\infty G(t)dt}{A_0}$, where A_0 is the mean age of childbearing in stationarity

- Using asymptotic stationarity, $N_S = e_0 \frac{\int_0^\infty G(t)dt}{A_0}$, where $e_0 = \int_0^\infty S(x)dx$ is life expectancy at birth

- Denote the current size of the population by N and define the (total) **momentum** as the ratio $M = N_S/N$.
  - Using the asymptotic stationarity: $M = \frac{N_S}{N} = \int_0^\infty \frac{c(x)}{c_0(x)} \int_x^\infty S(y)F(y)dydx / A_0$

#1

Decompositions

Decomposition techniques

- In demography, we use crude rates, averages and other convenient summary statistics to characterize population phenomena

- Ambiguities arise due to confounding influences, e.g., age, sex, socioeconomic status (compositional effects)
  - Example: Is a decline in the CDR due to a decline in mortality or to a change in the age-composition?

- Decomposition methods can be used to analyze confounding compositional effects
  - Basic idea: Separate a summary measure into its constituent parts
  - Typically: Arithmetic manipulation of differences or derivatives w.r.t. time

Example: Decomposing change in life expectancy

Table 1. Life Expectancy at Birth, $e^o(0,t)$, and the Decomposition of its Annual Change Around January 1, 1903, 1953 and 1998, in Sweden

t	1903	1953	1998
$e^o(0,t)$	53.383	71.858	79.262
$e^o(0,t-2.5)$	52.239	71.130	78.784
$e^o(0,t+2.5)$	54.527	72.586	79.740
$\dot{e}^o(0,t)$	0.458	0.291	0.191
$\bar{\rho}$ (%)	1.852	2.083	1.587
$e^\dagger$	22.362	11.988	10.053
$\bar{\rho}e^\dagger$	0.414	0.249	0.159
$\mathrm{Cov}_f(\rho,e^o)$	0.044	0.042	0.032
$\dot{e}^o(0) = \bar{\rho}e^\dagger + \mathrm{Cov}_f(\rho,e^o)$	0.458	0.291	0.191

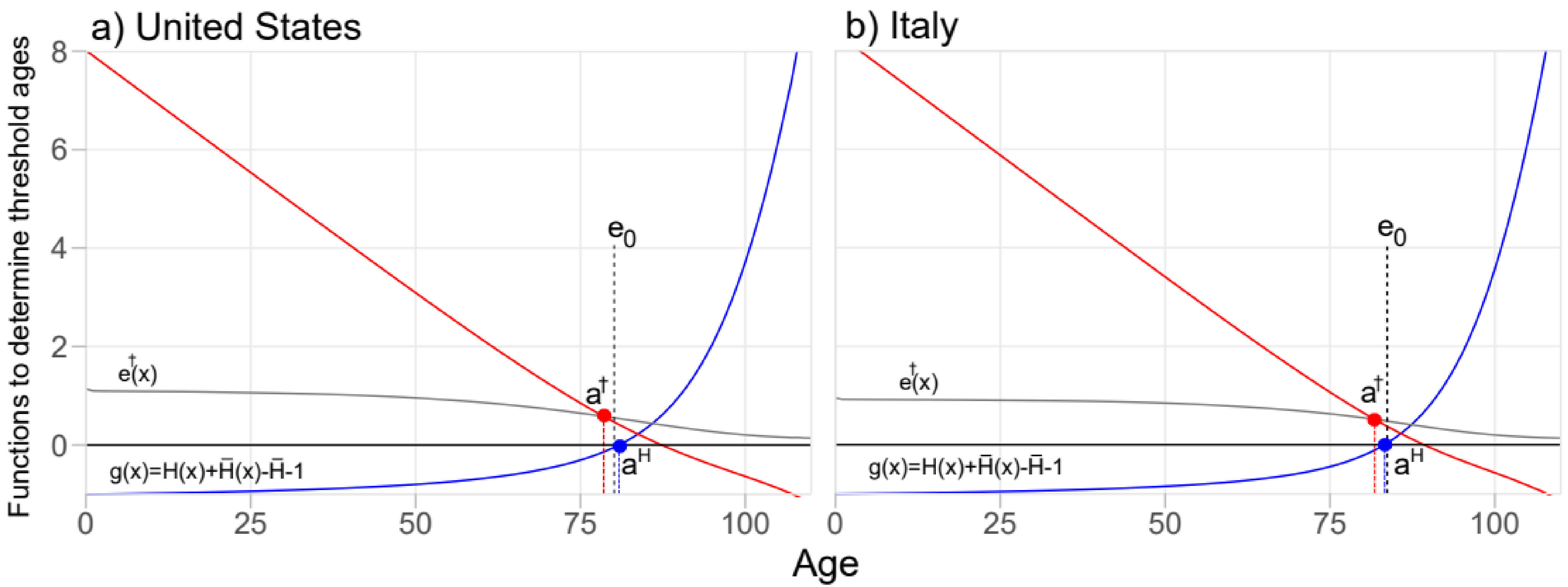
Sources: Authors' calculations described in Appendix B. Life table data are derived from the Human Mortality Database (2002). The data used pertain to single years of age. Life table values for the years 1900 and 1905, 1950 and 1955, and 1995 and 2000, were used to obtain results for the midpoints around January 1, 1903, 1953, and 1998, respectively.

From Vaupel & Canudas-Romo (2003)

#2

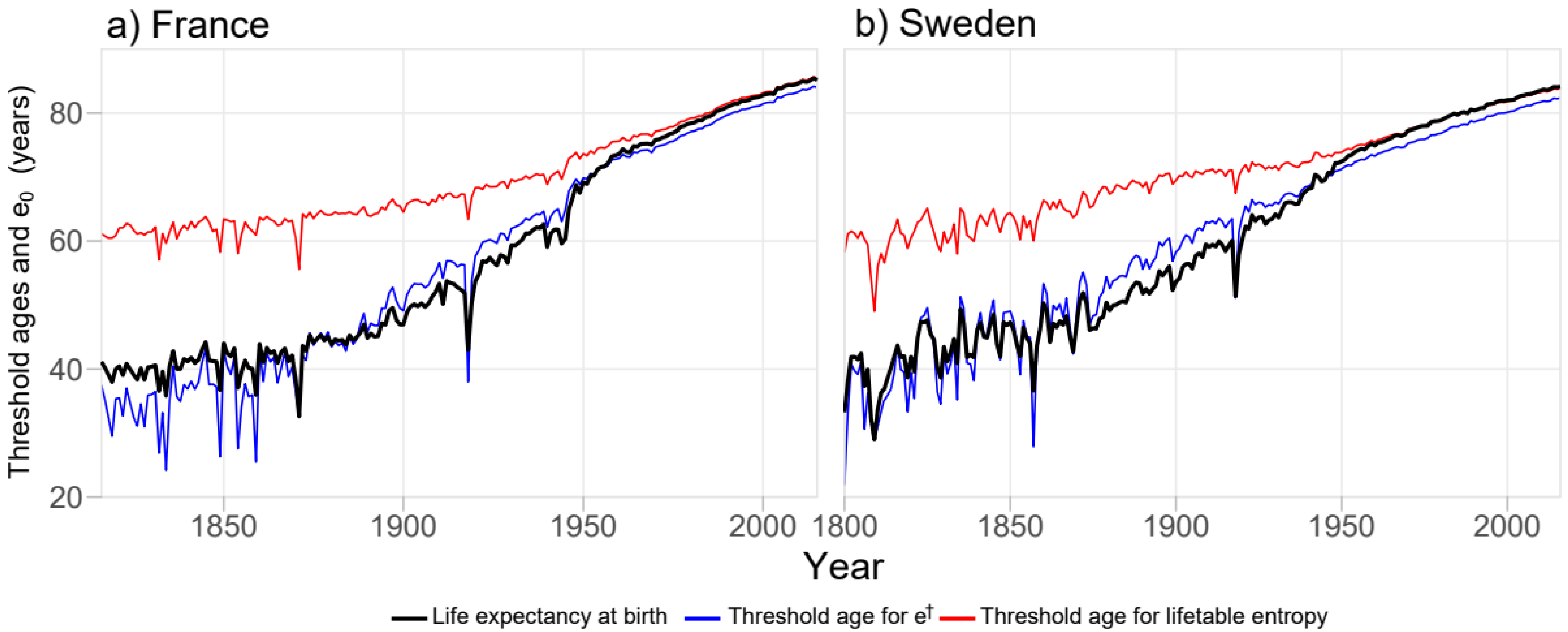
Threshold age

Figure 1: **Threshold ages for life disparity $(a^\dagger)$ and for the lifetable entropy (a^H), United States and Italy in 2005**



Note: Values in Panel a): $e_o = 80.13$, $a^\dagger = 78.51$, and $a^H = 80.86$. Values in Panel b): $e_o = 83.67$, $a^\dagger = 81.76$, and $a^H = 83.28$. Functions to determine the threshold age for $e^\dagger$ were rescaled by a factor of 1/10 for comparability.
Source: Human Mortality Database (2018).

Figure 2: Threshold ages for life disparity $(a^{\dagger})$ and for the lifetable entropy (a^{H}) compared to life expectancy at birth. French and Swedish females, 1800–2016



Source: Human Mortality Database (2018).