ESTIMATION OF NONLINEAR DC-MOTOR MODELS USING A SENSITIVITY APPROACH

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Abstract - A nonlinear model structure for a permanent magnet DC-motor, appropriate for simulation and controller design, is developed. The essential nonlinearities are due to coulomb friction and to voltage and velocity dependent brush resistance. The physical parameters of the nonlinear model are estimated directly, using a sensitivity approach for input design and evaluation of relative accuracy of the estimates. Experimental results demonstrate that linear models, as well as nonlinear models with the parameters determined from traditional static measurements, fit dynamic measurements poorly. A nonlinear model with the parameters estimated from dynamic measurements, however, fits the measurements very well, and the sensitivity measures combined with cross-validation results indicate that this model is robust and accurate.

Key words - Identification, physical parameters, nonlinear systems, DC-motor models.

1. INTRODUCTION

The DC-motor is still an attractive alternative to AC-motors in many high-performance motion control applications. Although the market share is predicted to fall, a future growth of the DC-drive market of 5-10% has been estimated [1]. To utilize the new and useful nonlinear model-based control algorithms, see e.g. [2], [3], an accurate nonlinear model of the DC-motor is essential.

The purpose of this paper is twofold:
- to present a nonlinear model of permanent magnet DC-motors, applicable for simulation and controller design
- to present a practical method for estimation of parameters in nonlinear dynamic systems, based on a sensitivity approach

The essential nonlinearities of permanent magnet DC-motors are due to coulomb friction and to variable brush contact resistance. Traditionally the parameters describing these nonlinearities are determined from blocked-rotor and constant-velocity measurements, and it is a common experience that the model so obtained fits dynamic measurement poorly. The reasons for this inconsistency and how to avoid it, is the main subject of the paper.

System identification is a powerful tool for building models of dynamic systems. In classical system identification the parameters of linear discrete-time models are determined, and assumptions about noise play an important part in the choice of model structure [4],[5]. Methods for design of optimal input signals are reported in numerous publications, e.g. [6], [7] and [8]. Most methods involve minimizing a scalar measure of Fisher Information matrix. In [9] input design is based on parameter sensitivity. In many practical applications, such as the one at hand, the parameters of primary interest are physical parameters in continuous-time, linear or nonlinear models. Methods for identification of linear continuous-time systems are described in [10].

In this paper a unified approach for input design, parameter estimation and accuracy evaluation, appropriate for nonlinear systems as well, is presented. It determines the physical parameters directly by fitting a linear or nonlinear simulation model to input-output measurement [11]. To design input signals with suitable frequency and amplitude distribution, and to evaluate the relative accuracy of the parameter estimates, a sensitivity approach, applying so called sensitivity ellipsoids in the parameter space, is used. This approach relies heavily on a physical insight in the device to be modelled, more than assumptions about the noise. Accordingly, it is quite user friendly, and it might help bridge the gab between theory and practice of system identification.

In the following sections the model structure of the DC-motor is first determined from theoretical considerations and experimental results, and a useful nonlinear simulation model is developed. Next the sensitivity approach for parameter estimation is described, and suitable input signals are designed from simulation studies. Finally the laboratory measuring system is described, and experimental results are presented and discussed.

2. DC-MOTOR MODELS

A linear low-frequency model of a permanent magnet DC-motor can be developed from basic physical laws. The electrical and mechanical properties are described by the following two differential equations, [12]

$$u(t) = R_a \cdot i\,(t) + K \cdot \omega(t) \tag{1}$$

$$K \cdot i(t) = J \cdot \frac{d\omega\,(t)}{dt} + B \cdot \omega\,(t) \tag{2}$$

For many applications this structure is, however, not sufficient. The major mechanical and electrical nonlinearities must be included in the model, e.g. as indicated in Fig.1.

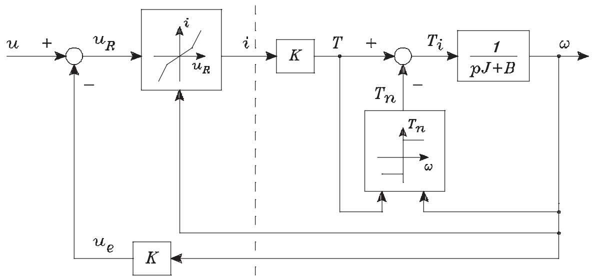

Fig.1 Motor model including nonlinear brush resistance and nonlinear friction. The nonlinearities are defined by the parameters: brush conductances G_0, G_1 and voltage U_1, and coulomb friction coefficient T_c.

The electrical nonlinearity consists of a voltage dependent brush resistance of the split ring commutator. Traditionally this is modelled as a constant brush voltage drop and a constant resistance, including the armature resistance, but a piecewise constant resistance appears to be a better approximation.

The relationship between u_R and i is normally measured with blocked rotor (ω=0). However, we have found that this relationship is substantially different for ω = 0 and ω ≠ 0. In Fig.2 results from measurements with blocked rotor as well as idle running are shown.

The difference displayed in Fig.2 is the main contributor to the poor performance of dynamic models based on static parameter measurements. The armature reaction, which can give rise to similar discrepancies for some DC-motors, has no noticeable effect for the motor type at hand.

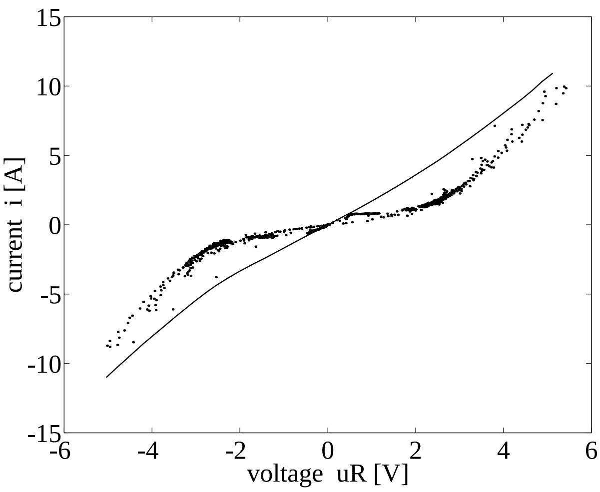

Fig.2 Nonlinear armature characteristic, $i=f(u_R)$ with blocked rotor: ____ and idle running: .....

3. PARAMETER ESTIMATION: THE SENSITIVITY APPROACH

The basic principle of the parameter estimation is quite simple: sequences of corresponding input and output signals are measured on the real system, in this case the DC-motor, (cf. section 5). The measured input signal is used as input to a simulation model, and with an initial choice (guess) of parameter values the model output is computed. A performance function for the fit is computed as the sum of squared deviations between the model output and the measured output, and finally the

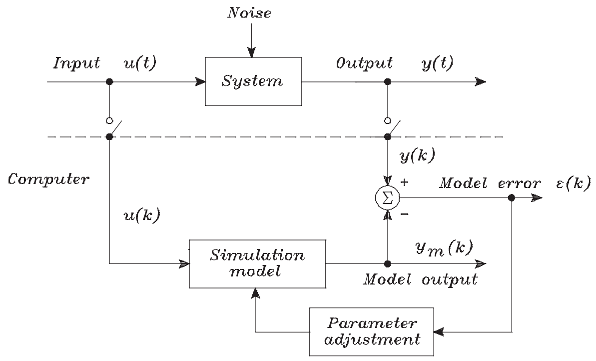

Fig.3 Parameter estimation principle: the parameters are adjusted until minimum model error

model parameters are adjusted until a minimum of the performance function is obtained. This principle is illustrated in Fig.3.

A characteristic feature of our method is that physical parameters, including the parameters describing the nonlinearities, are adjusted directly. The minimization is performed by a Gauss-Newton algorithm, operating directly on the physical parameters, and the model gradients are determined by numerical differentiation with respect to these parameters.

Various questions may, however, be pertinent, e.g. - are the parameter values thus obtained unique? - how accurate are they? - and what input signal should be used? Traditionally these questions are answered by checking if the model is "identifiable" and the experiment "informative" [4]. In this paper it is demonstrated how certain sensitivity measures, describing the model sensitivity to individual parameters as well as the correlation between the parameters, are very convenient for answering the questions.

The sensitivities are defined as the derivatives of a so called Root Mean Square Parameter Dependent Error, $\varepsilon_{p,RMS}(\theta)$ with respect to the relative parameters θ_r. As $\varepsilon_{p,RMS}$ is linear in the parameters, $\varepsilon_{p,RMS}(\theta_r)=1$ comprises an ellipsoid in the parameter space. In the following it is described how the shape of this sensitivity ellipsoid may be described by certain sensitivity measures, and the mathematical basis is briefly presented.

The simulation model may be in state-space form or a combination of linear dynamic transfer functions and static nonlinearities. In both cases the model output can be expressed as

$$y_m(k) = F(u_N, \theta) \tag{3}$$

where θ is a vector containing the physical parameters, and u_N is the input vector with N samples. F denotes a, normally nonlinear, function of u and θ.
The quadratic performance function $V(\theta)$ to be minimized is

$$V(\theta) = \frac{1}{N} \sum_{k=1}^{N} \varepsilon^2(k,\theta) \tag{4}$$

where the model error is

$$\varepsilon(k) = y(k) - y_m(k) \tag{5}$$

and the parameter estimate of θ based on N input-output data points, u_N and y_N is then the value θ_N minimizing $V(u_N, y_N, \theta)$

$$\theta_N = arg\,min V(u_N, y_N, \theta) \tag{6}$$

Using a Gauss-Newton algorithm for the minimization, the estimate θ_N of the unknown parameters can thus be cal culated from (4) and (6).

The model error, $\varepsilon(k)$ may be split up into two parts: the minimum value $\varepsilon_o(k,\theta_N)$, caused by noise and imperfect model structure, and a part, $\varepsilon_p(k,\theta)$ caused by θ being different from θ_N. The latter part, ε_p, which we shall denote the parameter dependent model error, is the crucial quantity for the sensitivity approach.

By a first order Taylor expansion around θ_N we obtain

$$\varepsilon_p(k,\theta) = y_m(k,\theta_N) - y_m(k,\theta) \cong \psi(k,\theta_N)(\theta_N - \theta) \tag{7}$$

where ψ is the model gradient

$$\psi(k,\theta_N) = \frac{dF(\theta)}{d\theta}\Big|_{\theta=\theta_N} \tag{8}$$

The root mean square of $\varepsilon_p(k,\theta)$ is next introduced

$$\varepsilon_{p,RMS}(\theta) = \sqrt{\frac{1}{N}\sum_{k=1}^{N} \varepsilon_p^2(k,\theta)} \cong \sqrt{(\theta-\theta_N)^T H(\theta_N)(\theta-\theta_N)} \tag{9}$$

where the approximation (7) is used, and the Hessian $H(\theta_N)$ is

$$H(\theta_N) = \frac{1}{N}\sum_{k=1}^{N} \psi(k,\theta_N)\psi^T(k,\theta_N) \tag{10}$$

As the values of the individual parameters may be very different, a comparison of absolute sensitivities has little implication. Relative parameter sensitivities are more meaningful, and they can be obtained by introducing relative parameters, θ_r and the corresponding relative H-matrix, H_r

$$\theta_r = L^{-1}\theta \qquad H_r(\theta_N) = L^T H(\theta_N) L \qquad where \quad L = diag(\theta_N) \tag{11}$$

The sensitivity of $\varepsilon_{p,RMS}(\theta)$ with respect to one relative parameter θ_{ri} - while the remaining relative parameters are equal to 1 - is then

$$S_i = \frac{d\varepsilon_{p,RMS}}{d\theta_{ri}} = \frac{d\varepsilon_{p,RMS}}{d\theta_i/\theta_{Ni}} = \sqrt{h_{rii}} \qquad h_{rii} = \{H_r(\theta_N)\}_{ii} \tag{12}$$

To obtain accurate parameter estimates, large sensitivities of the individual parameters is a necessary requirement. It is not sufficient, however, as the sensitivity of a combination of two or more parameters may be much smaller than the individual sensitivities, indicating a high correlation between the parameters. This problem can be illustrated in the two-dimensional parameter space, where iso-curves expressed by (9), $\varepsilon_{p,RMS}$=constant, are ellipses, see Fig.4. We will use the name, **sensitivity ellipse**, as *the distance from the centre to any point on the ellipse represent the reciprocal sensitivity in that direction.*

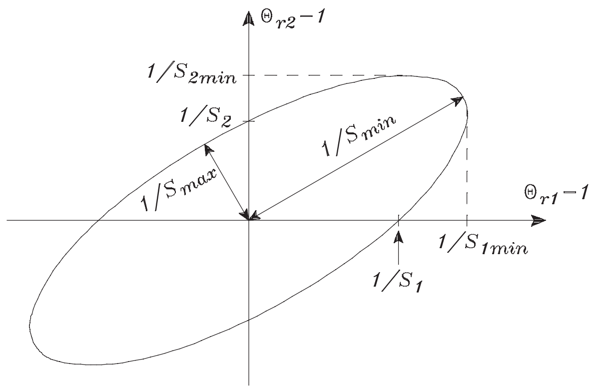

Fig.4 Sensitivity ellipse:
$\varepsilon_{p,RMS}(\theta_r) = sqrt((\theta_r\text{-}1)^T H_r(\theta_r\text{-}1)) = 1$
with some sensitivities indicated.

Four characteristic sensitivity measures are summarized in Table 1, together with an indication of their best values for accurate parameter estimation.

S_{min}: minimum sensitivity, inverse of major half axis
 - as large as possible.
$\mathbf{S_{i\,min}}$: minimum sensitivity of θ_i
 - as large as possible.
$\mathbf{R=S_{max}/S_{min}}$**:** ratio of maximum and minimum sensitivity in any direction or ratio of half axis
 - as close to 1 as possible.
$\mathbf{R_i=S_i/S_{i\,min}}$**:** ratio of sensitivity of θ_i alone and minimum sensitivity of θ_i - as close to 1 as possible.

Table 1. Characteristic parameter sensitivity measures

A large value of R_i indicates that the i'th parameter is correlated with one or more of the remaining parameters.

These sensitivities and ratios can be calculated from H_r, as

$$S_{\max} = \sqrt{\lambda_{\max}} \qquad S_{\min} = \sqrt{\lambda_{\min}} \tag{13}$$

where λ is an eigenvalues of H_r, and, according to [7]

$$S_{i\min} = \sqrt{\{H_r^{-1}(\theta_N)\}_{ii}^{-1}} \tag{14}$$

It may be emphasized that as the shape of the ellipse is determined by the Hessian $H(\theta_N)$, the values of the sensitivity measures are determined by the input signal.

Based on these considerations a procedure for design of a good input signal is:
i) *Obtain approximative parameter estimates or acquire à priori parameter values.*
ii) *Choose a class of preliminary input signals with feasible frequency and amplitude distribution*. The input signals shall depend on few (preferably just one) input signal parameters. One parameter, at least, shall control the spectrum; if the model is nonlinear, an additional input signal parameter shall control the amplitude. Use intuition and physical insight.
iii) *Optimize the input signal for best possible sensitivity ellipse (simulation)*. Calculate and plot some of the characteristic sensitivity measures as a function of input signal parameters, and choose best values of these according to Table 1. The ellipsoid shall approach a sphere with least possible radius.
iiii) *Use the determined input signal on the real system, obtaining an improved parameter estimate.*

4. DETERMINATION OF INPUT SIGNAL

An optimal input signal for estimating the 7 parameters of the nonlinear model shall now be determined from simulation studies.

Utilizing the procedure for input design, approximative parameter values must first be obtained (i). These values can be found in the manufacturers data sheets, or measured by traditional methods. Next, classes of input signals is chosen (ii). A square-wave signal with a suitable amplitude, characterized by the fundamental frequency ($f_1 = 1/T_1$, where T_1 is the period) for the linear models, and a variable amplitude signal, characterized by the maximum amplitude, a_p, see Fig.5, have been found to perform well.

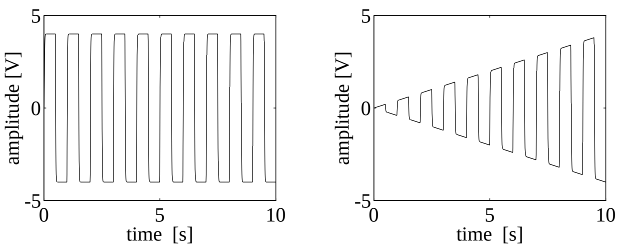

Fig.5 Input signals. Square wave for linear models and varying amplitude signal for nonlinear systems.

First the linear model is considered. The characteristic measures versus the fundamental frequency is shown in Fig.6. The optimal value is chosen as the one minimizing R, f1opt=0.5. This value also gives maximum value of the minimum sensitivity, Smin, and low correlations, Ri. As the expected accuracy of the friction B is lower than for the remaining parameters, the sensitivity of B is downweighted by a factor 4, [11].

For the nonlinear model, the same fundamental frequency, f1opt is chosen for the varying amplitude signal. In Fig.7 the characteristic measures are plotted as a function of the maximum amplitude a_p. Again the optimal value is chosen as the one minimizing R, apopt=7. Also with regards to the other characteristic sensitivity measures this is a good choice.

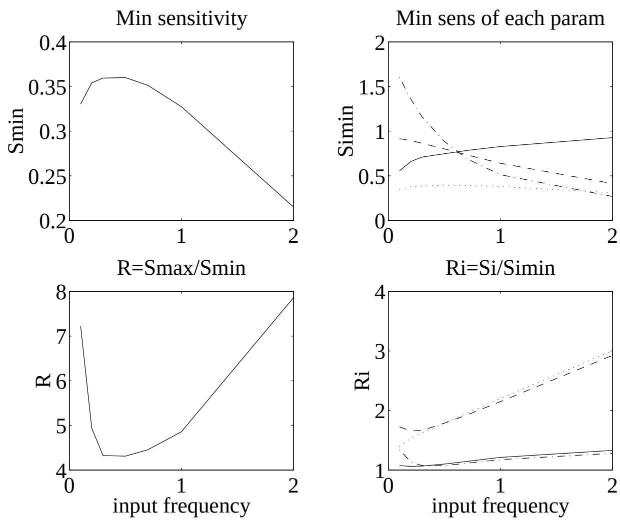

Fig.6 Sensitivity measures as a function of square wave frequency f_1, linear model.

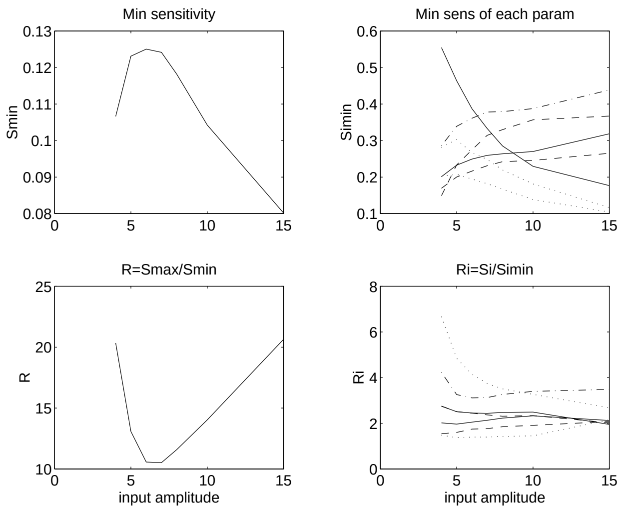

Fig.7 Nonlinear model. Characteristic measures versus maximum amplitude a_p,

The simulation study thus indicate, that with a proper input signal all 7 parameters can be estimated, and that the varying amplitude signal with $f_1 = 0.5$ and $a_p = 7$ is a good input signal.

5. EXPERIMENTAL RESULTS

The measuring system used to collect data for this investigation is shown in Fig.8. The input signal is generated by the computer and transmitted to the DC-motor through a low pass filter and a power amplifier. Three signals are measured: the voltage across the motor, the voltage across a measuring resistance R_m and hence the current, and the velocity, by an AXEM tachometer, F2TC . After low pass filtering, these signals are sampled with 12 bit A/D converters (DT2811) and stored by the computer. An induction, L (≈20 mH) is inserted to reduce noise in the current, generated by the commutation. The motor is an AXEM DC servo motor, F9M2 with rated power output 88 W and rated speed 3000 r.p.m. (=314 rad/s). The motor load consists of the tachometer only.

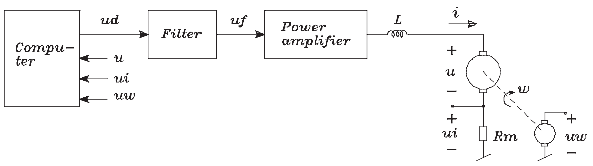

Fig.8 Laboratory set up for motor measurements.

The input voltage u_d is the varying amplitude signal, designed in the previous section. To evaluate the models, graphs of the measured signals and the corresponding model outputs are presented. To describe the fit with a single number the normed RMS output error is used

$$errn = \sqrt{\frac{\sum_{k=1}^{N} (y(k) - y_m(k,\theta_N))^2}{\sum_{k=1}^{N} y^2(k)}} \cdot 100 \ \% \qquad (15)$$

First the parameters of the nonlinear model are determined from traditional measurements and the manufacturers data sheet. The parameter values are listed in Table 2.
The total nonlinear model is simulated with these 'Trad' parameter values, and the model velocity output with input u is shown and compared to the measured velocity, Fig.9. The fit appears to be poor.

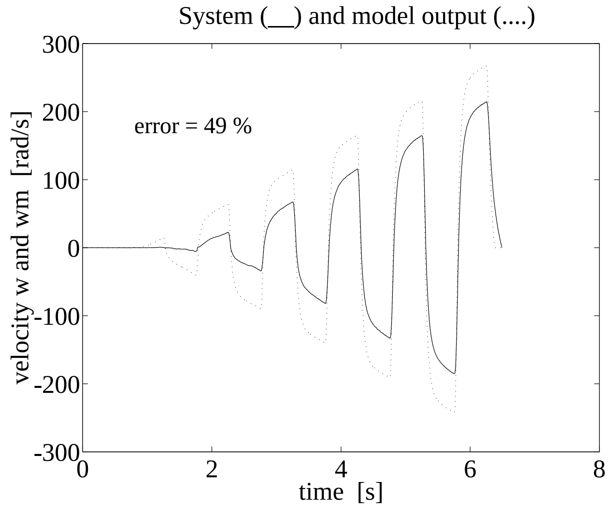

Fig.9 Total nonlinear model with traditionally measured parameters. Voltage u is input and velocity ω is output.

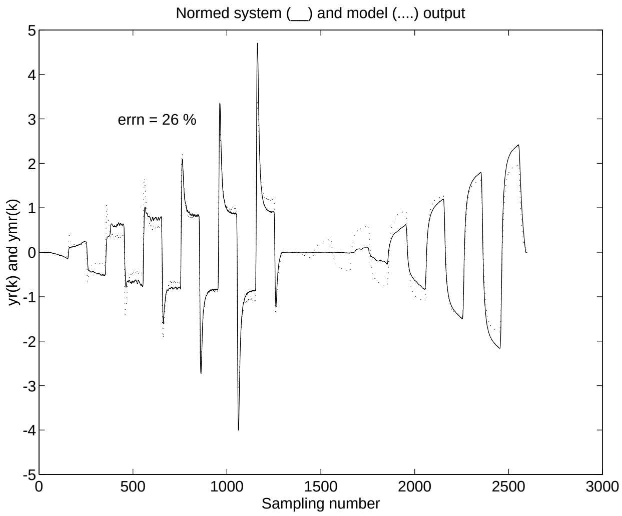

Fig.10 Measured system and *linear* model output: the first 1300 samples is current i, and the last 1300 samples is velocity ω.

	G_0 $[\Omega^{-1}]$	G_1 $[\Omega^{-1}]$	U_1 [V]	$K \cdot 10^{3}$ [V/rad/s]	$J \cdot 10^{6}$ [kg·m^2]	$B \cdot 10^{6}$ [N/rad/s]	$T_c \cdot 10^{3}$ [N]	errn [%]
Trad	1.6	2.3	4.0	29.6	70	66	39.6	49.0
Lin	0.8			35.5	82.5	416		24.7
Nlin	0.512	1.60	3.04	29.2	72.6	66.5	29.3	5.2
$S_{i\ min}$	0.29	0.23	0.24	0.46	0.26	0.041	0.15	

Table 2 Estimated parameter values and normed RMS output error for the total model. Trad: traditional measurements, Lin: linear model, Nlin: nonlinear model. $S_{i\ min}$, minimum parameter sensitivity for nonlinear model.

The parameters of the linear model are estimated from measured input-output data. Fig.10 shows how the current and velocity output of the linear model cannot possibly fit the measured current and velocity. In particular the absence of coulomb friction in the model is notable on ω.

Also, it appears from the estimated parameter values, Table 2, that the model attempts to compensate for the missing coulomb friction by increasing the viscous friction coefficient B.
The nonlinear model is then fitted to the same measurement data. Fig.11 shows substantial improvements in the model fit. The estimated parameter values and the sensitivities are listed in Table 2.

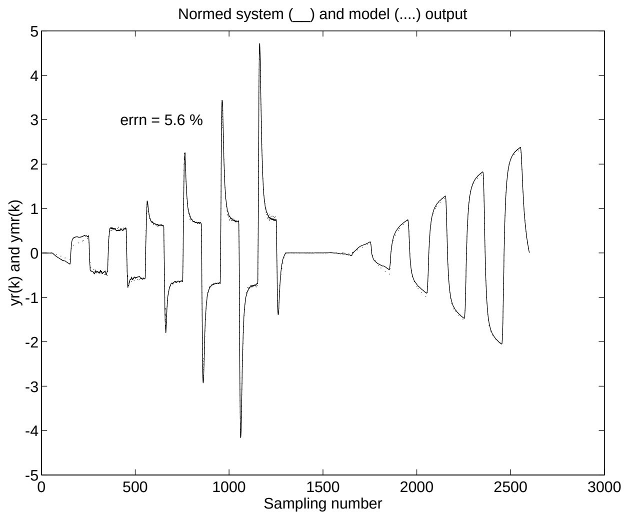

Fig.11 Measured system and *nonlinear* model output: the first 1300 samples is current i, and the last 1300 samples is velocity ω.

A cross validation of the model is performed comparing the output of a model based on the 'Nlin' parameters to the measured response of a different input signal, u_{new}. The obtained result, error = 4.3%, strongly supports the validity and robustness of the estimated model.

6. CONCLUSION

A nonlinear model structure for a permanent magnet DC-motor has been developed. Experiments demonstrated that this structure was able to fit measured input-output data much better than a linear model.

If the model parameters were determined by traditional measurements, however, the model fitted measured data very poorly. The reason is that the parameter values may depend on the measurement conditions. Accordingly, the parameters are best estimated under normal dynamic operating conditions, and a system identification technique is called for.

A sensitivity approach for determining identifiability of physical parameters in nonlinear systems and for design of good input signals was described. Using this approach an input signal with a suitable frequency and amplitude distribution was designed.

The parameters of the nonlinear motor model were estimated utilising voltage, current, and velocity measurements from a laboratory set-up. A relative output error of 5.6 % indicated that the nonlinear model structure is adequate. This was confirmed by a cross validation test using a different input signal.

In summary:
- the sensitivity approach, in combination with physical insight, was convenient for design of input signals
- the nonlinear model structure for the DC-motor could fit the voltage, current and velocity measurements well
- based on a good fit, high parameter sensitivity and low parameter correlation, and supported by solid cross-validation results, the estimates of the 7 motor parameters appear to be accurate.

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