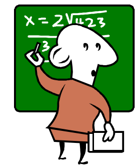
Parameter accuracy and sensitivity



Model verification: a reliable measure of the accuracy of the obtained model

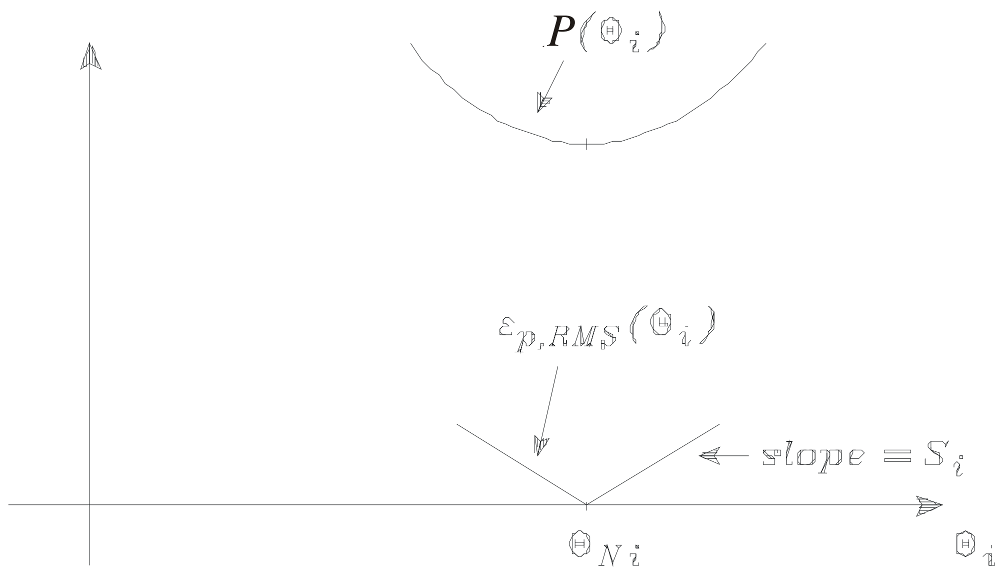
An accurate model require:

- A good fit between system- and model output conveyed by
    - normed root mean output error $erm = \sqrt{\dfrac{\sum_{k=1}^{N} (y(k) - y_m(k,\theta_N))^2}{\sum_{k=1}^{N} y(k)^2}} \times 100\ \%$
    - plot of system and model output
- Good parameter sensitivity measures
- Evaluation based on physical insight and common sense

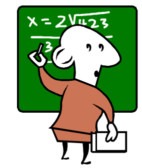
Parameter sensitivity

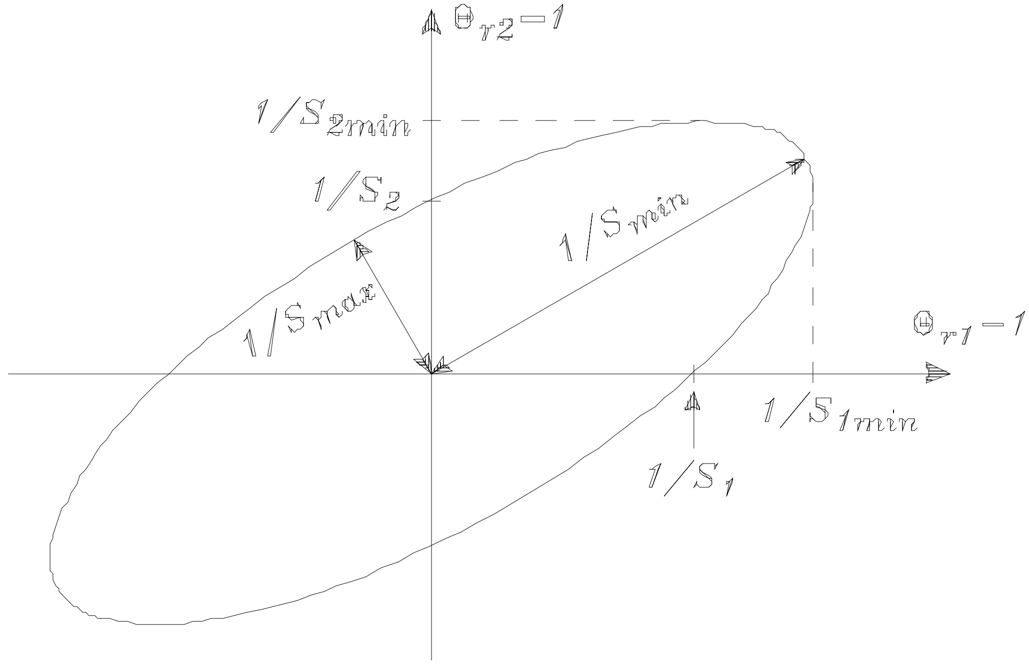
Parameter dependent model output error $\quad \varepsilon_p(k,\theta) = y_m(k,\theta_N) - y_m(k,\theta) \approx \psi(k,\theta_N)(\theta_N - \theta)$

Parameter sensitivity $\quad S_i = S_{ni} = \dfrac{\partial \varepsilon_{p,RMSn}}{\partial \theta_{ni}} = \sqrt{h_{mii}}$



Sensitivity ellipse





♑$_{p,RMSn}$ (🖹$_r$) = 1

S_i	sensitivity of θ_i alone
$S_{i\ min}$	minimum sensitivity of θ_i
S_{min}	minimum sensitivity in any direction
S_{max}	maximum sensitivity in any direction

Sensitivity measures and requirements

These characteristic measures are handy also – and in particular – for more than 2 parameters (ellipsoid)

S_{min}	minimum sensitivity, inverse of major half axis - as large as possible
$S_{i\ min}$	minimum sensitivity of θ_i - as large as possible
$R= S_{max} /S_{min}$	ratio of maximum and minimum sensitivity in any direction
	or ratio of half axis - as close to 1 as possible
$R_i= S_i/S_{imin}$	ratio of sensitivity of θ_i alone and minimum sensitivity of θ_i
	- as close to 1 as possible.
	R_i >>1 indicates correlation between two or more parameters

Parameter accuracy

Parameter errors can be divided into

- stochastic parameter uncertainty caused by noise
- deterministic error caused by error in the model structure (undermodelling)

Resumé – parameter accuracy and sensitivity

Use the sensitivity measures: $\mathbf{S_i}$ $\mathbf{S_{i\,min}}$ $\mathbf{R = S_{max} / S_{min}}$ $\mathbf{R_i = S_i / S_{imin}}$
for evaluating the situation, e.g. the need for another model structure og input

The parameter accuracy can be expressed by

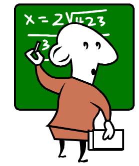
Noise only:

$$\sigma_{r,\theta i\%} \approx \frac{errn}{S_{i\min}} \frac{1}{\sqrt{N}} \quad [\%]$$

Undermodelling only:

$$\Delta_{eq,i} \approx \frac{errn}{S_{i\min}} \quad [\%]$$

Frequency-domain considerations



The frequency domain can be used for

- A better understanding of time-domain system identification methods
  - A time-domain fit is equal to a weighed frequency-domain fit

- Fitting a model to frequency-domain measurements
  - Requires time consuming measurements, but simple calculations

Resumé - Frequency-domain considerations

A least squares fit in the time domain is equal to a weighed least squares fit of the frequency function. The frequency weighting factor is

$$Q(\omega) = |U_N(\omega)|^2$$

that is, the fit is best in the frequency range, where the input signal power is high.

The fit can be further weighted by filtering the signals u(k) and y(k) by a so called prefilter L(q), giving a weighting factor

$$Q(\omega) = |L(\omega)|^2 |U_N(\omega)|^2$$

The model parameters can alternatively be estimated by fitting the model to a measured frequency function, using a frequency-domain performance function of the type

$$P_f(\theta) = (1/2N) \sum | G_{of}(j\omega_k) - G_f(j\omega_k, \theta) |^2$$

Again a frequency weighting can be obtained by choosing different input sine amplitudes.