1 | Question 1.b

1.1 Deterministic Sources

A model for the deterministic (periodic) sources can be done using a staircase curve or an affine model. These two give upper bound for the arrival packets to the networks and can be used to design based on upper bounds.

The staircase arrival model is given by

$$R(t) \leq Sc(t) = \left\lceil \frac{t - \text{offset}}{T} \right\rceil \cdot P \tag{1.1}$$

where $R(t)$ is the real number of packets that have already arrived, $Sc(t)$ is the staircase model arrival curve, offset is the time offset measured from $t = 0$ to first packet arrival, T is the period between packet arrivals and P is the packet size.

The worst case happens when the offset goes to zero, this means, that the first packet arrives at time zero.

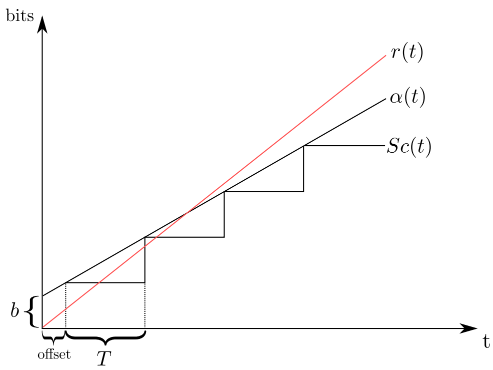

Figure 1.1: $Sc(t)$ is the staircase arrival curve, $\alpha(t)$ is the affine model arrival curve and $r(t)$ is the service curve defined by the capabilities of the CAN Bus.

The affine arrival model is given by

$$R(t) \leq \alpha(t) = b + \frac{P}{T}t \tag{1.2}$$

where α is the affine model arrival curve and b is the crossing of the affine curve with the y-axis.

The relation between both models and the real arrival number of packets that have arrived is then given by

$$R \leq Sc \leq \alpha \tag{1.3}$$

Wheels

The staircase arrival model for each of the wheel sensors is

$$Sc_{\mathrm{w}}(t) = \left\lceil \frac{t - \cancelto{0}{\text{offset}}}{T_{\mathrm{w}}} \right\rceil \cdot P_{\mathrm{w}} \tag{1.4}$$

$$Sc_{\mathrm{w}}(t) = \left\lceil \frac{t}{0.04} \right\rceil \cdot 160, \tag{1.5}$$

where $P_{\mathrm{w}} = 20 \cdot 8 = 160$ since the packet size is 20 B, which means that 160 b arrive at each time interval, $T = 0.04$ s. The time offset is set to zero in order to model the upper bound.

The affine arrival model for each wheel sensor data is

$$\alpha_{\mathrm{w}}(t) = b + \frac{P_{\mathrm{w}}}{T_{\mathrm{w}}} t \tag{1.6}$$

$$\alpha_{\mathrm{w}}(t) = 160 + \frac{160}{0.04} t = 160 + 6.4t \quad , \tag{1.7}$$

where $b = P_{\mathrm{w}} = 160$ since the time offset is set to zero.

Electronic Speed Control (ESC)

The staircase arrival model for the wheel sensor data is

$$Sc_{\mathrm{ESC}}(t) = \left\lceil \frac{t - \cancelto{0}{\text{offset}}}{T_{\mathrm{ESC}}} \right\rceil \cdot P_{\mathrm{ESC}} \tag{1.8}$$

$$Sc_{\mathrm{ESC}}(t) = \left\lceil \frac{t}{0.4} \right\rceil \cdot 64 \quad , \tag{1.9}$$

where $P_{\mathrm{ESC}} = 8 \cdot 8 = 64$ since the packet size is 8 B, which means that 64 b arrive at each time interval, $T = 0.4$ s. The time offset is again set to zero.

The affine arrival model for the wheel sensor data is

$$\alpha_{\mathrm{ESC}}(t) = b + \frac{P_{\mathrm{ESC}}}{T_{\mathrm{ESC}}} t \tag{1.10}$$

$$\alpha_{\mathrm{ESC}}(t) = 64 + \frac{64}{0.4} t = 64 + 25.6t \quad , \tag{1.11}$$

where $b = P_{\mathrm{ESC}} = 64$ since the time offset is set to zero to model the upper bound.

Models encoded in RTC

The stair case and the affine curves can be seen in Figure 1.2 and 1.3, as well as the service curve, which in this case is given by a 1MbpS CAN bus.

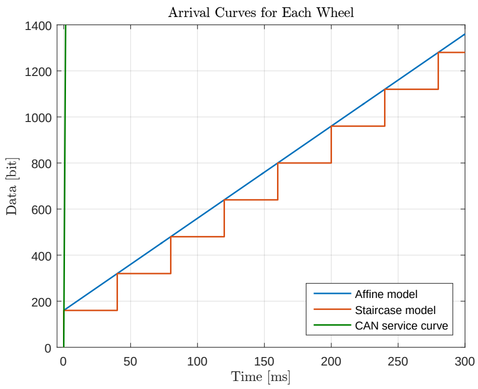

Figure 1.2: Arrival curves for the four wheels and service curve for the CAN-bus.

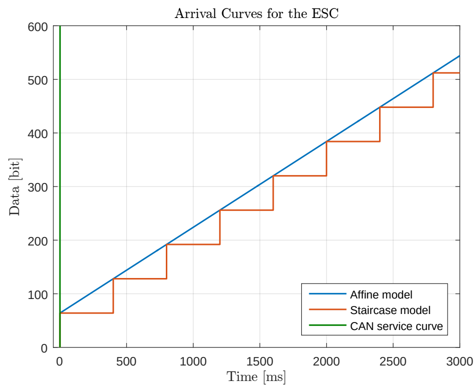

Figure 1.3: Arrival curves for the ESC and service curve for the CAN-bus.

1.2 Non-Periodic Sources

Some of the sources are non deterministic and are given by a Poisson distribution with parameter λ. To be ablo to create a deterministic model for them a token bucket filter is used. The main parameters are the queue length, L, the number of tokens that can be stored, M, and the token rate, r_t. A diagram of the filter including the parameters can be seen in Figure 1.4.

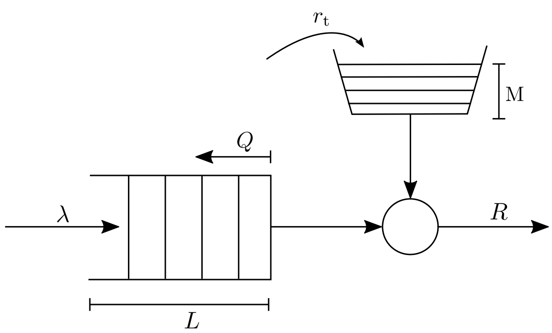

Figure 1.4: Diagram of the token bucket filter, including the parameters.

Once the parameters are known, an affine curve can be used to describe an upper bound for the arrival model similarly as in the case of periodic sources

$$\alpha(t) = M\ P + P\ r_\mathrm{t}\ t \tag{1.12}$$

where the burst is given by the maximum number of packets that can pass through if the bucket starts full of tokens.

Multimedia System

The parameters for the token bucket are selected looking at the simulation in True Time and finding a value that gives a good result in the number of packets in the queue and in the waiting time. In the case of the multimedia system the parameters are: $L = 5$ packets, $M = 6$ tokens and $r_\mathrm{t} = 50$ tokens per second.

The affine arrival model for the multimedia system is given by

$$\alpha_\mathrm{multimedia}(t) = MP_\mathrm{multimedia} + P_\mathrm{multimedia}\ r_\mathrm{t}\ t \tag{1.13}$$
$$\alpha_\mathrm{multimedia}(t) = 6 \cdot 11200 + 50 \cdot 11200t = 67200 + 560000t \tag{1.14}$$

where $P_\mathrm{multimedia} = 1400 \cdot 8 = 11200$ is the packet size in bits.

Rear Camera (RC)

In the case of the rear camera the parameters are: $L = 8$ packets, $M = 8$ tokens and $r_\mathrm{t} = 58$ tokens per second.

The affine arrival model for the multimedia system is given by

$$\alpha_\mathrm{multimedia}(t) = MP_\mathrm{RC} + P_\mathrm{RC}\ r_\mathrm{t}\ t \tag{1.15}$$
$$\alpha_\mathrm{multimedia}(t) = 8 \cdot 11200 + 58 \cdot 11200t = 89600 + 649600t \tag{1.16}$$

where $P_\mathrm{RC} = 1400 \cdot 8 = 11200$ is the packet size in bits.

1.3 Maximum Backlogs and Waiting Times of the Non-Periodic Sources

The models for the non-periodic sources and the service curve for the CAN bus can be plotted using RTC.

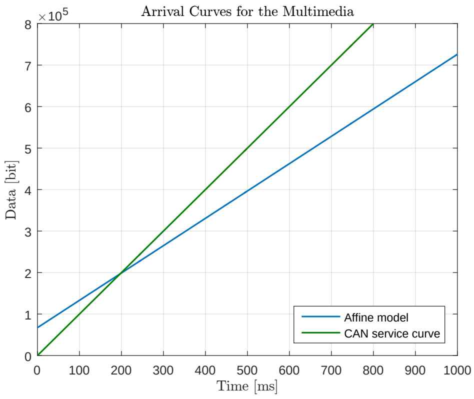

Figure 1.5: Arrival curves for the Multimedia and service curve for the CAN-bus.

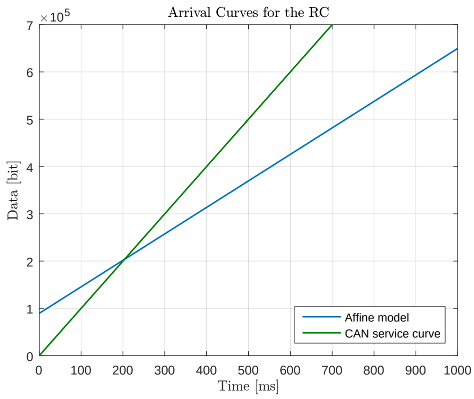

Figure 1.6: Arrival curves for the RC and service curve for the CAN-bus.

The maximum backlogs are given by the maximum vertical distance between the affine and the service curves before they cross, while the maximum waiting times by the maximum horizontal distance between the curves.

$$\begin{aligned}\text{max backlog for the multimedia} &= 67200 \text{ bits}\\ \text{max waiting for the multimedia} &= 0.0672 \text{ s}\\ \text{max backlog for the RC} &= 89600 \text{ bits}\\ \text{max waiting for the RC} &= 0.0896 \text{ s}\end{aligned}$$

1.4 Token Bucket Dynamics

The dynamics of the filter can be described using the matrix H as

$$P_n = P_{n-1}H \tag{1.17}$$

where P_n contains the probabilities of being in each of the states of the queue at each time in the queue and P_{n-1} contains the probabilities in the previous time.

As time approaches infinity, P_n converges. This converged vector of probabilities is called Π and fulfills the balance equation

$$\Pi = \Pi H \tag{1.18}$$

To be able to calculate the mean queue length, mean waiting time and the packet loss probability, it is necessary to find Π, that gives the converged probabilities of each of the queue.

This is done solving Equation 1.18 and forcing $\sum_{i=0}^{L} \Pi$ to be 1 (L is the queue length).

The first step is to calculate H. It contains the probabilities of moving among the different states in the queue. Since the queue lengths are sampled each token period, $\frac{1}{r_t}$, the probabilities for a number of packets arriving can be calculated as

$$A_j = P(a_n = j) = \frac{(\lambda\frac{1}{r_t})^j}{j!}\exp(-\lambda\frac{1}{r_t}) \tag{1.19}$$

where a_n is the number of arrivals from a Poisson stream in the service period, λ is the arrival rate of packets to the filter.

$$H = \begin{bmatrix} A_0 & A_1 & A_2 & A_3 & \dots & \dots & 1-\sum_0^{L-1} A_j \\ A_0 & A_1 & A_2 & A_3 & \dots & \dots & 1-\sum_0^{L-1} A_j \\ 0 & A_0 & A_1 & A_2 & \dots & \dots & 1-\sum_0^{L-2} A_j \\ 0 & 0 & A_0 & A_1 & \dots & \dots & 1-\sum_0^{L-3} A_j \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & A_0 & 1-A_0 \end{bmatrix} \tag{1.20}$$

Once it is calculated it is possible to solve for Π in Equation 1.18

The mean queue length, mean waiting time and packet loss probability are given by:

$$\bar{Q} = \sum_{i=0}^{L} \Pi_i i \tag{1.21}$$

$$\bar{W} = \frac{\bar{Q}}{\lambda} \tag{1.22}$$

$$PLP = \Pi_L \tag{1.23}$$

Rear Camera

The H matrix in the case of the rear camera is 9×9 as the queue can hold a maximum of 8 packets. The mean queue length, mean waiting time and PLP is found following the procedure above.

The estimated queue lengths and mean delay have been found using the TrueTime Simulation by averaging their values in a 50 seconds simulation. These are seen below.

$$\bar{Q} = 0.0391 \text{ packets}$$
$$\bar{W} = 0.00073 \text{ s}$$
$$Q_{\max} = 5 \text{ packets}$$
$$W_{\max} = 0.0717 \text{ s}$$

The figures below show how the queue and the delay evolved in the simulation.

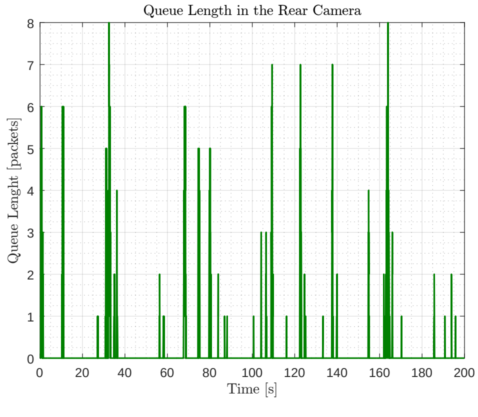

Figure 1.7: Queue length of the rear camera in the simulation.

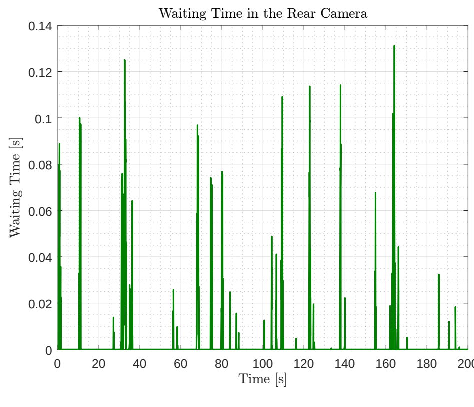

Figure 1.8: Waiting time of the rear camera in the simulation.

Multimedia System

In the multimedia case, the H matrix is 6×6 as the queue length is 5. The analytical values for the mean queue length, mean waiting time and PLP are found similarly as for the multimedia.

The estimated queue lengths and mean delay have been found using the TrueTime Simulation following the same procedure as for the rear camera.

$$\begin{aligned} \bar{Q} &= 0.1389 \text{ packets} \\ \bar{W} &= 0.0032 \text{ s} \\ Q_{\max} &= 5 \text{ packets} \\ W_{\max} &= 0.1 \text{ s} \end{aligned}$$

The figures below show how the queue and the delay evolved in the simulation.

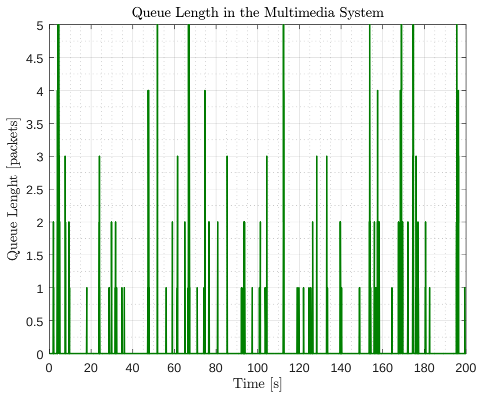

Figure 1.9: Queue length of the rear camera in the simulation.

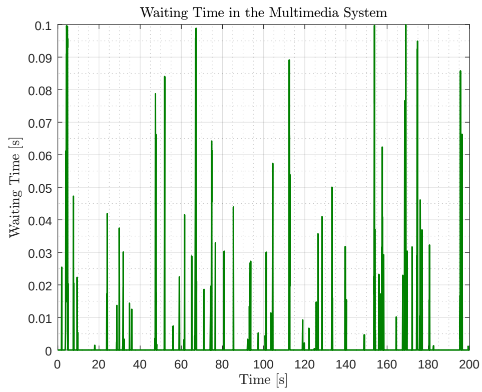

Figure 1.10: Waiting time of the rear camera in the simulation.

2 | Question 2

The failure rate can be translated to be in fails/year. This is done in Equation 2.1.

$$\lambda = \frac{1\text{fails}}{2 \cdot 10^6\text{h}} \cdot \frac{8760\text{h}}{1\text{year}} = \frac{1\text{fails}}{288.3\text{year}} \tag{2.1}$$

The lifetime of the car can be expressed by its probability density function seen in Equation 2.2.

$$f_{\text{car}}(t) = \begin{cases} \frac{1}{10} & \text{if } t \in [5, 15] \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

Case 1

For the first case, the reliability, cumulative and probability functions for the lifetime of the network can be found. The first one obtained is the reliability function and it is found by multiplying the individual reliabilities for the two switches as they are connected in series, see Figure 2.1, and they have independent probabilities of failing.

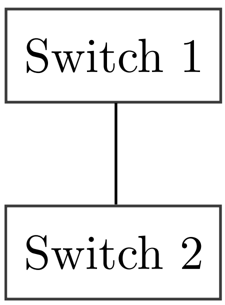

Figure 2.1: Switch diagram for case 1.

$$R_{\text{n}}(t) = R_1(t)R_2(t) = e^{-\lambda t}e^{-\lambda t} = e^{-2\lambda t} \tag{2.3}$$

$$F_{\text{n}}(t) = 1 - R_{\text{n}}(t) = 1 - e^{-2\lambda t} \tag{2.4}$$

$$f_{\text{n}}(t) = F'_{\text{n}} = 2\lambda e^{-2\lambda t} \tag{2.5}$$

To find the probability of the network failing before the rest of the car does, a double integration is performed from 5 to 15 for and from 0 to t_{c}, where t_{c} is the time in which the car fails. Equation 2.6 shows the performed computation.

$$P(t_{\mathrm{n}} \leq t_{\mathrm{c}}) = \int_{5}^{15} \left[\int_{0}^{t_{\mathrm{c}}} f_{\mathrm{n}}(t) f_{\mathrm{c}}(t) dt_{\mathrm{n}} \right] dt_{\mathrm{c}} \tag{2.6}$$

The result of this integral gave a probability of 0.0668 of the network failing before the rest of the car.

Case 2 a

In case 2 a, one of the switches is duplicated as seen in Figure 2.2. Even though the method is the same, the reliability, cumulative and probability functions change as now there is one more switch in the network.

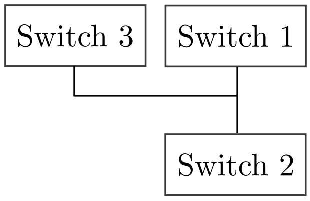

Figure 2.2: Switch diagram for case 2 a.

$$R_{\mathrm{n}}(t) = R_2(R_1(t) + R_3(t) - R_1(t)R_3(t)) = e^{-\lambda t}\left(2e^{-\lambda t} - e^{-2\lambda t}\right) = 2e^{-2\lambda t} - e^{-3\lambda t} \tag{2.7}$$

$$F_{\mathrm{n}}(t) = 1 - R_{\mathrm{n}}(t) = 1 - 2e^{-2\lambda t} + e^{-3\lambda t} \tag{2.8}$$

$$f_{\mathrm{n}}(t) = F'_{\mathrm{n}} = 4\lambda e^{-2\lambda t} - 3\lambda e^{-3\lambda t} \tag{2.9}$$

The integral is done in the same way as seen in Equation 2.6 but in this case the expression for f_{n} is different. The result obtained is 0.00346.

Case 2 b

In the last case, both switches are duplicated. The structure is shown in Figure 2.3. Again, the reliability, cumulative and probability functions change and they need to be re-calculated taking into account the new combination of series and parallel connection. This is shown in Equation 2.10 and 2.11 and 2.12.

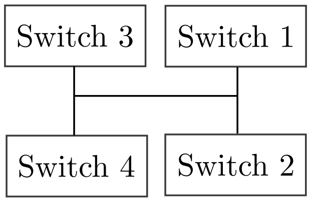

Figure 2.3: Switch diagram for case 2 b.

$$R_{\mathrm{n}}(t) = (R_1(t) + R_3(t) - R_1(t)R_2(t))(R_2(t) + R_4(t) - R_2(t)R_4(t)) = \left(2e^{-\lambda t} - e^{-2\lambda t}\right)\left(2e^{-\lambda t} - e^{-2\lambda t}\right) = \left(4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t}\right) \tag{2.10}$$

$$F_{\mathrm{n}}(t) = 1 - R_{\mathrm{n}}(t) = 1 - 4e^{-2\lambda t} + 4e^{-3\lambda t} - e^{-4\lambda t} \tag{2.11}$$

$$f_{\mathrm{n}}(t) = F'_{\mathrm{n}} = 8\lambda e^{-2\lambda t} - 12\lambda e^{-3\lambda t} + 4\lambda e^{-4\lambda t} \tag{2.12}$$

The new probability density function is used to find the probability of the network failing before the car does in the same way as with the previous two cases. See Equation 2.6. The result obtained for this case is 0.0025.

It can be seen that the failure probability is reduced as the number of redundant components in the network increases.