Networked control for water distribution

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***Abstract*—Water Distribution networks are play a key infrastructure role for cities and industrial areas around the world. Pressure control in these networks is vital to ensure end-users receive a desirable pressure. Other benefits of pressure control include reduction of water leakages and energy savings. The nature of water distribution networks means the actuators, sensors and control systems are geographically separated, thus, it is necessary to communicate measurements over a network. This project aims to design a pressure controller for a water distribution network. This entails modelling the network by applying electrical circuit theory to the network. Since the chosen control method is a linear control, the system model needs to be linearized. The linear controller is designed using state-space methods.**

Keywords— Water Distribution Networks; State-space; LQR Control; Linearization

I. Introduction

The aim of this project is to design and implement a networked control strategy for controlling pressure in a water distribution system. Improved pressure control in water distribution systems can contribute to saving water by decreasing water leakage and the number of pipe bursts [1].

In developing cities and industrial areas around the world, water distribution networks play a key role in infrastructure. One of the important factors in water distribution systems is controlling the pressure in water distribution networks (WDN) so all consumers have a satisfying water flow in their homes. Addition of users in a network, leakages in pipes and many other reasons can cause pressure drop in distribution system. On the other hand, having high pressure can cause problems in pipes according to Barlow's formula [2]. So it is vital to have the pressure in whole distribution network between maximum and minimum allowed pressure. In any different network this maximum and minimum can be change according to consumer's need and components available in that network. This project aims to find a solution for small pressure change around a desired operating pressure via a control system based on a Linear Quadratic Regulator (LQR) controller. The system used in this project consists of two pressure management areas (PMA) and the pressures in the system are regulated using three pumps. Each PMA has two outlets. The water distribution network has nonlinear characteristics [3].

In Section II, the water distribution network is linearized and state space model is derived from linearized model. In Section III, the parameters of the system model are estimated using experimental data from the physical system. In Section IV, delay in the communication network is modelled. In Section V, the LQR (linear quadratic regulator) controller is designed. The data obtained from network after implementing the controller is compared to simulated network in Section VI. Section VII is the conclusion. In Section VIII, recommendations on the project are made.

II. System Model

A. Water Distribution Network

A Water Distribution Network is a hydraulic network comprised of many two terminal components such as pipes, pumps and valves. The objective of the network is to distribute water from a source, to various outputs in Pressure Management Areas while maintaining a desirable pressure at these outputs of the network. The network may have multiple water sources, height differences within the network and multiple pumping stations. To analyze such a network, the various system components need to be modelled.

These components are characterized by algebraic or dynamic relationships between two variables, the pressure drop, Δp, across a component, and the flow, q, flowing through that component [4].

1) Pipe Model: A large portion of the network comprises of pipes. Thus, is important to derive a dynamic model that describes relation between the differential pressure across the pipe and the flow through it. The compact notation for the pipe model which can be applied to the k^{th} pipe component in the hydraulic network is expressed as

$$J_k \dot{q}_k = \Delta p_k - R_k |q_k| q_k - \zeta_k \quad (1)$$

In (1) J_k represents the inertia in the k^{th} pipe, Δp_k is the pressure drop across the pipe. R_k represents the resistance in the pipe which includes both the surface resistance and from resistance. These resistances can be calculated using the equations in [5]. The ζ_k term, represents the pressure difference due to the change in height.

Lastly, the units used to express the flow [m^3/s] and pressure [Pa] are converted to [m^3/h] and [Bar], respectively. These are the chosen units used throughout the paper.

2) Valve Model: The valves used in this project have a variable opening degree (OD). The pressure loss across the valve can be described as a function of the OD.

The valve model is derived similarly to the pipe model. The valve can be modelled as a pipe with its length and height difference assumed as zero. This results in the following equation

$$0 = \Delta p_k - R_k |q_k| q_k \quad (2)$$

While is possible to estimate the R_k parameter for the k^{th}, valve manufacturers provide a more accurate k_v value in the datasheets. The k_v-factor specifies the water flow, in m^3/h, through the valve at a pressure drop across the valve of 1 Bar [6]. This relation is for a fully open valve. This relation is given by the equation [6]

$$q = k_v \sqrt{\Delta p} \quad (3)$$

3) Pump Model: In a hydraulic network, a pump creates flow by providing a positive pressure difference. A model that describes this positive pressure difference is derived in [7] and is given in equation (4). The pressure increase is given as a function of the flow, q, through the pump and the rotational speed, ω, of the pump. The pump parameters a_{h0}, a_{h1}, a_{h2} are found experimentally.

$$\Delta p = a_{h0}\omega^2 + a_{h1} q \omega - a_{h2} q^2 \quad (4)$$

B. Linearization

This project investigates an application of linear control theory for the water distribution network. The pipe and valve are described by nonlinear models that need to be linearized to perform linear control techniques. The model that describes the network were linearized using the Taylor Series approximation to the first order.

Let a first order, nonlinear differential model, be given by

$$\dot{x} = f(x) + u$$

To linearize this equation at a suitable operating point $\{x_0; u_0\}$, the Taylor Series Expansion is used to describe $f(x)$ to the first derivative, such that values $f(x)$ can be approximated in the neighborhood of x_0 as shown in (5) below

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0) \quad (5)$$

At the steady-state operating point the differential model is in the form

$$0 = f(x_0) + u_0$$

$$f(x_0) = -u_0$$

Using a linearized model about a given operating point, only the small changes are considered. The vectors, x and u can be rewritten in a small signal form such that $x = x_0 + \tilde{x}$ and $u = u_0 + \tilde{u}$. Substituting equations into the differential model, the following linearized model is achieved

$$\dot{x} \approx f(x_0) + f'(x_0)\tilde{x} - f(x_0) + \tilde{u}$$

$$\dot{x} \approx f'(x_0)\tilde{x} + \tilde{u}$$

C. Water Distribution Network Model

To model the network, the equations for the components above are used. The component equations describe a relationship between the flows through the components and the differential pressures across the components.

The diagram symbol for the k^{th} pipe, valve and pump showing the pressure difference, $\Delta p = p_{in} - p_{out}$, across it and the flow, q, through it is given in Fig. 1 below

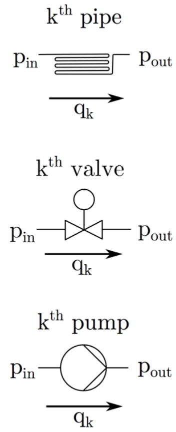

Fig 1. Pipe, valve and pump diagram symbol

The method used to analyze the network uses an implementation of Kirchhoff's Circuit Laws (KCL). KCL can be used on hydraulic networks by replacing voltage and currents with pressures and flows, respectively. Kirchhoff's Current Law is used to describe continuity of nodal flow and Kirchhoff's Voltage Law is used to represent that the energy in a closed loop in conserved [8]. Pressure drops in pipes and valves are seen as inductors and nonlinear resistors, hence having an impedance.

The analogy between electrical circuits and hydraulic circuits is illustrated below. For the given hydraulic network in Figure 2 below, C_k represents the k^{th} component in the network and the components are in reference to Figure 1. The hydraulic network can be translated to the electrical circuit shown in Figure 3.

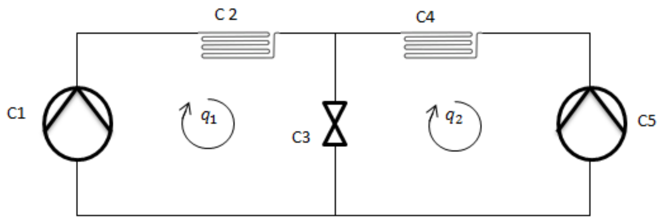

Fig 2. Two loop hydraulic network

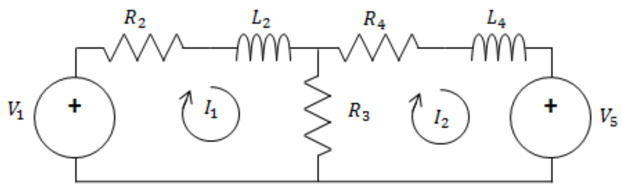

Fig 3. Two loop electrical circuit

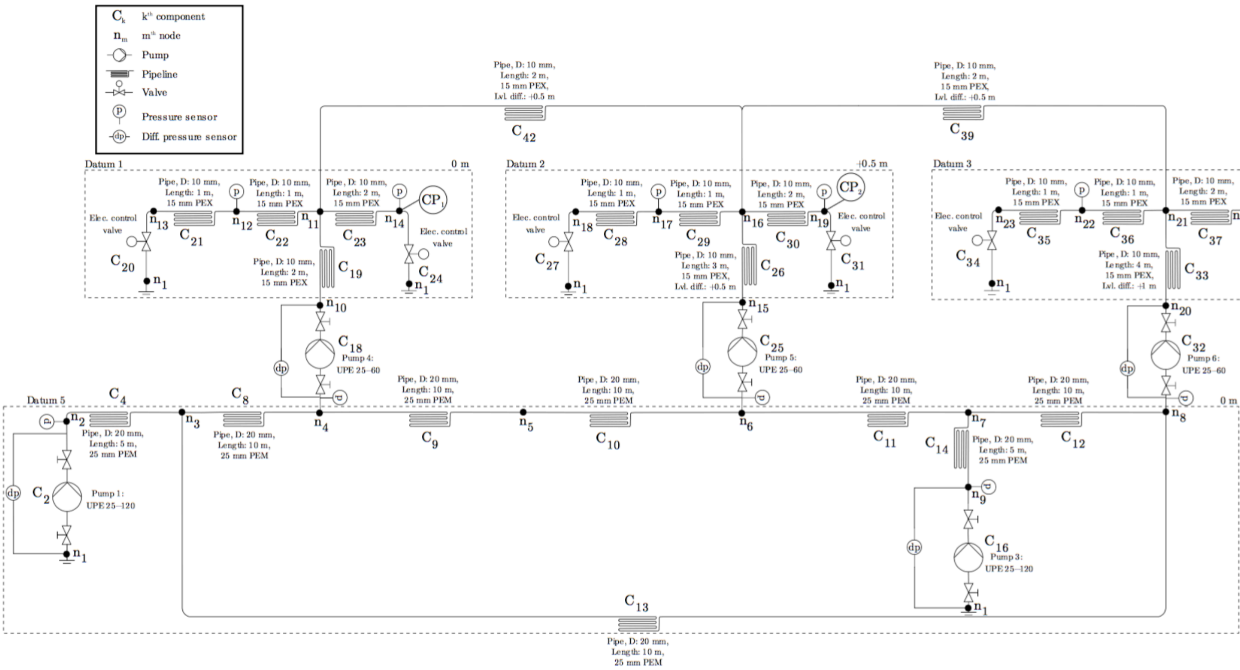

Fig 4. Test Water Distribution Network Diagram

In the two loop electrical circuit shown in Figure 3, KCL is applied and the following equations are derived.

For the first loop,

$$V_1 - I_1 R_2 - L_2 \frac{dI_1}{dt} - (I_1 - I_2) R_3 = 0$$

and for the second loop

$$-V_5 - I_2 R_4 - L_4 \frac{dI_2}{dt} - (I_2 - I_1) R_3 = 0$$

In matrix form,

$$\begin{bmatrix} L_2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \frac{dI_1}{dt} \\ \frac{dI_2}{dt} \end{bmatrix} = - \begin{bmatrix} R_2 + R_3 & -R_3 \\ -R_3 & R_4 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ -V_5 \end{bmatrix}$$

This matrix can be translated back using the component models for the pipe, valve from Section II.A. The linearized models for the pipes and valve were used. The loop equations are shown below.

Thus, in matrix form,

$$\begin{bmatrix} J_2 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \dot{\tilde{q}}_1 \\ \dot{\tilde{q}}_2 \end{bmatrix} = - \begin{bmatrix} R_2 + R_3 & -R_3 \\ -R_3 & R_4 + R_3 \end{bmatrix} \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{bmatrix} + \begin{bmatrix} \Delta p_1 \\ -\Delta p_5 \end{bmatrix} \quad (6)$$

D. Test Network System

The test water distribution network system in Figure 4 shows the system that was modelled. The input pumps of the system that were chosen were C_2, C_{18} and C_{25}. The output pressures measured were using the pressure sensors located at n_{14} and n_{19}.

E. State-Space Representation

The system equations derived from the loop equations can be expressed in the following form:

$$J\dot{\tilde{q}} = -R\tilde{q} + B\tilde{u}$$

Such that J matrix represents the inertia in the system and the R matrix represents the damping. Both J and R are positive definite and symmetric matrices i.e. $J_{ii} > 0$ and $J_{ij} = J_{ji}$. Similarly, for R. This relation is a property from using the KCL analysis and is also shown in (6).

To design a pressure based controller for the system, the system needs to be in the form

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

For the system dynamics equation, the J matrix must be invertible, therefore nonsingular. For a properly designed system this condition will be met as described in [9]. With this condition being met, the state equation is given by

$$\dot{\tilde{q}} = -J^{-1}R\tilde{q} + J^{-1}B\tilde{u} \quad (7)$$

The output of the system is determined by differential pressures across valves only, hence only a resistive term as shown in (2). Thus, the output equation is given by

$$\tilde{y} = C\tilde{q} \quad (8)$$

III. Parmeter Estimation

The state-space model derived for the system is not complete until all the parameters of the components are well stated. Model parameters that have great uncertainty in its values may need to be estimated using parameter estimation techniques.

To fit the state-space model to the physical systems response, the MATLAB System Identification Toolbox was used. The inertia matrix, J, is determined using the length and cross-sectional area of the pipe and the density of water. These values were well defined, hence the matrix J, is modelled sufficiently well. Thus, only the steady-state response was used in the parameter estimation technique. The steady-state model is given as

$$0 = -R\tilde{x} + B\tilde{u}$$

To estimate the correct R parameters, the system model was treated as a Grey-Box Model and the Gauss-Newton Least Squares method was used.

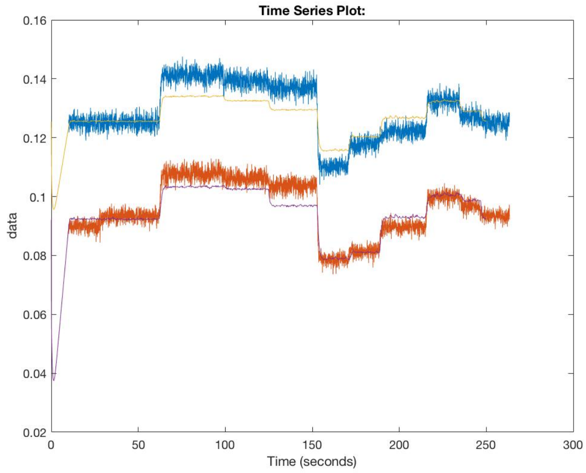

Fig 5. Comparison of state-space model and physical system

Figure 5. above, the system's output is compared with the linearized model's output. It can be seen that for small deviations from the operating point, the model tracks the system but from time 50 [s] to 100 [s] there is a larger error, due to a larger deviation from the operating point.

IV. NETWORKING & DELAY

The system used in this project is a network control system where LQR controller is connected as feedback from sensors to pumps. This introduced the delays from sensors to controlling part and also from controlling parts to actuators. In water distribution networks, delay is occurred in system by controllers, actuators and network controlling part exchanging data. Delays due to internal signal processing in the actuators are significantly high compared to other. The measured delay, T_d, in the communication between the control unit and the pump of the system was 0.7 [s].

Padè's Approximation was used to obtain a transfer function of the delay, such that the delayed input to the system is given by the equation:

$$Z(s) = \frac{1 - \frac{T_d}{2}s}{1 + \frac{T_d}{2}s} U(s)$$

where Z(s) and U(s) are actuators signals with and without delay in Laplace domain. Padè's approximation is used to linearize the delay function and that is why first order approximation is used.

The delay was transformed into a state-space model so that it could be extended to the original state-space system. By taking $X = Z + U$, the following equation for each input that is connected to the system separately is obtained by

$$\dot{X} = -\frac{2}{T_d}X + \frac{4}{T_d}U$$

$$Y_d = X - U$$

The pump delayed model of the three pumps have therefore a state-space model with four diagonal matrices where their values are:

$$A_c = -\frac{1}{\frac{T_d}{2}} \;; B_c = \frac{2}{\frac{T_d}{2}} \;; C_c = 1 \;; D_c = -1$$

The original state space model was extended, thus giving the model

$$\begin{bmatrix} \dot{q} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_c \end{bmatrix} \begin{bmatrix} q \\ x \end{bmatrix} + \begin{bmatrix} B \\ B_c \end{bmatrix} Y_d$$

The final extended state-space model with the inputs, without delay, has the matrices

$$A_d = \begin{bmatrix} A & B \\ 0 & A_c \end{bmatrix}; B_d = \begin{bmatrix} -B \\ B_c \end{bmatrix}; C_d = [C\ 0]$$

such that the extended state-space model has the form

$$\dot{\tilde{x}}_d = A_d\tilde{x}_d + B_d\tilde{u}_d \tag{9}$$

$$y_d = C_d x_d$$

V. CONTROLLER DESIGN

The control method chosen for the state-space system is state feedback control. For the state-space system represented in (9) the control law is given by

$$\tilde{u}_d = -K\tilde{x}_d$$

where K is the gain matrix of the controller.

A. LQR Control

The state feedback control method chosen was a Linear Quadratic Regulator (LQR). The LQR method is designed such that the gains for a state feedback controller are chosen to optimize a cost function. This is done to balance system performance and the magnitude of inputs required to achieve a certain level of performance [10].

For the given system the quadratic cost function shall be minimized.

$$\mathcal{J} = \int_0^\infty \tilde{x}_d^T Q_x \tilde{x}_d + \tilde{u}_d^T Q_u \tilde{u}_d \, dt$$

where Q_x and Q_u are symmetric, positive definite weight matrices. The choice for the weight matrices determines the trade-off between the states and the control inputs [10].

To choose the specific values for the cost function weights, Q_x and Q_u, *Bryson's Rule* is used.

Bryson's Rule is used to choose the diagonal elements of the Q_x and Q_u matrices such that

$$Q_{x_{ii}} = \frac{1}{(maximum\ acceptable\ number\ of\ x_{ii})^2}$$

$$Q_{u_{ii}} = \frac{1}{(maximum\ acceptable\ number\ of\ u_{ii})^2}$$

For an LQR controller, the gain K is given by

$$K = Q_u^{-1} B^T P$$

where P is symmetric, positive definite matrix, with the same dimensions as A. The matrix, P, satisfies the *algebraic Riccati equation*

$$PA + A^T P - PBQ_u^{-1} B^T P + Q_x = 0$$

Once Q_x and Q_u are designed, the gain matrix, K, can be solved using MATLABs *lqr* command.

The systems closed loop response can be tuned by making adjustments to the Q_x and Q_u weight matrices.

B. Observer Design

The state feedback controller designed in Section IV.A is only realizable if the states of the system can be measured.

To design an observer for such system, the system must be observable. It is done by placing the eigenvalues of the matrix $[A - LC]$. These eigenvalues can be arbitrarily placed.

This means the system states must be able to be recovered from the inputs and measured outputs of the system. The state equation of the observer is in the form

$$\dot{\hat{x}}_d = A\hat{x}_d + Bu_d + L(y_d - C\hat{x}_d)$$

VI. Results

The LQR controller has been simulated in MATLAB and the following results have been obtained. The LQR weight matrices were tuned to penalize the inputs more than the states. The response due to a disturbance can be seen in Figure 6. below.

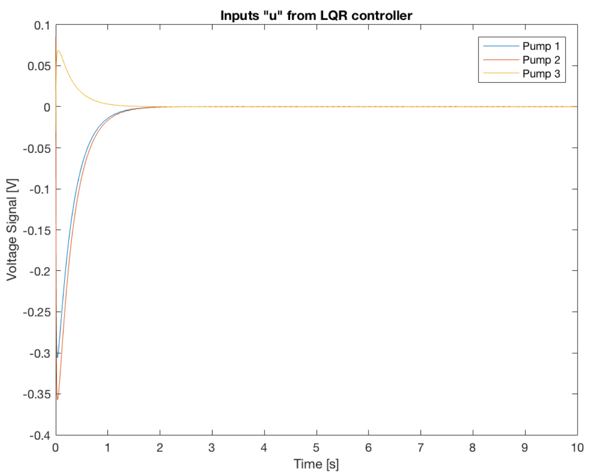

Fig 6. System inputs from LQR controller

The outputs of the system are shown in the Figure 7. below

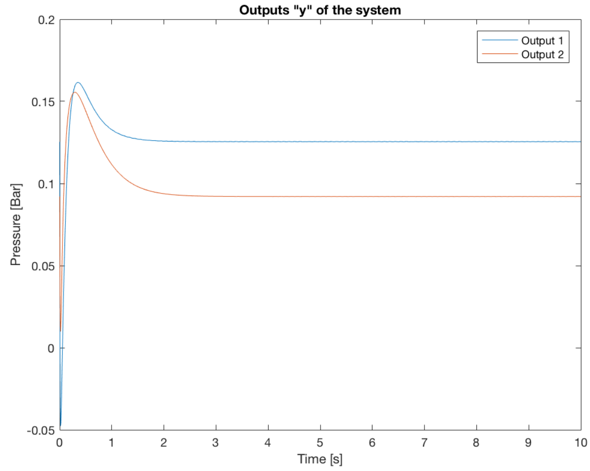

Fig 7. System outputs based on LQR controller

It can be seen that the response stabilizes after 3 [s] and there is an initial overshoot.

VII. Conclusion

The designed controller's performance, shown in Figure 6, has an adequate settling time but the overshoot is undesirable. Since the controller is based on a linearized model, the physical system may not perform as expected as with the same controller.

VIII. Recommendations

The controller shall be tuned for future testing on the physical setup. The networking problem can be investigated further by using a switch connecting two computers, one running the control system and the other interfacing to the physical system.

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