Control of Distributed Heat Exchanger Systems with Model Based Optimal references

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***Abstract*—This paper examines the possibilities of optimizing the control of a heat exchanger system (HES) in an office building. Initially a model is made for a simple HES with 2 heaters, 3 pumps and a control valve. The modelling contains an individual model for the hydraulic and thermal part of the system which is combined into a full model of the HES. To optimize the system a cost function of the running costs of the HES is derived based on information from Skive district heating plant (DHP). Based on the derived model and cost function a linear control solution with model based optimal reference generation is proposed. The controller solution is implemented and compared to real measurements on a test set-up. The test shows that the system still fulfills user demands, but with a possible lack of efficiency due to model errors and disturbances. In the end possible solutions is proposed.**

I. Introduction

THE energy used for heating buildings represents big part of its total energy consumption. Among the heating supply systems, DHP is one of the most used in real life. The pricing in a DHP supply system uses to be composed of the energy consumption cost and a punishment/reward rate dependent on the temperature of the water going back to the DHP. In this scenario a potential saving can be achieved if this return temperature is minimized.

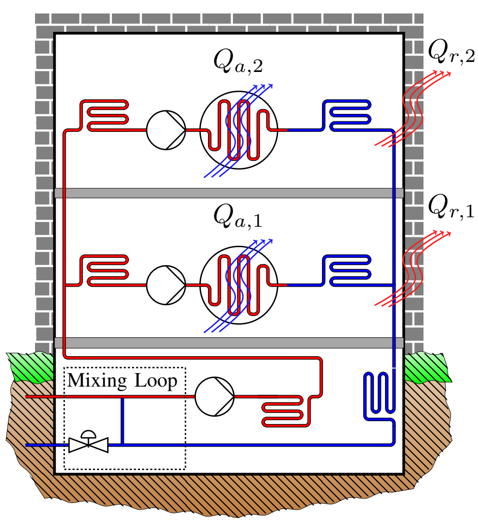

Fig. 1. Sketch of a simplified HES in an building composed of two rooms with individual heat exchangers. The HES is getting heat energy from a DHP, which supplies hot water into the system at a constant temperature, T_{in}. The hot water is blended in the mixing loop with the water returning from the heat exchangers.

Danfoss One® is a commercial solution for efficient heating of buildings. This solution mainly saves energy by varying the temperature of the building throughout the day [1]. If the punishment/reward policy and the cost of using the actuators in the system is considered, additional money can potentially be saved as well. Traditionally, the heating of rooms is controlled by a thermostat. In this project these thermostats are replaced by pumps due to inspiration from the report [2]. This report is investigating potential ways of saving energy in typical Danish district heat networks. At Fig. 1 a sketch of the system investigated is presented. The control objective in this set-up is to deliver the heat-energy flows $Q_{a,1}$ and $Q_{a,2}$, into the rooms that equals the heat-energy flows $Q_{r,1}$ and $Q_{r,2}$ out of the rooms at user defined room temperatures. The total price paid for running such a system is composed of the price paid for the heat energy delivered by the DHP, and the price paid for the electricity needed to drive the actuators in the system. Since this kind of systems mostly operate in steady state, the cost of the transient response of the system is small compared to the cost of steady state operation. Thus the minimization problem (MP) to consider is

$$\min_{\chi,u} C_T\left(\chi,u\right) = \min_{\chi,u} C_H\left(\chi\right) + C_A\left(u\right) \tag{1}$$

s.t.

$$\dot{\chi} = f_\chi(\chi,u) = 0$$
$$f_y(\chi) = Q_{\text{ref}} = \begin{bmatrix} Q_{a,1,\text{ref}} \\ Q_{a,2,\text{ref}} \end{bmatrix}$$
$$u_{\text{min}} \leq_e u \leq_e u_{\text{max}}$$

Where C_T is the running cost of the system, C_H is the price of the heat energy, C_A is the price of running the actuators, χ is the states of the system, $Q_{a,i,\text{ref}}$ is the reference for the desired heat-energy flow needed to keep the i'th room temperature at a user defined reference and u_{min} and u_{max} are the minimum and maximum values for the input signals to the actuators.

In this paper this MP is analyzed and a MIMO model for this non-linear system together with a linear control solution that takes into account the MP are presented and tested.

Notation: In the following, $f(x)$ donates that f is a function of x, $|f(x)|$ donates the absolute value of $f(x)$, $f(x) \in C^2$ donates that $f(x)$ has continuous second-order partial derivatives, and $x \in R_c$ donates that x is an element of the set R_c. Furthermore, the dot notation is used to denote time derivatives and v^T donates the transpose of the vector v.

II. Modelling

The system shown in Fig. 1 is a non-linear system containing long transport delays. To make a model of the system the modelling is split into three parts: a model of the hydralic

part of the system, a model of the thermal part and finally a combined model to yield a complete model of the system.
The hydraulic part of the system is composed of pipes, fittings, heat exchangers (HE), controllable pumps and a controllable valve. The hydraulic model is based on the following assumptions:

H1) The DHP delivers a constant pressure that can be modelled as a pump and piping.
H2) The water flow is turbulent.
H3) The length of the pipes is sufficiently long, meaning that pressure drops due to form resistance, such as pipe bends, elbows and pipe fitting, can be lumped into the pipe model.
H4) There is no leakage in the system, thus there is conservation of the mass (continuity equation).
H5) The water in the system is incompressible.
H6) The heater and HE have the same hydraulic behaviour as pipes.

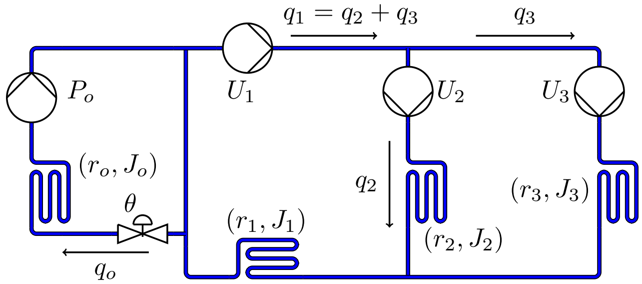

Fig. 2. A lumped model of the hydraulic network of the HES shown in Fig. 1. P_o is the pressure drop across the DHP pump, U_i are actuator inputs for the system pumps, θ is the opening degree of the controllable mixing loop valve and r_i, J_i are the parameters defining the long pipes and HEs which are lumped together.

As the HES is a closed network pressure drops caused by height differences of the pipe-ends can be neglected. Under these assumptions a lumped model can be composed of the three components: pipes, valves, and pumps, with the model dynamics described by (2), (3) and (4) [3]. Fig. 2 shows the lumped model of the hydraulic system. The DHP part of the model shown in the left of Fig. 2, is modelled as a pump generating the constant P_o pressure and some piping to include any dynamics to the otherwise assumed constant P_o. The dynamic equations for the different parts of the model is shown below. Pipes are modelled by

$$J\dot{q} = \Delta p - r\,|q|\,q \tag{2}$$

where J and r are parameters describing the pipe section $\left[\mathrm{h^2 mbar/m^3}, \mathrm{h^2 bar/m^6}\right]$, q is the volumetric flow rate of water through the pipe $\left[\mathrm{m^3/h}\right]$, and Δp is the pressure drop across the pipe $[\mathrm{bar}]$. Valves are modelled by

$$0 = \Delta p - \left[K_1 e^{K_2 \theta}\right]^{-2} |q|\,q \tag{3}$$

where K_1 is the conductivity of the valve $\left[\mathrm{m^3/bar^{0.5}h}\right]$, K_2 is a parameter for the valve [1], and θ is the opening degree of the controllable valve $[\%]$. Pumps are modelled by

$$\Delta p = a_{h2}\,|q|\,q + a_{h1} q U + a_{h0} U^2 \tag{4}$$

where U is the applied voltage to the pump [V], and a_h are parameters describing the pump.
Due to assumption H4, Kirchhoff's laws can be used to combine (2), (3) and (4). Kirchhoff's laws applied to hydralic modelling states that

- The sum of inflows equals the sum of outflows in a node.
- The sum of the pressure drop along any loop of the network is zero.

Solving the resulting equations using Kirchhoff's laws yields the dynamic equations of the hydraulic model given in a form of

$$\dot{q}_o = f_1\left(q_o, \theta\right) \tag{5}$$
$$\dot{q}_2 = f_2\left(q_2, q_3, U_1, U_2, U_3\right) \tag{6}$$
$$\dot{q}_3 = f_3\left(q_2, q_3, U_1, U_2, U_3\right) \tag{7}$$

Fig. 3 shows the model of the thermal system. The thermal

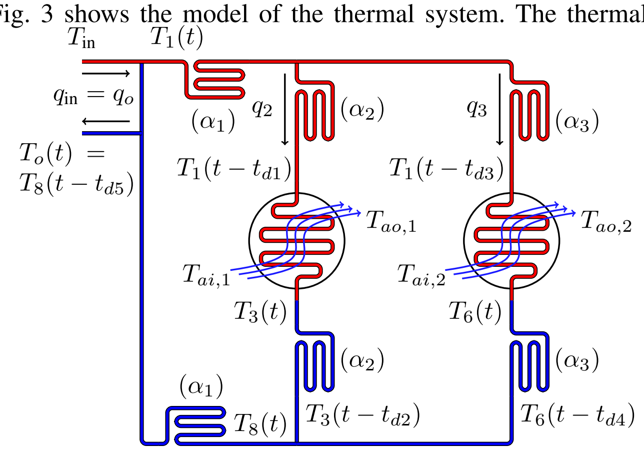

Fig. 3. Thermal model of the HES shown in Fig. 1. Identifiers for the flow directions, the temperatures at the different nodes and their delayed counterparts, due to long pipes, are marked.

model is based on the following assumptions:

T1) The water temperature delivered by the DHP is constant.
T2) No loss of heat energy in pipes, fittings and pumps. Any non-accounted heat energy loss will still contribute to the heating of the room and will hence still affect the energy output, $Q_{a,i}$.
T3) The spatially varying temperatures of the air and water inside the HE is the same as the ones coming out of it.
T4) The thermal resistances averaged across an entire HE, R, the density of the water, ρ_w and the air mass flow rate, w_a are constant.
T5) The length and form of the pipes from the mixing loop out to a HE is the same as the length of the pipes running from a HE back to the mixing loop.
T6) Pipes are the only components resulting in time delays.
T7) The air temperature in the rooms varies slowly and thus can be modelled as being piecewise constant.

Based on assumption T3 and T4 the dynamics of a HE can be modeled as [4]

$$C_w \dot{T}_{wo} = c_w \rho_w q\left[T_{wi} - T_{wo}\right] - R^{-1}\left[T_{wo} - T_{ao}\right] \tag{8}$$
$$C_a \dot{T}_{ao} = R^{-1}\left[T_{wo} - T_{ao}\right] - w_a c_a\left[T_{ao} - T_{ai}\right] \tag{9}$$

where T_{wi} and T_{wo} are the water temperatures in and out of the HE respectively [°C], T_{ai} and T_{ao} are the air temperatures in and out of the HE respectively [°C], R is the thermal resistance of the HE [K/J], C_a and C_w are the thermal capacities of the air and water inside the HE respectively [J/K], w_a is the mass flow rate of air thru the HE [kg/s], c_a and c_w are the specific heat capacities of the air and water respectively [J/(g K)], q is the water flow thru the HE $\left[\mathrm{m}^3/\mathrm{h}\right]$ and ρ_w is the density of the water $\left[\mathrm{kg}/\mathrm{m}^3\right]$.

Furthermore based on assumption T2 and T6 the rest of the system does only contribute to the thermal model through blending of water and time delays. Based on the mass conservation principle the temperature of the water running out of a node, resulting from multiple flows of water running into the same node is given by [3]

$$T_{\text{out}} = \frac{\sum_{i=1}^{n} q_i T_i}{q_{\text{out}}} \tag{10}$$

where T_{out} is the temperature of the water running out of the node [°C], n is the number of input flows into the node [1], q_i is the i'th flow into the node $\left[\mathrm{m}^3/\mathrm{h}\right]$, T_i is the temperature of i'th water flow into the node [°C] and q_o is the flow out of the node $\left[\mathrm{m}^3/\mathrm{h}\right]$. Time delays are modelled by The time delays in the transportation of the hot water depend on the water flows in the pipes. In this project the time delays is modelled as shown in (11) [3].

$$t_{d,j} = \frac{\alpha_i}{q_i(t)} \tag{11}$$

where $t_{d,j}$ is the thermal delay [s] thru a piping section, j, α_i is a constant parameter that mainly depends on the pipe characteristics linked to the flow, q_i, thru that pipe section. Under assumption T5 we get the relation $t_{d,1} = t_{d,2} + t_{d,5}$ and $t_{d,3} = t_{d,4} + t_{d,5}$. Furthermore we get $t_{d,2} = \frac{\alpha_2}{q_2(t)}$, $t_{d,4} = \frac{\alpha_3}{q_3(t)}$ and $t_{d,5} = \frac{\alpha_1}{q_2(t)+q_3(t)}$.

Combining (8), (9), (10) and (11) and assuming that $T_{ai,1}$ and $T_{ai,2}$ is constant, yields the dynamic equations of the thermal model given in the form of

$$\dot{T}_3 = f_4(q_2, T_3, T_{ao,1}, \lambda) \tag{12}$$

$$\dot{T}_{ao,1} = f_5(T_3, T_{ao,1}, T_{ai,1}) \tag{13}$$

$$\dot{T}_6 = f_6(q_3, T_6, T_{ao,2}, \lambda) \tag{14}$$

$$\dot{T}_{ao,2} = f_7(T_6, T_{ao,2}, T_{ai,2}) \tag{15}$$

where λ is a funtion of different delayed versions of T_3, T_6, q_o, q_2, and q_3. Thus the full system can be expressed as

$$\dot{\chi}(t) = f(\chi, u, \lambda, T_{ai}) = [f_1, f_2, f_3, f_4, f_5, f_6, f_7]^T \tag{16}$$

where $\chi(t) = [q_o, q_2, q_3, T_3, T_{ao,1}, T_6, T_{ao,2}]^T$, $u = [U_1, U_2, U_3, \theta]^T$ and $T_{ai} = [T_{ai,1}, T_{ai,2}]^T$. Furthermore the heat energy flows out of the i'th heater can be expressed as [4]

$$Q_{a,i} = f_{y,i}(\chi) = w_{a,i} c_a [T_{ao,i} - T_{ai,i}] \tag{17}$$

where $Q_{a,i}$ is the energy output of the i'th heater. Thus $f_y(\chi)$ can be expressed as $f_y(\chi, T_{ai}) = [f_{y1}, f_{y2}]^T$.

III. Minimization Problem

The total cost of running the HES is composed of the cost for the heat energy, C_H and the cost for running the actuators, C_A. It is chosen to consider steady state of the system when minimizing the cost. The heat energy being pulled out of the hot inflow water from the DHP can be expressed as

$$E_w = \left[3.6 \cdot 10^3\right]^{-1} c_w \rho_w V [T_{in} - T_o] \tag{18}$$

where E_w is the energy pulled out of the water [kW h] in a given time period, Δt [s] , and V is the volume of the water the has run through the system in the same time period $\left[\mathrm{m}^3\right]$. By expressing the volume of water as a flow, that is $\frac{V}{\Delta t} = \frac{1}{60 \cdot 60} q_o$, the price of the heat energy per second is expressed by

$$C_H(T_o, q_o) = \frac{C_W(T_o) E_w}{\Delta t} = c_H C_E(T_o) q_o [T_{in} - T_o] \tag{19}$$

where C_H is the cost of the heat energy that the HES delivers every second [DKK/s], C_W is the price of heat energy from the DHP depending on the return water temperature T_o [DKK/kWh], and $c_H = \frac{1}{3.6^2 \cdot 10^9} c_w \rho_w$.

Based on the information given in Section II it should be apparent that $T_o(\chi) = \frac{q_2 T_3 + q_3 T_6}{q_2 + q_3}$ in steady state and thus $C_H(T_o, q_o) \Rightarrow C_H(\chi)$.

The actuators in a HES are the pumps and the controllable valve. The valve is only consuming energy when it turns and thus does not consume any energy in steady state operation. The valve is therefore omitted from the cost model.

If the energy consumed by the pumps can be approximated by an affine function, then the cost of running the pumps can be expressed as

$$C_A(u) = C_E \sum_{i=1}^{N} [a_{p,i} U_i + b_{p,i}] \tag{20}$$

Where C_A is the cost of the electrical energy that the actuators use every second in steady state [DKK/s], N is the total amount of pumps in the HES [1], C_E is the price of electrical energy [DKK/kWh], and $a_{p,i}$ and $b_{p,i}$ are parameters describing the affine approximation of the energy consumption of the i'th pump. Substituting (19) and (20) into the MP stated in (1) yields the complete cost function, C_T. If $C_T \in C^2$ then C_T is convex in the set R_c if and only if the Hessian of C_T, $H_{C_T}(\chi, u)$, is positive semidefinite for $x \in R_c$ [5].

IV. Control Solution

A commonly used linear MIMO control scheme is the LQR controller that yields an optimal controller in the sense of state errors and actuation energy being minimized according to the infinite time integral $\int_0^\infty \chi^T Q \chi + u^T R u + 2\chi^T N u$. Since the cost function, $C_T(\chi, U_1, U_2, U_3)$, can only be written on this form if the MP is convex and quadratic it is chosen to take the MP out of the control loop to generate optimal references for a feedback controller. The model based optimal reference generator minimizes the cost function to yield the most cost efficient set of states while still reaching the desired references. Based on these references a feedback controller should force the HES into the optimal combination of χ and u that minimizes the steady state cost of running the HES. The suggested control solution is shown in Fig. 4.

The feedback loop is based on measurements of the states, χ, instead of $Q_{a,1}$ and $Q_{a,2}$ directly, since these variables

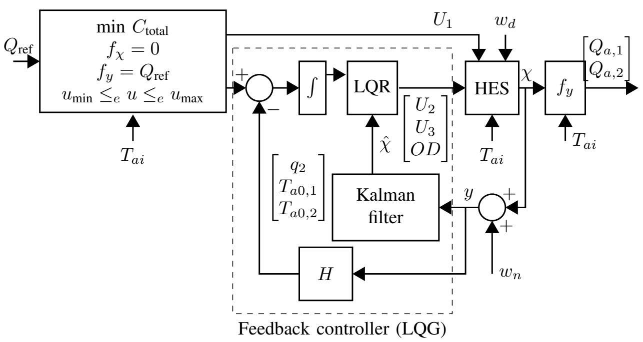

Fig. 4. The suggested control scheme incorporating an LQG controller with integral action and an optimal reference generator that generates references for q_2, $T_{ao,1}$ and $T_{ao,2}$.

cannot be measured directly. From (5), (6), (7), (12), (13), (14), and (15), it should be clear that T_3, $T_{ao,1}$, T_6 and $T_{ao,2}$ has to be controlled through control of q_o, q_2 and q_3. Thus only 3 states can be controlled independently. If more than 3 states are controlled then the system would be forced to find a compromise between the references, and thus a steady state error would occur. This is not desirable since the system will not necessarily deliver the desired $Q_{a,1}$ and $Q_{a,2}$. From (17) it is known that $Q_{a,1}$ and $Q_{a,2}$ are closely related to the states $T_{ao,1}$ and $T_{ao,2}$, thus it is deemed a good idea to set references for these states. To identify how all the 7 states of the HES can be forced into the optimal by only putting references to one more state besides the $T_{ao,1}$ and $T_{ao,2}$ references, (8) and (9) are considered in steady state where

$$\begin{gathered} \dot{T}_{ao,1} = \dot{T}_{ao,2} = \dot{T}_3 = \dot{T}_6 = 0 \\ T_{ao,1} = T_{ao,1,ref} \quad \text{and} \quad T_{ao,2} = T_{ao,2,ref} \end{gathered} \tag{21}$$

is valid and $T_{ai,1}$ and $T_{ai,2}$ can be assumed constant, since the energy will even out at some point, $Q_{a,1} = Q_{r,1}$ and $Q_{a,2} = Q_{r,2}$. Thus (9) shows that there can be only one solution to T_3 and T_6. Under assumption T2 it should furthermore be evident from Fig. 3 that $T_{wi,1} = T_{wi,2} = T_1$ in steady state. Based on these facts (8) for both heaters can be solved for T_1 and combined resulting in (22), where q_2 and q_3 are the only variables left.

$$\begin{gathered} T_{wo,1} + T_{wo,1}\left[R_1 c_w \rho_w q_2\right]^{-1} - T_{ao,1}\left[R_1 c_w \rho_w q_2\right]^{-1} \\ T_{wo,2} + T_{wo,2}\left[R_2 c_w \rho_w q_3\right]^{-1} - T_{ao,2}\left[R_2 c_w \rho_w q_3\right]^{-1} \end{gathered} \tag{22}$$

As multiple combinations of q_2 and q_3 fulfils the references for $T_{ao,1}$ and $T_{ao,2}$ in (22) the controller should force either q_2 or q_3 into a reference to make sure that both states reaches the optimal value. It has arbitrarily been chosen to fix q_2 to a reference consequently yielding one possible value of q_3. With the flows being fixed (12) or (14) will in steady state yield only one solution for q_o hence (5) yields only one solution for θ.

From (6) and (7) it can be seen that multiple combinations of U_1, U_2 and U_3 will fulfil these three flows. To overcome this, it is chosen to fix U_1 to the optimal solution of the MP. Using (6) and (7) this results in four possible combinations of U_2 and U_3 due to the quadratic terms of the pump model (4). The combinations includes both positive and negative values for the pump voltages whereof only the solution with positive voltages is feasible. This results in only one possible combination.

Based on these decisions the controller scheme is summarized into fixing U_1 directly to the optimal value found from the MP and making a feedback controller that tracks the optimal references for q_2, $T_{ao,1}$ and $T_{ao,2}$ by controlling θ, U_2 and U_3. Under the assumption of no modelling errors tracking these references should force the system into the optimal combination of states and inputs, χ and u, in steady state thereby yielding the minimum running costs.

It is assumed that a linear approximation on the form shown in (23) will yield a good approximation of the non-linear model when running near the operating point. To linearize the model all time time-varying transport delays is omitted.

$$\begin{gathered} \dot{\chi} = A\chi + Bu + \omega_d \\ y = \chi + \omega_n \end{gathered} \tag{23}$$

where ω_n and ω_d are the measurement noise inputs and disturbance/process noise respectively, which are assumed to be uncorrelated zero-mean Gaussian stochastic processes with constant power spectral density matrices. Based on this assumption it is chosen to utilize a Linear-Quadratic-Gaussian (LQG) controller with integral action to make sure that the HES tracks the references for q_2, $T_{ao,1}$ and $T_{ao,2}$. The LQG includes a Kalman filter that gives an optimal estimate of the system states, $\hat{\chi}$, based on the linearised model even in case of sensor loss or random loss of sensor measurements. The LQR controller is tuned by adjusting the weights in the Q and R matrices. The initial weight are based on the Bryson's rule defining the weights using the actuator saturations and maximum state values.

V. Results

For modelling and simulations it is assumed that:

c_w = 4.2 J/(g K)	ρ_w = 1000 kg/m^3	c_a = 1 J/(g K)

TABLE I

Assumed constants used for the simulation model

The test setup has been excited in smaller parts to make sure that each individual part and parameter was excited sufficiently to allow proper parameter estimation. Using the non-linear model and measurements from the test setup, Senstools and Simulink Parameter Estimation toolbox has been used to find the parameters of the system given in Tabel II.

r_o+$a_{h2,0}$=−352 h^{2}bar/m^6	K_2=16.6 m^3	a_{h1}=420 mbar h V/m^3
r_1=7.98 h^{2}mbar/m^6	α_1=16.8 m^3	a_{h2}=641 h^{2}mbar/m^6
r_2=808 h^{2}mbar/m^6	α_2=6.06 m^3	$a_{h0,0}$=mbar/V^2
r_3=627 h^{2}mbar/m^6	α_3=14.1 m^3	$a_{h1,0}$=2 mbar h V/m^3
J_o=1.36 h^{2}bar/m^3	$w_{a,1}$=801 kg/h	R_1=64.7 °C/MW
J_1=55.6 h^{2}mbar/m^3	$w_{a,2}$=916 kg/h	R_2=289 °C/MW
J_2=379 h^{2}mbar/m^3	$C_{a,1}$=1.51 J/°C	$C_{w,1}$=46.2 kJ/°C
J_3=288 h^{2}mbar/m^3	$C_{a,2}$=1.41 J/°C	$C_{w,2}$=70.1 kJ/°C
K_1=221 m^3/bar$^{0.5}$h	a_{h0}=120 mbar/V^2	

TABLE II

Estimated coefficients used for the simulation model

To specify C_T for an optimal reference generator the tariffs of Skive DHP is used as an example. As a user of heat energy from Skive DHP one pays $\mathrm{DKK}0.620/(\mathrm{kW\,h})$, furthermore the punishment/reward policy of Skive DHP is as follows [6]:

- You pay 2% less for each degree that the return temperature is below the expected return temperature.
- You are not punished, nor rewarded, if the return temperature is 0-5 degree higher than the expected return temperature.
- You pay 1% extra for each degree that the return temperature is higher than the expected return temperature.

To make the analysis easier this piecewise continuous function is approximated with an affine function

$$C_W = a_W T_o + b_W \tag{24}$$

Where a_W and b_W are stated Table III, and the approximation is shown in Figure 6. Now (24) can be substituted into (19), forming the C_H used for the optimal reference generator.
Based on the datasheets it is assumed that the relationship between the input signal to the pumps and the electrical energy that they consume is affine. Thus the formula in (20) can be used. All the pumps consume 70 W when running at full speed (9 V) and 6 W when idle (0 V). This gives the values of $a_{p,i}$ and $b_{p,i}$, shown in Table III, used for C_A in this project, defined by (20).

a_W = 0.008 267 DKK/kWh°C	b_W = 0.227 34 DKK/kWh
$a_{p,i}$ = 0.0071 kW h/(s V)	$b_{p,i}$= 0.006 kW h/s

TABLE III
COEFFICIENTS USED FOR THE MP, IN THE OPTIMAL REFERENCE GENERATOR

It has not been a focus in this project to examine if C_T is convex or not. Nevertheless a linear combination of functions that are convex on the set R_c is also convex [5]. C_T is a linear combination of functions. One of these functions is $c_4\left(T_o, q_o\right) = -c_H b_H q_o T_o \in C^2$ and has the Hessian

$$H_{c_4}\left(T_o, q_o\right) = \begin{bmatrix} 0 & -c_H b_H \\ -c_H b_H & 0 \end{bmatrix} \tag{25}$$

It can easily be verified that

$$v^T H_{c_4}\left(T_o, q_o\right) v = -2 c_H b_H v_1 v_2 \tag{26}$$

where $v = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$, is not positive or zero for all values of v_1 and v_2 which is required for the Hessian to be positive semidefinite. Thus it must be concluded that $c_4\left(T_o, q_o\right)$ is not convex, and therefore it should neither be expected that $c_4\left(\chi\right)$ nor C_T is convex. Finding a global minima in the MP can thereby not be guaranteed.
To test the control solution a set of changing and feasible Q_{ref} references are given to the system throughout a test period of 3.5 hours. Whenever the reference is changing a new optimal set of states is found by the reference-based minimizer. Based on the reference-based minimizer optimal references for q_2, $T_{ao,1}$ and $T_{ao,2}$ are provided for the LQG controller. This test is performed both for the real world test set-up, and a simulation with the non-linear model. Fig. 5 shows the results of three test.

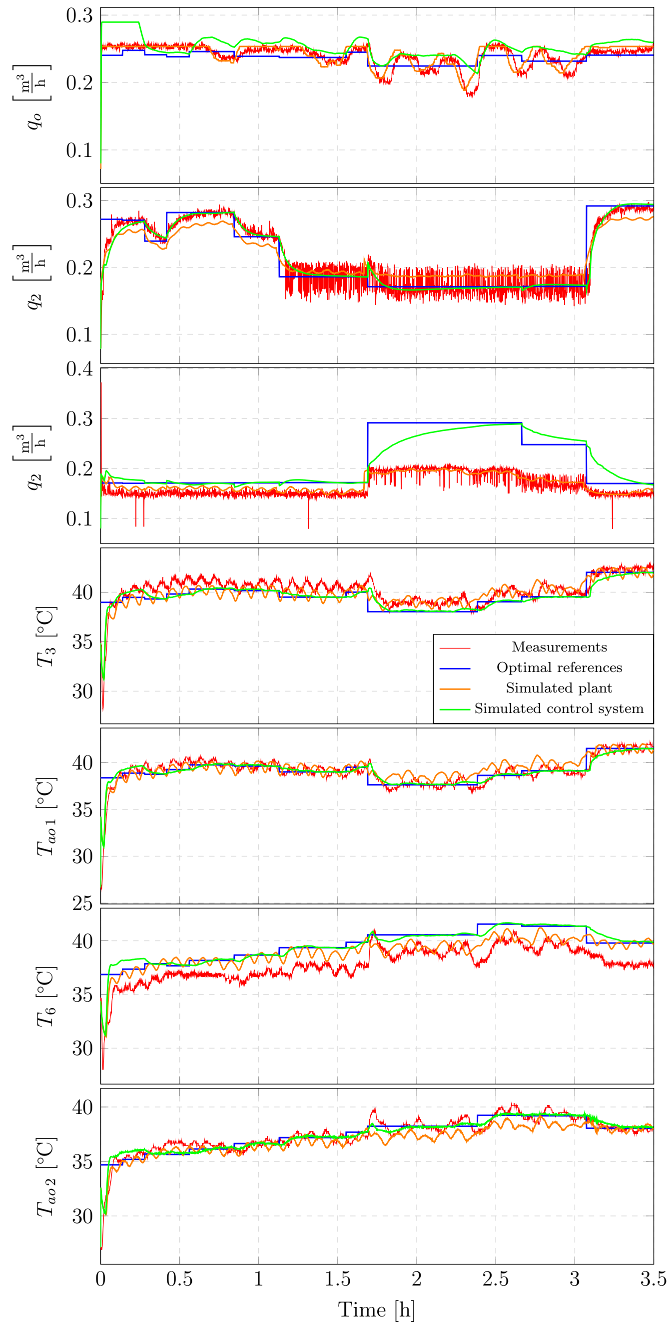

Fig. 5. Comparison between measured states, non-linear simulation based on measured inputs, non-linear simulation and optimal references obtained from the MP. The comparison is based on a 3.5 hour test with randomized Q_1 and Q_2 references.

1) The control solution running in a simulation with a non-linear model,
2) The control solution running on the test set-up,
3) and the output of the non-linear model exposed to the same actuator signals, T_{in}, $T_{ao,1}$ and $T_{ao,2}$ as the test set-up experienced in the test described in item 1.

The simulation of the non-linear model exposed to the the same actuator signals as the test set-up follows the actual measurements, confirming that the non-linear model is suited for simulation-based analysis.

The control solution running in a simulation with a non-linear model, shows output that follows the optimal references for all the states, as expected.
The test on the test set-up shows that the solution is tracking the optimal references for q_2, $T_{ao,1}$ and $T_{ao,2}$, but also that the states q_0, q_3, T_3, and T_6 differs slightly from the optimal values, which should be expected in the case model errors or disturbances. Furthermore it can be seen both on the test metioned in item 2 and 3, that the temperatures is fluctuating alot. This is proberbly due to the test set-up not delivering a constant T_in as assumed in the modelling.

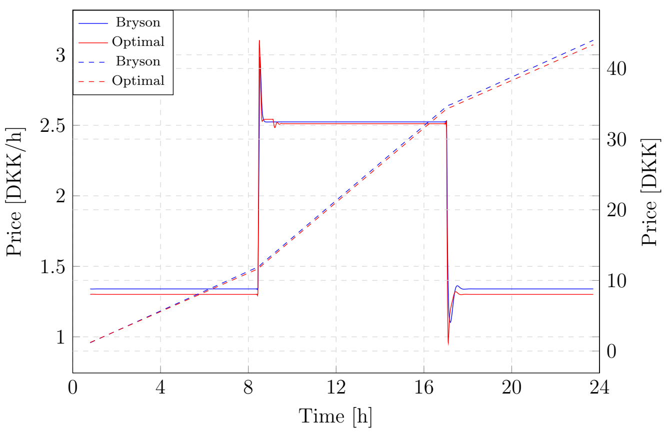

Fig. 6. Comparison of the cost of running the system with an LQG controller tunned with Bryson's rule and the optimal controller designed. The comparison is made on a simulation where the system has been running for 24h in an office building where the HES increases the temperature of the rooms at 8 AM and decreases it again at 4 PM.

To compare the efficiency of the system a similare LQG controller is implemented, capable of freely controlling all four actuators with neither U_1 being fixed and without a q_2 reference. To simulate a days usage of the HES a set of references throughout a day of 24 hours is designed for an office building. It is assumed that the heat demand is increased at 8 o'clock when work starts and that the heat demand decreases again at 16 o'clock. In Fig. 6 the references given to the controllers are shown. The total cost of running the HES after 24 hours is DKK 43.35 for the optimal reference controller and DKK 43.99 for the non-optimal controller. Hence the cost of running the HES with a controller given optimal references is cheaper than using a non-optimal controller mainly due to the steady state cost optimization of the reference optimizer. Notice that the transient response of the non-optimal controller is less aggressive and more dampened resulting in less expensive transient periods compared to the optimal reference controller. If the amount of reference changes throughout a day is only a few the running costs of the system is dominated by the steady state costs why the transient response does not matter.

VI. Discussion

This control solution, like every control solution, of course does have some limitations. Some of these limitations are listed below.

- L1) The highly nonlinear nature of the system could give problems with the performance of the linear LQG controller.
- L2) The solution to the MP cannot be better then the precision of the model allows the solution to be.
- L3) The LQG controller gives no guarantees of satisfactory robustness properties. [7]

Limitations L1 immediately suggests some kind of gain scheduling to improve the system performance in a greater range. Limitations L2 should be evident from the test on the test set-up where the controller did not force the system into the calculated optimal state. This must be due to modeling errors, and therefore it cannot really be concluded if the controller actually reaches the optimal state or not. This limitation could potentially be overcome if the MP could be updated with a new model on the fly, e.g. like it is done in adaptive control solutions, ensuring that the MP always operates on as good a model as possible. As stated in limitations L3 the LQR controller could give problems with robustness of the system. The robustness of the system with an LQR controller could be improved by a loop transfer recovery (LTR) procedure [7]. If a LTR procedure does not yield satisfactory robustness properties the LQG could simply be replaced with another controller that tracks the optimal references for $T_{ao,1}$, $T_{ao,2}$ and either q_2 or q_3.

VII. Conclusion

This paper presents a linear control solution for a distributed heat exchanger system based on a cost-minimization problem. Due to the non-convex nature of the MP the controller solution it is not guaranteed to find the global minima. It is proven that given an ideal system with cost-optimal references it is possible to reach all optimal states if U_1 is fixed and the optimal references for q_2 and $T_{ao,1}$ and $T_{ao,2}$ are tracked. It is shown in a test set-up that, even with possible modeling errors and disturbances, the system is controllable and the references can be reached with a controller based on a linearized model that does not take the time delays into account. Although, with modelling errors one cannot guarantee that all optimal states are reached thereby not guaranteeing optimal running cost.

References

[1] Danfoss, "Optimalt indeklima. perfekt kontrol. 1 løsning." [Online]. Available: http://heating.danfoss.com/PCMPDF/VBIOF201_Danfoss_One_L.pdf

[2] H. Kristjansson, F. Bruss, B. Bøhm, N. K. Vejen, J. Rasmussen, K. P. Christensen, and N. Bidstrup, "Fjernvarmeforsyning af lavenergiområder," 2004.

[3] J. Val, "Control of district heating system with input-dependent state delays," Master's thesis, Aalborg University, Aalborg, 2016.

[4] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 7th ed. Pearson, 2010.

[5] A. Antoniou and W.-S. Lu, *Practical Optimization - Algorithms and Engineering Applications*. Springer, 2007.

[6] Skive fjernvarme. (2016) Ny fair afregning af din fjernvarme. [Online]. Available: http://www.skivefjernvarme.dk/media/2279418/Skive-Fjernvarme-Motivationstarif.pdf

[7] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control - Analysis and Design*, 2nd ed. JOHN WILEY AND SONS, 2001.