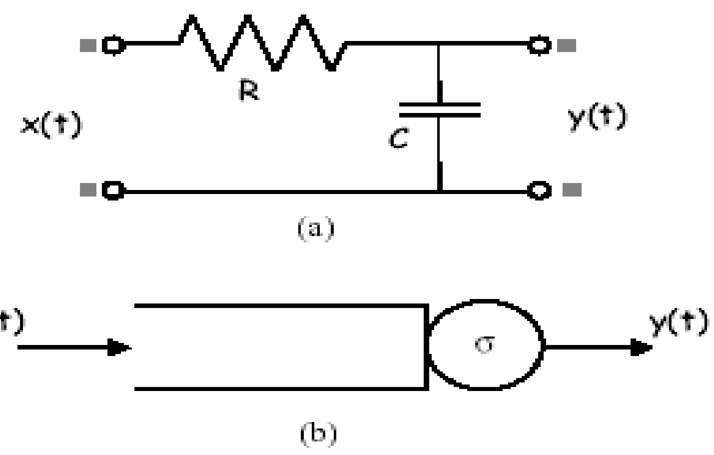
Deterministic Network Calculus

Background

- Queueing Theory gives probabilistic results
- Critical applications need hard bounds
- Queueing theory extends only partially to networks.
- Scheduling theory accounts only for CPU sharing and bounded blocking.
- Periodic studies do not account for bursty traffic.

Cirquit analysis

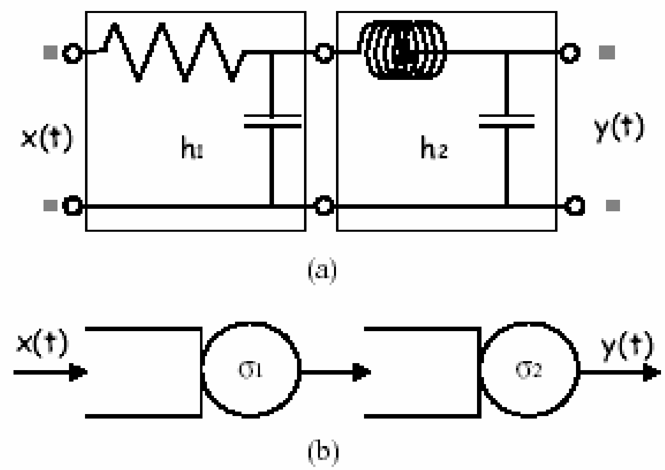
$$y(t) = (h \otimes x)(t) = \int_0^t h(t-s)x(s)ds.$$



$$y(t) = (\sigma \otimes x)(t) = \inf_{s \in \mathbb{R} \text{ such that } 0 \leq s \leq t} \{\sigma(t-s) + x(s)\}.$$

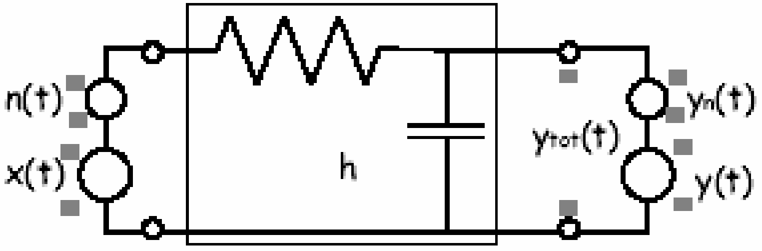
Concatenation of Network Elements

$$h(t) = (h_1 \otimes h_2)(t) = \int_0^t h_1(t-s)h_2(s)ds.$$

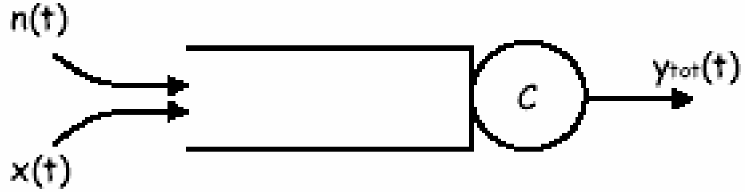


$$\sigma(t) = (\sigma_1 \otimes \sigma_2)(t) = \inf_{s \in \mathbb{R} \text{ such that } 0 \le s \le t} \{\sigma_1(t-s) + \sigma_2(s)\}.$$

Converging Flows

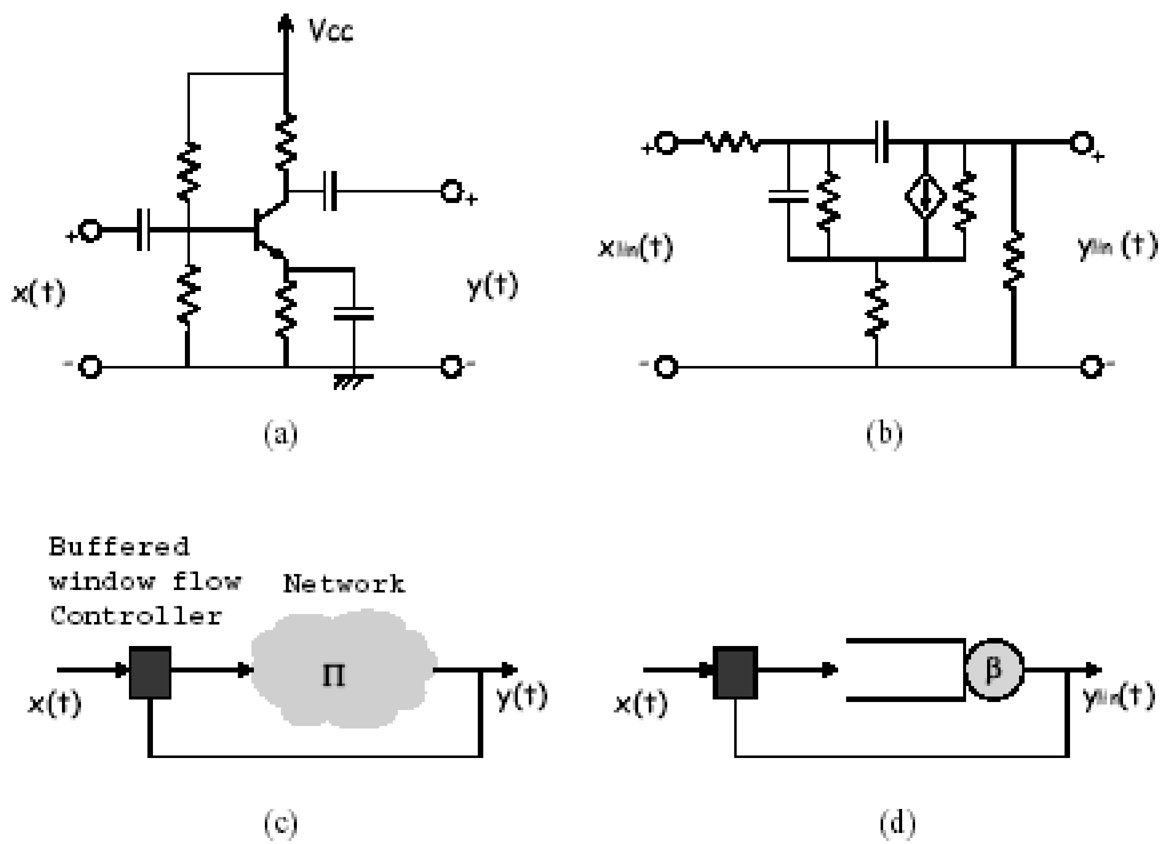


(a)



(b)

Feedback

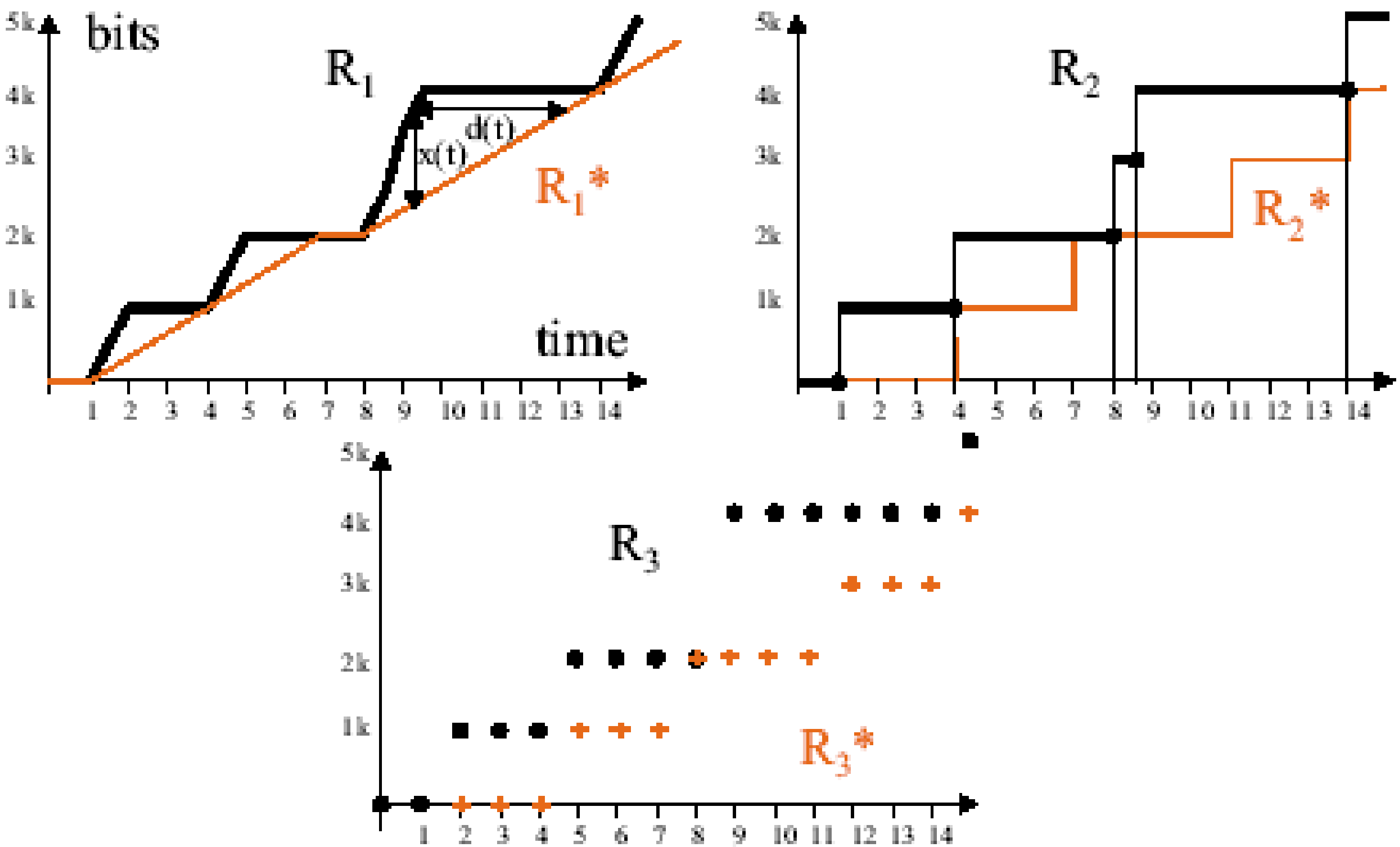

Vcc
x(t)
y(t)
(a)
$x_{lin}(t)$
$y_{lin}(t)$
(b)
Buffered
window flow
Controller
Network
Π
x(t)
y(t)
(c)
β
x(t)
$y_{lin}(t)$
(d)

Definition and Range Spaces

Convention: A flow is described by a wide-sense increasing function $R(t)$; unless otherwise specified, in this book, we consider the following types of models:

- discrete time: $t \in \mathbb{N} = \{0, 1, 2, 3, ...\}$
- fluid model: $t \in \mathbb{R}^+ = [0, +\infty)$ and R is a continuous function
- general, continuous time model: $t \in \mathbb{R}^+$ and R is a left- or right-continuous function

Definition and Range Spaces



The Playout Buffer

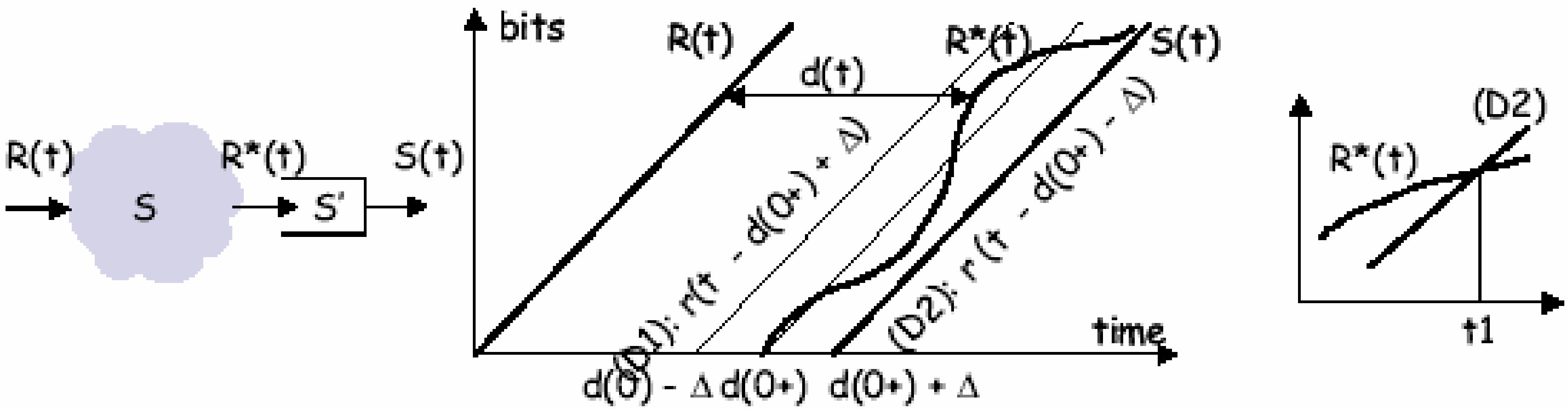


Figure 1.2: A Simple Playout Buffer Example

Arrival Curves

Definition 1.3.1 (Arrival Curve). *Given a wide-sense increasing function α defined for $t \geq 0$ (namely, $\alpha \in \mathcal{F}$), we say that a flow R is constrained by α if and only if for all $s \leq t$:*

$$R(t) - R(s) \leq \alpha(t - s)$$

We say that R has α as an arrival curve, or also that R is α-smooth.

Arrival Curves

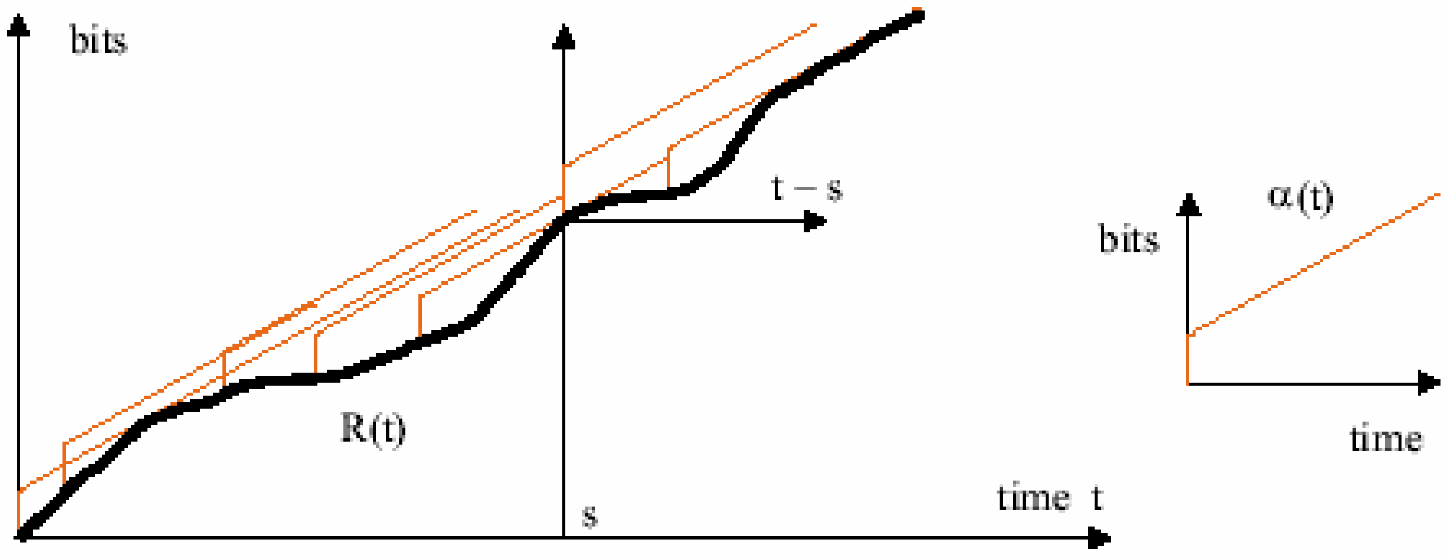


Figure 1.3: Example of Constraint by arrival curve, showing a cumulative function $R(t)$ constrained by the arrival curve $\alpha(t)$.

Spacing and Staircases

Proposition 1.3.1 (Spacing as an arrival constraint). *Consider a flow, with cumulative function $R(t)$, that generates packets of constant size equal to k data units, with instantaneous packet arrivals. Assume time is discrete or time is continuous and R is left-continuous. Call t_n the arrival time for the nth packet. The following two properties are equivalent:*

1. *for all m, n, $t_{m+n} - t_m \geq nT - \tau$*
2. *the flow has $kv_{T,\tau}$ as an arrival curve*

An Equivalence

Proposition 1.3.2. *Consider either a left- or right- continuous flow* $R(t), t \in \mathbb{R}^+$, *or a discrete time flow* $R(t), t \in \mathbb{N}$, *that generates packets of constant size equal to* k *data units, with instantaneous packet arrivals. For some* T *and* τ, *let* $r = \frac{k}{T}$ *and* $b = k(\frac{\tau}{T} + 1)$. *It is equivalent to say that* R *is constrained by* $\gamma_{r,b}$ *or by* $kv_{T,\tau}$.

The Leaky Bucket in Words

Definition 1.3.2 (Leaky Bucket Controller). *A Leaky Bucket Controller is a device that analyzes the data on a flow $R(t)$ as follows. There is a pool (bucket) of fluid of size b. The bucket is initially empty. The bucket has a hole and leaks at a rate of r units of fluid per second when it is not empty.*

Data from the flow $R(t)$ has to pour into the bucket an amount of fluid equal to the amount of data. Data that would cause the bucket to overflow is declared non-conformant, otherwise the data is declared conformant.

Leaky Bucket in Graphics

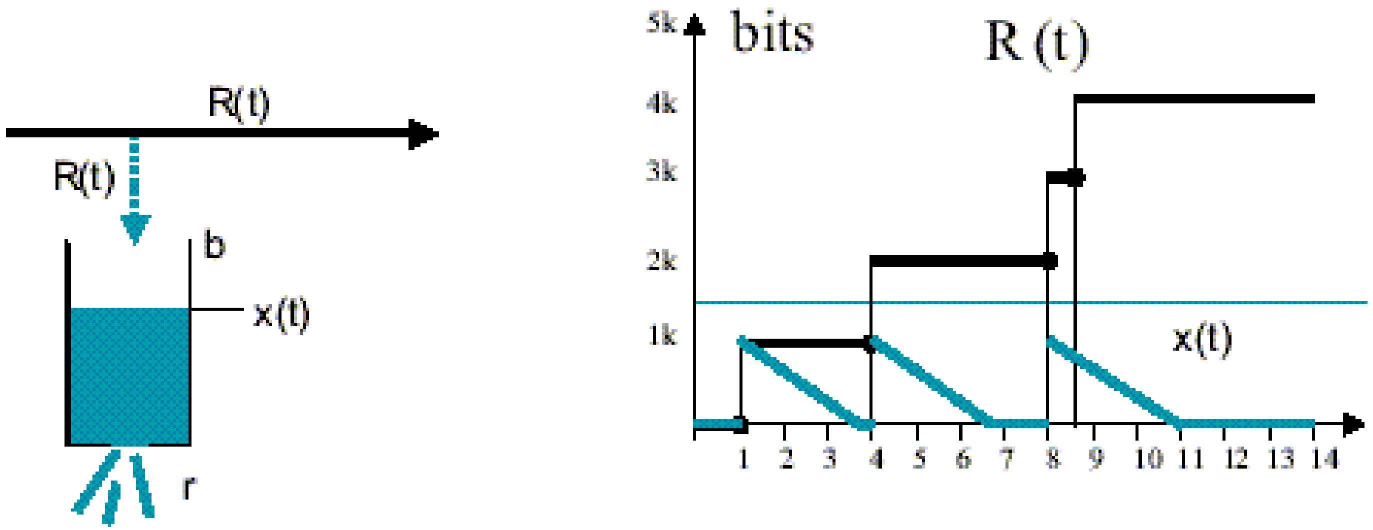


Figure 1.4: A Leaky Bucket Controller. The second part of the figure shows (in grey) the level of the bucket $x(t)$ for a sample input, with $r = 0.4$ kbits per time unit and $b = 1.5$ kbits. The packet arriving at time $t = 8.6$ is not conformant, and no fluid is added to the bucket. If b would be equal to 2 kbits, then all packets would be conformant.

Nice to know about Leaky Buckets

Proposition 1.3.3. *A leaky bucket controller with leak rate r and bucket size b forces a flow to be constrained by the arrival curve $\gamma_{r,b}$, namely:*

1. *the flow of conformant data has $\gamma_{r,b}$ as an arrival curve;*
2. *if the input already has $\gamma_{r,b}$ as an arrival curve, then all data is conformant.*

Some DN-Calculus in Play

Lemma 1.3.2. *Consider a buffer served at a constant rate* r. *Assume that the buffer is empty at time* 0. *The input is described by the cumulative function* $R(t)$. *If there is no overflow during* $[0, t]$, *the buffer content at time* t *is given by*

$$x(t) = \sup_{s:s \leq t} \{R(t) - R(s) - r(t - s)\}$$

Henrik should prove this on the blackboard

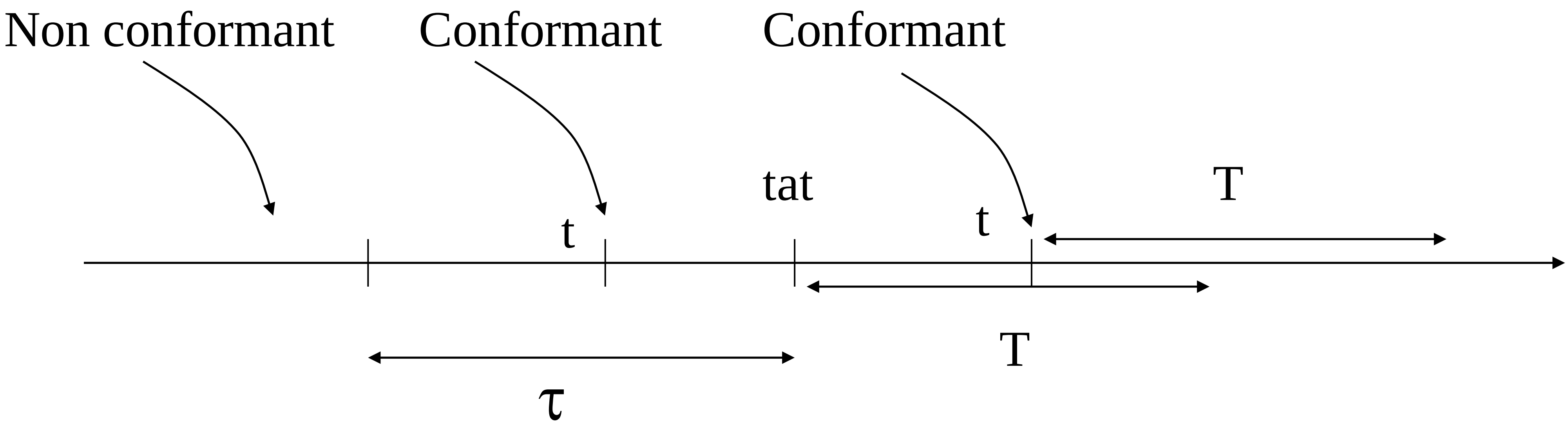
The Generic Cell Rate Algorithm

Definition 1.3.3 (GCRA (T, τ)). *The Generic Cell Rate Algorithm (GCRA) with parameters (T, τ) is used with fixed size packets, called cells, and defines conformant cells as follows. It takes as input a cell arrival time* `t` *and returns* `result`*. It has an internal (static) variable* `tat` *(theoretical arrival time).*

- *initially,* `tat = 0`
- *when a cell arrives at time* `t`*, then*

```
if (t < tat - tau)
   result = NON-CONFORMANT;
else {
     tat = max (t, tat) + T;
     result = CONFORMANT;
     }
```

GCRA



GCRA facts

Proposition 1.3.4. *Consider a flow, with cumulative function $R(t)$, that generates packets of constant size equal to k data units, with instantaneous packet arrivals. Assume time is discrete or time is continuous and R is left-continuous. The following two properties are equivalent:*

1. *the flow is conformant to GCRA(T, τ)*
2. *the flow has $(k\, v_{T,\tau})$ as an arrival curve*

Leaky Buckets and GCRAs

Corollary 1.3.1. *For a flow with packets of constant size, satisfying the GCRA(T, τ) is equivalent to satisfying a leaky bucket controller, with rate r and burst tolerance b given by:*

$$b = (\frac{\tau}{T} + 1)\delta$$

$$r = \frac{\delta}{T}$$

In the formulas, δ is the packet size in units of data.

Subadditivity

- We assume α (0) = 0
- Subadditive if $\alpha(t+s) <= \alpha(t) + \alpha(s)$
- Subadditive closure $\overline{\alpha}$ of α:

  The largest subadditive function

  less than or equal to α

Subadditivity

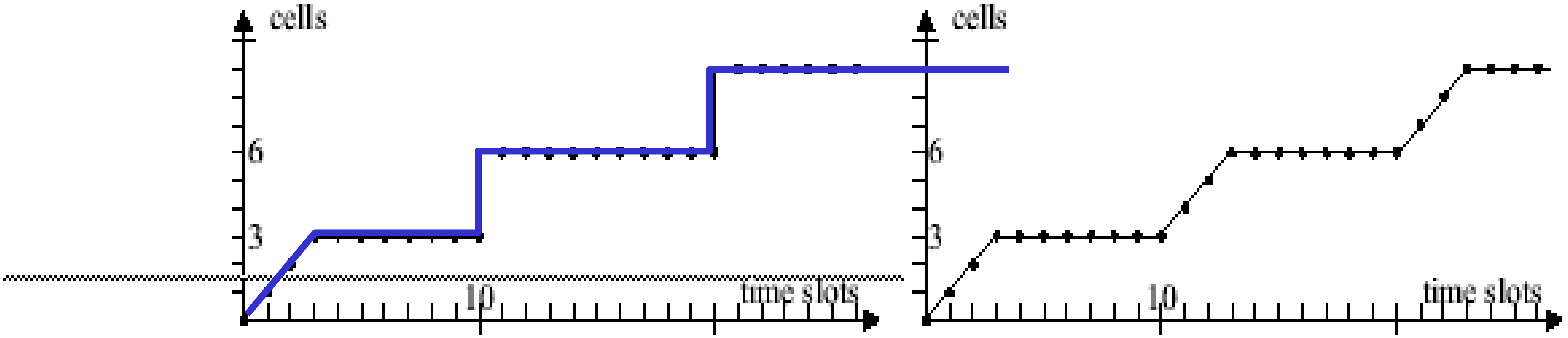


Figure 1.6: The arrival curve $\alpha_1 = \min(3v_{10,0}, v_{1,0})$ on the left, and its sub-additive closure ("good" function) $\bar{\alpha_1}$ on the right. Time is discrete, lines are put for ease of reading.

Sufficiency of Subadditive Arrivals

Theorem 1.3.1 (Reduction of Arrival Curve to a Sub-Additive One). *Saying that a flow is constrained by a wide-sense increasing function* α *is equivalent to saying that it is constrained by the sub-additive closure* $\bar{\alpha}$.