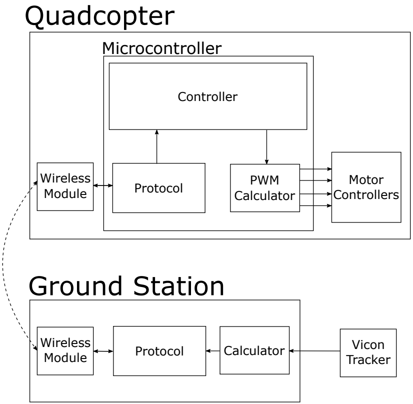
Attitude and Position Control of a Quadcopter in a Networked Distributed System

Alejandro Alonso García, Amalie Vistoft Petersen, Andrea Victoria Tram Løvemærke, Niels Skov Vestergaard, Noelia Villarmarzo Arruñada

Agenda

Introduction

- Surveillance and inspection
- Rescue
- Aerial photography
- ...

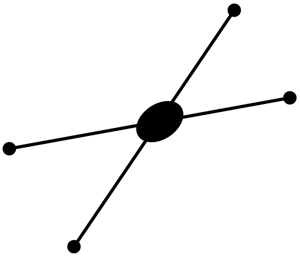


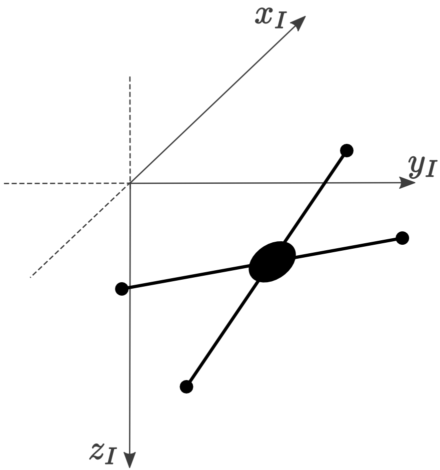
Quadcopter
Microcontroller
Controller
Wireless Module
Protocol
PWM Calculator
Motor Controllers
Ground Station
Wireless Module
Protocol
Calculator
Vicon Tracker

Introduction
Prototype

- Motors and propellers
- ESCs
- Vicon markers
- Battery
- Processor
- XBee modules

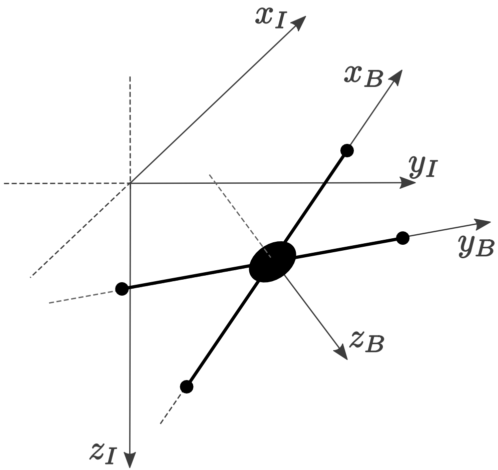
Model
Attitude Model

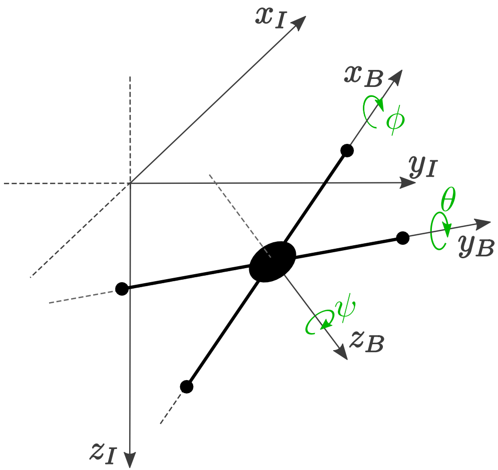




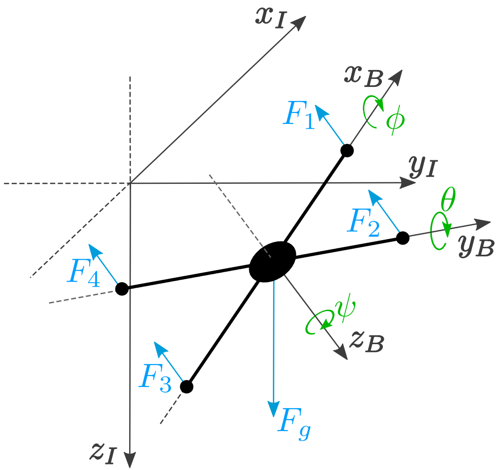
x_I
y_I
z_I

Model
Attitude Model

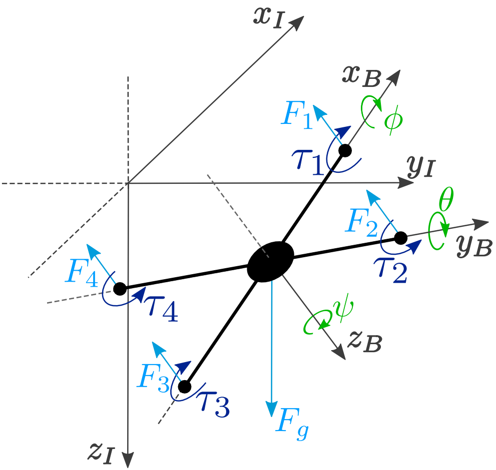




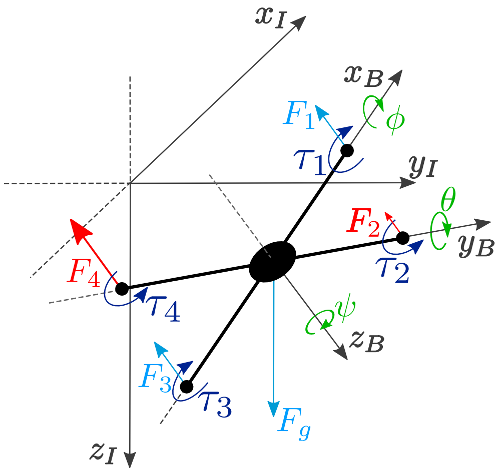
x_I
x_B
ϕ
y_I
θ
y_B
ψ
z_B
z_I



x_I
x_B
F_1
ϕ
y_I
θ
F_2
y_B
F_4
ψ
z_B
F_3
F_g
z_I



x_I
x_B
ϕ
F_1
τ_1
y_I
θ
F_2
y_B
τ_2
F_4
τ_4
ψ
z_B
F_3
τ_3
F_g
z_I


x_I
x_B
F_1
ϕ
τ_1
y_I
θ
F_2
y_B
τ_2
F_4
τ_4
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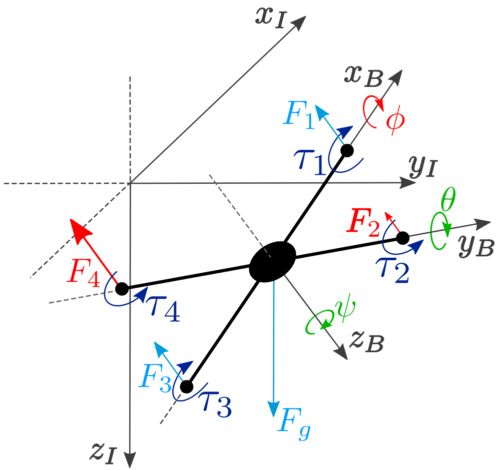
z_B
F_3
τ_3
F_g
z_I



x_I
x_B
F_1
ϕ
τ_1
y_I
θ
F_2
y_B
τ_2
F_4
τ_4
ψ
z_B
F_3
τ_3
F_g
z_I

- Dynamic Equations

- Dynamic Equations

$$J\alpha = \sum \tau$$

- Dynamic Equations

$$J\alpha = \sum \tau$$

$$J_x\ddot{\phi} = (F_4 - F_2)L$$
$$J_y\ddot{\theta} = (F_1 - F_3)L$$
$$J_z\ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

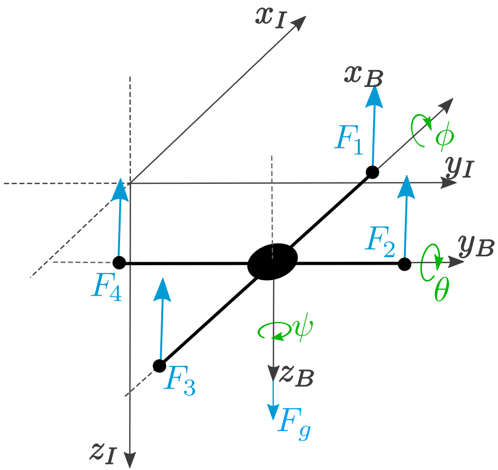
- Dynamic Equations

$$J\alpha = \sum \tau$$

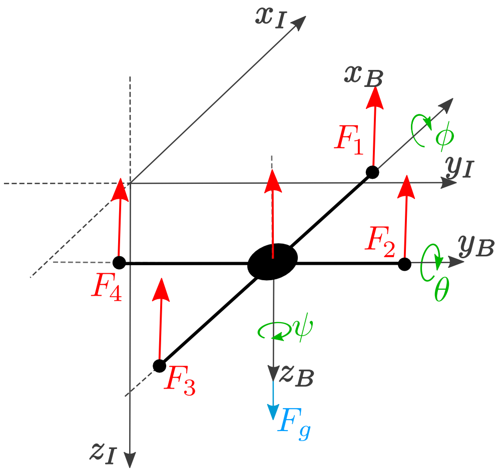
$$J_x\ddot{\phi} = (F_4 - F_2)L$$
$$J_y\ddot{\theta} = (F_1 - F_3)L$$
$$J_z\ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

$$J_x\ddot{\phi} = k_{\mathrm{th}}(\omega_4^2 - \omega_2^2)L$$
$$J_y\ddot{\theta} = k_{\mathrm{th}}(\omega_1^2 - \omega_3^2)L$$
$$J_z\ddot{\psi} = k_{\mathrm{d}}(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$$

Agenda



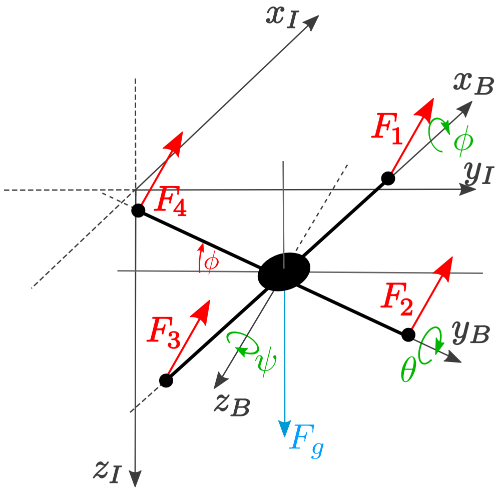
x_I
x_B
F_1
ϕ
y_I
F_2
y_B
θ
F_4
ψ
F_3
z_B
F_g
z_I



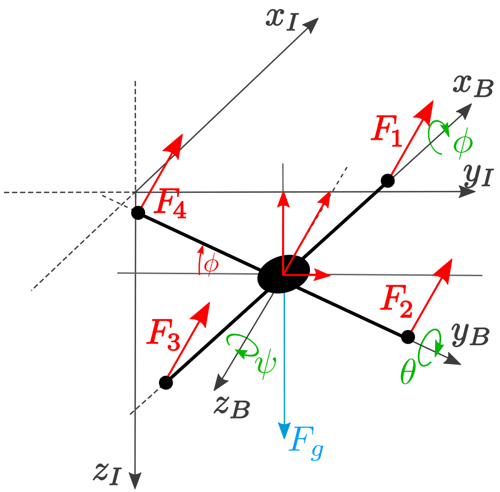
x_I
x_B
F_1
ϕ
y_I
F_2
y_B
θ
F_4
ψ
F_3
z_B
F_g
z_I

Model

Translational Model



Model
Translational Model



Translational Model

- Rotation Matrix

$$R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \quad R_Y = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \quad R_Z = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_Z R_Y R_X$$

$$v_{\mathrm{I}} = R v_{\mathrm{B}}$$

- Dynamic Equations

- Dynamic Equations

$$ma = \sum F$$

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$$ma = \sum F$$

$$
\begin{aligned}
m\ddot{x}_{\mathrm{I}} &= -(F_1 + F_2 + F_3 + F_4)(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \\
m\ddot{y}_{\mathrm{I}} &= -(F_1 + F_2 + F_3 + F_4)(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \\
m\ddot{z}_{\mathrm{I}} &= F_g - (F_1 + F_2 + F_3 + F_4)\cos\phi \cos\theta
\end{aligned}
$$

- Dynamic Equations

$$ma = \sum F$$

$$\begin{aligned}
m\ddot{x}_{\mathrm{I}} &= -(F_1 + F_2 + F_3 + F_4)(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \\
m\ddot{y}_{\mathrm{I}} &= -(F_1 + F_2 + F_3 + F_4)(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \\
m\ddot{z}_{\mathrm{I}} &= F_g - (F_1 + F_2 + F_3 + F_4) \cos\phi \cos\theta
\end{aligned}$$

$$\begin{aligned}
m\ddot{x}_{\mathrm{I}} &= -k_{\mathrm{th}}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2)(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \\
m\ddot{y}_{\mathrm{I}} &= -k_{\mathrm{th}}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2)(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \\
m\ddot{z}_{\mathrm{I}} &= F_g - k_{\mathrm{th}}({\omega_1}^2 + {\omega_2}^2 + {\omega_3}^2 + {\omega_4}^2) \cos\phi \cos\theta
\end{aligned}$$

- First order Taylor approximation

Model
Linearization

- First order Taylor approximation

$$f(x) \approx f(\overline{x}) + f'(\overline{x})(x - \overline{x}) \rightarrow \; \Delta f(x) \approx f'(\overline{x})\Delta x$$

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$$m\ddot{\overline{z}}_{\mathrm{I}} = F_g - k_{\mathrm{th}}({\overline{\omega}_1}^2 + {\overline{\omega}_2}^2 + {\overline{\omega}_3}^2 + {\overline{\omega}_4}^2)\cos\overline{\phi}\cos\overline{\theta}$$

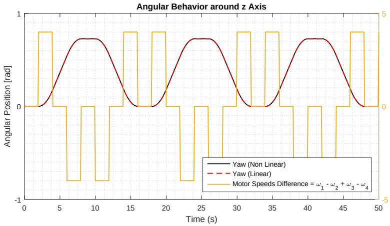
- First order Taylor approximation

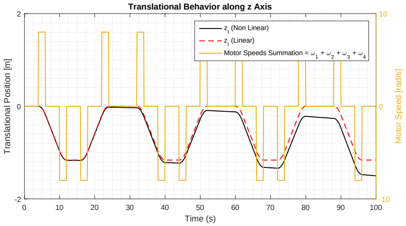
$$f(x) \approx f(\overline{x}) + f'(\overline{x})(x - \overline{x}) \rightarrow \; \Delta f(x) \approx f'(\overline{x})\Delta x$$

$$m\overline{\ddot{z}}_{\mathrm{I}} = F_g - k_{\mathrm{th}}(\overline{\omega}_1{}^2 + \overline{\omega}_2{}^2 + \overline{\omega}_3{}^2 + \overline{\omega}_4{}^2)\cos\overline{\phi}\cos\overline{\theta}$$

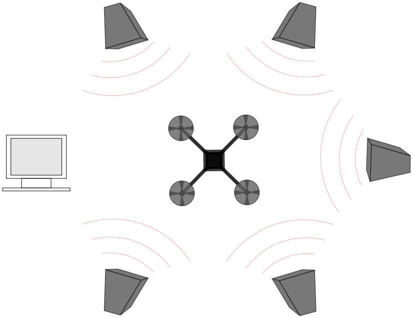
$$\overline{\omega}_i = \sqrt{\frac{F_{\mathrm{g}}}{4k_{th}}}$$

Model
Linearization

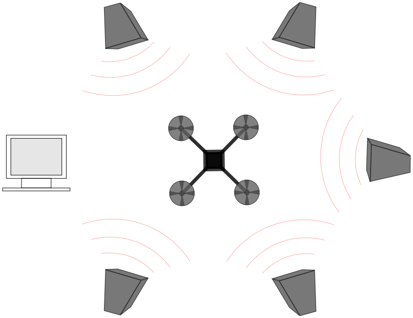




Network

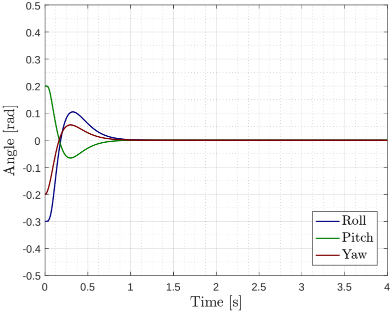


Network

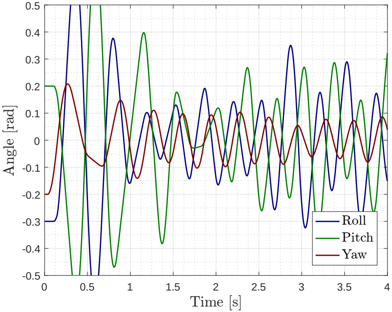


- Delay
- Missed packets

Network



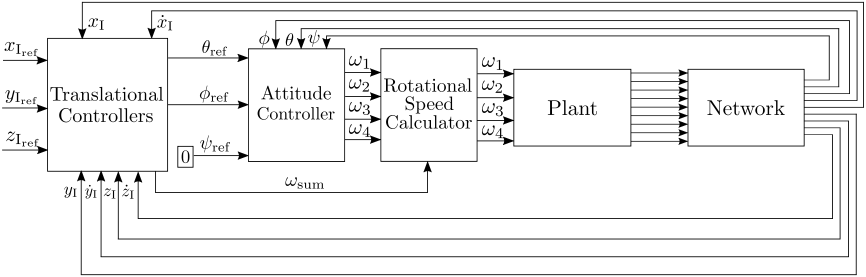
Control design only taking the model into account



Same controller with the effect of the network

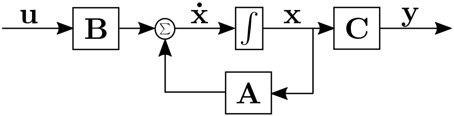
Agenda

Control Solution



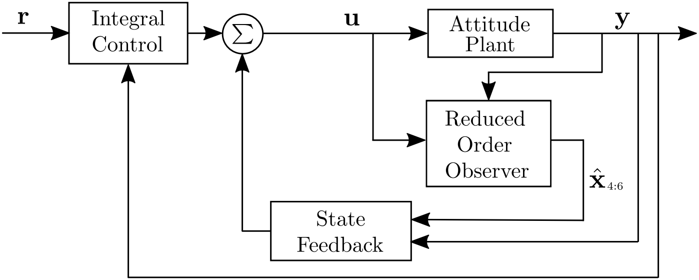
- System Representation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$



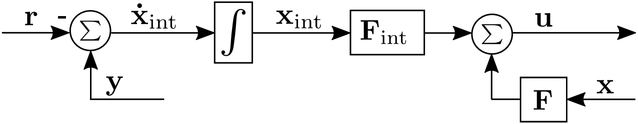
$$\mathbf{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

Control Solution
Attitude Controller



- State Feedback with Integral Control

$$\dot{\mathbf{x}}_{\mathrm{Int}}(t) = \mathbf{y}(t) - \mathbf{r}(t)$$



$$\mathbf{u}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{F}_{\mathrm{Int}}\mathbf{x}_{\mathrm{Int}}(t)$$

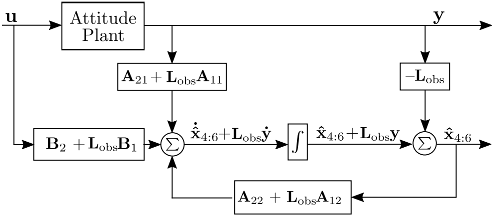
- LQR

$$\mathcal{J} = \int_0^\infty \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} \; dt$$

- Bryson's Rule

$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } [x_i^2]}$$
$$R_{ii} = \frac{1}{\text{maximum acceptable value of } [u_i^2]}$$

- Reduced Order Observer



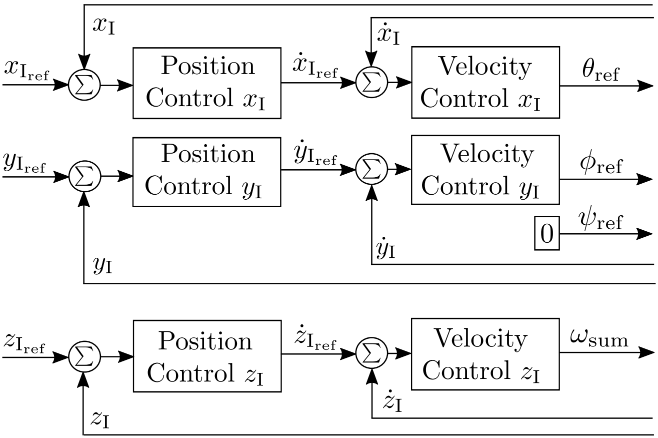
$$\mathbf{x}_{4:6} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

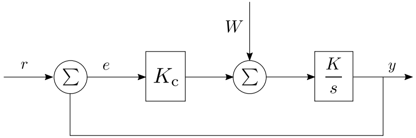
$$\mathbf{A}_{22} + \mathbf{L}_{\text{obs}}\mathbf{A}_{12}$$

Agenda

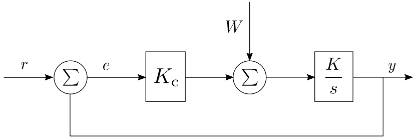
Control Solution

Translational Controller

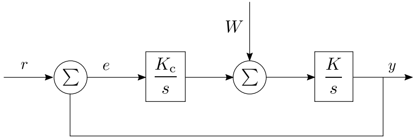



W
r
Σ
e
K_c
Σ
$\frac{K}{s}$
y

Control Solution
Translational Controller

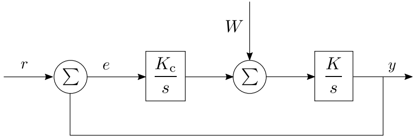


$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \to 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \to 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$

Control Solution
Translational Controller



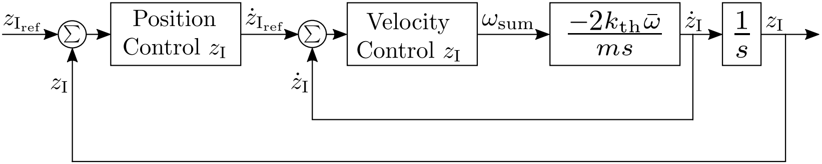
$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + \frac{K_c}{s}\frac{K}{s}} = \frac{Ks}{s^2 + K_c K} \Rightarrow \lim_{s \to 0} \frac{Ks}{s^2 + K_c K} = 0$$

Control Solution
Translational Controller

$$\frac{\dot{x}_\mathrm{I}}{\theta} = \frac{-k_\mathrm{th} 4\overline{\omega}^2}{ms} \qquad \frac{\dot{y}_\mathrm{I}}{\phi} = \frac{k_\mathrm{th} 4\overline{\omega}^2}{ms} \qquad \frac{\dot{z}_\mathrm{I}}{\omega_\mathrm{sum}} = \frac{-2k_\mathrm{th}\overline{\omega}}{ms}$$

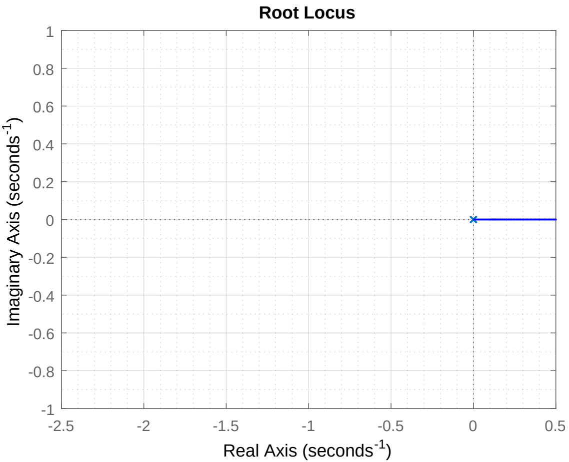
Control Solution
Translational Controller

$$\frac{\dot{x}_{\mathrm{I}}}{\theta} = \frac{-k_{\mathrm{th}} 4\overline{\omega}^2}{ms} \qquad \frac{\dot{y}_{\mathrm{I}}}{\phi} = \frac{k_{\mathrm{th}} 4\overline{\omega}^2}{ms} \qquad \frac{\dot{z}_{\mathrm{I}}}{\omega_{\mathrm{sum}}} = \frac{-2k_{\mathrm{th}}\overline{\omega}}{ms}$$

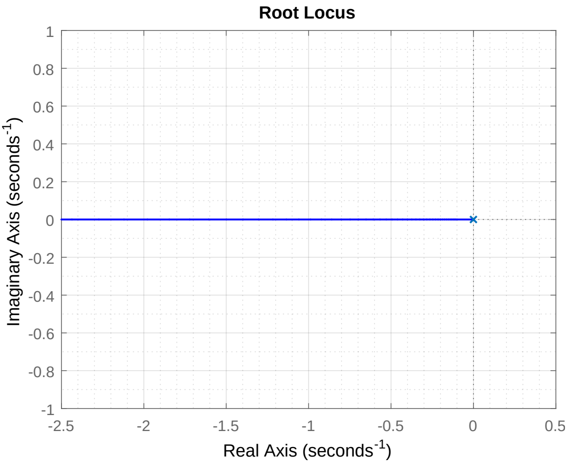


Control Solution

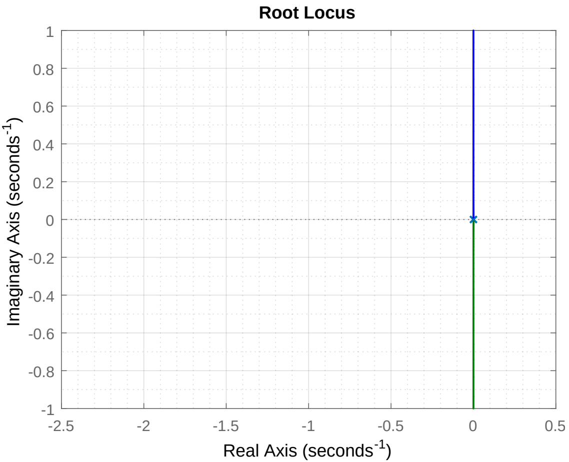
Translational Controller



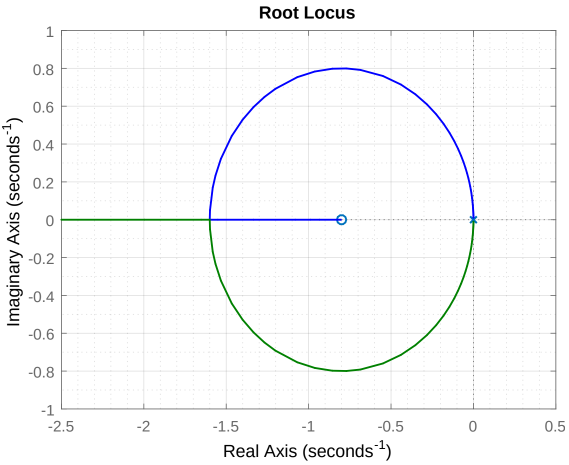
Control Solution
Translational Controller



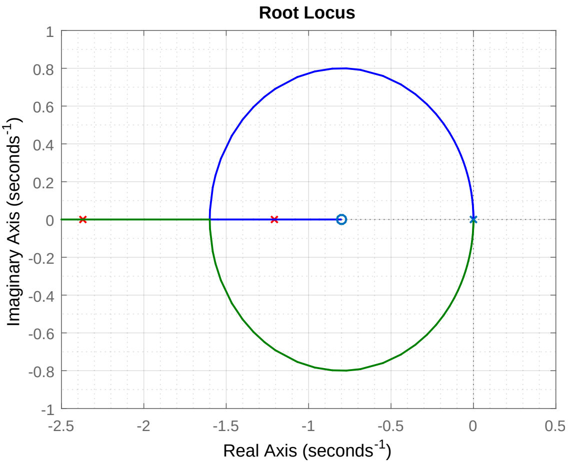
Control Solution
Translational Controller



Control Solution
Translational Controller



Control Solution
Translational Controller



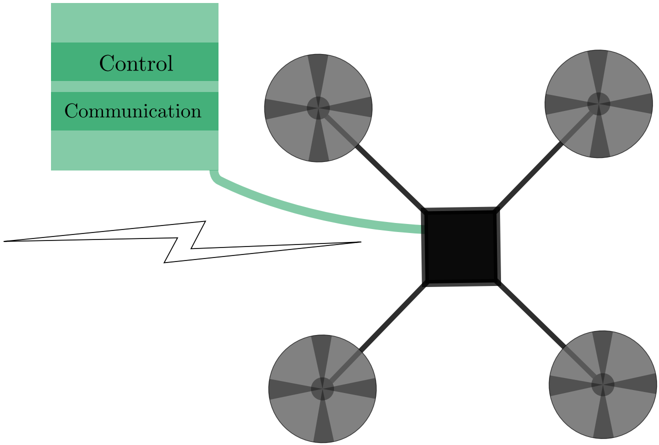
Control Solution
Translational Controller

$$C_{\dot{x}_{\mathrm{I}}} = -0.0038\frac{1+20s}{s} \qquad C_{x_{\mathrm{I}}} = 0.3$$

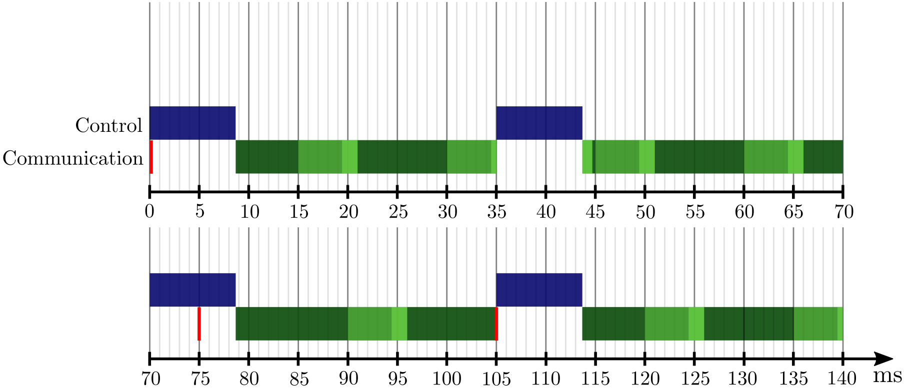
$$C_{\dot{y}_{\mathrm{I}}} = 0.0038\frac{1+20s}{s} \qquad C_{y_{\mathrm{I}}} = 0.3$$

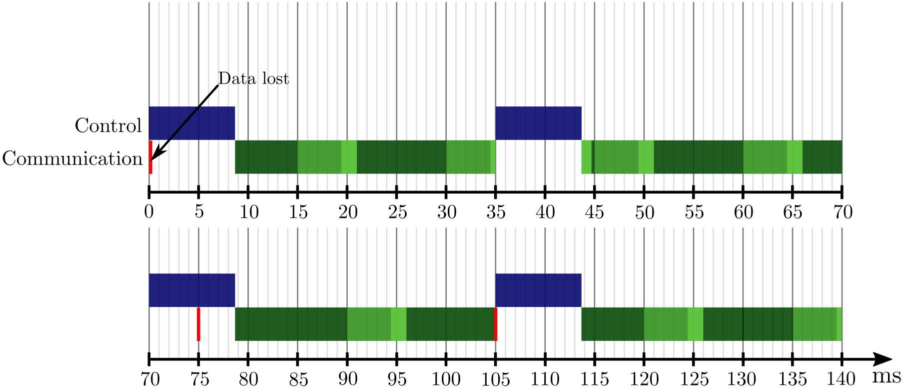
$$C_{\dot{x}_{\mathrm{I}}} = -280\frac{s+0.8}{s} \qquad C_{z_{\mathrm{I}}} = 1$$

Implementation



Implementation
Schedule

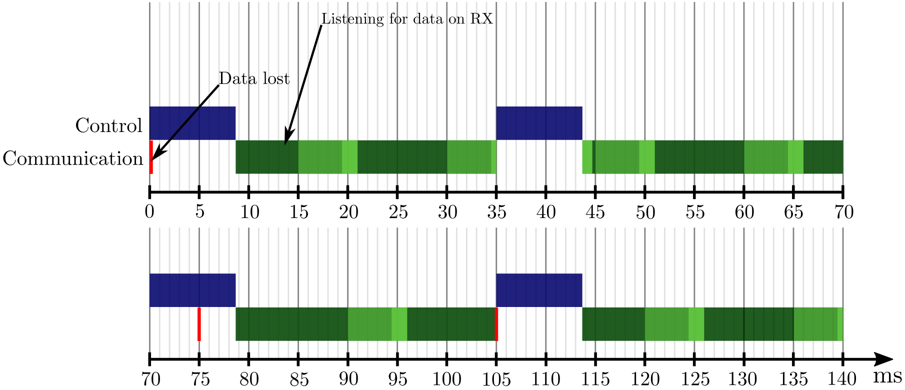


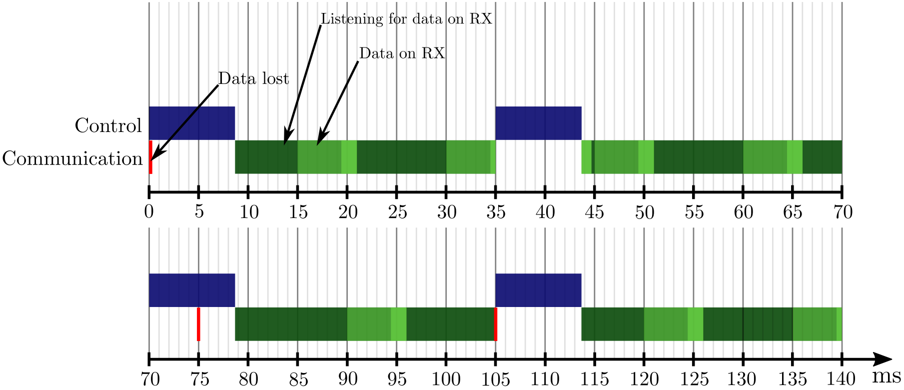


Data lost
Control
Communication
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
125
130
135
140
ms

Implementation

Schedule

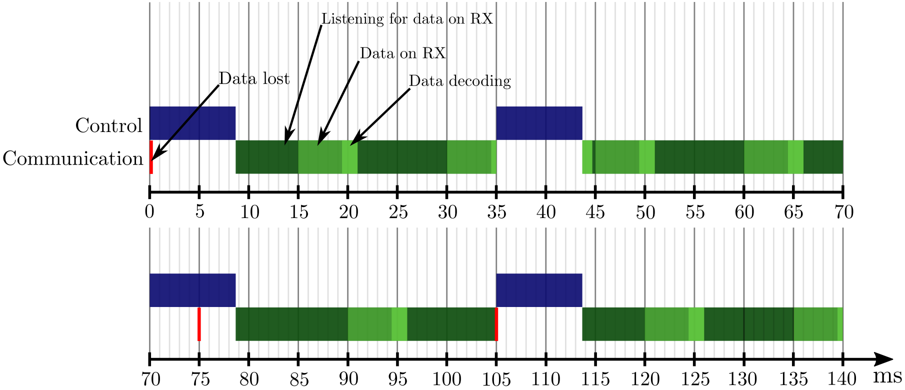




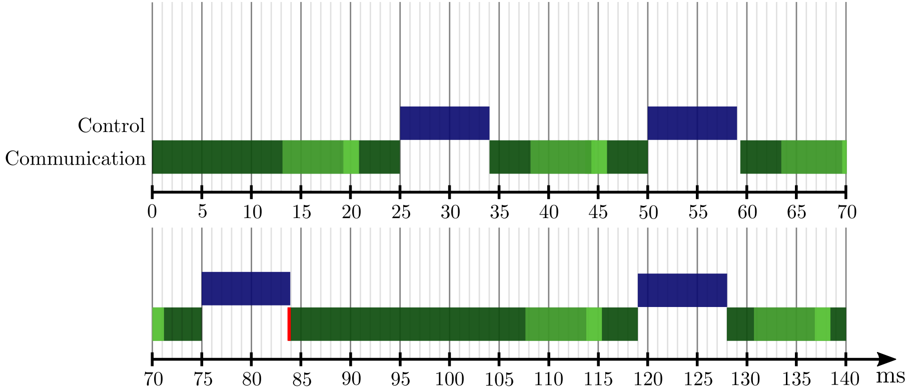
Listening for data on RX
Data on RX
Data lost
Control
Communication
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70
70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
ms

Implementation

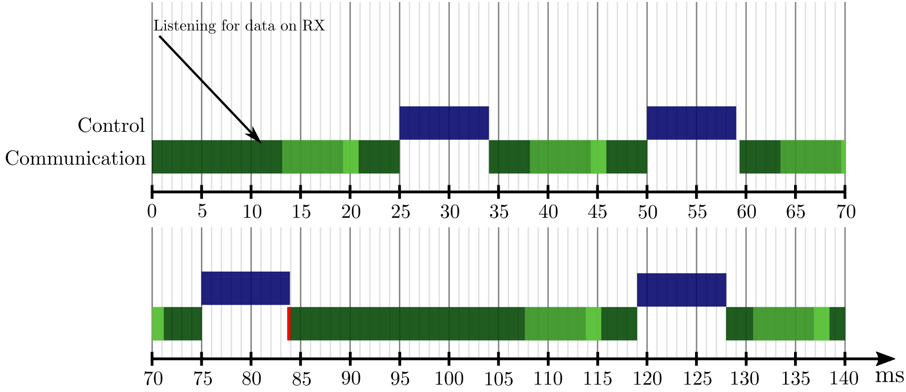
Schedule



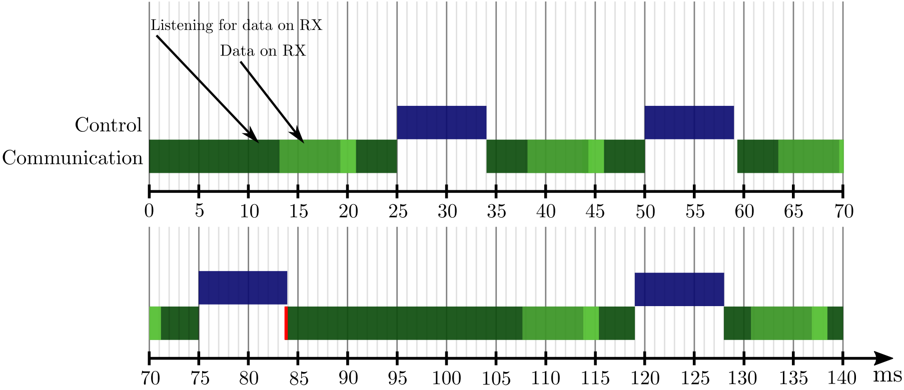
Implementation
Schedule

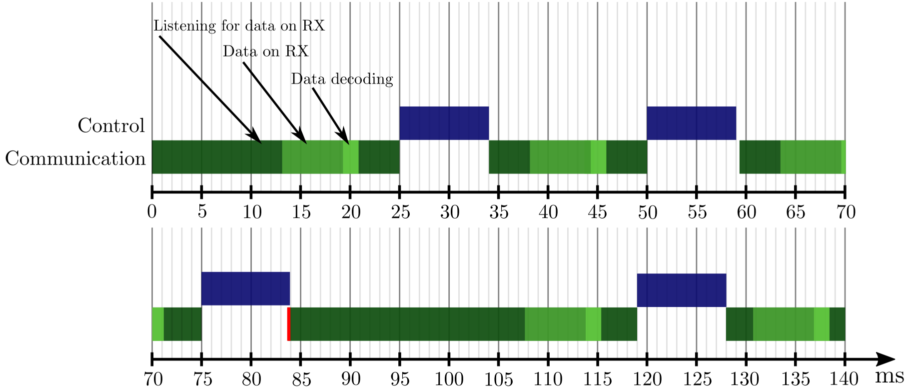


Implementation
Schedule



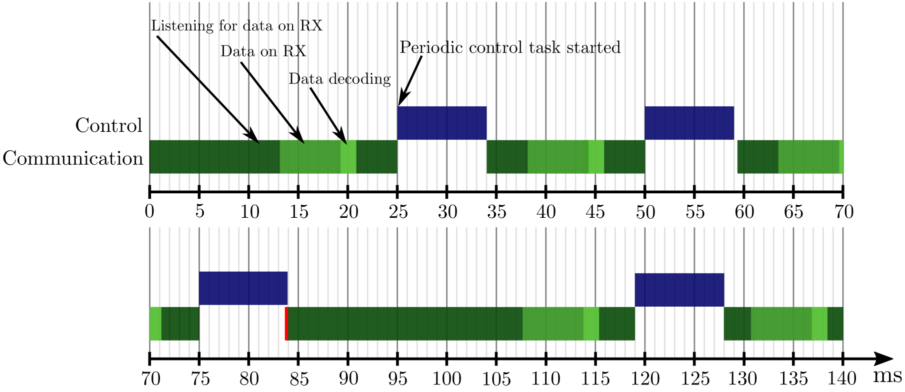
Implementation
Schedule



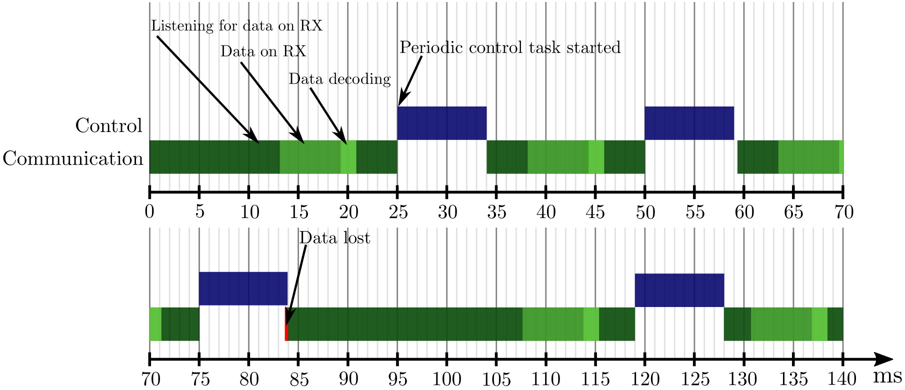


Listening for data on RX
Data on RX
Data decoding
Control
Communication
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
125
130
135
140
ms

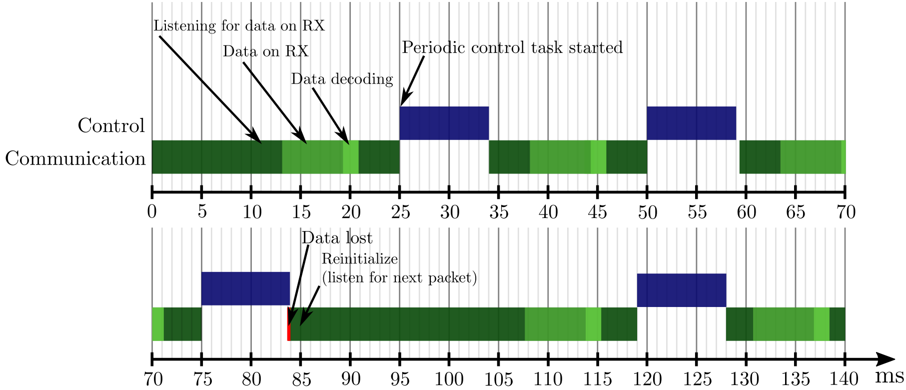
Implementation
Schedule



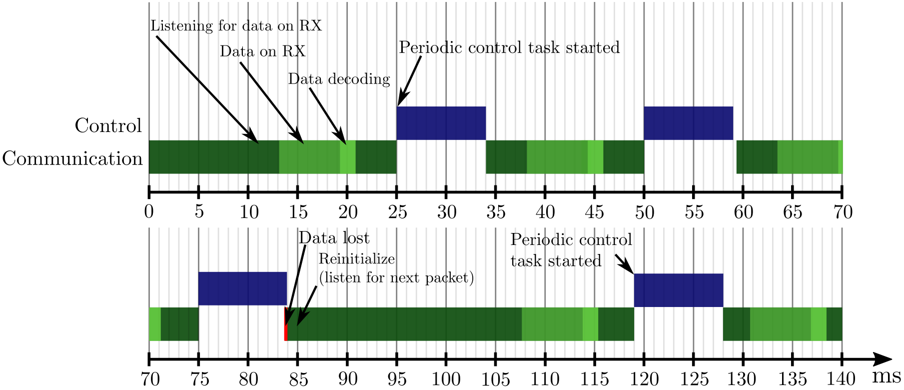
Implementation
Schedule



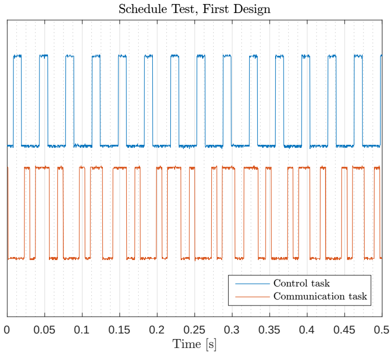
Implementation
Schedule

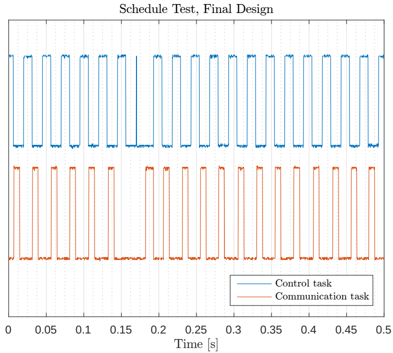


Implementation
Schedule



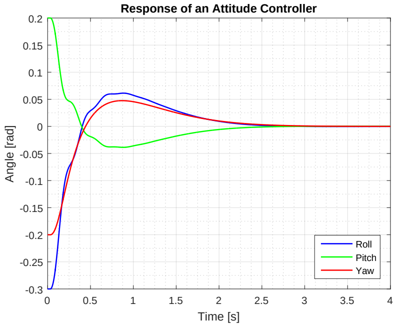
Implementation
Schedule

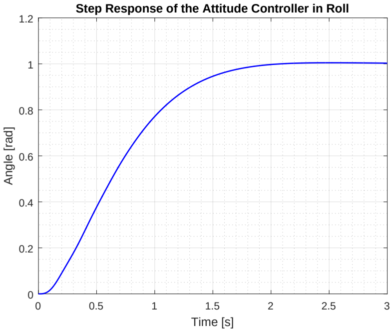




Agenda

Results
Attitude Controller Simulations

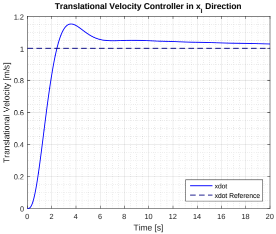


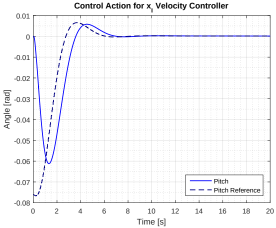


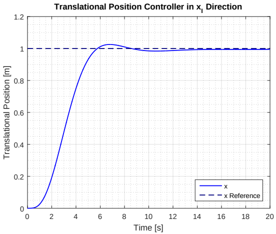
Results

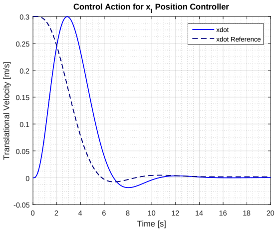
Translational Controllers Simulations

- x_{I} (and y_{I}) controllers for velocity and position

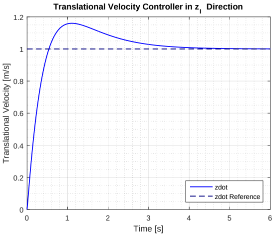


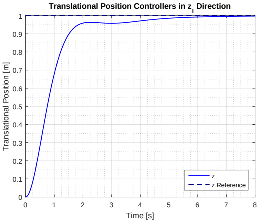


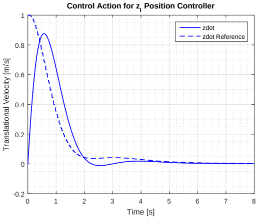




- z_{I} controllers for velocity and position

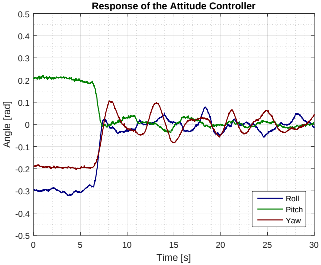


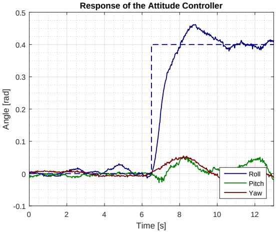


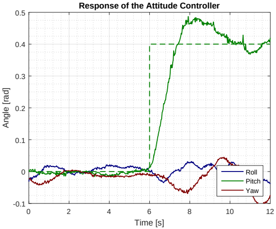


Results

Attitude Controller Functional Tests







Final Statements

- Limited bandwidth of controllers due to sampling.
- Control design working in simulation for attitude and translational controller.
- Implementation/Hardware issue when testing the translational controllers.
- Attitude controller tests carried out successfully.

Attitude and Position Control of a Quadcopter in a Networked Distributed System