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Backstepping based Nonlinear Flight Control Strategy for 6 DOF Aerial Robot

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Abstract: In this paper a nonlinear model of a 6-DOF quad rotor aerial robot is derived, based on Newton-Euler formalism, and backstepping based PID flight control strategy is implemented for motion control of the derived model. The derivation comprises determining equations of motion of quad rotor aerial robot in three dimensions and seeking to approximate actuation forces through modeling of the aerodynamic coefficients and electric motor dynamics.
The derived MIMO model, constituted of translational and rotational subsystem, is dynamically unstable. A nonlinear control strategy that includes integrator backstepping control for the translational subsystem and backstepping based PID control for the rotational subsystem is implemented for the quad rotor aerial robot. The stability of the control design is ensured by Lyapunov global stability theorem. The performance of the nonlinear control algorithm is evaluated using nonlinear simulation. Results from nonlinear simulation validate effectiveness of the designed control strategy for quad rotor aerial robot near quasi stationary (hover or near hover) flight.

Keywords: Quad rotor aerial robot, Newton-Euler formalism, MIMO Model, Integrator backstepping, Backstepping based PID control.

1. INTRODUCTION

Quad rotor aerial robots exhibit a number of important physical effects such as aerodynamic effects, inertial counter torques, gravity effect, gyroscopic effects and friction etc. Due to these effects it is difficult to design a real-time control for aerial robots.

Quad rotor aerial robot is a highly nonlinear, multivariable, strongly coupled and underactuated system since they have six degrees of freedom (position (x, y, z), pitch, roll and yaw) and only four control inputs (pitching, rolling and yaw moments and main thrust).

The free body diagram and axes of quad rotor aerial robot is as shown in Fig. 1.

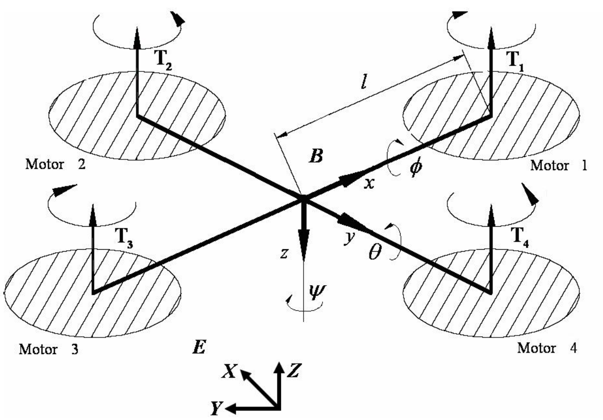

Fig. 1: Forces and moments acting on quad rotor aerial robot.

Where l represents distance of motor from pivot centre. ϕ, θ and ψ represent Euler angles about x, y, z body axis respectively. T_n represents Thrust force produced by each propeller, for $n = [1,2,3,4]$. Earth fixed frame is represented by $E = \{X, Y, Z\}$ and body fixed frame is represented by $B = \{x, y, z\}$.

Increasing or decreasing speed of the four motors together generates vertical motion. When motor pair (3, 1) is allowed to operate independently then the pitch angle θ (rotation about the y-axis) can be controlled along with the indirect control of motion along the same axis. Similarly when motor pair (2, 4) is allowed to operate independently then the roll angle ϕ (rotation about the x-axis) can be controlled along with the indirect control of motion along the same axis. Finally when motor pair (3, 1) is rotating clockwise and motor pair (2, 4) rotating counter-clockwise, the yaw angle ψ (rotation about the z-axis) can be controlled. The quad rotor aerial robot has now six degrees of freedom.

In recent works, McKerrow [1] provided a theoretical analysis of a 6 DOF quad rotor. Pounds et al. [2] designed a control structure based on internal linearization while Tayebi et al. [3] developed a quaternion based PD feedback control scheme for attitude stabilization of quad rotor. However most of the authors treated the underactuated quad rotor in such a manner that the underactuated control problem is degenerated to a full actuation one.

When the roll and yaw angles are set to zero, a hovering quad rotor can be viewed as a Planer Vertical Takeoff and Landing (PVTOL) aircraft. Therefore based on the dynamics of a PVTOL aircraft, Castillo et al. [4] and Hamel et al. [5] designed controller for yaw angular displacement and pitch and roll movements of a hovering quad rotor aerial robot. Mian et al. [6] provided a complete mathematical model for a 6 DOF underactuated quad rotor aerial robot and used affine nonlinear control method for the aerial robot. In this paper backstepping based PID flight control strategy is implemented for rotational subsystem of the quad rotor aerial robot. Integrator action in backstepping was proposed for linear systems by Kanellakopoulos et al. [7] and Krstic et al. [8].

The main contribution of this paper is to obtain a complete dynamic model of quad rotor aerial robot, based on the works of Koo et al. [9], and design integrator backstepping control for translational subsystem and backstepping based PID control for the rotational subsystem of quad rotor aerial robot. Lyapunov stability theorem is used to ensure negative definiteness of the system. Results from the nonlinear simulation validate the effectiveness of the proposed control strategy for quad rotor aerial robot near quasi stationary flight.

The main advantages of quad rotor aerial robot are that they have small rotors and can be enclosed, making them safer for indoor flights. Quad rotors have higher payload capacity and better maneuverability. The main disadvantages of quad rotor are high energy requirements and size.

2. QUAD ROTOR DYNAMICS

The aerodynamic forces and moments are derived using a combination of momentum and blade element theory, Castillo et al. [10] and Prouty [11]. The main forces and moments acting on quad rotor are those produced by propellers. Two propellers in the system are counter rotating propellers such that total torque of the system is balanced. Quad rotor has four motors with propellers. A voltage applied to each motor results in a net torque being applied to the rotor shaft, Q_i, which results in a thrust, T_i. If the rotor disk is moving, there is a difference in relative velocity between the blade and air when moving through the forward and backward sweep, resulting in a net moment about the roll axis, R_i. Forward velocity also causes a drag force on the rotor that acts opposite to the direction of travel, D_i. Thrust and drag can be defined in terms of aerodynamic coefficients C_T and C_D as,

$$T = C_T \rho A r^2 \Omega^2 \tag{1}$$

$$D = C_D \rho A r^2 \Omega^2 \tag{2}$$

where A is blade area, ρ is density of air, r is radius of the blade and Ω is the angular velocity of propeller.

Similarly torque Q and rolling moment R, in terms of aerodynamic coefficients C_Q and C_R, are defined as,

$$Q = C_Q \rho A r^2 \Omega^2 r \tag{3}$$

$$R = C_R \rho A r^2 \Omega^2 r \tag{4}$$

Then total forces, $\mathbf{f}_{\mathbf{total}}$, and moments, $\boldsymbol{\tau}_{\mathbf{total}}$, acting on quad rotor in body frame are given by,

$$\mathbf{f}_{\mathbf{total}} = -\frac{1}{2} C_{x,y,z} A_c \rho \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}^2 - mgZ + \sum_{i=1}^{4} T_i z - \sum_{i=1}^{4} D_i (x \;\; y) \tag{5}$$

$$\boldsymbol{\tau}_{\mathbf{total}} = \begin{bmatrix} (-1)^i \sum_{i=1}^{4} Q_i z + (-1)^{i+1} \sum_{i=1}^{4} R_i (x \;\; y) + \\ h \sum_{i=1}^{4} D_i (-y \;\; x) + (T_4 - T_2) l x + (T_3 - T_1) l y \\ + [((D_2 - D_4) + (D_3 - D_1)) l] z \end{bmatrix} \tag{6}$$

where the first term in Eq. (5) represents the friction force on quad rotor body during horizontal motion with $C_{x,y,z}$ representing longitudinal drag coefficients, A_c representing fuselage area and $\dot{x}, \dot{y}, \dot{z}$ representing speeds in x, y and z direction respectively, Z defines the vertical axis in inertial coordinates and vector $(x \;\; y)$ defines the direction of velocity. h is vertical distance between propeller centre and CG of quad rotor, m represents total mass of quad rotor while g represents force due to gravity.

Now consider quad rotor as a single rigid body with 6 DOF. Assuming that earth is flat and neglecting ground effect, the equations of motion for a rigid body subject to body force, $\mathbf{f}^{\mathrm{b}} \in \Re^3$, and body moment, $\boldsymbol{\tau}^{\mathrm{b}} \in \Re^3$, applied at the center of mass and expressed in Newton-Euler formalism, as shown in [9], are given by,

$$\begin{bmatrix} m\mathbf{I} & 0 \\ 0 & \mathbf{J} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}^b \\ \dot{\boldsymbol{\omega}}^b \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^b \times m\mathbf{v}^b \\ \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b \end{bmatrix} = \begin{bmatrix} \mathbf{f}^b \\ \boldsymbol{\tau}^b \end{bmatrix} \tag{7}$$

where $\mathbf{v}^b \in \Re^3$ is body velocity vector, $\boldsymbol{\omega}^b \in \Re^3$ is body angular velocity vector, $m \in \Re$ specifies total mass, $\mathbf{I} \in \Re^{3\times3}$ is an identity matrix, and $\mathbf{J} \in \Re^{3\times3}$ is an inertial matrix.

2.1 Rotational Dynamics

Assuming that the inertia tensor is diagonal (symmetric design of quad rotor), the moment equations governing the quad rotor are given by,

$$\boldsymbol{\tau}^b = \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b + \boldsymbol{\tau}_{\mathbf{total}} \tag{8}$$

From Eq. (8) and Eq. (6), rotational dynamics of quad rotor in body axis are given by,

$$J_x \ddot{\phi} = \begin{bmatrix} \dot{\theta}\dot{\psi}(J_y - J_z) + l(T_4 - T_2) + \\ \sum_{i=1}^{4} (-1)^{i+1} R_{xi} - h \sum_{i=1}^{4} D_{yi} \end{bmatrix} \tag{9}$$

$$J_y\ddot{\theta} = \begin{bmatrix} \dot{\phi}\dot{\psi}(J_z - J_x) + l(T_3 - T_1) + \\ \sum_{i=1}^{4}(-1)^{i+1}R_{yi} + h\sum_{i=1}^{4}D_{xi} \end{bmatrix} \quad (10)$$

$$J_z\ddot{\psi} = \begin{bmatrix} \dot{\phi}\dot{\theta}(J_x - J_y) + \sum_{i=1}^{4}(-1)^i Q_i + \\ \left((D_{x2} - D_{x4}) + (D_{y3} - D_{y1})\right)l \end{bmatrix} \quad (11)$$

where R_x, R_y represent rolling moment due to forward and sideward flight while $h(D_x), h(D_y)$ represent drag moment due to forward and sideward flight respectively, $(D_{x2} - D_{x4})$ and $(D_{y3} - D_{y1})$ represent drag force unbalance in forward and sideward flight respectively.

2.2 Translational Dynamics

Effect of body moments on the translational dynamics is neglected. Then the translational dynamics governing quad rotor in inertial axes are given by,

$$m\ddot{X} = \begin{bmatrix} (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\sum_{i=1}^{4}T_i - \\ \sum_{i=1}^{4}D_i - \frac{1}{2}C_x A_c \rho\dot{x}|\dot{x}| \end{bmatrix} \quad (12)$$

$$m\ddot{Y} = \begin{bmatrix} (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\sum_{i=1}^{4}T_i - \\ \sum_{i=1}^{4}D_i - \frac{1}{2}C_y A_c \rho\dot{y}|\dot{y}| \end{bmatrix} \quad (13)$$

$$m\ddot{Z} = -mg + (\cos\phi\cos\theta)\sum_{i=1}^{4}T_i - \frac{1}{2}C_z A_c \rho\dot{z}|\dot{z}| \quad (14)$$

3. ENGINE MODEL

On the electrical side of DC motor, a current I_a flows through the armature according to drive voltage V_a, the motor's inductance L_m, resistance R_m and back emf voltage V_{emf}, then,

$$V_a - V_{emf} = L_m\frac{dI_a}{dt} + R_m I_a \quad (15)$$

Motor converts electrical armature current into a mechanical torque applied to shaft, $T_m = K_{tm}I_a$. The applied torque produces angular velocity ω_m according to inertia J_m and motor load T_l, given by,

$$T_m = J_m\frac{d\omega_m}{dt} + T_l \quad (16)$$

Defining $V_{emf} = K_e\omega_m$, neglecting inductance of the small motor and introducing propeller and gearbox models, then from Eq. (15) and Eq. (16) we have,

$$\dot{\omega}_m = -\frac{K_{tm}K_e}{R_m J_m}\omega_m - \frac{d}{\eta r_g^3 J_m}\omega_m^2 + \frac{K_{tm}}{R_m J_m}V_a \quad (17)$$

where η is gear box efficiency, d is drag factor and r_g is gear box reduction ratio.

4. CONTROL STRATEGY

A nonlinear control strategy is implemented to stabilize the quad rotor near quasi stationary flight. Since we are considering flight near hover so some of the terms, in translational and rotational dynamics, associated with vehicular velocity can be neglected. Furthermore thrust and torque coefficients are supposed to be constant i.e.,

$$\begin{aligned} T_i &= b\Omega_i^2 \\ Q_i &= d\Omega_i^2 \end{aligned} \quad (18)$$

where b and d are thrust and drag factors respectively.

Thus the inputs to quad rotor i.e. vertical force input u_1, roll input u_2, pitch input u_3 and yaw input u_4 are defined as,

$$\left.\begin{aligned} u_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ u_2 &= b(\Omega_4^2 - \Omega_2^2) \\ u_3 &= b(\Omega_3^2 - \Omega_1^2) \\ u_4 &= d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{aligned}\right\} \quad (19)$$

The control strategy is implemented such that the altitude of the quad rotor is stabilized by using the vertical force input u_1. The desired roll and pitch angles are generated to the rotational controller, from position subsystem. The rotational controller is used to stabilize the quad rotor near quasi stationary conditions with control inputs u_2, u_3 and u_4.

4.1 Altitude control

The altitude subsystem of the quad rotor is given by Eq. (14). An integrator backstepping control technique is implemented for altitude subsystem.

Altitude tracking error is defined as:

$$e_{a1} = Z_d - Z \quad (20)$$

where Z_d represents desired altitude.

Taking derivate of tracking error,

$$\dot{e}_{a1} = \dot{Z}_d - \dot{Z} \quad (21)$$

There is no control input in Eq. (21). $\dot{Z}$ represents altitude rate of the quad rotor and has its own dynamics. Let us consider $\dot{Z}$ be the virtual control. Defining the desired virtual control $(\dot{Z})_d$ as,

$$(\dot{Z})_d = c_{a1}e_{a1} + K_{a1}\Gamma_1 + \dot{Z}_d \quad (22)$$

where c_{a1} and K_{a1} are positive constants for increasing the convergence speed of the altitude tracking loop and Γ_1 represents integral of altitude error, given by, $\Gamma_1 = \int e_{a1}dt$.

Now the virtual control $(\dot{Z})_d$ represents the altitude rate of quad rotor and has its own error given by,

$$e_{a2} = (\dot{Z})_d - \dot{Z} = (c_{a1}e_{a1} + K_{a1}\Gamma_1 + \dot{Z}_d) - \dot{Z} \tag{23}$$

From Eq. (21) and Eq. (22),

$$\dot{e}_{a1} = -c_{a1}e_{a1} - K_{a1}\Gamma_1 + e_{a2} \tag{24}$$

Taking derivative of Eq. (23),

$$\dot{e}_{a2} = \begin{bmatrix} \left[c_{a1}(-c_{a1}e_{a1} - K_{a1}\Gamma_1 + e_{a2}) + K_{a1}e_{a1} + \ddot{Z}_d\right] - \\ \left[-g + \left(\dfrac{\cos\phi\cos\theta}{m}\right)u_1\right] \end{bmatrix} \tag{25}$$

The desirable dynamics of $\dot{e}_{a2}$ are,

$$\dot{e}_{a2} = -c_{a2}e_{a2} - e_{a1} \tag{26}$$

Eq. (25) will be negative if the real control input u_1 is then given by,

$$u_1 = \frac{m}{\cos\phi\cos\theta}\begin{bmatrix} g + e_{a1} - c_{a1}^2 e_{a1} + K_{a1}e_{a1} + \\ c_{a1}e_{a2} + c_{a2}e_{a2} + \ddot{Z}_d - c_{a1}K_{a1}\Gamma_1 \end{bmatrix} \tag{27}$$

Stability analysis of the proposed method is performed using Lyapunov theory. The candidate Lyapunov function chosen is,

$$V = K_1\frac{1}{2}\Gamma_1^2 + \frac{1}{2}e_{a1}^2 + \frac{1}{2}e_{a2}^2 \tag{28}$$

Taking derivative of Eq. (28) and from Eq. (26) and Eq. (24), we obtain,

$$\dot{V} = -c_{a1}e_{a1}^2 - c_{a2}e_{a2}^2 \le 0 \tag{29}$$

Thus Eq. (29) is negative semi-definite. In order to assert global asymptotic stability of a system, consider Lyapunov global stability theorem, as in Li [12]. With the help of Lyapunov theorem we can ensure an asymptotical stability starting from a point in a set around the equilibrium. To ensure global asymptotic stability sufficient conditions are fulfilled in our case.

4.2 Position control

Let $\dot{x}_d$ and $\dot{y}_d$ be the desired speeds in x and y direction respectively. Then the error in desired and actual speeds is given by,

$$e_x = \dot{x}_d - \dot{x} \tag{30}$$

$$e_y = \dot{y}_d - \dot{y} \tag{31}$$

Desired roll and pitch angles, in term of the error between actual and desired speed, are thus given by,

$$\phi_d = \sin^{-1}(u_{ex}\sin\psi - u_{ey}\cos\psi) \tag{32}$$

$$\theta_d = \sin^{-1}\left[\frac{u_{ex}}{\cos\phi\cos\psi} - \frac{\sin\phi\sin\psi}{\cos\phi\cos\psi}\right] \tag{33}$$

where u_{ex} and u_{ey} are given by,

$$u_{ex} = \frac{K_x e_x m}{u_1}, \quad u_{ey} = \frac{K_y e_y m}{u_1}$$

where K_x and K_y are positive constants and u_1 is the desired vertical force input from altitude control.

4.3 Rotational control

Backstepping based PID control technique is implemented for rotational subsystem in which the control inputs u_2, u_3 and u_4 control the quad rotor at hover.

Let the roll tracking error be defined as,

$$e = \phi - \phi_d \tag{34}$$

The first error to be considered in the backstepping design is,

$$z_1 = K_1 e + K_2\int e\,dt \tag{35}$$

where K_1 and K_2 are positive tuning parameters and $\int e\,dt$ represents integral of roll error.

Lyapunov theorem is considered by using the Lyapunov function z_1 positive definite and its time derivative negative semi definite as,

$$V_1 = \frac{1}{2}z_1^2 \tag{36}$$

The derivative of Eq. (36) is given by,

$$V_1 = z_1\dot{z}_1 = z_1(K_1\dot{\phi} - K_1\dot{\phi}_d + K_2 e) \tag{37}$$

There is no control input in Eq. (37). Let us consider $\dot{\phi}$ as the virtual control. Then the desired virtual control $(\dot{\phi})_d$ is defined as,

$$(\dot{\phi})_d = \dot{\phi}_d - \frac{K_2}{K_1}e - \frac{c_1 z_1}{K_1} \tag{38}$$

where c_1 is positive constant for increasing the convergence speed of the roll tracking loop.

Now the virtual control $\dot{\phi}$ represents the roll rate of quad rotor and has its own error given by,

$$z_2 = \dot{\phi} - (\dot{\phi})_d = \frac{1}{K_1}[\dot{z}_1 + c_1 z_1] \tag{39}$$

The augmented Lyapunov function for the second step is given by,

$$V_2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{40}$$

And its derivative is given by,

$$\dot{V}_2 = z_1\dot{z}_1 + z_2\dot{z}_2 \tag{41}$$

Putting $\dot{z}_1$ and $\dot{z}_2$ in Eq. (41) we have,

$$\dot{V}_2 = \begin{bmatrix} z_2\begin{bmatrix} e\left(K_1^2 + \dfrac{c_1K_2}{K_1}\right) + \int edt(K_1K_2) + \dot{e}\left(\dfrac{K_2}{K_1} + c_1\right) + \\ \dot{\theta}\dot{\psi}\left(\dfrac{J_y - J_z}{J_x}\right) + \dfrac{l}{J_x}u_2 - \ddot{\phi}_d \end{bmatrix} \\ -z_1\left[c_1K_1e + c_1K_2\int edt\right] \end{bmatrix} \tag{42}$$

The desirable dynamics are,

$$\dot{V}_2 = -c_2 z_2 = -\frac{c_2}{K_1}[\dot{z}_1 + c_1 z_1] \tag{43}$$

where c_2 is a positive tuning parameter.

From Eq. (43),

$$\dot{V}_2 = -\dot{e}(c_2) - e\left(\frac{c_2K_2}{K_1} + c_1c_2\right) - \int edt\left(\frac{c_2c_1K_2}{K_1}\right) \tag{44}$$

The desirable dynamics ensure negative definiteness of position tracking error, its integration and velocity tracking error.

The derivate of Eq. (42) is negative if,

$$u_2 = \frac{J_x}{l}\begin{bmatrix} -e\left(\frac{c_2}{K_1}K_2 + c_2c_1 + K_1^2 + \frac{K_2c_1}{K_1}\right) - \\ \int edt\left(\frac{c_2c_1K_2}{K_1} + K_1K_2\right) - \\ \dot{e}\left(c_2 + \frac{K_2}{K_1} + c_1\right) + \ddot{\phi}_d - \dot{\theta}\dot{\psi}\left(\frac{J_y - J_z}{J_x}\right) \end{bmatrix} \tag{45}$$

In regulation, Eq. (45) is a PID where the gains of each mode are given by, $P = \left(\frac{c_2}{K_1}K_2 + c_2c_1 + K_1^2 + \frac{K_2c_1}{K_1}\right)$, $D = \left(c_2 + \frac{K_2}{K_1} + c_1\right)$ and $I = \left(\frac{c_2c_1K_2}{K_1} + K_1K_2\right)$.

Let us consider the characteristic equation of regulation dynamics. The rotational subsystem is both observable and controllable, therefore, pole placement technique is used by placing the poles at desired location to solve for roots of the characteristic equation. Selecting larger values for c_1 and c_2 makes derivative of the Lyapunov function more negative and thus making the regulation dynamics faster.

Similarly u_3 and u_4 are obtained as,

$$u_3 = \frac{J_y}{l}\begin{bmatrix} -e\left(\frac{c_4K_4}{K_3} + c_4c_3 + K_3^2 + \frac{K_4c_3}{K_3}\right) - \\ \int edt\left(\frac{c_4c_3K_4}{K_3} + K_3K_4\right) - \\ \dot{e}\left(c_4 + \frac{K_4}{K_3} + c_3\right) + \ddot{\theta}_d - \dot{\phi}\dot{\psi}\left(\frac{J_z - J_x}{J_y}\right) \end{bmatrix} \tag{46}$$

$$u_4 = \frac{J_z}{l}\begin{bmatrix} -e\left(\frac{c_6K_6}{K_5} + c_6c_5 + K_5^2 + \frac{K_6c_5}{K_5}\right) - \\ \int edt\left(\frac{c_6c_5K_6}{K_5} + K_5K_6\right) - \dot{e}\left(c_6 + \frac{K_6}{K_5} + c_5\right) + \ddot{\psi}_d \end{bmatrix} \tag{47}$$

Table 1 summarizes different system parameters of the prototype quad rotor aerial robot.

Table 1: Physical parameters of quad rotor

Parameter	Value	Units
l	0.3050	m
J_x	0.0154	Kg m^2
J_y	0.0154	Kg m^2
J_z	0.0309	Kg m^2
m	0.6150	kg

5. RESULTS AND DISCUSSION

The closed loop system is simulated with nonlinear control algorithm. The angles and their time derivatives of rotational subsystem do not depend on translational components, as evident from 6-DOF equations, however the translations depend on the angles.

Rotational control keeps the 3D orientation of the quad rotor aerial robot to the desired value. Roll and pitch angles are usually forced to zero which permits hovering flight. The rotational controller task is to compensate the initial error, stabilize roll, pitch and yaw angles and maintain them at zero. This is accomplished with the backstepping based PID control strategy.

The initial conditions for nonlinear simulation are $\phi = \theta = \psi = 30°$ and $z = 1$ meters. The reference input to the controller are, $\dot{x}_d = \dot{y}_d = 0$, $z_d = 1$ and $\psi_d = 0$.

Fig. 2 shows the response of the nonlinear controller to stabilize the quad rotor at hover.

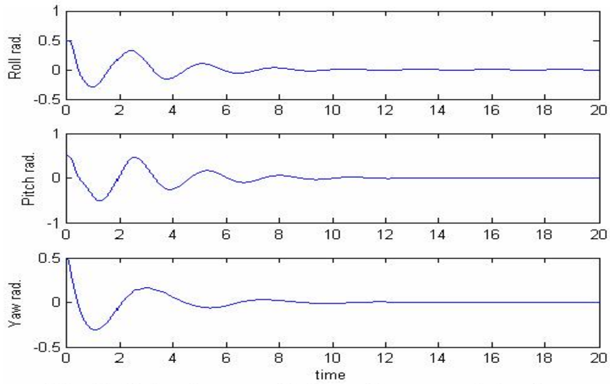

Fig. 2: Attitude control of quad rotor aerial robot

Although very hard initial conditions are used yet it can be seen that the controller effectively controls the attitude angles of quad rotor in less than 7 seconds.

The roll rate, pitch rate and yaw rate plots are as shown in Fig. 3.

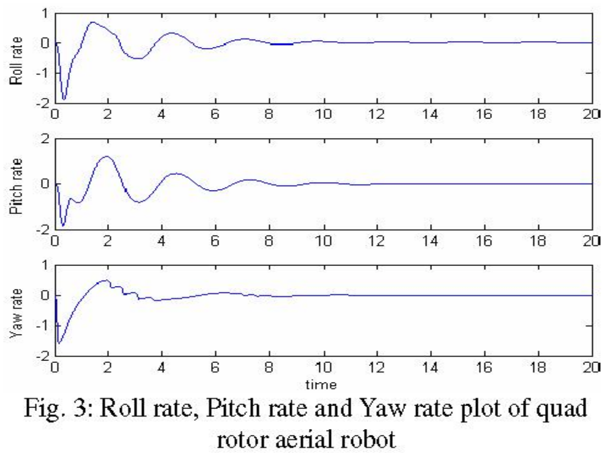

Fig. 3: Roll rate, Pitch rate and Yaw rate plot of quad rotor aerial robot

The altitude and position (x, y) control response of quad rotor aerial robot is given by Fig. 4.

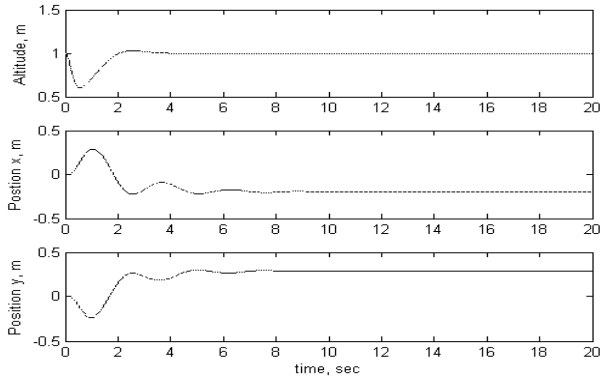

Fig. 4: Altitude and position control of quad rotor aerial robot

The altitude rate and position rate response of the quad rotor aerial robot is as shown in Fig. 5.

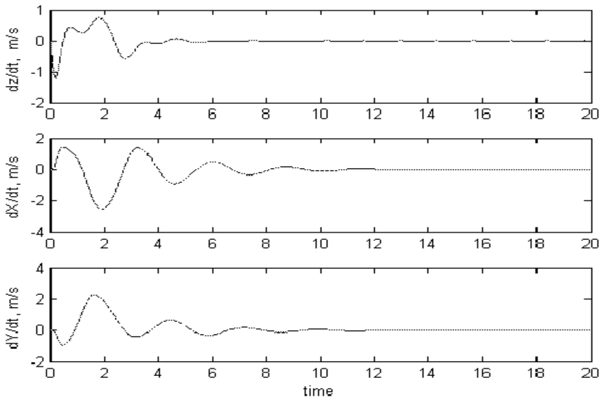

Fig. 5: Altitude rate and Position rate of quad rotor aerial robot

The control vector response of quad rotor aerial robot is as shown in Fig. 6.

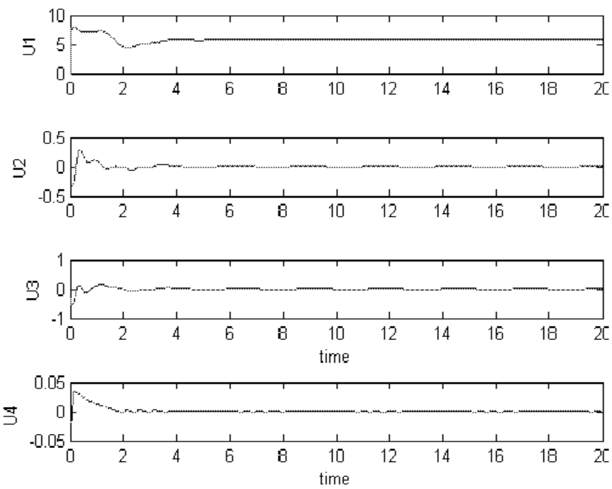

Fig. 6: control vector response of quad rotor aerial robot

6. CONCLUSION

We presented a stabilization nonlinear control method for quad rotor aerial robot. The modeling of the quad rotor is based on Newton-Euler formalism. A novel control strategy, in which the integral of the tracking error considered in the first step of the backstepping procedure, is applied to rotation subsystem of the quad rotor aerial robot. The control law derived for nonlinear quad rotor aerial robot is thus a backstepping based PID for regulation dynamics. The stabilization ability of the nonlinear controller is examined through nonlinear simulation and through Lyapunov global stability theorem. Results indicate effectiveness of the proposed control strategy for the quad rotor aerial robot.

REFERENCES

[1] P. McKerrow, "Modelling the Draganflyer four-rotor helicopter" Proceedings of the IEEE International Conference on Robotics and Automation, USA, pp. 25~53, 2004.

[2] P. Pounds, R. Mahony, P. Hynes and J. Roberts, "Design of a Four-Rotor Aerial Robot" Proceedings of Australian Conference on Robotics and Automation, Auckland, pp. 145~150, 2002.

[3] A. Tayebi, S. McGilvray, "Attitude stabilization of a four-rotor aerial robot" 43rd IEEE Conference on Decision and Control, Bahamas, pp 1216~1221, 2004.

[4] P. Castillo, A. Dzul, R. Lozaro, "Real-time stabilization and tracking of a four rotor mini rotorcraft, IEEE Transactions on Cotnrol System Technology, 2004.

[5] T. Hamel, R. Mahony, A. Chriette, "Visual servo trajectory tracking for a four rotor VTOL aerial vehicle:, Proceedings of IEEE International Conference on Robotics and Automation, 2002.

[6] A. A. Mian, D. Wang, "Dynamic Modeling and nonlinear control strategy for an underactuated quad rotor rotorcraft" doi: 10.1631/jzus. A071434, 2008.

[7] I. Kanellakopoulos and P. Krein, "Integral-action nonlinear control of induction motors," in Proceedings of the 12th IFAC World Congress, Sydney, pp. 251~254, 1993.

[8] M. Krstic, I. Kanellakopoulos and P. Kokotovic, Nonlinear and Adaptive Control Design, John Wiley & Sons, New York, pp. 87~122, 1995.

[9] T. J. Koo, Y. Ma and S. S. Sastry, "Nonlinear Control of a helicopter based Unmanned Aerial Vehicle Model" 37th IEEE, Conference on Decision and Control, Florida, pp. 1~25, 1998.

[10] P. Castillo, R. Lozano, A. Dzul, Modeling and Control of Mini-Flying Machines, Springer-Verlag, New York, pp. 23~59, 2005.

[11] R.W. Prouty, Rotorcraft Performance, Stability and Control, Krieger Publishing Company, Florida, pp. 443~639 1996.

[12] J.-J. E. Slotine, W. Li, Applied Nonlinear Control, Prentice Hall, New Jersey, pp. 40~126, 1991.