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Dynamic modeling and nonlinear control strategy for an underactuated quad rotor rotorcraft

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Abstract: In this paper, a nonlinear dynamic MIMO model of a 6-DOF underactuated quad rotor rotorcraft is derived based on Newton-Euler formalism. The derivation comprises determining equations of motion of the quad rotor in three dimensions and seeking to approximate the actuation forces through modeling of the aerodynamic coefficients and electric motor dynamics. The derived model is dynamically unstable, so a sequential nonlinear control strategy is implemented for the quad rotor. The control strategy includes exact feedback linearization technique, using the geometric methods of nonlinear control. The performance of the nonlinear control algorithm is evaluated using simulation and the results show the effectiveness of the proposed control strategy for the quad rotor rotorcraft near quasi-stationary flight.

Key words: Quad rotor rotorcraft, MIMO model, Underactuated systems
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INTRODUCTION

Control of underactuated mechanical systems has attracted many researchers in recent years. This is due to the broad range of real life applications of underactuated systems in Robotics (e.g., mobile robots, exible-link robots, snake-type robots, walking robots), Aerospace Vehicles (e.g., aircraft, spacecraft, helicopters, and satellites), and Marine Vehicles (e.g., surface vessels and underwater vehicles).

Quad rotor rotorcraft is a highly nonlinear, multivariable, strongly coupled and underactuated system (6 DOFs and only 4 actuators). Quad rotor has 4 propellers driven by motors. The main forces and moments acting on quad rotor are those produced by the propellers. The propellers generate a lift force that can be used to control pitch and roll angles. Total torque generated by the propeller motors causes a yaw to the body. Two propellers in the system are counter rotating propellers such that the total torque of the system is balanced.

Quad rotor rotorcraft achieves stable hovering and precise flight by balancing the forces (McKerrow, 2004). In (Pounds *et al*., 2002), the control structure was based on internal linearization while in (Tayebi and McGilvray, 2004), a quaternion based PD feedback control scheme is implemented for quad rotor. In this paper a complete dynamic model of quad rotor is derived based on the work of (Koo *et al*., 1998). A sequential nonlinear control strategy is implemented for the derived 6-DOF model, constituted of translational and rotational subsystems. The controller for translational subsystem stabilizes the altitude and generates the desired roll and pitch angles to the rotational subsystem controller. The rotational controller stabilizes the quad rotor near hover.

The rotational subsystem of the model is transformed into affine nonlinear system. Affine nonlinear system possesses the feature of being nonlinear to state vector but linear to control variables (Lu *et al*., 2001). Exact feedback linearization, using Lie derivates and Lie brackets, is implemented for the rotational subsystem. The proposed control strategy shows good results for quad rotor rotorcraft near hover.

The advantage of quad rotor over conventional helicopters is that they have small rotors and can be enclosed, making them safer for indoor flights. They also have higher payload capacity and better maneuverability in comparison with the conventional ones. It is also possible to achieve more stationary hovering with four thrust forces acting at a distance from the centre of gravity than with one force acting through the centre of gravity, as is the case with conventional helicopters. The main disadvantage of quad rotor is high energy requirement because of four motors.

QUAD ROTOR DYNAMICS

Aerodynamic forces and moments are derived using a combination of momentum and blade element theory (Prouty, 1995; Castillo *et al*., 2005). A quad rotor has four motors with propellers. A voltage applied to each motor results in a net torque being applied to the rotor shaft, $\boldsymbol{Q}_i$, which results in a thrust, $\boldsymbol{T}_i$. If the rotor disk is moving, there is a difference in relative velocity between the blade and air when moving through the forward and backward sweeps, which results in a net moment about the roll axis, $\boldsymbol{R}_i$. Forward velocity also causes a drag force on the rotor that acts opposite to the direction of travel, $\boldsymbol{D}_i$. The thrust can be defined in terms of aerodynamic coefficients C_T as

$$\boldsymbol{T} = \frac{1}{2}\rho A C_T r^2 \boldsymbol{\Omega}^2, \tag{1}$$

where A is the blade area, ρ the density of air, r the radius of the blade and $\boldsymbol{\Omega}$ the angular velocity of the propeller.

At hover, it can be assumed that the thrust and drag are proportional to the square of the propellers' rotation speed. Thus the thrust and drag forces, as in (McKerrow, 2004), are given by

$$\boldsymbol{T}_i \approx K_T \boldsymbol{\Omega}_i^2, \tag{2}$$

$$\boldsymbol{D}_i \approx K_D \boldsymbol{\Omega}_i^2, \tag{3}$$

where K_T and K_D are constants and $\boldsymbol{\Omega}_i$ is the propeller's rotation speed.

The free body diagram and axes of quad rotor rotorcraft are shown in Fig.1.

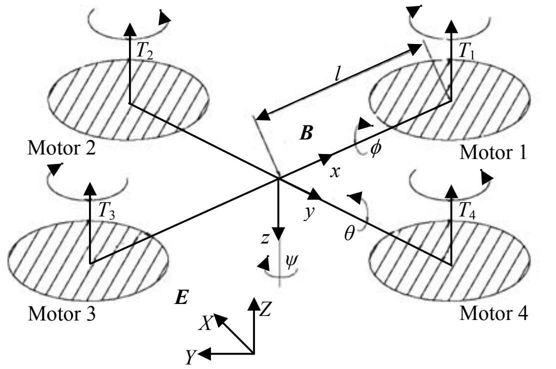

Fig.1 Forces and moments acting on the quad rotor rotorcraft

In Fig.1, l represents the distance of motor from the pivot centre. ϕ, θ and ψ represent Euler angles about x, y, z body axes, respectively. $\boldsymbol{T}_n$ (n=1, 2, 3, 4) represent the thrust force produced by each propeller. The earth fixed frame and the body fixed frame are represented by $\boldsymbol{E}=\{X, Y, Z\}$ and $\boldsymbol{B}=\{x, y, z\}$, respectively.

Increasing or decreasing speed of the four motors together generates the vertical motion. When motor pair (3, 1) is allowed to operate independently, then the pitch angle θ can be controlled along with the indirect control of motion along the same axis. Similarly when motor pair (2, 4) is allowed to operate independently, the roll angle ϕ can be controlled along with the indirect control of motion along the same axis. Finally when motor pair (3, 1) is rotating clockwise and motor pair (2, 4) rotating counterclockwise, the yaw angle ψ can be controlled. The quad rotor has now 6 DOFs.

The control inputs to quad rotor are defined as:

Vertical force input: $u_1 = K_T \sum_{i=1}^{4} \boldsymbol{\Omega}_i^2$.

Roll moment input: $u_2 = K_T(\boldsymbol{\Omega}_4^2 - \boldsymbol{\Omega}_2^2)$.

Pitch moment input: $u_3 = K_T(\boldsymbol{\Omega}_1^2 - \boldsymbol{\Omega}_3^2)$.

Yaw moment input: $u_4 = K_D(\boldsymbol{\Omega}_2^2 + \boldsymbol{\Omega}_4^2 - \boldsymbol{\Omega}_1^2 - \boldsymbol{\Omega}_3^2)$.

Consider the quad rotor as a single rigid body with 6 DOFs. Assuming the earth is flat and neglecting the ground effect, the equations of motion for a rigid body subject to body force $\boldsymbol{f}^b$ and body moment $\boldsymbol{\tau}^b$ ($\boldsymbol{f}^b$, $\boldsymbol{\tau}^b \in \mathbb{R}^3$), applied at the center of mass and expressed in Newton-Euler formalism, as shown in (Koo *et al*., 1998), are given by

$$\begin{bmatrix} m\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{v}}^{\mathrm{b}} \\ \dot{\boldsymbol{\omega}}^{\mathrm{b}} \end{bmatrix}+\begin{bmatrix} \boldsymbol{\omega}^{\mathrm{b}}\times(m\boldsymbol{v}^{\mathrm{b}}) \\ \boldsymbol{\omega}^{\mathrm{b}}\times(\boldsymbol{J}\boldsymbol{\omega}^{\mathrm{b}}) \end{bmatrix}=\begin{bmatrix} \boldsymbol{f}^{\mathrm{b}} \\ \boldsymbol{\tau}^{\mathrm{b}} \end{bmatrix}, \tag{4}$$

where $\boldsymbol{v}^{\mathrm{b}}$ and $\boldsymbol{\omega}^{\mathrm{b}}$ ($\boldsymbol{v}^{\mathrm{b}}, \boldsymbol{\omega}^{\mathrm{b}}\in\mathbb{R}^3$) are the body velocity and body angular velocity, respectively, $m\in\mathbb{R}$ specifies the total mass, $\boldsymbol{I}\in\mathbb{R}^{3\times3}$ is an identity matrix, and $\boldsymbol{J}\in\mathbb{R}^{3\times3}$ is an inertial matrix.

Translational dynamics

Total forces acting on quad rotor are given by

$$\boldsymbol{f}^{\mathrm{b}}=\boldsymbol{\omega}^{\mathrm{b}}\times(m\boldsymbol{v}^{\mathrm{b}})+\boldsymbol{f}_{\mathrm{tot}}, \tag{5}$$

where

$$\boldsymbol{f}_{\mathrm{tot}}=-C_{x,y,z}((\boldsymbol{v}^{\mathrm{b}})^2)+m\boldsymbol{g}Z+\sum_{i=1}^{4}[-\boldsymbol{T}_i z-\boldsymbol{D}_i(x,y)]. \tag{6}$$

The first term in the right hand side of Eq.(6) represents the friction force on the quad rotor body during horizontal motion with $C_{x,y,z}$ representing longitudinal drag coefficients in x, y and z directions, respectively. Z defines the vertical axis in inertial coordinates, and x, y and z define the body axes. The vector (x, y) defines the direction of velocity. At hover, $C_{x,y,z}((\boldsymbol{v}^{\mathrm{b}})^2)$ and $\boldsymbol{D}_i$ (i=1, 2, 3, 4) are all zero.

Neglecting friction force and the effect of body moments on the translational dynamics, an expression of translational dynamics on the quad rotor expressed in inertial axis is given by

$$\boldsymbol{R}(\phi,\theta,\psi)(\boldsymbol{f}^{\mathrm{b}})=m\boldsymbol{g}Z+\boldsymbol{R}(\phi,\theta,\psi)\sum_{i=1}^{4}(-T_i z), \tag{7}$$

where $\boldsymbol{R}(\phi,\theta,\psi)$ represents the complete rotation matrix, also called direct cosine matrix, given by

$$\boldsymbol{R}(\phi,\theta,\psi)=\boldsymbol{R}(z,\psi)\boldsymbol{R}(y,\theta)\boldsymbol{R}(x,\phi), \tag{8}$$

where

$$\boldsymbol{R}(z,\psi)=\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{R}(y,\theta)=\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix},$$

$$\boldsymbol{R}(x,\phi)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}.$$

From Eqs.(7) and (2), the translational dynamics of quad rotor are given by

$$\begin{cases} m\ddot{X}=(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi)u_1, \\ m\ddot{Y}=(\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi)u_1, \\ m\ddot{Z}=mg-(\cos\phi\cos\theta)u_1. \end{cases} \tag{9}$$

Rotational dynamics

Assuming that the inertia tensor is diagonal (symmetric design of the quad rotor), the moment equations governing quad rotor are given by

$$\boldsymbol{\tau}^{\mathrm{b}}=\boldsymbol{\omega}^{\mathrm{b}}\times(\boldsymbol{J}\boldsymbol{\omega}^{\mathrm{b}})+\boldsymbol{\tau}_{\mathrm{total}}, \tag{10}$$

where

$$\begin{aligned} \boldsymbol{\tau}_{\mathrm{total}}=&\sum_{i=1}^{4}\left[\boldsymbol{Q}_i z+\boldsymbol{R}_i(x,\ y)+\boldsymbol{D}_i h(-y,\ x)\right] \\ &+u_2 lx+u_3 ly+u_4 z, \end{aligned} \tag{11}$$

h is the height above the CG of the blade plane. At hover, $\boldsymbol{D}_i$ and $\boldsymbol{R}_i$ (i=1, 2, 3, 4) are all zero.

The rotational dynamics of quad rotor in the body axes are given by

$$\begin{cases} J_x\ddot{\phi}=\dot{\theta}\dot{\psi}(J_y-J_z)+lu_2, \\ J_y\ddot{\theta}=\dot{\phi}\dot{\psi}(J_z-J_x)+lu_3, \\ J_z\ddot{\psi}=\dot{\phi}\dot{\theta}(J_x-J_y)+u_4. \end{cases} \tag{12}$$

ENGINE MODEL

On the electrical side of DC motor, a current I flows through the armature according to drive voltage V_{a}, the motor's inductance L, resistance R and back emf voltage V_{emf}, then

$$V_{\mathrm{a}}-V_{\mathrm{emf}}=L\frac{\mathrm{d}I}{\mathrm{d}t}+RI. \tag{13}$$

The motor converts the electrical armature current into a mechanical torque applied to shaft by

$$T_{\mathrm{m}}=K_{\mathrm{T}}I. \tag{14}$$

The applied torque produces angular velocity ω_m according to inertia J and motor load T_l, given by

$$T_m = J\frac{d\omega_m}{dt} + T_l. \quad (15)$$

Defining $V_{emf}=K_e\omega_m$, neglecting the inductance of the small motor and introducing the propeller and gearbox models, then from Eqs.(13) and (15) we have

$$\dot{\omega}_m = -\frac{K_m^2}{RJ}\omega_m - \frac{d}{\eta r_g^3 J}\omega_m^2 + \frac{K_m}{RJ}V_a, \quad (16)$$

where η is the gear box efficiency, d the drag factor and r_g the gear box reduction ratio.

CONTROL STRATEGY

In (Fantoni and Lazano, 2005), control laws for underactuated helicopter are derived based on passivity and partial feedback linearization. In this paper exact feedback linearization using Lie derivative and Lie brackets is used for quad rotor. A sequential nonlinear control strategy is adopted, in which the altitude of the quad rotor is controlled by the vertical force input u_1 and the required roll and pitch angles of the quad rotor are extracted from the translational subsystem. Rotational controller stabilizes the quad rotor at quasi-stationary (hover or near hover) flight with control inputs u_2, u_3 and u_4.

The complete 6-DOF dynamic model of the quad rotor is given by

$$\ddot{x} = (\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)u_1/m, \quad (17)$$

$$\ddot{y} = (\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)u_1/m, \quad (18)$$

$$\ddot{z} = g - (\cos\phi\cos\theta)u_1/m, \quad (19)$$

$$\ddot{\phi} = \dot{\theta}\dot{\psi}\left(\frac{J_y - J_z}{J_x}\right) + \frac{l}{J_x}u_2, \quad (20)$$

$$\ddot{\theta} = \dot{\phi}\dot{\psi}\left(\frac{J_z - J_x}{J_y}\right) + \frac{l}{J_y}u_3, \quad (21)$$

$$\ddot{\psi} = \dot{\phi}\dot{\theta}\left(\frac{J_x - J_y}{J_z}\right) + \frac{u_4}{J_z}. \quad (22)$$

Altitude control

The altitude subsystem of the quad rotor is represented by Eq.(19) and can be linearized by selecting input u_1 as

$$u_1 = \frac{mg}{\cos\phi\cos\theta} + \frac{\nu}{\cos\phi\cos\theta}. \quad (23)$$

The necessary condition for Eq.(23) is $\cos\phi\cos\theta\neq 0$. ν can be a PD controller, given by

$$\nu = K_d\dot{z} + K_p(z - z_d), \quad (24)$$

where K_p and K_d are proportional and derivative positive gains, respectively, and z_d is the desired altitude.

Position control

The position subsystem is represented by Eqs.(17) and (18). Let $\dot{x}_d$ and $\dot{y}_d$ be the desired speeds in x and y directions, respectively. Then the errors in desired and actual speeds are given by

$$e_x = \dot{x}_d - \dot{x}, \quad (25)$$

$$e_y = \dot{y}_d - \dot{y}. \quad (26)$$

The desired roll and pitch angles, in terms of the error between the actual and desired speeds, are thus given by

$$\phi_d = \arcsin(u_{e_x}\sin\psi - u_{e_y}\cos\psi), \quad (27)$$

$$\theta_d = \arcsin\left(\frac{u_{e_x} - \sin\phi\sin\psi}{\cos\phi\cos\psi}\right), \quad (28)$$

where

$$u_{e_x} = \frac{K_1 e_x m}{u_1}, \quad u_{e_y} = \frac{K_2 e_y m}{u_1},$$

K_1 and K_2 are positive constants and u_1 is the desired vertical force input.

When the error between actual and desired speeds in x and y directions is zero, then $\phi_d=\theta_d=0$ and $u_1=mg$.

Rotation control

The rotational subsystem of the quad rotor is given by Eqs.(20)~(22). For the rotational subsystem, exact feedback linearization technique is applied. The objective of exact feedback linearization is to control the nonlinear model by applying the nonlinear control law, as illustrated in Fig.2.

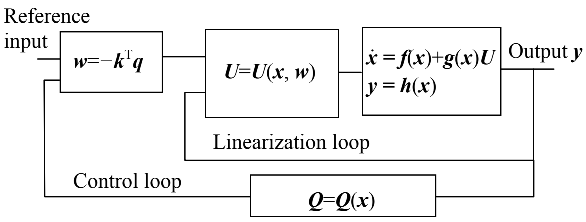

Fig.2 Exact linearization for the quad rotor system

This can be described by linear state space equations via states transformation $\boldsymbol{Q}=\boldsymbol{Q}(\boldsymbol{x})$ and nonlinear state feedback $\boldsymbol{U}=\boldsymbol{U}(\boldsymbol{x}, \boldsymbol{w})$, where $\boldsymbol{x}=(x_1, x_2, \ldots, x_6)^{\mathrm{T}}$ is the state vector of the nonlinear system. $\boldsymbol{q}=(q_1, q_2, \ldots, q_6)^{\mathrm{T}}$ is the state vector and $\boldsymbol{w}=(w_1, w_2, w_3)^{\mathrm{T}}$ is the input of the linear system resulting from the transformation. The transformation of nonlinear form in linear and controllable form is given by

$$\dot{\boldsymbol{q}} = \boldsymbol{A}\boldsymbol{q} + \boldsymbol{B}\boldsymbol{w},\ \ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{q}, \tag{29}$$

where $\boldsymbol{A}\in\mathbb{R}^{6\times6}$ and $\boldsymbol{B}\in\mathbb{R}^{6\times3}$ are control matrices and $\boldsymbol{C}\in\mathbb{R}^{3\times6}$ is the output matrix.

Let the states be defined as $\boldsymbol{x} = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})^{\mathrm{T}}$. Then the rotational subsystem is given by

$$\begin{cases} \dot{x}_1 = x_2,\ \ \dot{x}_2 = (x_4 x_6)(J_y - J_z)/J_x + l u_2 / J_x, \\ \dot{x}_3 = x_4,\ \ \dot{x}_4 = (x_2 x_6)(J_z - J_x)/J_y + l u_3 / J_y, \\ \dot{x}_5 = x_6,\ \ \dot{x}_6 = (x_2 x_4)(J_x - J_y)/J_z + l u_4 / J_z. \end{cases} \tag{30}$$

The rotational subsystem of the quad rotor is transformed into affine nonlinear form.

1. Affine nonlinear model

Affine nonlinear model of the quad rotor is given by

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \sum_{i=1}^{3} \boldsymbol{g}_i u_i, \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), \end{cases} \tag{31}$$

where $\boldsymbol{x}$ is a 6×1 state vector, u_i (i=1, 2, 3) are control variables, $\boldsymbol{y}$ is a 3×1 output function vector, $\boldsymbol{f}$ and $\boldsymbol{h}$ are smooth vector fields.

The function vectors $\boldsymbol{f}(\boldsymbol{x})\in\mathbb{R}^6$, $\boldsymbol{g}_i\in\mathbb{R}^6$ and output vector $\boldsymbol{y}$ are given by

$$\boldsymbol{f}(\boldsymbol{x}) = \big[x_2 \ \ x_4 x_6 (J_y - J_z)/J_x \ \ x_4 \ \ x_2 x_6 (J_z - J_x)/J_y \ \ x_6 \ \ x_2 x_4 (J_x - J_y)/J_z\big]^{\mathrm{T}},$$

$$\boldsymbol{g}_1(\boldsymbol{x}) = \begin{bmatrix}0 & l/J_x & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{g}_2(\boldsymbol{x}) = \begin{bmatrix}0 & 0 & 0 & l/J_y & 0 & 0\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{g}_3(\boldsymbol{x}) = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1/J_z\end{bmatrix}^{\mathrm{T}},$$

$$\boldsymbol{y}(\boldsymbol{x}) = \begin{bmatrix}x_1 & x_3 & x_5\end{bmatrix}^{\mathrm{T}}.$$

The controllability and involutivity of the system are given by the controllability matrix:

$$\big[\boldsymbol{g} \ \ ad_{\boldsymbol{f}}\boldsymbol{g} \ \ ad_{\boldsymbol{f}}^2\boldsymbol{g}\big],$$

where $ad_{\boldsymbol{f}}\boldsymbol{g}$ represents the Lie bracket of the two vector fields $\boldsymbol{f}$ and $\boldsymbol{g}$.

Next the relative degree of rotational subsystem is explored. By definition, if

(1) The Lie derivative of the function $L_{\boldsymbol{f}}^k \boldsymbol{h}(\boldsymbol{x})$ along $\boldsymbol{g}$ equals zero in a neighborhood of x_o, i.e., $L_{\boldsymbol{g}} L_{\boldsymbol{f}}^k \boldsymbol{h}(\boldsymbol{x}) = 0,\ k < r_i - 1$;

(2) The Lie derivative of the function $L_{\boldsymbol{f}}^{k-1} \boldsymbol{h}(\boldsymbol{x})$ along the vector field $\boldsymbol{g}(\boldsymbol{x})$ is not equal to zero, i.e.,

$$L_{\boldsymbol{g}} L_{\boldsymbol{f}}^{k-1} \boldsymbol{h}(\boldsymbol{x}) \neq 0.$$

Then this system is said to have relative degree r'.

For the rotational subsystem, the output y_i (i=1, 2, 3) are given by

$$\begin{bmatrix} y_1^{(2)} \\ y_2^{(2)} \\ y_3^{(2)} \end{bmatrix} = \begin{bmatrix} L_{\boldsymbol{f}}^2 h_1(\boldsymbol{x}) \\ L_{\boldsymbol{f}}^2 h_2(\boldsymbol{x}) \\ L_{\boldsymbol{f}}^2 h_3(\boldsymbol{x}) \end{bmatrix} + \boldsymbol{E} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \tag{32}$$

where $\boldsymbol{E}$ is a 3×3 invertible matrix, also called the decoupling matrix, given by

$$\boldsymbol{E}(\boldsymbol{x}) = \begin{bmatrix} l/J_x & 0 & 0 \\ 0 & l/J_y & 0 \\ 0 & 0 & 1/J_z \end{bmatrix}.$$

The linearizing control law $\boldsymbol{U}=\boldsymbol{U}(\boldsymbol{x}, \boldsymbol{w})$, by which the nonlinear feedback exactly compensates the system nonlinearities, is defined as

$$\boldsymbol{U}=\boldsymbol{E}^{-1}\left(\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}-\begin{bmatrix} L_f^2 h_1(\boldsymbol{x}) \\ L_f^2 h_2(\boldsymbol{x}) \\ L_f^2 h_3(\boldsymbol{x}) \end{bmatrix}\right). \quad (33)$$

Using the control law defined by Eq.(33), a decoupled set of equations are obtained, given by

$$y_i^{(2)}=w_i, \quad i=1,\ 2,\ 3. \quad (34)$$

For a decoupled set of equations, the MIMO pole placement technique is used to place the poles in the desired locations. The two necessary conditions for pole placement technique are satisfied, i.e., the actuators are capable of driving the system in a manner that allows control of all modes of behavior and that the sensors measure sufficient system parameters to enable construction of a complete state estimate.

Table 1 summarizes different system parameters of the prototype quad rotor. Mass moments of inertia are determined, by experiments and CAD simulation, as in (Wu *et al.*, 2002).

Table 1 Physical parameters of the quad rotor

Parameter	Value
Distance of motor from the pivot centre (l)	0.3 m
Mass moments of the airframe inertia along the x, y, z axes	
J_x	0.0154 kg·m^2
J_y	0.0150 kg·m^2
J_z	0.0309 kg·m^2
Total mass of the airframe (m)	0.6 kg

SIMULATION RESULTS

Let us simulate the closed loop system with nonlinear control algorithm. The initial conditions used are $\phi=\theta=0.5$ rad, $\dot{\phi}=\dot{\theta}=0.5$ rad/s and $z=1$ m. All other initial conditions are zero. The reference inputs to the controller are $z_d=1$, $\dot{x}_d=\dot{y}_d=0$ and $\psi_d=0$. Fig.3 shows the response of the nonlinear controller to stabilize the quad rotor at hover.

The rotational subsystem of the quad rotor is completely controllable and observable. The total relative degree of the rotational subsystem (i.e., the sum of relative degrees of all subsystems) is equal to the system order. Thus in this case, there are no internal dynamics.

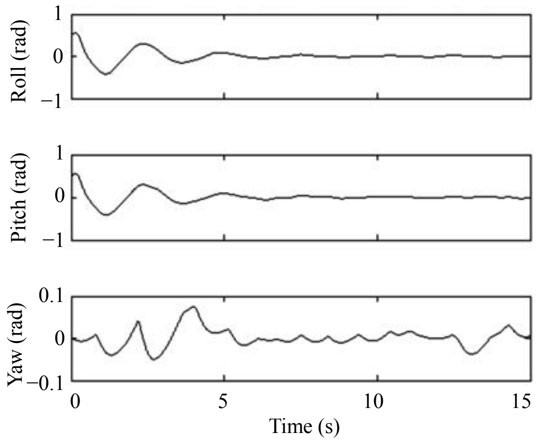

Fig.3 Attitude control response of the quad rotor rotorcraft for $\phi=\theta=0.5$ rad

The angles and their time derivatives of the rotational subsystem do not depend on the translation components, as evident from 6-DOF equations governing the quad rotor rotorcraft. However the translations depend on the angles. Ideally it can be imagined as two subsystems, the angular rotations and the linear translations. Due to its complete independence from the other subsystem, the angular rotations subsystem is tuned first.

Rotational control keeps the 3D orientation of the quad rotor rotorcraft to the desired value. Roll and pitch angles are usually forced to zero which permits hovering flight. Simulation results shown in Fig.3 are achieved by performing the simulation with a model which includes actuators' dynamics. The task of rotational controller is to compensate the initial error, stabilize the roll, pitch and yaw angles and maintain them at zero. This is accomplished with a nonlinear control law u_2, u_3 and u_4 as given by

$$\begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}=\begin{bmatrix} 0.0530x_6x_4+0.0513w_1 \\ -0.0517x_6x_2+0.0500w_2 \\ -0.0004x_2x_4+0.0309w_3 \end{bmatrix}.$$

Fig.4 shows the plots of quad rotor control vector output. Fig.5 shows the response of the altitude controller with $K_d=1.96$ and $K_p=3.98$.

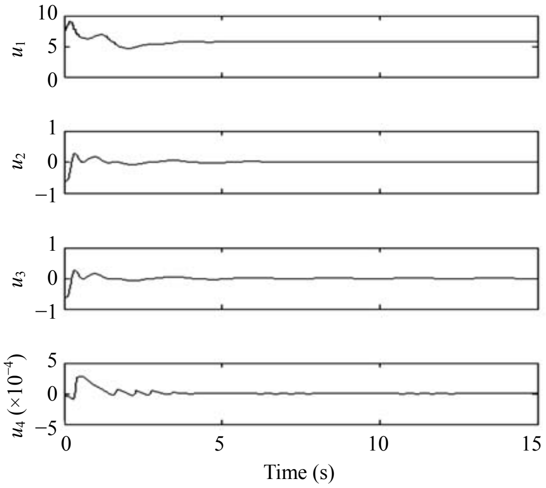

Fig.4 Control vector response of the quad rotor rotorcraft

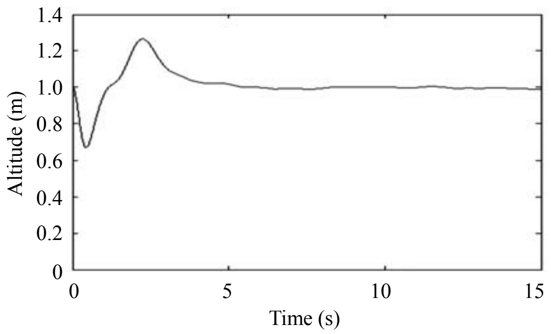

Fig.5 Altitude control of the quad rotor

CONCLUSION

We have presented a stabilization nonlinear control method for quad rotor rotorcraft. The modeling of the quad rotor is based on Newton-Euler formalism. The linearization accomplished for rotational subsystem is an "exact feedback linearization" as opposed to the conventional "Jacobian linearization" (Taylor-series expansion).

One of the advantages of the nonlinear controller for rotational subsystem of quad rotor system lies in that MIMO nonlinear system is decoupled under proper state transformation. The stabilization ability of the nonlinear controller is examined and simulation results indicate the effectiveness of the proposed control strategy for the quad rotor rotorcraft.

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