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Robust Feedback Linearization and GH_∞ Controller for a Quadrotor Unmanned Aerial Vehicle

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***Abstract*— In this paper, a mixed robust feedback linearization with linear GH_∞ controller is applied to a non linear quadrotor unmanned aerial vehicle. An actuator saturation and constrain on state space output are introduced to analyse the worst case of control law design.The results show that the overall system becomes robust when weighting functions are chosen judiciously . Performance issues of the controller are illustrated in a simulation study that takes into account parameter uncertainties and external disturbances as well as measurement noise.**

***Index Terms*— GH_∞ Controller, Sensitivity, Quadrotor.**

I. INTRODUCTION

The H_∞ control problem with continuous-time measurement output for linear systems has been studied by many researchers such as Kwakernaak [1] and Grimble [2] in polynomial form and by van der Schaft [3] and Ball [4] for nonlinear systems. For the hovering control of helicopters, many control methods have been proposed including linear approaches such as LQG [5], H_∞ design [6],[7] and the nonlinear approaches such as sliding mode [8], backstepping technique [9], and input/output linearization [10]. Eventough the design of controllers to achieve a linear input-output response for nonlinear systems has been well researched [11], the conventional input-output linearization techniques will perform very poorly when it comes to output tracking as it will render the unstable internal dynamics unobservable [12]. Following this context, classical feedback linearization may have poor robustness properties and cannot be easily combined with a H_∞ control law . So a robust nonlinear feedback is proposed to robustly control an uncertain nonlinear system around an operating point on using an appropriate approach for stability and robustness, namely "W-stability" [13] . The Inertial Navigation System (INS) and GPS are used to calculate position and orientation of the vehicle. The Quadrotor to be controlled is described by six-degree-of-freedom nonlinear dynamics with plant uncertainties due to the variation of moments of inertia and payload operation. The successful application of the autonomous quadrotor depends on their level of controllability and flying qualities. The overall inner outer controller should improve tracking performance and disturbance rejection capability. Using the uncertainty bounds step can be used to define simple sensitivity and complementary sensitivity weights. These weights are chosen to maximize the disturbance attenuation properties. The disturbances attempt to maximize a performance index, while the control attempts to minimize it. The analysis of this problem has been primarily in the frequency domain. This analysis can be carried out in time domain, and naturally extends H_∞ theory to finite-time and non-linear systems [14]. In this work we mention the polynomial solution based on Diophantine equations [15].

II. DYNAMIC QUADROTOR

Using Newton law and referring to V. Misler at. al [16][17] , the general MIMO nonlinear system can be represented into the form:

$$\begin{aligned} \dot{x} &= F(x) + G_1(x)w + G_2(x)\bar{u} \qquad (1)\\ y_1 &= H_1(x) + K_{12}(x)\bar{u} \\ y_2 &= H_2(x) + K_{21}(x)w \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state vector, $\bar{u} \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^n$ is the noise and unknown perturbation vector, $y_1 \in \mathbb{R}^l$ is the controlled output, $y_2 \in \mathbb{R}^p$ is the available measurement vector. The following hypothesis hold:

(A1) the functions $F(x)$, $G_1(x)$, $G_2(x)$, $H_1(x)$, $H_2(x)$, $K_{12}(x)$, $K_{21}(x)$ are piecewise continuous .

(A2) $F(0) = 0$, $H_1(0) = 0$ and $H_2(0) = 0$ for almost every t.

(A3) $H_1^T(x)K_{12}(x) = 0$, $K_{12}^T(x)K_{12}(x) = I$, $K_{21}(x)G_1^T(x) = 0$, $K_{21}(x)K_{21}^T(x) = I$.

where
$$w = \begin{bmatrix} w_b \\ w_p \end{bmatrix}$$

w_b is the noise vector of size 14. w_p is composed of aerodynamic forces disturbances $[A_x, A_y, A_z]^T$and aerodynamic moment disturbances $[A_p, A_q, A_r]^T$. They act on the UAV and are computed from the aerodynamic coefficients C_i as $A_i = \frac{1}{2}\rho_{air}C_iW^2$ (ρ_{air} is the air density, W is the velocity of the UAV with respect to the air), (C_i depend on several parameters like the angle between airspeed and the body fixed reference system, the aerodynamic and geometric form of the wing). The state vector and other parameters are defined as:

$$x = (x_0, y_0, z_0, \psi, \theta, \phi, \dot{x}_0, \dot{y}_0, \dot{z}_0, \zeta_1, \xi, \dot{\psi}, \dot{\theta}, \dot{\phi})^T$$

$$F(x) = [f_1(x), .., f_{14}(x)]^T$$

$$G_1(x) = \begin{bmatrix} 0_{6\times14} & 0_{6\times3} & 0_{6\times3} \\ 0_{3\times14} & M_1 & 0_{3\times3} \\ 0_{2\times14} & 0_{2\times3} & 0_{2\times3} \\ 0_{3\times14} & 0_{3\times3} & P_1 \end{bmatrix}$$

$$G_2(x) = \begin{bmatrix} 0_{10\times4} \\ P_4 \end{bmatrix}$$

$$H_1(x) = [0, 0, 0, 0, x_0, y_0, z_0, \psi]^T$$

$$H_2(x) = x$$

$$K_{12}(x) = \begin{bmatrix} I_{4\times4} \\ 0_{4\times4} \end{bmatrix}$$

$$K_{21}(x) = [I_{14\times14}]$$

The absolute position is described by three coordinates (x_0, y_0, z_0),and its attitude by Euler angles (ψ, θ, ϕ), under the conditions $(-\pi \leq \psi < \pi)$ for yaw, $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ for pitch and $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$ for roll. The real control signals (u_1, u_2, u_3, u_4) have been replaced by $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4)$ to avoid singularity in lie transformation matrices when using feedback linearization [16]. In that case u_1 has been delayed by double integrator. The other control signals will keep unchanged.

$$\begin{aligned} u_1 &= \zeta_1 + mg \\ \dot{\zeta} &= \xi \\ \dot{\xi} &= \bar{u}_1 \\ u_2 &= \bar{u}_2 \\ u_3 &= \bar{u}_3 \\ u_4 &= \bar{u}_4 \end{aligned} \tag{2}$$

Let the state vector be written into the form:

$$x = (x_1, x_2, ..., x_{14})^T \tag{3}$$

so one can have:

$$\begin{aligned} &f_1(x) = x_7,\ f_2(x) = x_8,\ f_2(x) = x_9 \\ &f_4(x) = x_{12},\ f_5(x) = x_{13},\ \ f_6(x) = x_{14} \\ &f_7(x) = g_1^7 x_{10},\ \ f_8(x) = g_1^8 x_{10}, \\ &f_9(x) = g + g_1^9(x_{10} + mg) \\ &f_{10}(x) = x_{11}, f_{11}(x) = 0 \end{aligned}$$

$$\begin{bmatrix} f_{12}(x) \\ f_{13}(x) \\ f_{14}(x) \end{bmatrix} = \mathbf{P}_2 \begin{bmatrix} x_{12}^2 \\ x_{13}^2 \\ x_{14}^2 \end{bmatrix} + \mathbf{P}_3 \begin{bmatrix} x_{12}x_{13} \\ x_{12}x_{14} \\ x_{13}x_{14} \end{bmatrix}$$

with

$$M_1 = \frac{1}{m} I_{3\times3}$$

$$P_1 = \frac{1}{d} \begin{bmatrix} 0 & g_3^{12} & g_4^{12}d \\ 0 & g_3^{13} & g_4^{13}d \\ g_2^{14} & g_3^{14} & g_4^{14}d \end{bmatrix}$$

$$P_2 = \begin{bmatrix} p_{211} & 0 & 0 \\ p_{221} & 0 & 0 \\ p_{231} & p_{232} & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} p_{311} & p_{312} & p_{313} \\ p_{321} & p_{322} & p_{323} \\ p_{331} & p_{332} & p_{333} \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & g_3^{12} & g_4^{12} \\ 0 & 0 & g_3^{13} & g_4^{13} \\ 0 & g_2^{14} & g_3^{14} & g_4^{14} \end{bmatrix}$$

$$\begin{aligned} g_1^7 &= -\frac{1}{m} \left(Cx_6 Cx_4 Sx_5 + Sx_6 Sx_4 \right) \\ g_1^8 &= -\frac{1}{m} \left(Cx_6 Sx_5 Sx_4 - Cx_4 Sx_6 \right) \\ g_1^9 &= -\frac{1}{m} (Cx_5 Cx_6) \end{aligned}$$

$$\begin{aligned} g_2^{14} &= \frac{d}{I_x} \\ g_3^{12} &= \frac{dSx_6}{I_y Cx_5};\ g_3^{13} = \frac{dCx_6}{I_y};\ g_3^{14} = \frac{dSx_5 Sx_6}{I_y Cx_5} \\ g_4^{12} &= \frac{Cx_6}{I_z Cx_5};\ g_4^{13} = -\frac{Sx_6}{I_z}; g_4^{14} = \frac{Sx_5 Cx_6}{I_z Cx_5} \end{aligned}$$

$$\begin{aligned} p_{211} &= Sx_5 Sx_6 Cx_6 \left(I_1 - I_3 \right) \\ p_{221} &= Sx_5 Cx_5 \left(Cx_6 \right)^2 \left(I_1 - I_3 \right) - Cx_5 Sx_6 \left(I_1 \right) \\ p_{231} &= Sx_6 Cx_6 \left(Cx_5 \right)^2 \left(I_2 + I_3 - I_1 \right) + Sx_6 Cx_6 \left(I_1 + I_3 \right) \\ p_{232} &= -I_2 Sx_6 Cx_6 \end{aligned}$$

$$\begin{aligned} p_{311} &= T(x_5) \left(Cx_6 \right)^2 \left(I_1 - I_3 \right) + T(x_5) \left(1 + I_3 \right) \\ p_{312} &= Sx_6 Cx_6 \left(I_3 - I_1 \right) \\ p_{313} &= \left(Cx_6 \right)^2 S_e(x_5) \left(I_3 - I_1 \right) + S_e(x_5)(1 - I_3) \\ p_{321} &= Sx_6 Cx_6 Sx_5 \left(I_1 + I_3 \right) \\ p_{322} &= Cx_5 \left(Cx_6 \right)^2 \left(I_3 - I_1 \right) - Cx_5 \left(1 - I_1 \right) \\ p_{323} &= Sx_6 Cx_6 \left(I_1 - I_3 \right) \\ p_{331} &= S_e(x_5) \left(1 + I_3 \right) + \left(Cx_6 \right)^2 S_e(x_5) \left(I_1 - I_3 \right) + \\ &\quad \left(Cx_6 \right)^2 \left(2I_2 + I_3 - I_1 \right) - Cx_5 \left(I_2 + I_3 \right) \\ p_{332} &= Sx_6 Cx_6 Sx_5 \left(-I_1 + I_3 \right) \\ p_{333} &= T(x_5) \left(Cx_6 \right)^2 \left(I_3 - I_1 \right) + T(x_5)(1 - I_3) \end{aligned}$$

$$I_1 = \frac{I_y - I_x}{I_z};\ I_2 = \frac{I_y - I_z}{I_x};\ I_3 = \frac{I_z - I_x}{I_y}$$

- g is the gravity constant ($g = 9.81ms^{-2}$);
- d is the distance from the center of mass to the rotors;
- u_1 is the resulting thrust of the four rotors defined as $u_1 = (F_1 + F_2 + F_3 + F_4)$
- u_2 is the difference of thrust between the left rotor and the right rotor defined as $u_2 = d\,(F_4 - F_2)$
- u_3 is the difference of thrust between the front rotor and the back rotor defined as $u_3 = d(F_3 - F_1)$
- u_4 is the difference of torque between the two clockwise turning rotors and the two counter-clockwise turning rotors defined as $u_4 = C(F_1 - F_2 + F_3 - F_4)$
- $C$is the force to moment scaling factor

- $I_x,\ I_y,\ I_z$ represent the diagonal coefficient of inertia matrix of the system.
- $S(.) = \sin(.),\ C(.) = \cos(.),\ T(.) = \tan(.),\ S_e(.) = \sec(.)$.

III. Robust feedback linearization (inner controller)

The robust feedback linearization method used in this context is based on Sobolev norm defined as

$$\|h\|_W = \left[\int_0^\infty h^T(t)h(t)dt + \int_0^\infty \dot{h}^T(t)\dot{h}(t)dt\right]^{\frac{1}{2}} \tag{4}$$

it transform a nonlinear system into its tangent linearized system around an operating point. Then, under state feedback

$$\bar{u}(x,v) = \alpha(x) + \beta(x)v$$

and change of coordinates

$$z = \phi(x)$$

defined by

$$\begin{aligned}\alpha(x) &= \alpha_c(x) + \beta_c(x)LT\phi_c(x)\\ \beta(x) &= \beta_c(x)R^{-1}\\ \phi(x) &= T^{-1}\phi_c(x)\end{aligned} \tag{5}$$

where $L = -\Delta.\frac{\partial\alpha_c}{\partial x}\mid_{x=0}, T = \frac{\partial\phi_c}{\partial x}\mid_{x=0}, R = \Delta^{-1},\ \alpha_c(x) = -\Delta^{-1}(x)b(x),\ \beta_c(x) = \Delta^{-1}(x)$ then the nonlinear system is transformed into a following one

$$\dot{z} = Az + B_2 v + \left[\frac{\partial\phi}{\partial x}G_1(x)\right]_{x=\phi^{-1}(z)} \tag{6}$$

With $A = \frac{\partial F(x)}{\partial x}\mid_{x=0}, B_2 = G_2(0)$. Note that the equation(5) satisfy $\frac{\partial\alpha}{\partial x}\mid_{x=0}= 0,\ \frac{\partial\phi}{\partial x}\mid_{x=0}= I_{14\times14},\ \beta(0) = I_{4\times4}$. For the quadrotor helicopter the input-output decoupling problem is solvable for the nonlinear system by means of static feedback. The vector relative degree $\{r_1, r_2, r_3, r_4\}$ is given by

$$r_1 = r_2 = r_3 = 4;\ r_4 = 2$$

and we have

$$b(x) = \begin{bmatrix} L_f^{r_1}h_1(x) & L_f^{r_2}h_2(x) & L_f^{r_3}h_3(x) & L_f^{r_4}h_4(x)\end{bmatrix}^T$$

where

$$\begin{aligned}L_f h(x) &= \sum_{i=1}^{n}\frac{\partial h}{\partial x_i}f_i(x)\\ L_f^k h(x) &= L_f(L_f^{k-1}h(x))\end{aligned}$$

and

$$\phi_c(x) = [\phi_{c1}(x), \phi_{c2}(x), \phi_{c3}(x), \phi_{c4}(x)]^T$$

$$\phi_{c1}(x) = \begin{bmatrix} h_1(x) = x_0\\ L_f h_1(x) = x_7 = \dot{x}_0\\ L_f^2 h_1(x) = \frac{A_x}{m} + g_1^7 x_{10} = \ddot{x}_0\\ L_f^3 h_1(x) = \dddot{x}_0\end{bmatrix}$$

$$\phi_{c2}(x) = \begin{bmatrix} h_2(x) = y_0\\ L_f h_2(x) = x_8 = \dot{y}_0\\ L_f^2 h_2(x) = \frac{A_y}{m} + g_1^8(x_4, x_5, x_6)x_{10} = \ddot{y}_0\\ L_f^3 h_2(x) = \dddot{y}_0\end{bmatrix}$$

$$\phi_{c3}(x) = \begin{bmatrix} h_3(x) = z_0\\ L_f h_3(x) = x_9 = \dot{z}_0\\ L_f^2 h_3(x) = \frac{A_z}{m} + g + g_1^9 x_{10} = \ddot{z}_0\\ L_f^3 h_3(x) = \dddot{z}_0\end{bmatrix}$$

$$\phi_{c4}(x) = \begin{bmatrix} h_4(x) = x_4\\ L_f h_4(x) = \dot{x}_4\end{bmatrix}$$

$$\Delta(x) = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24}\\ \Delta_{31} & \Delta_{32} & \Delta_{33} & \Delta_{34}\\ \Delta_{41} & \Delta_{42} & \Delta_{43} & \Delta_{44}\end{bmatrix}$$

with

$$\begin{aligned}\Delta_{11} &= L_{g1}L_f^{r_1-1}h_1(x)\\ &= -\frac{1}{m}(Cx_6Cx_4Sx_5 + Sx_6Sx_4)\\ \Delta_{12} &= L_{g2}L_f^{r_1-1}h_1(x)\\ &= \frac{d}{mI_x}(x_{10}Sx_6Cx_4Sx_5 - x_{10}Cx_6Sx_4)\\ \Delta_{13} &= L_{g3}L_f^{r_1-1}h_1(x) = \frac{d}{mI_y}(-x_{10}Cx_4Cx_5)\\ \Delta_{14} &= 0\end{aligned}$$

$$\begin{aligned}\Delta_{21} &= L_{g1}L_f^{r_2-1}h_2(x)\\ &= -\frac{1}{m}(Cx_6Sx_5Sx_4 - Cx_4Sx_6)\\ \Delta_{22} &= L_{g2}L_f^{r_2-1}h_2(x)\\ &= \frac{d}{mI_x}(x_{10}Sx_6Sx_4Sx_5 + x_{10}Cx_6Cx_4)\\ \Delta_{23} &= L_{g3}L_f^{r_2-1}h_2(x) = \frac{d}{mI_y}(-x_{10}Sx_4Cx_5)\\ \Delta_{24} &= L_{g4}L_f^{r_2-1}h_2(x) = 0\end{aligned}$$

$$\begin{aligned}\Delta_{31} &= L_{g1}L_f^{r_3-1}h_3(x) = -\frac{1}{m}(Cx_5Cx_6)\\ \Delta_{32} &= L_{g2}L_f^{r_3-1}h_3(x) = \frac{d}{mI_x}(x_{10}Sx_6Cx_5)\\ \Delta_{33} &= L_{g3}L_f^{r_3-1}h_3(x) = \frac{d}{mI_y}(x_{10}Sx_5)\\ \Delta_{34} &= L_{g4}L_f^{r_3-1}h_3(x) = 0\end{aligned}$$

$$\begin{aligned}\Delta_{41} &= L_{g1}L_f^{r_4-1}h_4(x) = 0\\ \Delta_{42} &= L_{g2}L_f^{r_4-1}h_4(x) = 0\\ \Delta_{43} &= L_{g3}L_f^{r_4-1}h_4(x) = \frac{d}{I_y}(Sx_6S_ex_5)\\ \Delta_{44} &= L_{g4}L_f^{r_4-1}h_4(x) = \frac{1}{I_z}(Cx_6S_ex_5)\end{aligned}$$

in fact the system in equation(6) is still nonlinear because of w vector. one seeks a controller which ensures the compensated system to be internally asymptotically stable and its output to tend asymptotically toward a desired trajectory even in the presence of external disturbance. In this context the linear GH_∞ is proposed.

IV. H_∞ OPTIMAL CONTROL (OUTER CONTROLLER)

H_∞ synthesis methods take into account in explicit manner some specification of robustness. The problematic here is to take a maximum of guarantee for a synthesized control law on a chosen model work effectively on the physical system. For that a transfer function family is considered where the nominal model $W_{nom} = A_0^{-1}B_0$ constitutes the “center”. We assume that is possible to choose these sets of transfer function contain the real system. Hence if the stability and performance of the closed loop system are obtained and demonstrated for all W_i elements ,then it will be also for the real system.

Let the transfer function of the uncertain system be

$$\tilde{W} = (A_0 + D_p\Delta_2 P_p)^{-1}(B_0 + D_p\Delta_1 F_p) \tag{7}$$

where $D_p\Delta_2 P_p$ and $D_p\Delta_1 F_p$ are modelling errors on A_0 and B_0. D_p, P_p and F_p are characterized by low and pass filter respectively and Δ_1, Δ_2 are the non structured uncertainty . It is assumed that disturbances are bounded and there exists a function V which verify

$$\left\|V^{-1}\Delta_1\right\|_\infty^2 + \left\|V^{-1}\Delta_2\right\|_\infty^2 < 1 \tag{8}$$

Hence the minimization criteria is written as :

$$J_\infty = \left\|(P_pS + F_pM)^* \Phi_{ff}(P_pS + F_pM)\right\|_\infty \tag{9}$$

The cost function of GH_∞(generalized H_∞) leads to eigenvalue problem which lead to the minimization of $\left\|(P_pS + F_pM)A^{-1}D_f\right\|$ where $\Phi_{ff} = A^{-1}D_fD_f^*A^{*-1}$.The difference between H_∞ and GH_∞ is based on the representation of the weighting functions in the minimization criteria. The weighting functions in GH_∞ can be represented as :

$$P_p(z^{-1}) = P_d^{-1}P_n,\ F_p(z^{-1}) = P_d^{-1}F_n \tag{10}$$

where P_d is strictly schur and $P_n(0) \neq 0$. The polynomials P_n and F_n are chosen to assure that the polynomial

$$L_c = P_nB - F_nA \tag{11}$$

verify $L_cL_c^* > 0$ on $|z| = 1$. if we can write $L_c = L_1L_2$ with L_1 strictly minimal phase, L_2 a non-minimal phase and L_{2s} is schur polynomial satisfying $L_{2s} = L_2^* z^{-n2}$ where $n_2 = deg(L_2)$, then the control law procedure is summarized as follow:

- compute (G_2, H_2, F_2) through Diophantine equations:

$$F_2AP_d + L_2G_2 = L_{2s}P_nD_f \tag{12}$$

$$F_2BP_d - L_2H_2 = L_{2s}F_nD_f \tag{13}$$

- compute the eigenvalue/eigenvector equation $(N_1,\ F_1,\ F_{1s},\ \lambda)$ with F_{1s} is schur polynomial satisfying $F_{1s} = F_1^* z^{-n1}$ where $n_1 = deg(F_1)$.

$$L_2N_1 + F_1\lambda L_{2s} = -F_{1s}F_2 \tag{14}$$

- compute the control law

$$C_0 = (H_2 + KB)^{-1}(G_2 - KA), \quad K = F_{1s}^{-1}N_1P_d \tag{15}$$

Finally the H_∞ controller has been computed with the following constrain:

1) Forces must be greater than or equal to zero and less than 10 $(0 \leq F_i \leq 10N)$ which systematically leads to $(u_1 \geq 0)$. This is due to actuator output limits.
2) The altitude z_0 must be less than or equal zero $(z_0 \leq 0)$ since the reference frame is upside down.

V. APPLICATION TO QUADROTOR

The nominal transfer matrices computed for $I_{y0} = 1.2416; I_{z0} = 1.2416; I_{x0} = 1.2416; m_0 = 2; d = 0.1$ with inputs v_1, v_2, v_3, v_4 and outputs x_0, y_0, z_0, ψ. $W_{nom} =$

$$\begin{bmatrix} 0 & 0 & \frac{-.7269652062}{s^4} & 0 \\ 0 & \frac{0.7269652062}{s^4} & 0 & 0 \\ \frac{-0.5}{s^4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{0.8054123711}{s^2} \end{bmatrix}$$

The reference trajectory is chosen to have the steady states $x_{0d} = 1m$; $y_{0d} = 1m$; $z_{0d} = 1m$; $\psi_d = 1rad$; The driving noise signals are assumed to be white, zero mean, uncorrelated and of 0.1 variance.

With a chosen weighting function F_n, P_n and P_d , the controller C_0 is given. The sensitivity S, the control sensitivity M, the complementary sensitivity T and the cost function are represented:

- **Output** x_0, y_0, z_0

Poles of C_0	Zeros of C_0
-27.2518 ±4.2603i	613.34
-27.1501 ± 4.2764i	-186.0725 ± 394.6909i
-20.6711	-353.7157
-9.6508 ± 4.1499i	-27.2518 ± 4.2603i
-2.5778 ± 4.2796i	-3.5256
-3.5256	-2.2263
-2.2262	-0.1676 ± 0.1779i
-0.1676 ± 0.1779i	-0.1301 ± 0.1325i
	-0.0388

$|\lambda_{x0}| = |\lambda_{y0}| = |\lambda_{z0}| = 0.89617$

$\gamma_{x_0} = \gamma_{y_0} = \gamma_{z_0} = 1.1159$

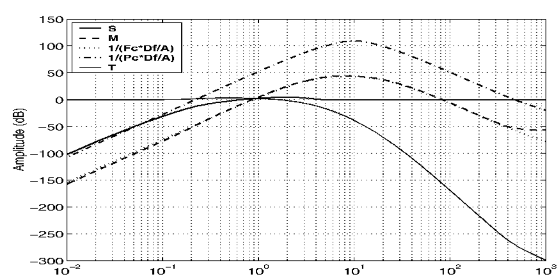

Fig. 1. Sensitivity S, M and T and inverse of weightings for x_0, y_0, z_0

- **Output** ψ

Poles de $C_{0\psi}$	Zeros de $C_{0\psi}$
-27.6616 ± 2.2323i	-24.6533
-20.0710	-24.3742 ±0.4382i
-17.4163	-23.7707
-6.6884	-3.6203
-3.6203	-1.8308
-1.8308	-0.8156
-0.8160	-0.3817

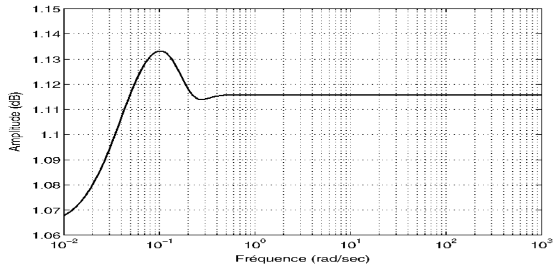

Fig. 2. Cost function for x_0, y_0, z_0

$|\lambda_\psi| = 2.0135; \quad \gamma_\psi = \frac{1}{|\lambda_\psi|} = 0.49665$

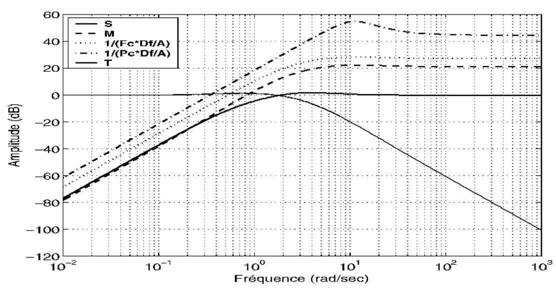

Fig. 3. Sensitivity S, M, T and inverse weightings for ψ

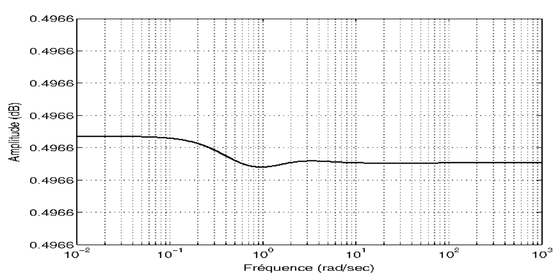

Fig. 4. Cost function for ψ

A. Case for Aerodynamic forces disturbances:

An aerodynamic force disturbance has been taken for $A_x = 0.5N$,$A_y = 0.5N$ and $A_z = 1.5N$ occurring at 20sec, 40 sec and 60sec respectively, the results of (x_0, y_0, z_0, ψ) and (θ, ϕ) are shown in figure-(5,6). The behavior of the applied forces and aerodynamic force disturbances is shown in figure-(7). The tracking errors are shown in figure-(8).

B. Case for Aerodynamic moments disturbances:

For the aerodynamic moments $A_p = 0.08N.m$, $A_q = 0.08N.m$ and $A_r = 0.5N.m$ occurring at 20sec, 40sec and 60sec respectively the results of the applied forces are represented in figure-(9). Tracking errors are represented in figure-(10).

- It is noted from figures 5 to 8 that the system when subjected to aerodynamic force disturbance $[A_x, A_y, A_z]$ and 20% uncertainties on mass and inertia,the mixed inner-outer controller give good results even without block disturbance estimation. This can be shown from tracking error trajectories which vanished after a finite time with a perfect convergency. This is due not only to the robustness of the H_∞ controller but also to robust feedback linearization which preserve the good robustness properties. This is shown at the time 20sec on x_0, 40sec on y_0, and at the time 60sec on z_0 trajectory when the disturbances occurs. The robustness of the system can be confirmed by tracking error trajectories (figure-8). The magnitude disturbances are limited by actuator saturation between 0 and $10N$. However despite the overshoots on forces in figure(7) which lead to saturation the system remain stable.
- However when subjected to aerodynamic moments disturbances $[A_P, A_q, A_r]$ and 20% uncertainties on mass and inertia, the results are shown in figure-9, 10. It is seen that the inner-outer controllers show their efficiency to overcome easily disturbances on z and ψ, better than on x and y. The forces in figure-9 reflect perfectly the relation between control input $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4)$ and (F_1, F_2, F_3, F_4). The computed sensitivity S in figure-1 and figure-3 show a low gain at low frequency and a gain oscillating near 0 dB at high frequency , however the complementary sensitivity T shows a gain of 0 dB at low frequency and a low gain at high frequency. This confirms that the resulting design is appropriate.

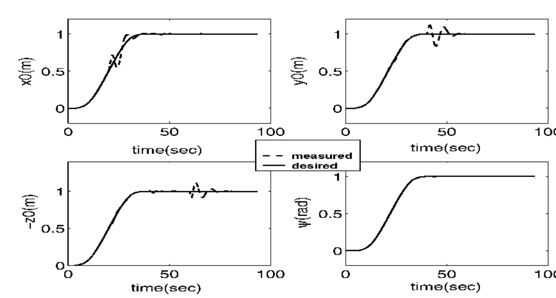

Fig. 5. Trajectories x_0, y_0, z_0 and ψ

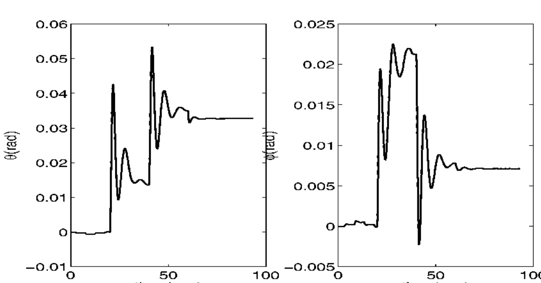

Fig. 6. Trajectories θ and ϕ

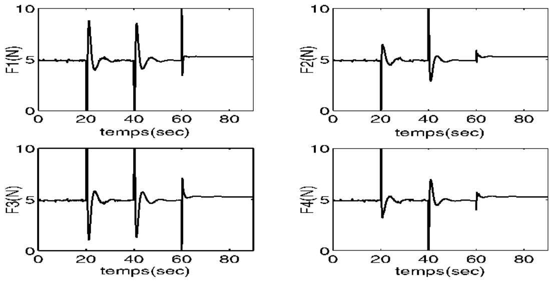

Fig. 7. Applied forces

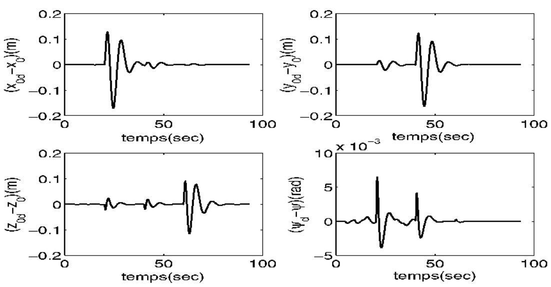

Fig. 8. Tracking errors

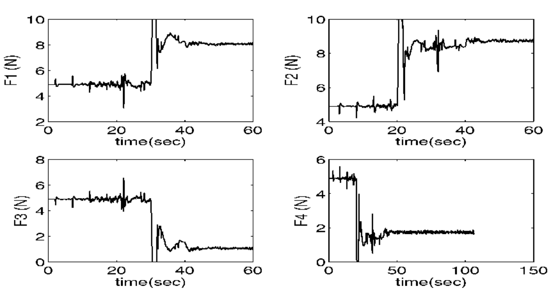

Fig. 9. Applied forces

VI. Conclusion

GH_∞ method is introduced to compute a controller which is mixed with robust feedback linearization to control altogether a non linear UAV system. A dual criterion involving mixed weighted sensitivity and weighted control sensitivity is analyzed. The introduction of robust feedback linearization is seen to be useful to transform the MIMO system into a non interacting one without breaking robustness. The convergence of output state vector is obtained when uncertainties on system parameters and disturbances occur. However the sensitivity and complementary sensitivity are presented to confirm the performance and robustness theory and validate the efficiency of results. The weighting function choices are analyzed through frequency domain. The results obtained show the convergence in finite time and a satisfying tracking error of desired trajectories. Further investigation will be on using an observer to reconstruct the velocity of the motions rather than using sensors. figure

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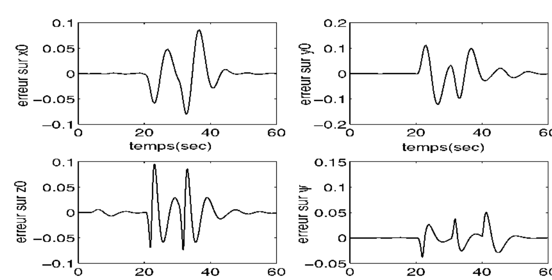

Fig. 10. Tracking errors