

Attitude and Position Control of a Quadcopter in a Networked Distributed System



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Agenda



Introduction

Model

- Attitude Model
- Translational Model
- Linearization

Network

Control Solution

- Attitude Controller
- Translational Controller

Implementation

Results

Final Statements

Introduction



- ▶ Surveillance and inspection
- ▶ Rescue
- ▶ Aerial photography
- ▶ ...

Introduction

Prototype



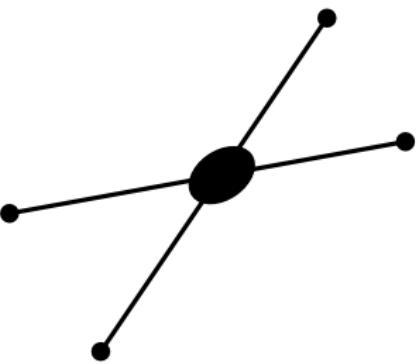
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- ▶ Motors and propellers
- ▶ ESCs
- ▶ Vicon markers
- ▶ Battery
- ▶ Processor
- ▶ XBee modules

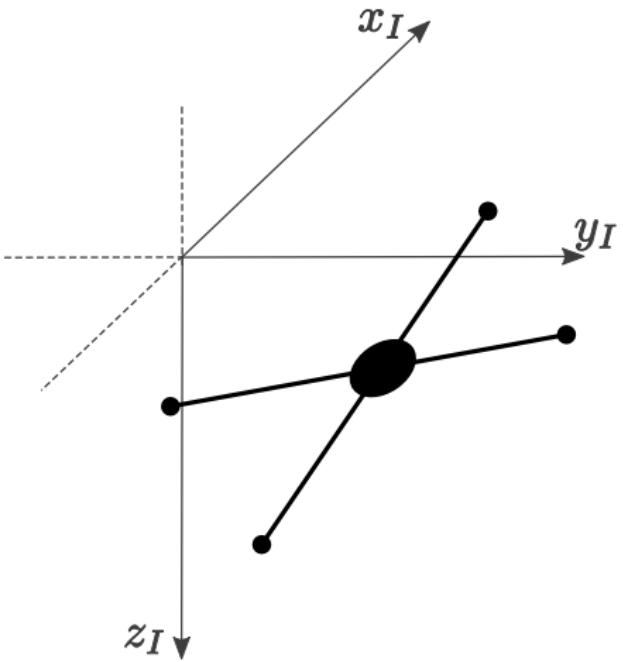
Model

Attitude Model



Model

Attitude Model

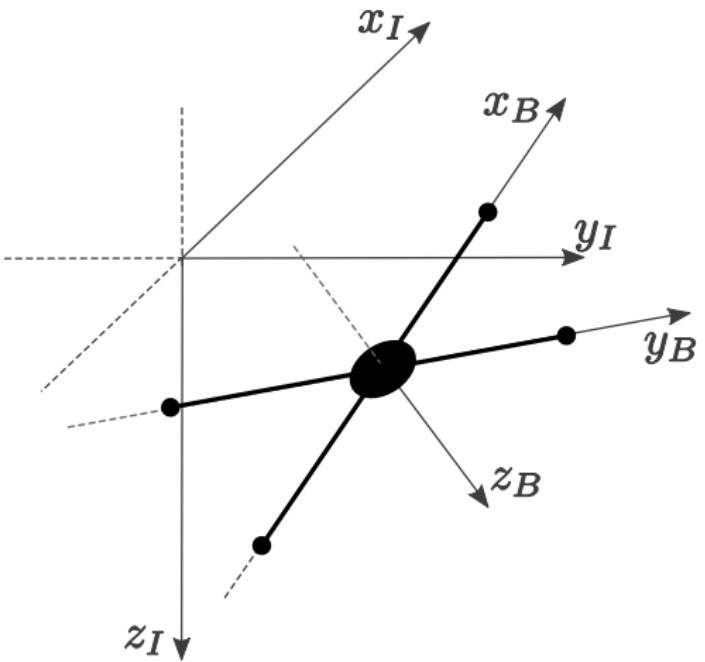


Model

Attitude Model

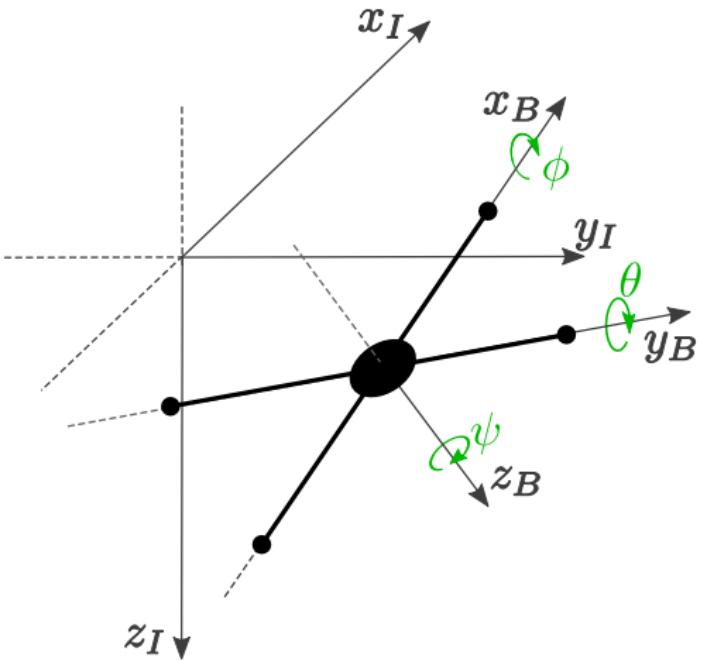


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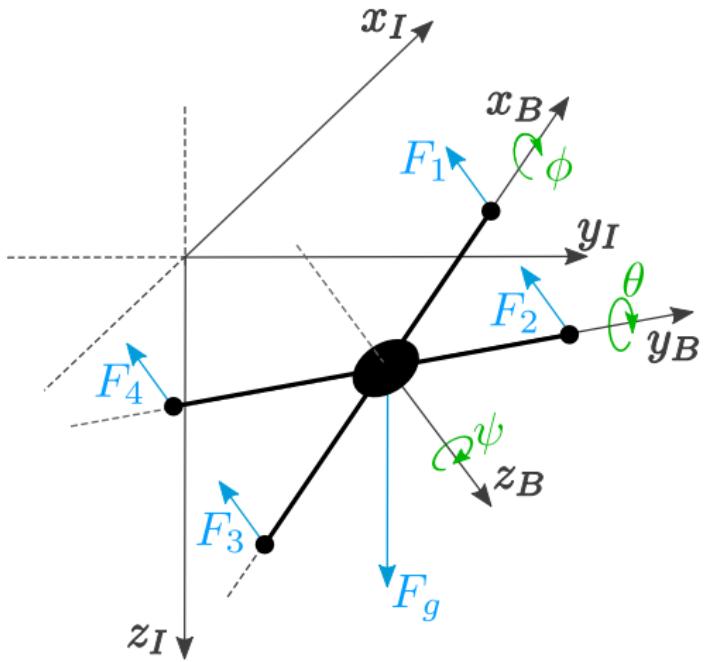
Model

Attitude Model



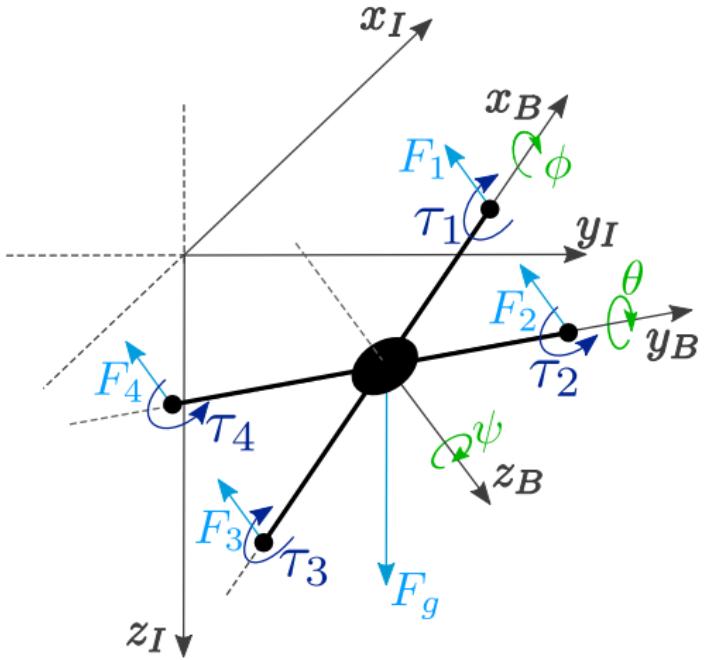
Model

Attitude Model



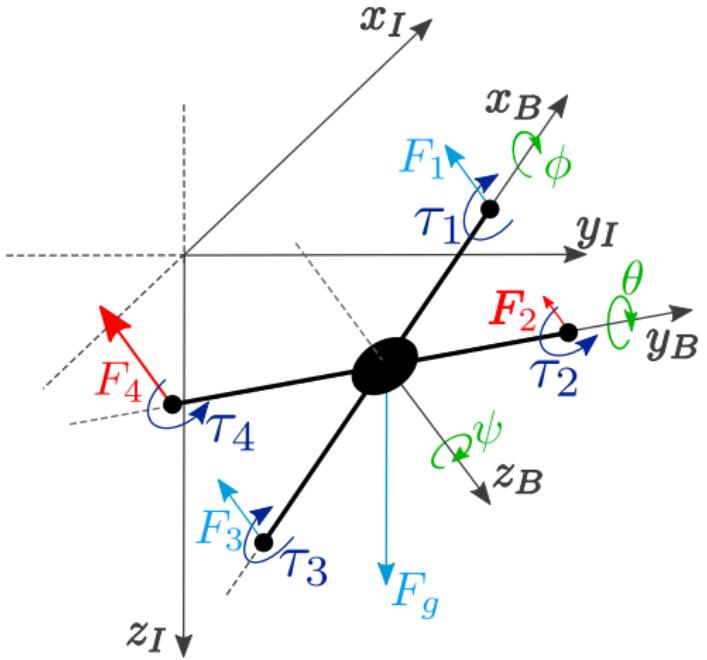
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Attitude Model



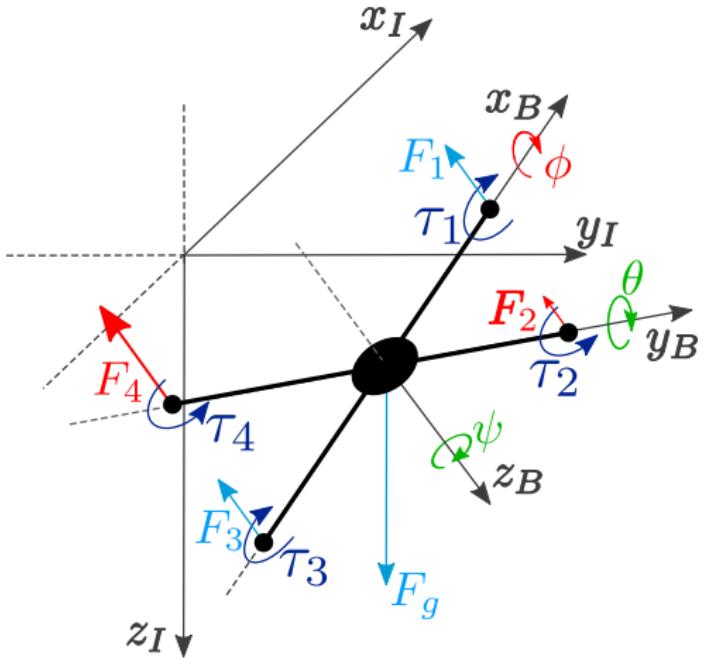
Model

Attitude Model



Model

Attitude Model



Model

Attitude Model



- ▶ Dynamic Equations

Model

Attitude Model



- ▶ Dynamic Equations

$$J\alpha = \sum \tau$$

Model

Attitude Model



► Dynamic Equations

$$J\alpha = \sum \tau$$

$$J_x \ddot{\phi} = (F_4 - F_2)L$$

$$J_y \ddot{\theta} = (F_1 - F_3)L$$

$$J_z \ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

Model

Attitude Model



► Dynamic Equations

$$J\alpha = \sum \tau$$

$$J_x \ddot{\phi} = (F_4 - F_2)L$$

$$J_y \ddot{\theta} = (F_1 - F_3)L$$

$$J_z \ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

$$J_x \ddot{\phi} = k_{\text{th}}(\omega_4^2 - \omega_2^2)L$$

$$J_y \ddot{\theta} = k_{\text{th}}(\omega_1^2 - \omega_3^2)L$$

$$J_z \ddot{\psi} = k_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$$

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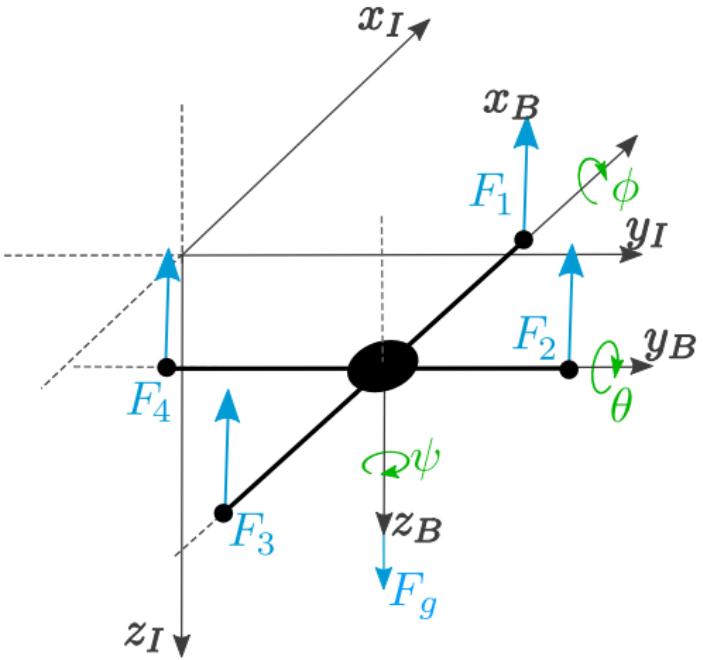
Implementation

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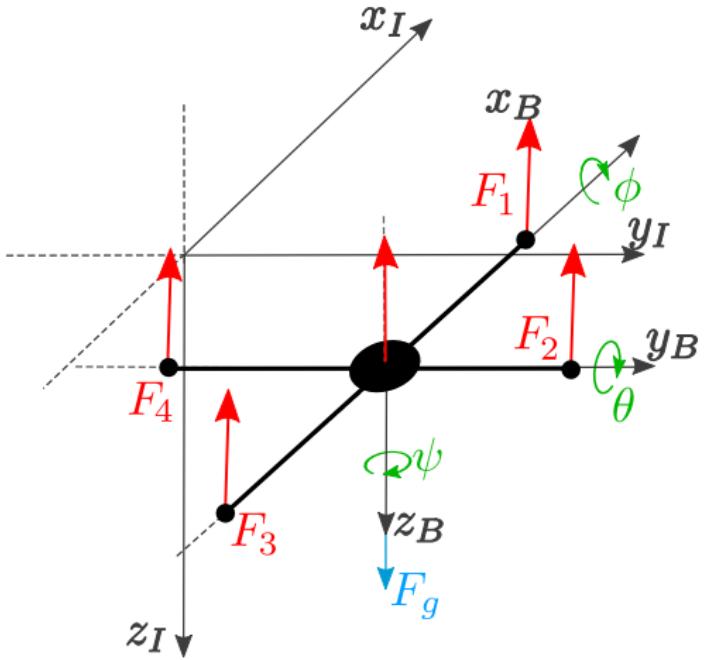
Model

Translational Model



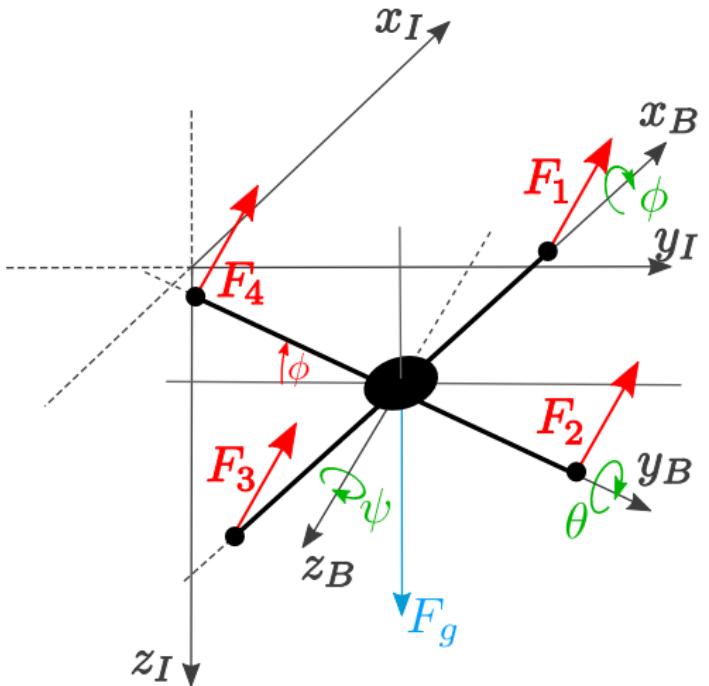
Model

Translational Model



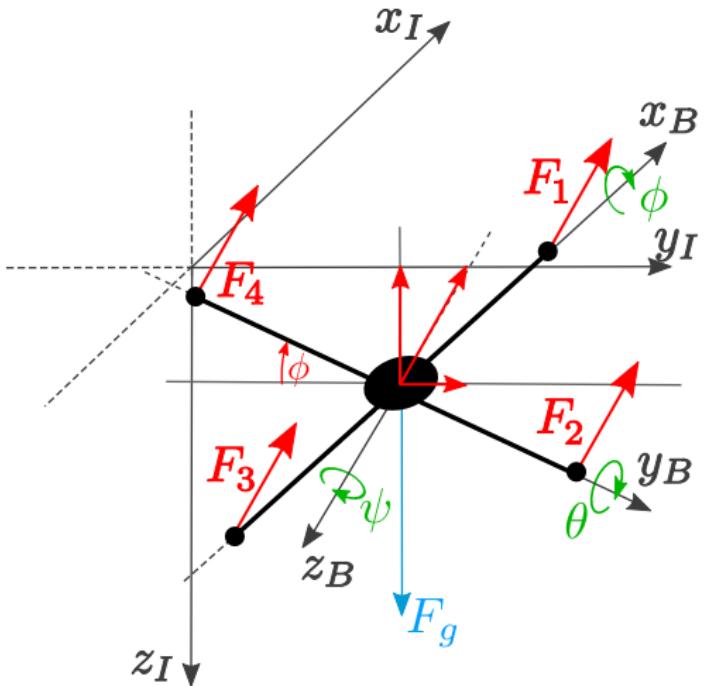
Model

Translational Model



Model

Translational Model



Translational Model



► Rotation Matrix

$$R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \quad R_Y = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \quad R_Z = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = R_Z R_Y R_X$$

$$v_I = R v_B$$

Model

Translational Model



- ▶ Dynamic Equations

Model

Translational Model



- ▶ Dynamic Equations

$$ma = \sum F$$

Model

Translational Model



► Dynamic Equations

$$ma = \sum F$$

$$m\ddot{x}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - (F_1 + F_2 + F_3 + F_4) \cos \phi \cos \theta$$

Model

Translational Model



► Dynamic Equations

$$ma = \sum F$$

$$m\ddot{x}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - (F_1 + F_2 + F_3 + F_4) \cos \phi \cos \theta$$

$$m\ddot{x}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \cos \phi \cos \theta$$

Model

Linearization



- ▶ First order Taylor approximation

Model

Linearization



- ▶ First order Taylor approximation

$$f(x) \approx f(\bar{x}) + f'(\bar{x})(x - \bar{x}) \rightarrow \Delta f(x) \approx f'(\bar{x})\Delta x$$

Model

Linearization



- ▶ First order Taylor approximation

$$f(x) \approx f(\bar{x}) + f'(\bar{x})(x - \bar{x}) \rightarrow \Delta f(x) \approx f'(\bar{x})\Delta x$$

$$m\ddot{\bar{z}}_I = F_g - k_{th}(\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \cos \bar{\phi} \cos \bar{\theta}$$

Model

Linearization



- ▶ First order Taylor approximation

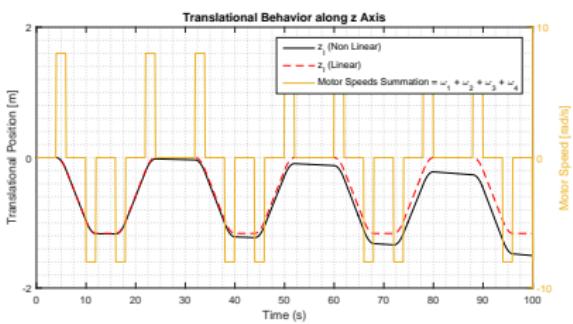
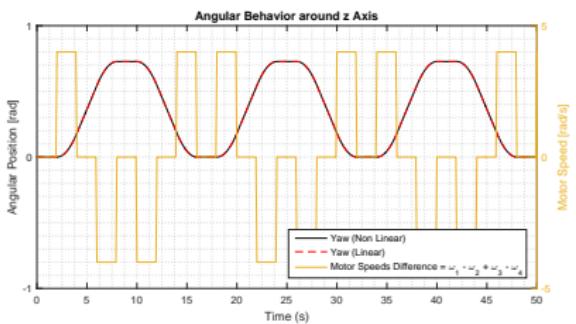
$$f(x) \approx f(\bar{x}) + f'(\bar{x})(x - \bar{x}) \rightarrow \Delta f(x) \approx f'(\bar{x})\Delta x$$

$$m\ddot{\bar{z}}_I = F_g - k_{th}(\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \cos \bar{\phi} \cos \bar{\theta}$$

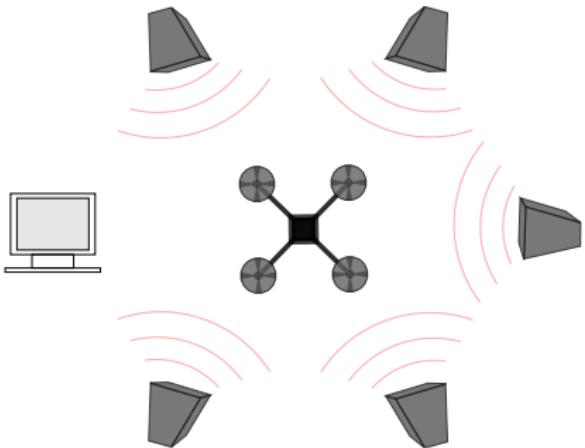
$$\bar{\omega}_i = \sqrt{\frac{F_g}{4k_{th}}}$$

Model

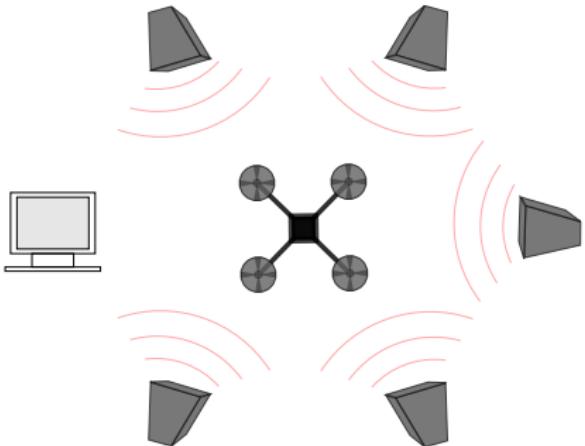
Linearization



Network



Network

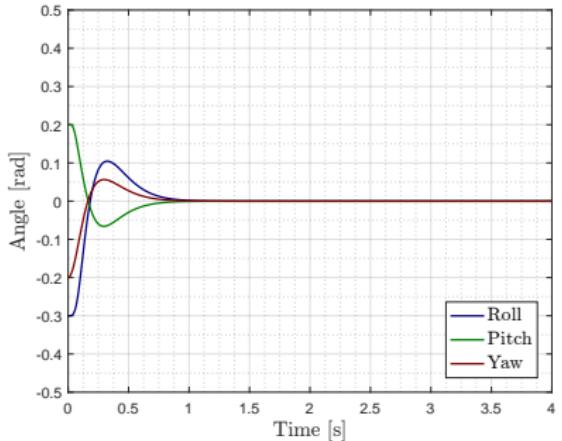


- ▶ Delay
- ▶ Missed packets

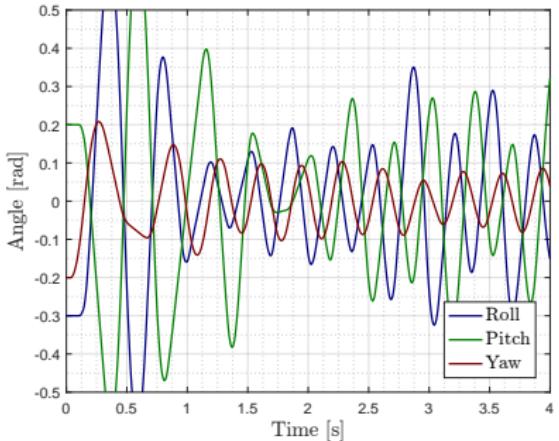
Network



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Control design only taking the model into account



Same controller with the effect of the network

Agenda



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Control Solution

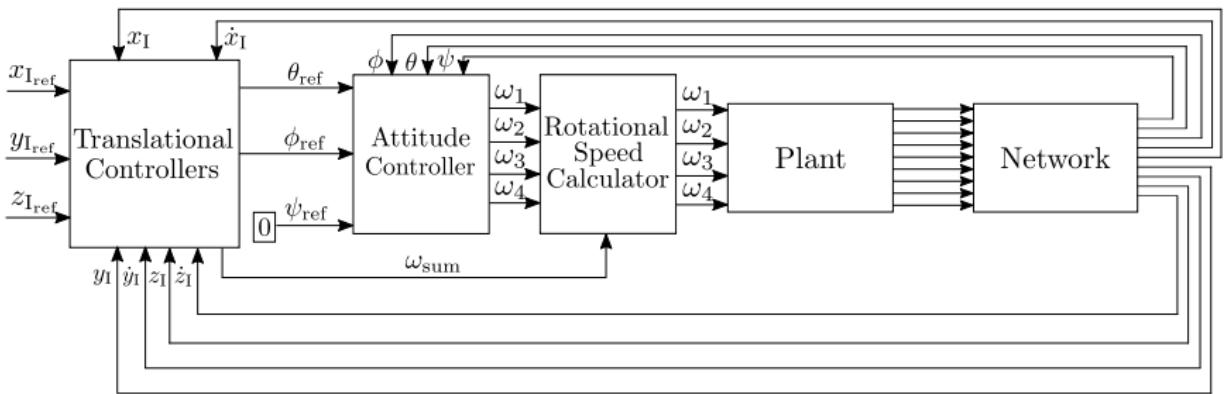
- Attitude Controller
- Translational Controller

Implementation

Results

Final Statements

Control Solution



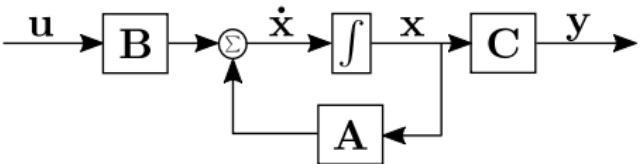
Control Solution

Attitude Controller



► System Representation

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned}$$



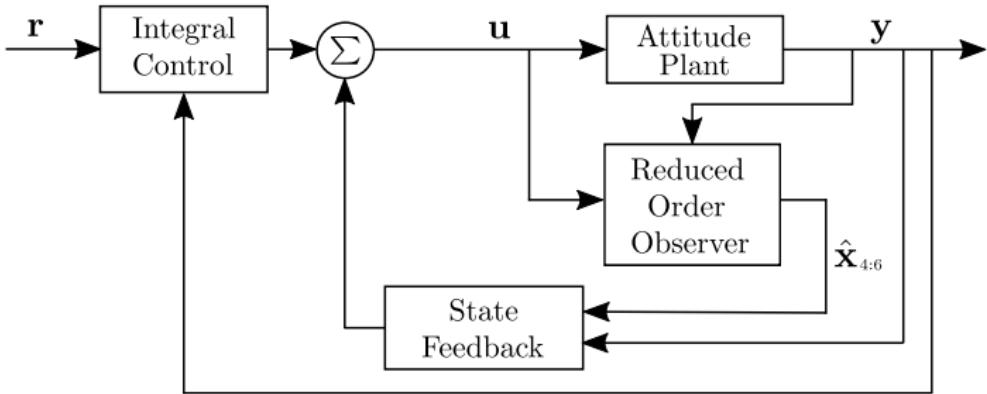
$$\mathbf{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

Control Solution

Attitude Controller



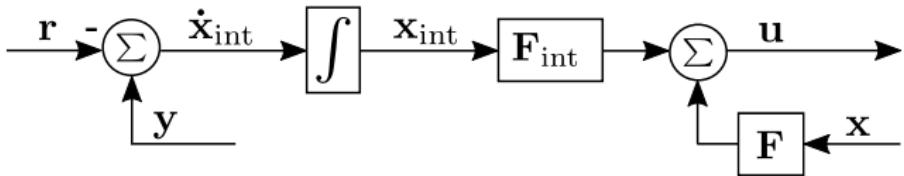
Control Solution

Attitude Controller



- ▶ State Feedback with Integral Control

$$\dot{\mathbf{x}}_{\text{Int}}(t) = \mathbf{y}(t) - \mathbf{r}(t)$$



$$\mathbf{u}(t) = \mathbf{Fx}(t) + \mathbf{F}_{\text{Int}}\mathbf{x}_{\text{Int}}(t)$$

Control Solution

Attitude Controller



- ▶ LQR

$$\mathcal{J} = \int_0^{\infty} \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} \, dt$$

- ▶ Bryson's Rule

$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } [x_i^2]}$$

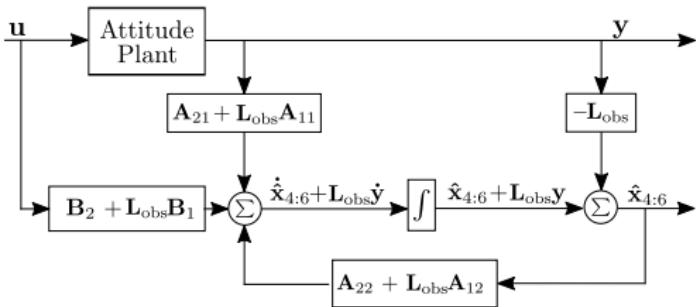
$$R_{ii} = \frac{1}{\text{maximum acceptable value of } [u_i^2]}$$

Control Solution

Attitude Controller



► Reduced Order Observer



$$\mathbf{x}_{4:6} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$A_{22} + L_{obs}A_{12}$$

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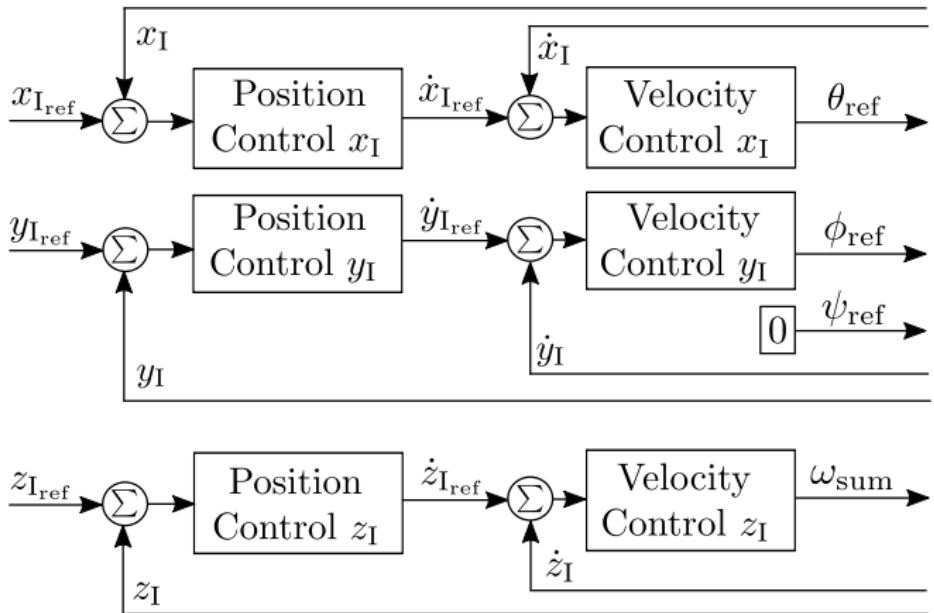
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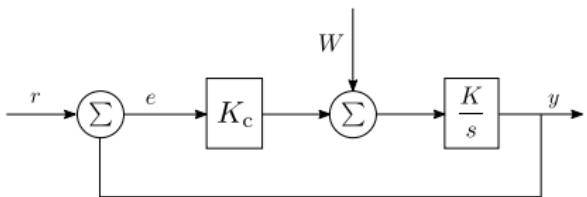
Control Solution

Translational Controller



Control Solution

Translational Controller

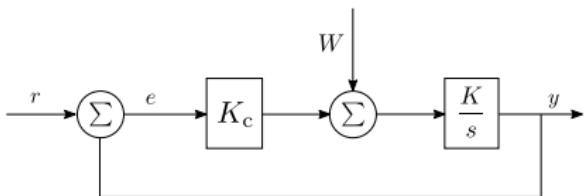


Control Solution

Translational Controller



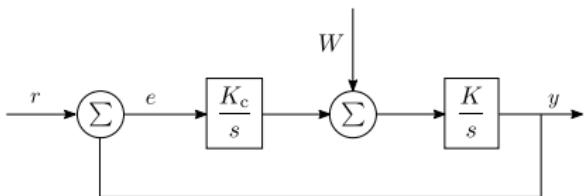
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$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$

Control Solution

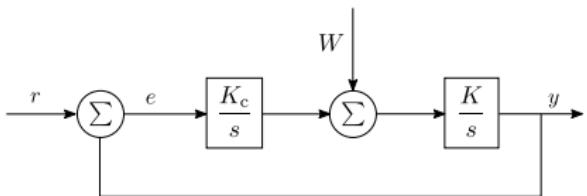
Translational Controller



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$

Control Solution

Translational Controller



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + \frac{K_c}{s} \frac{K}{s}} = \frac{Ks}{s^2 + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{Ks}{s^2 + K_c K} = 0$$

Control Solution

Translational Controller



$$\frac{\dot{x}_I}{\theta} = \frac{-k_{th}4\bar{\omega}}{ms} \quad \frac{\dot{y}_I}{\phi} = \frac{k_{th}4\bar{\omega}}{ms} \quad \frac{\dot{z}_I}{\omega_{sum}} = \frac{-2k_{th}\bar{\omega}}{ms}$$

Control Solution

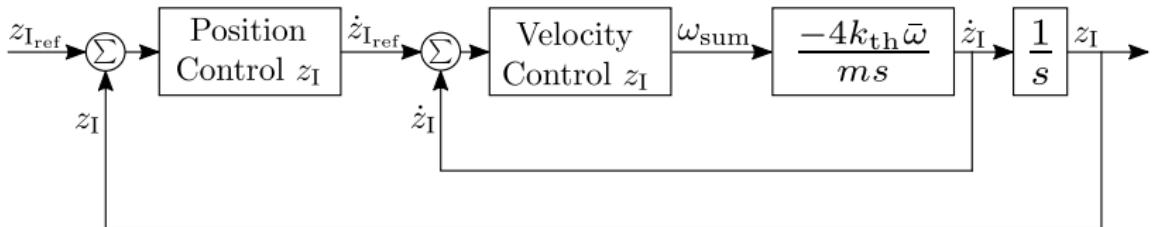
Translational Controller



$$\frac{\dot{x}_I}{\theta} = \frac{-k_{th} 4 \bar{\omega}}{ms}$$

$$\frac{\dot{y}_I}{\phi} = \frac{k_{th} 4 \bar{\omega}}{ms}$$

$$\frac{\dot{z}_I}{\omega_{sum}} = \frac{-2k_{th} \bar{\omega}}{ms}$$



Control Solution

Translational Controller



put three root locus

Control Solution

Translational Controller



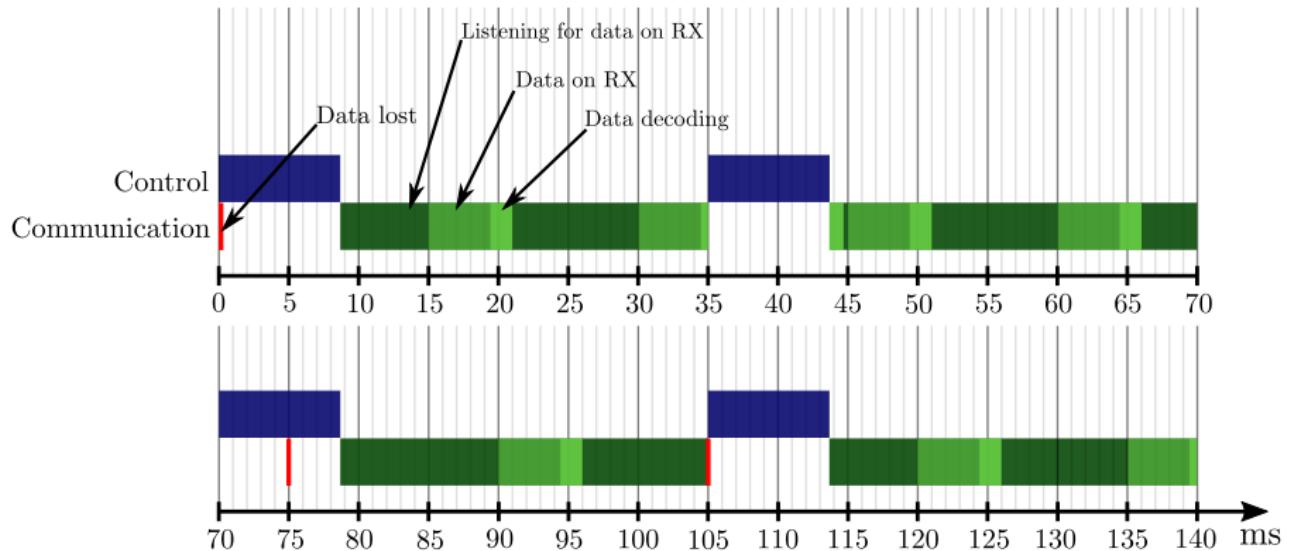
consideration of bandwidth equations for velocity controllers
equations for position controllers

Implementation



FreeRTOS, tasks

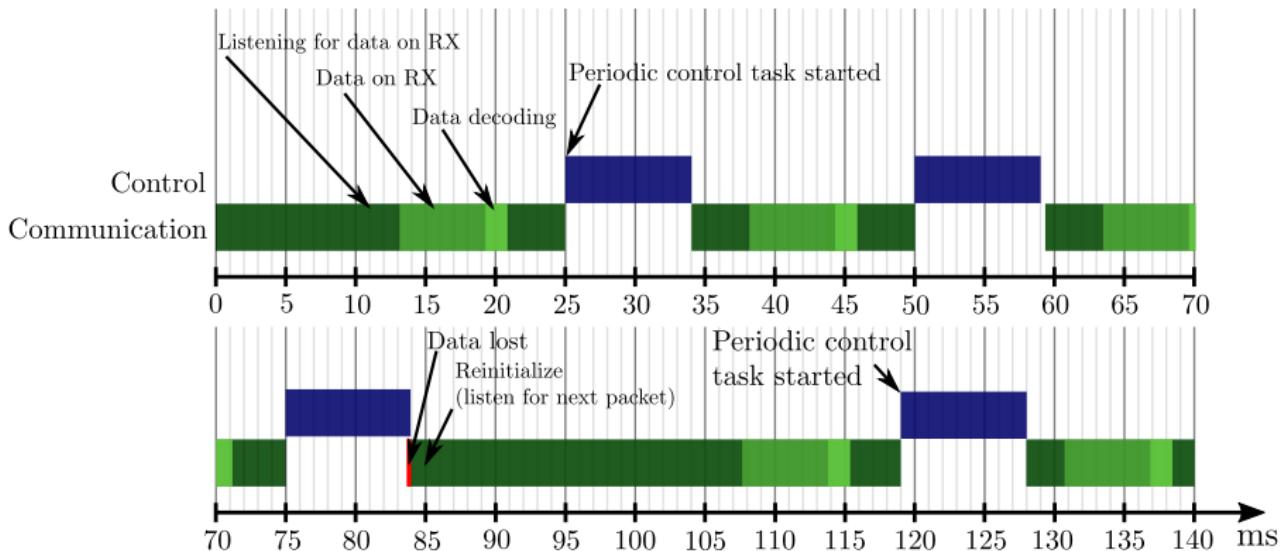
Implementation Schedule



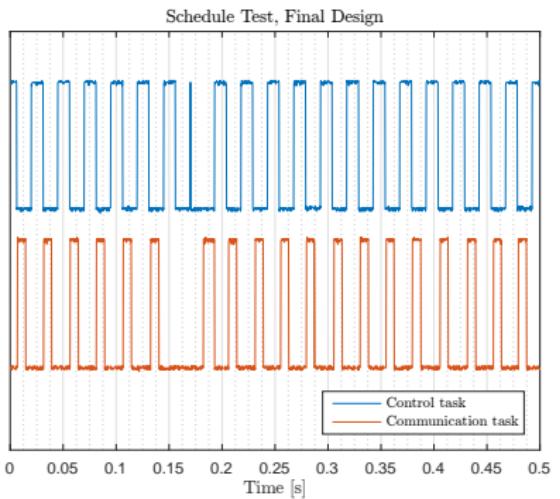
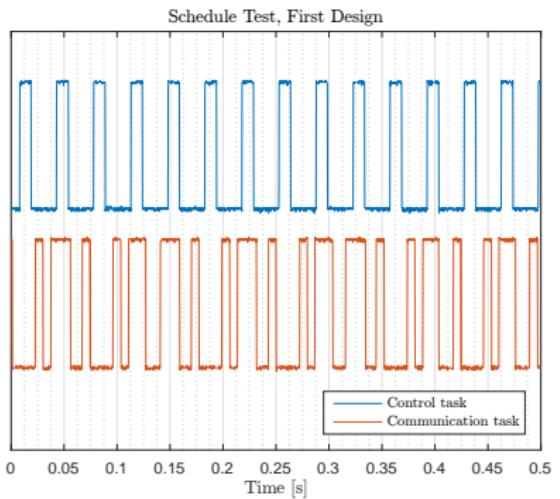
Implementation Schedule



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Implementation Schedule



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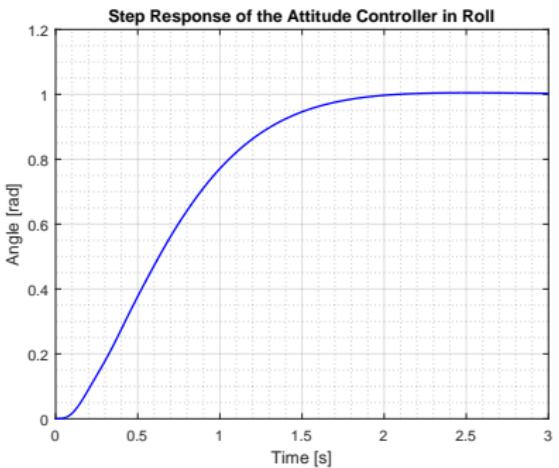
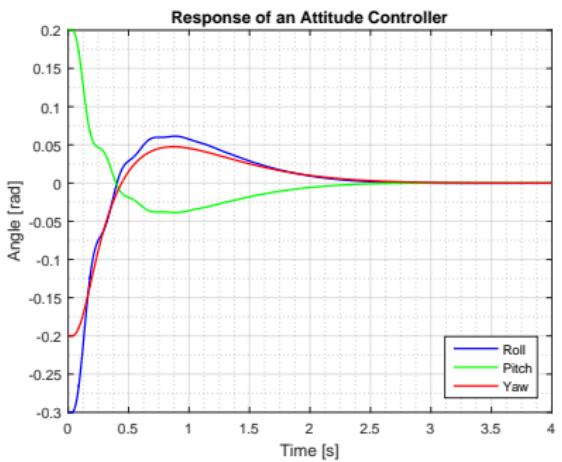
Implementation

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Results

Attitude Controller Simulations

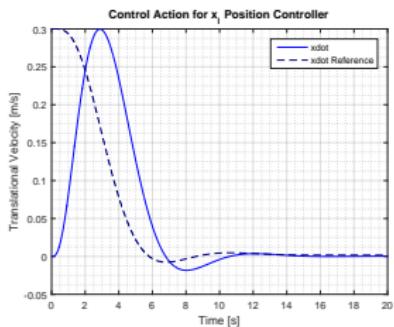
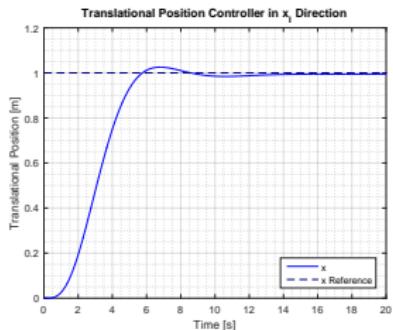
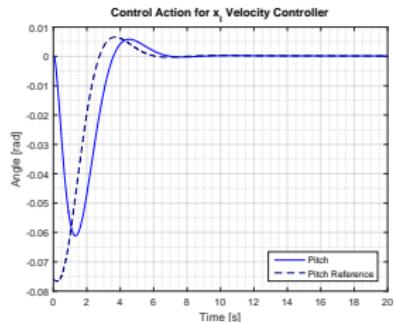
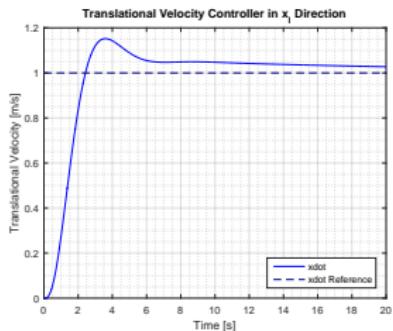


Results

Translational Controllers Simulations



- x_1 (and y_1) controllers for velocity and position

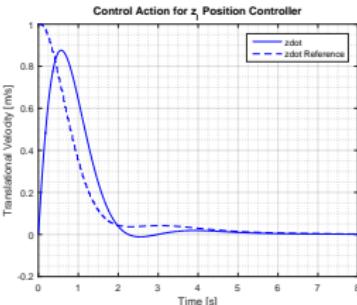
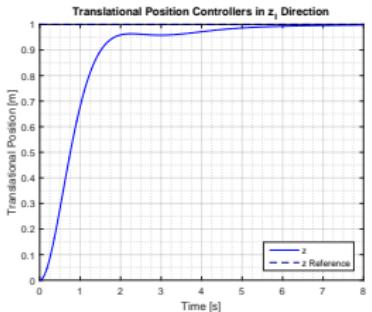
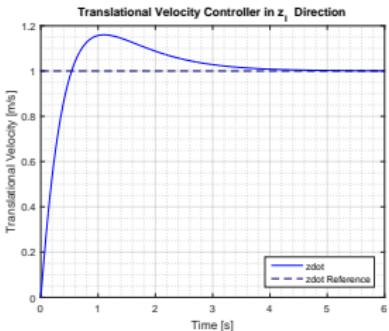


Results

Translational Controllers Simulations

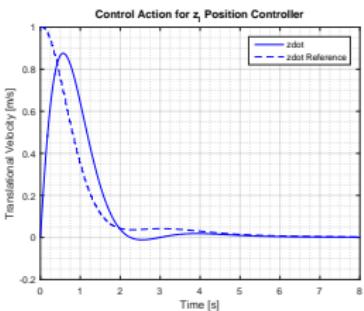
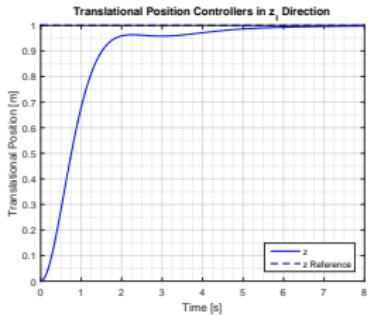
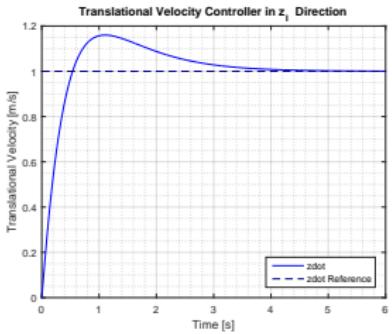


- z_I controllers for velocity and position



Results

Attitude Controller Functional Tests



Final Statements



- Limited bandwidth of controllers due to sampling.
- Control design working in simulation for attitude and translational controller.
- Implementation/Hardware issue when testing the translational controllers.
- Attitude controller tests carried out successfully.

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