

# Attitude and Position Control of a Quadcopter in a Networked Distributed System



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Løvemærke, Niels Skov Vestergaard, Noelia Villamarzo Arruñada

# Agenda



Introduction

Model

- Attitude Model
- Translational Model
- Linearization

Network

Control Solution

- Attitude Controller
- Translational Controller

Implementation

Results

Final Statements

# Introduction

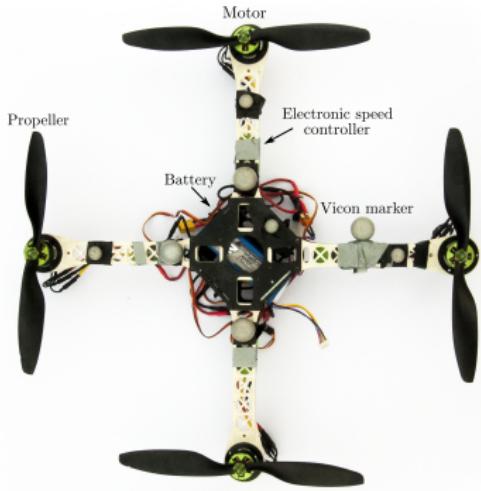


# Introduction



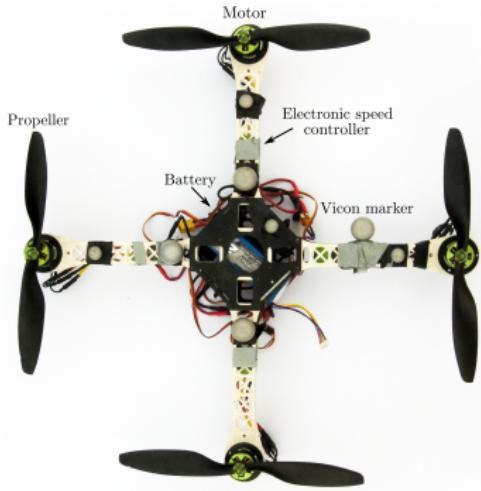
# Introduction

## Prototype



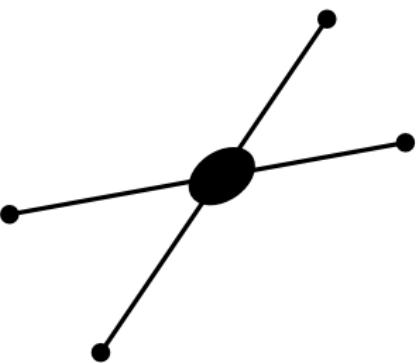
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## Prototype



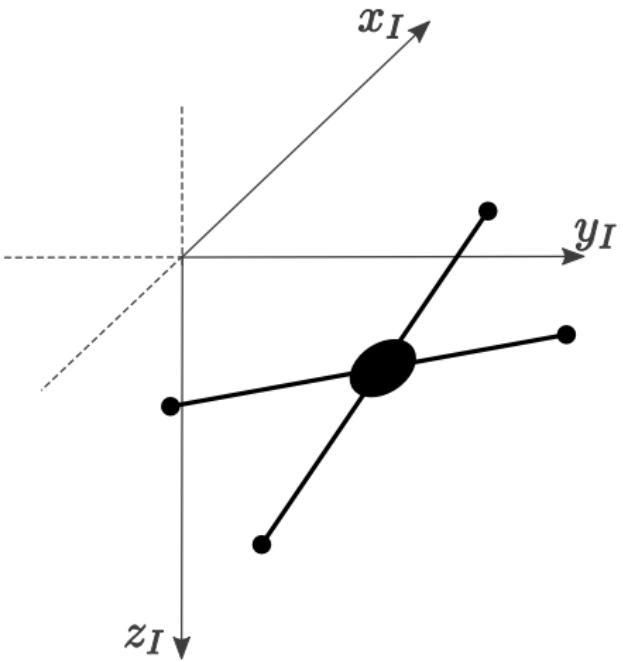
# Model

## Attitude Model



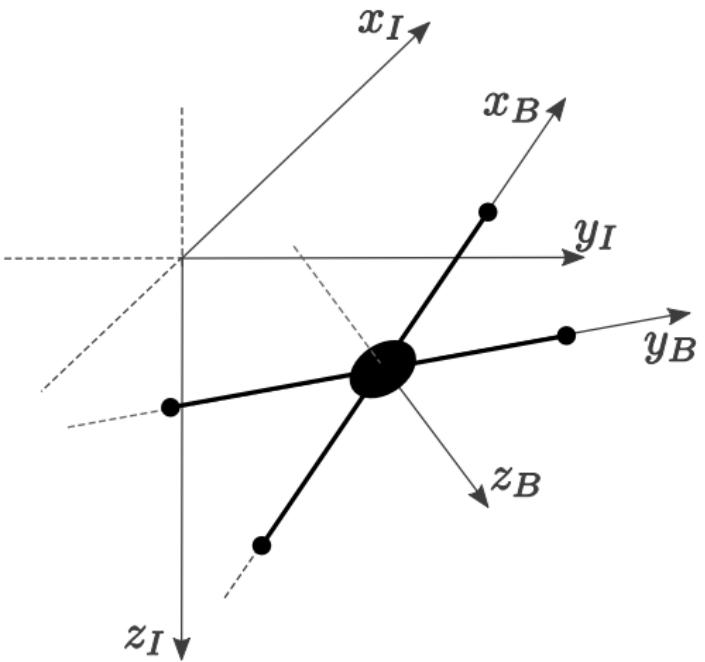
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## Attitude Model



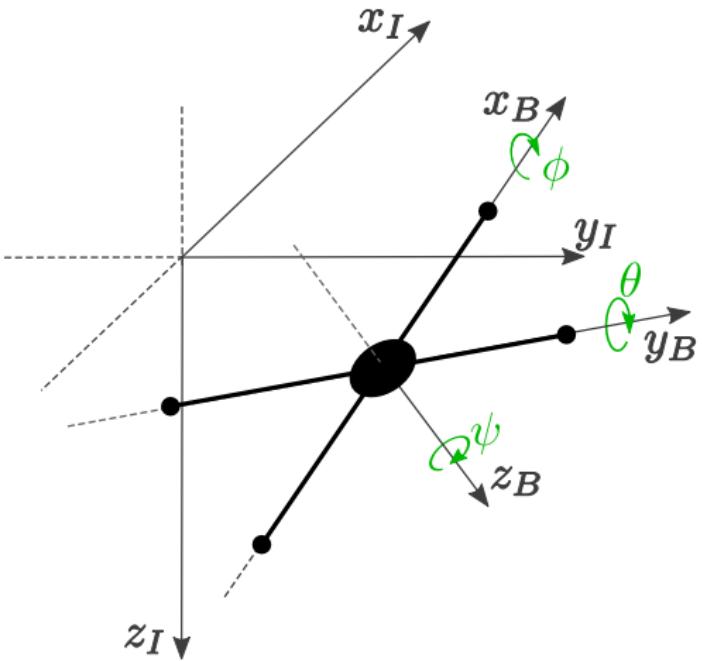
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## Attitude Model



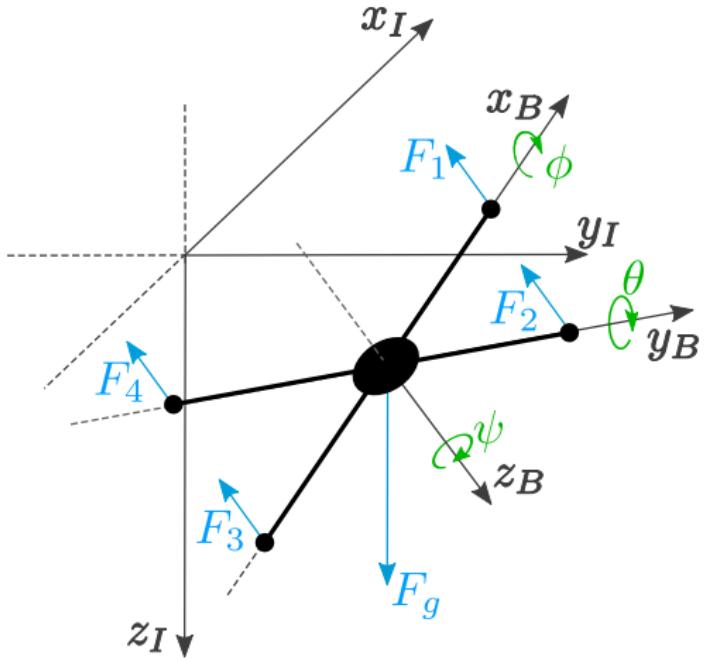
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## Attitude Model



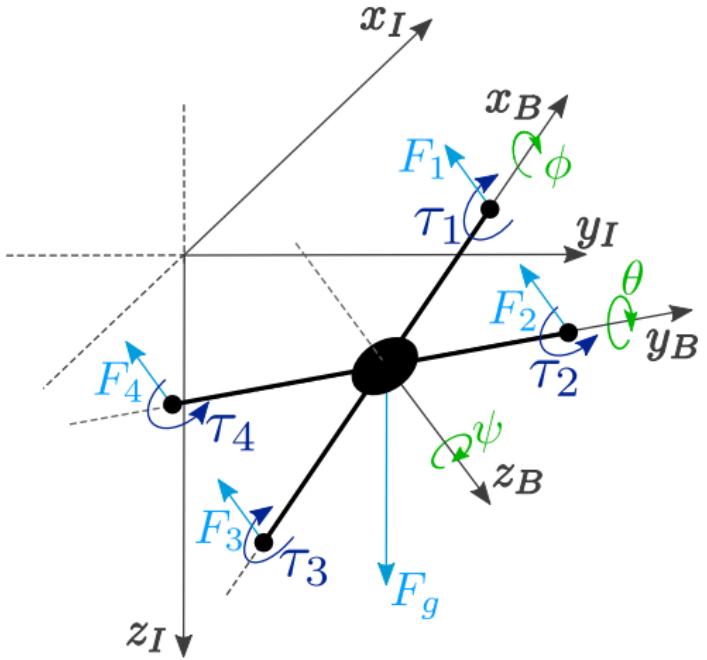
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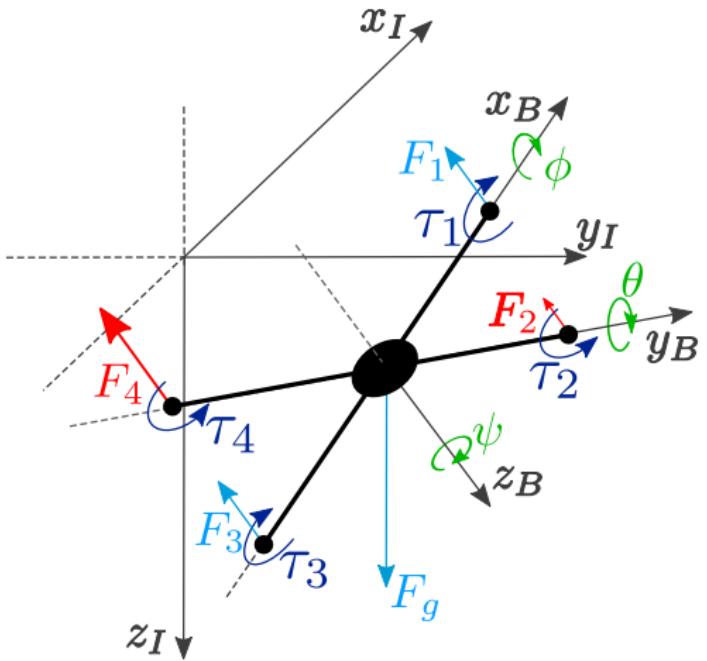
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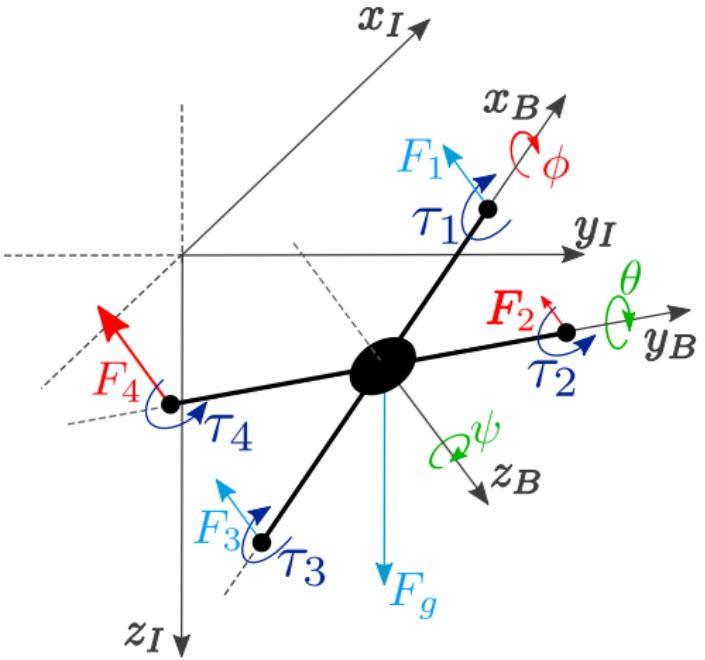
# Model

## Attitude Model



# Model

## Attitude Model



# Model

## Attitude Model



- ▶ Dynamic Equations

# Model

## Attitude Model



- ▶ Dynamic Equations

$$J\alpha = \sum \tau$$

# Model

## Attitude Model



### ► Dynamic Equations

$$J\alpha = \sum \tau$$

$$J_x \ddot{\phi} = (F_4 - F_2)L$$

$$J_y \ddot{\theta} = (F_1 - F_3)L$$

$$J_z \ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

# Model

## Attitude Model



### ► Dynamic Equations

$$J\alpha = \sum \tau$$

$$J_x \ddot{\phi} = (F_4 - F_2)L$$

$$J_y \ddot{\theta} = (F_1 - F_3)L$$

$$J_z \ddot{\psi} = \tau_1 - \tau_2 + \tau_3 - \tau_4$$

$$J_x \ddot{\phi} = k_{\text{th}}(\omega_4^2 - \omega_2^2)L$$

$$J_y \ddot{\theta} = k_{\text{th}}(\omega_1^2 - \omega_3^2)L$$

$$J_z \ddot{\psi} = k_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)$$

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Network

Control Solution

- Attitude Controller
- Translational Controller

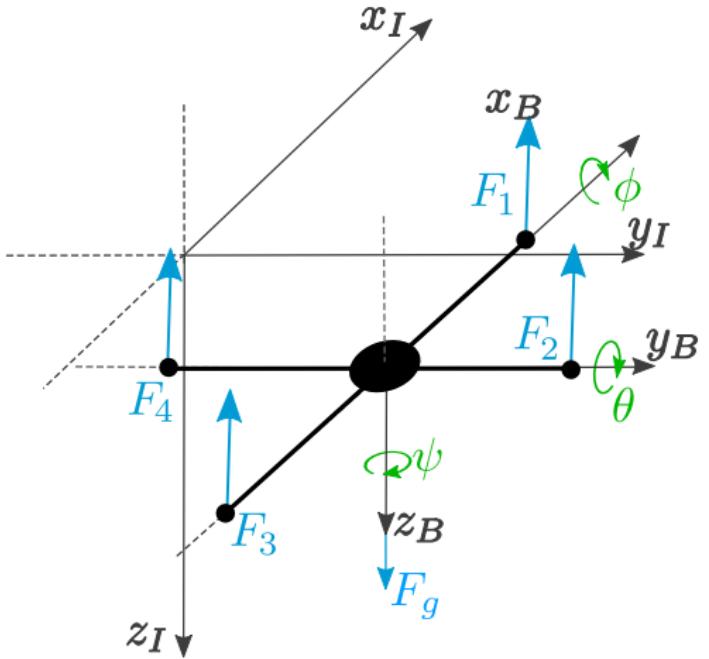
Implementation

Results

Final Statements

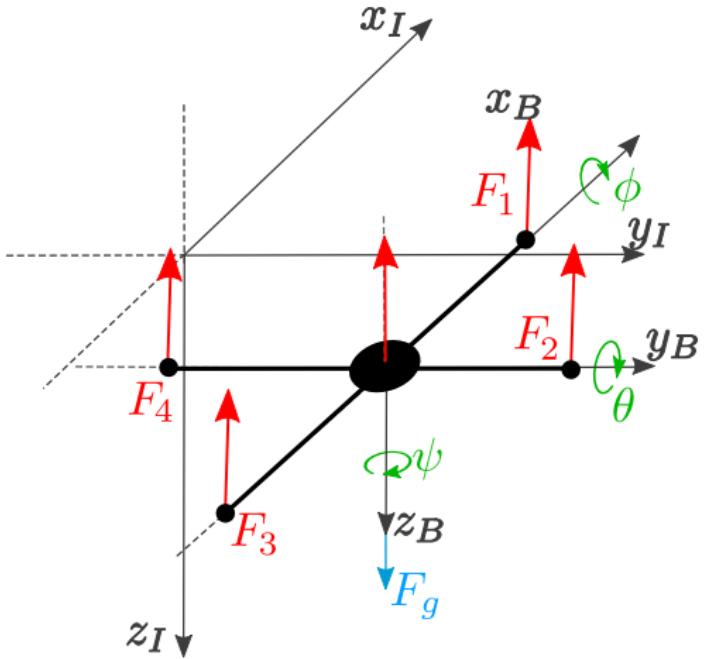
# Model

## Translational Model



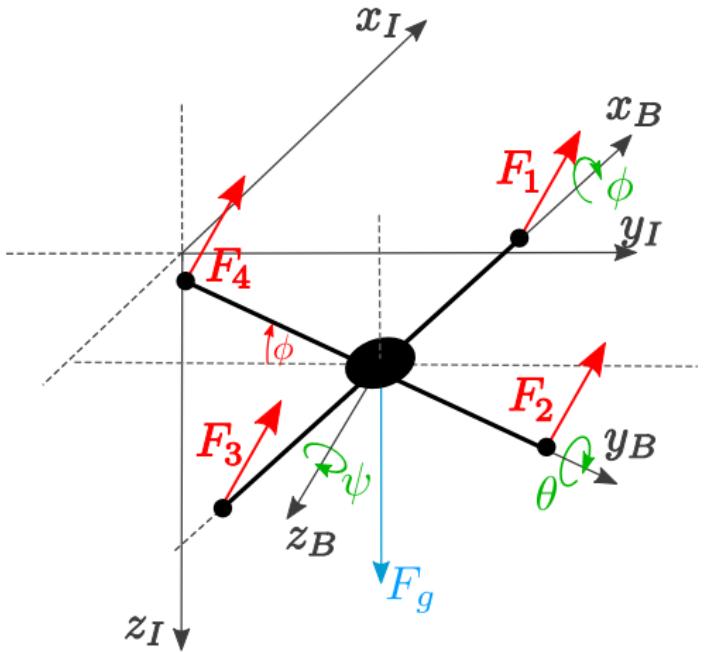
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## Translational Model



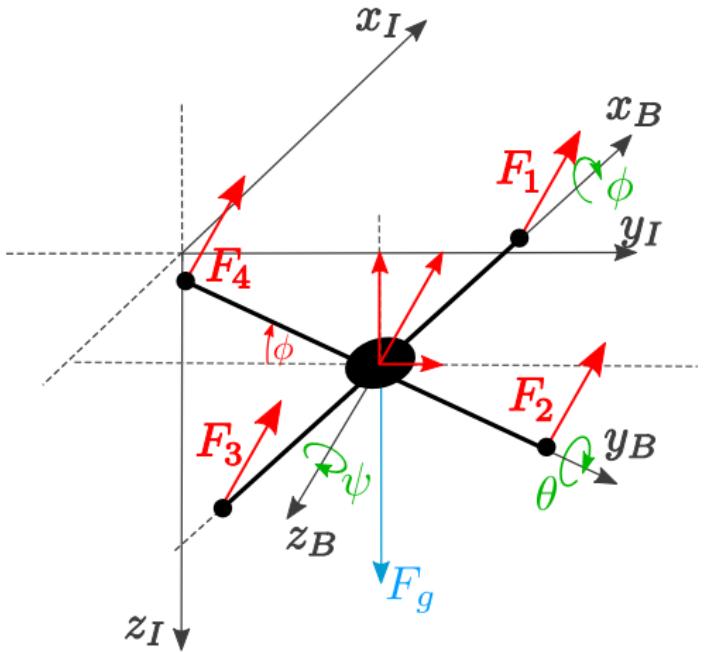
# Model

## Translational Model



# Model

## Translational Model



# Translational Model



- ▶ Rotation Matrix

$$R = R_Z R_Y R_X$$

$$v_I = R v_B$$

# Model

## Translational Model



- ▶ Dynamic Equations

# Model

## Translational Model



- ▶ Dynamic Equations

$$ma = \sum F$$

# Model

## Translational Model



### ► Dynamic Equations

$$ma = \sum F$$

$$m\ddot{x}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - (F_1 + F_2 + F_3 + F_4) \cos \phi \cos \theta$$

# Model

## Translational Model



### ► Dynamic Equations

$$ma = \sum F$$

$$m\ddot{x}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -(F_1 + F_2 + F_3 + F_4)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - (F_1 + F_2 + F_3 + F_4) \cos \phi \cos \theta$$

$$m\ddot{x}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \cos \phi \cos \theta$$

# Model Linearization



# Model

## Linearization



$$m\ddot{\bar{z}}_I = F_g - k_{th}(\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \cos \bar{\phi} \cos \bar{\theta}$$

# Model

## Linearization



$$m\ddot{\bar{z}}_I = F_g - k_{th}(\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \cos \bar{\phi} \cos \bar{\theta}$$

$$\bar{\omega}_i = \sqrt{\frac{F_g}{4k_{th}}}$$

# Model Linearization



# Network



# Network



- ▶ Delay
- ▶ Missed packets

# Network

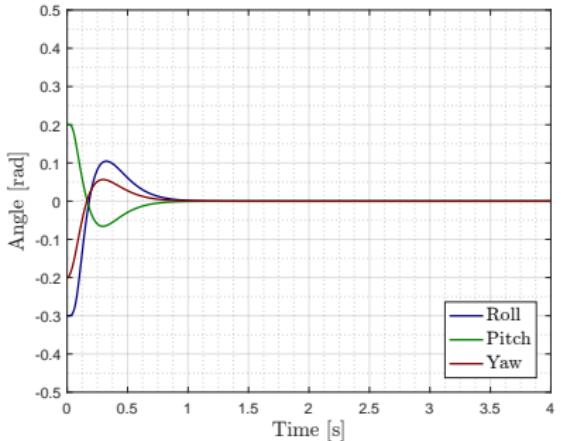


- ▶ Delay
- ▶ Missed packets

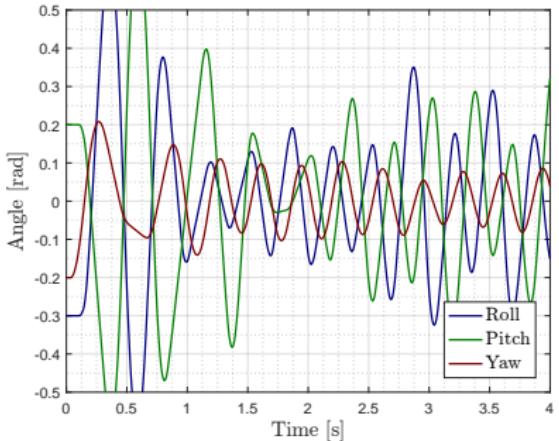
# Network



12



Control design only taking the model into account



Same controller with the effect of the network

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Control Solution

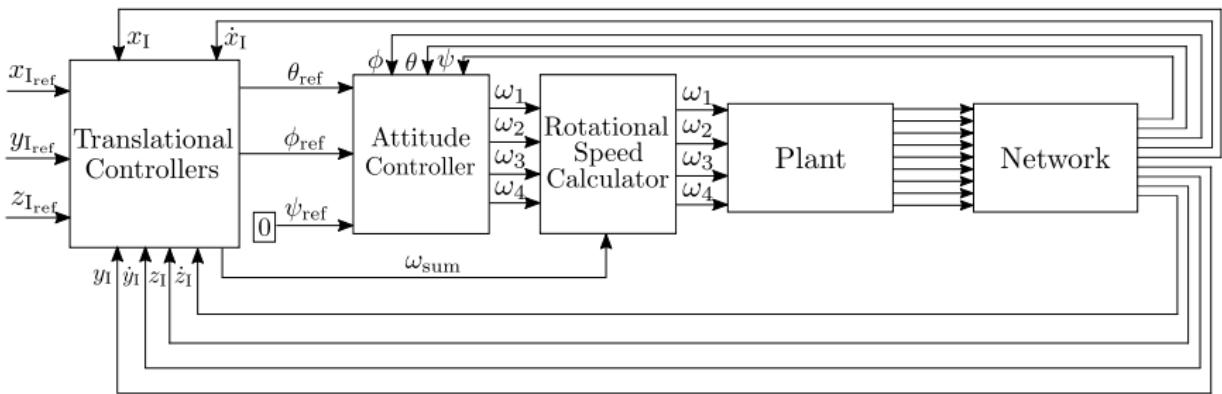
- Attitude Controller
- Translational Controller

Implementation

Results

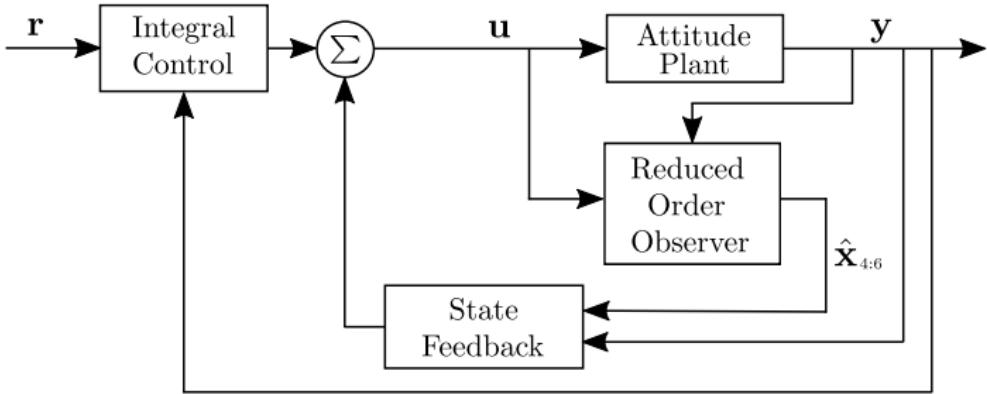
Final Statements

# Control Solution



# Control Solution

## Attitude Controller



# Control Solution

## Attitude Controller



- ▶ System Representation

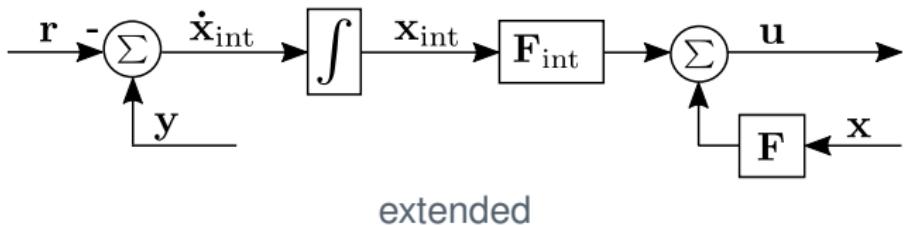
# Control Solution

## Attitude Controller



17

- ▶ State Feedback with Integral Control



# Control Solution

## Attitude Controller



- ▶ LQR

$$J = \int_0^{\infty} \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} \, dt$$

- ▶ Bryson's Rule

$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } [x_i^2]}$$

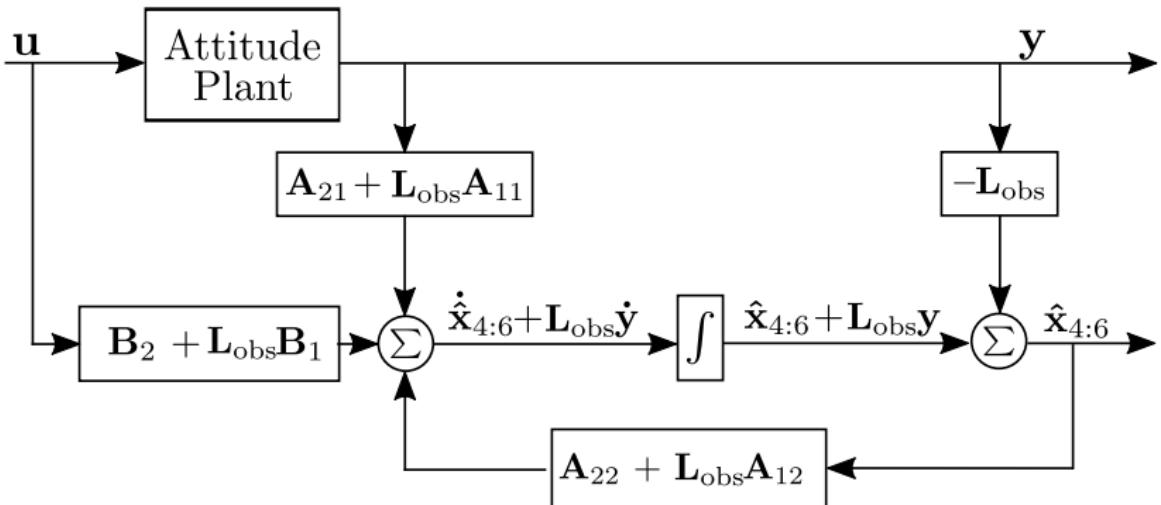
$$R_{ii} = \frac{1}{\text{maximum acceptable value of } [u_i^2]}$$

# Control Solution

## Attitude Controller



- Reduced Order Observer



equation what are the states we estimate

$$A_{22} + L_{obs}A_{12}$$

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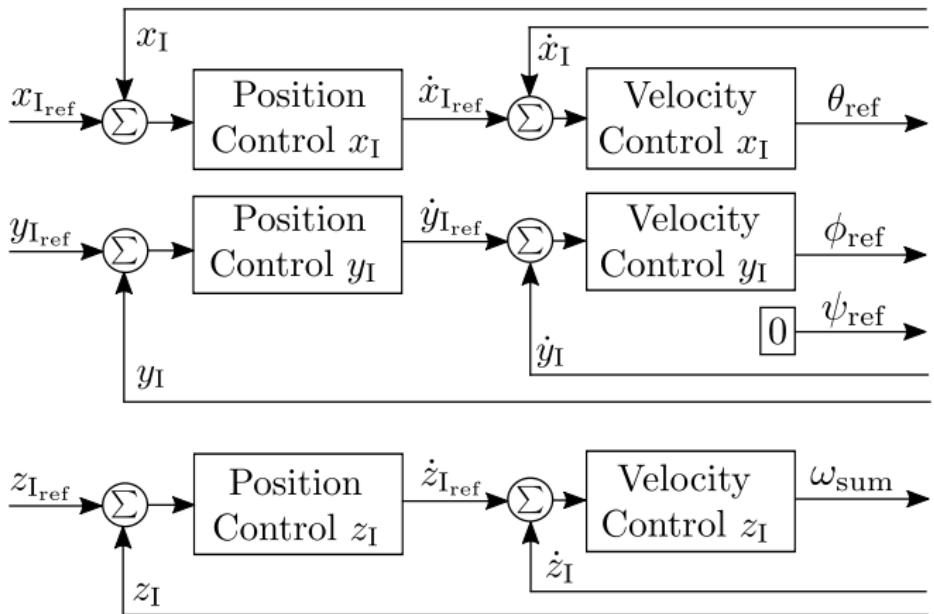
Final Statements

# Control Solution

## Translational Controller

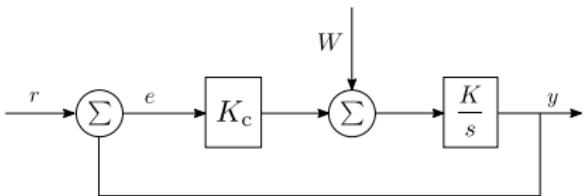


21



# Control Solution

## Translational Controller

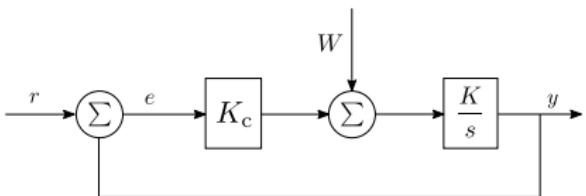


# Control Solution

## Translational Controller



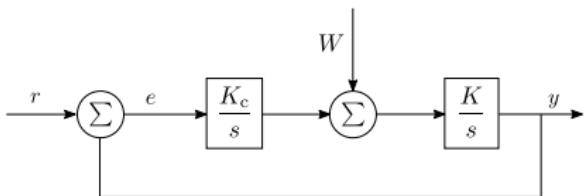
22



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$

# Control Solution

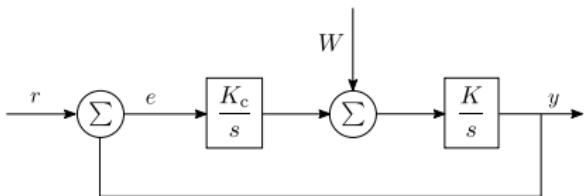
## Translational Controller



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + K_c \frac{K}{s}} = \frac{K}{s + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{K}{s + K_c K} = \frac{1}{K_c}$$

# Control Solution

## Translational Controller



$$\frac{y}{W} = \frac{\frac{K}{s}}{1 + \frac{K_c}{s} \frac{K}{s}} = \frac{Ks}{s^2 + K_c K} \Rightarrow \lim_{s \rightarrow 0} \frac{Ks}{s^2 + K_c K} = 0$$

# Control Solution

## Translational Controller



$$\frac{\dot{x}_I}{\theta} = \frac{-k_{th}4\bar{\omega}}{ms} \quad \frac{\dot{y}_I}{\phi} = \frac{k_{th}4\bar{\omega}}{ms} \quad \frac{\dot{z}_I}{\omega_{sum}} = \frac{-2k_{th}\bar{\omega}}{ms}$$

# Control Solution

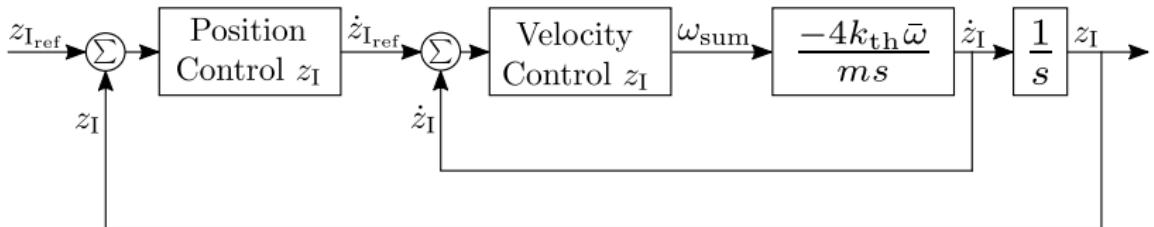
## Translational Controller



$$\frac{\dot{x}_I}{\theta} = \frac{-k_{th} 4 \bar{\omega}}{ms}$$

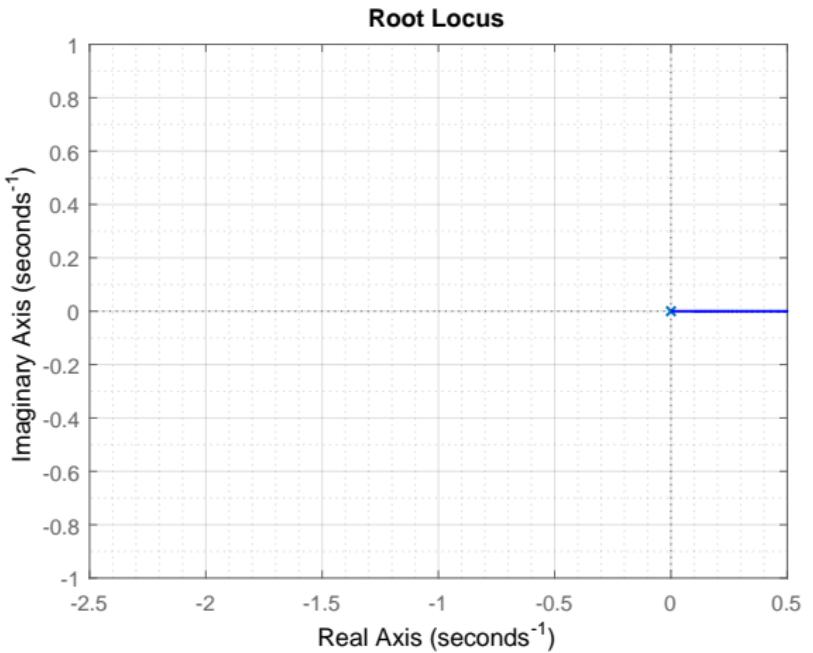
$$\frac{\dot{y}_I}{\phi} = \frac{k_{th} 4 \bar{\omega}}{ms}$$

$$\frac{\dot{z}_I}{\omega_{sum}} = \frac{-2k_{th} \bar{\omega}}{ms}$$



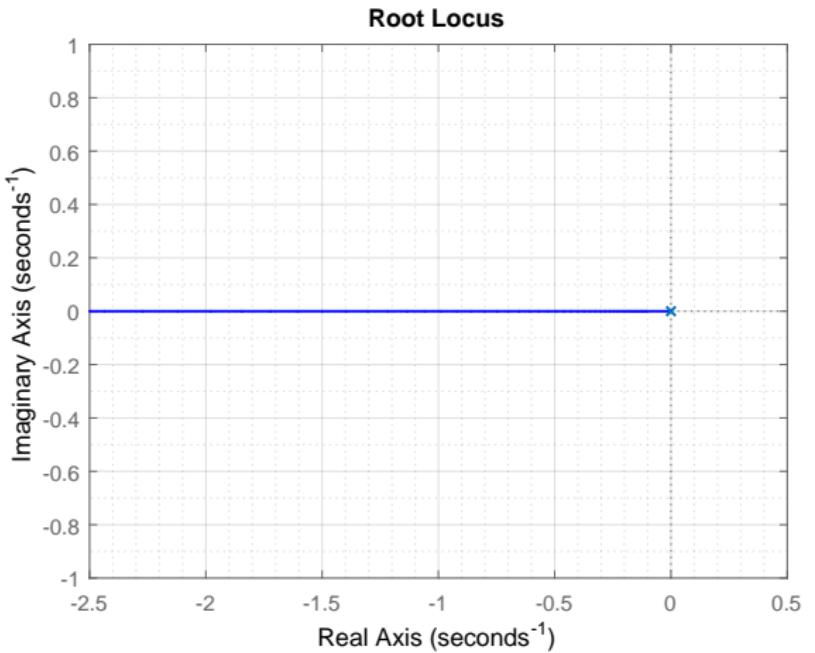
# Control Solution

## Translational Controller



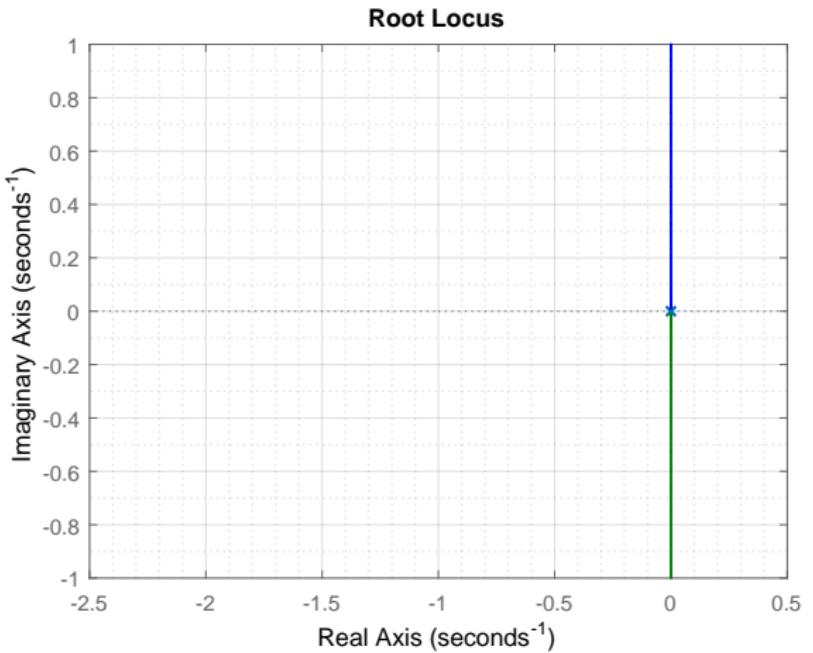
# Control Solution

## Translational Controller



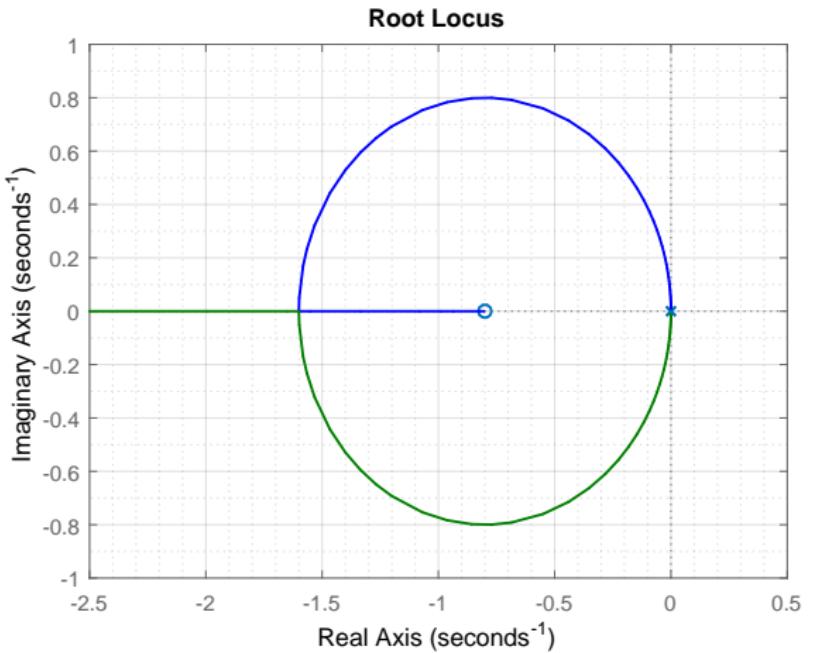
# Control Solution

## Translational Controller



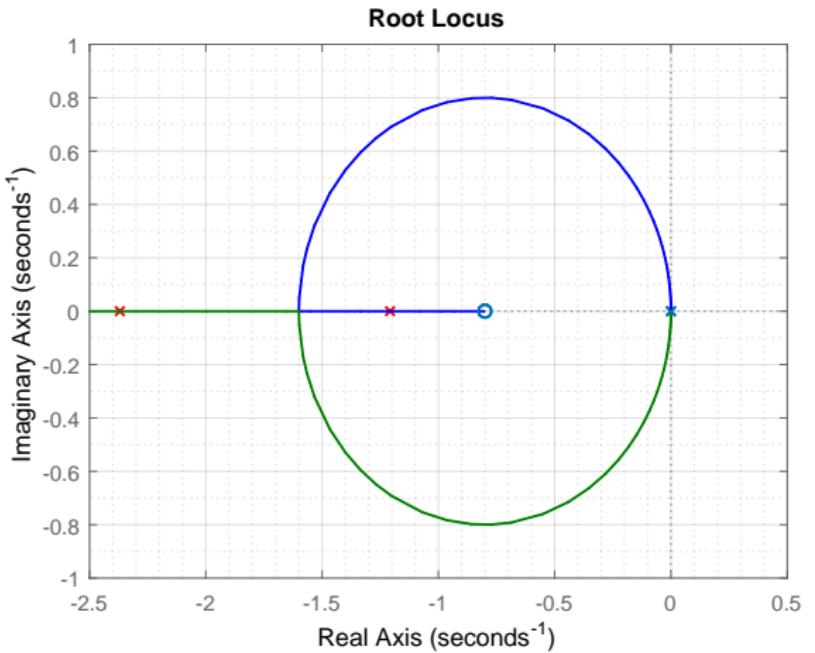
# Control Solution

## Translational Controller



# Control Solution

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# Control Solution

## Translational Controller

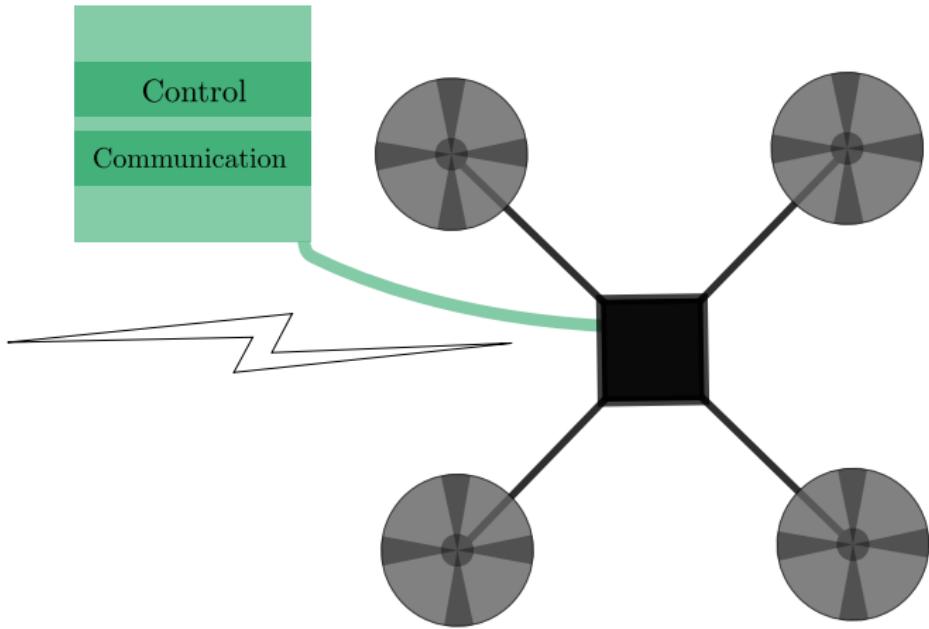


$$C_{\dot{x}_I} = -0.0038 \frac{1 + 20s}{s} \quad C_{x_I} = 0.3$$

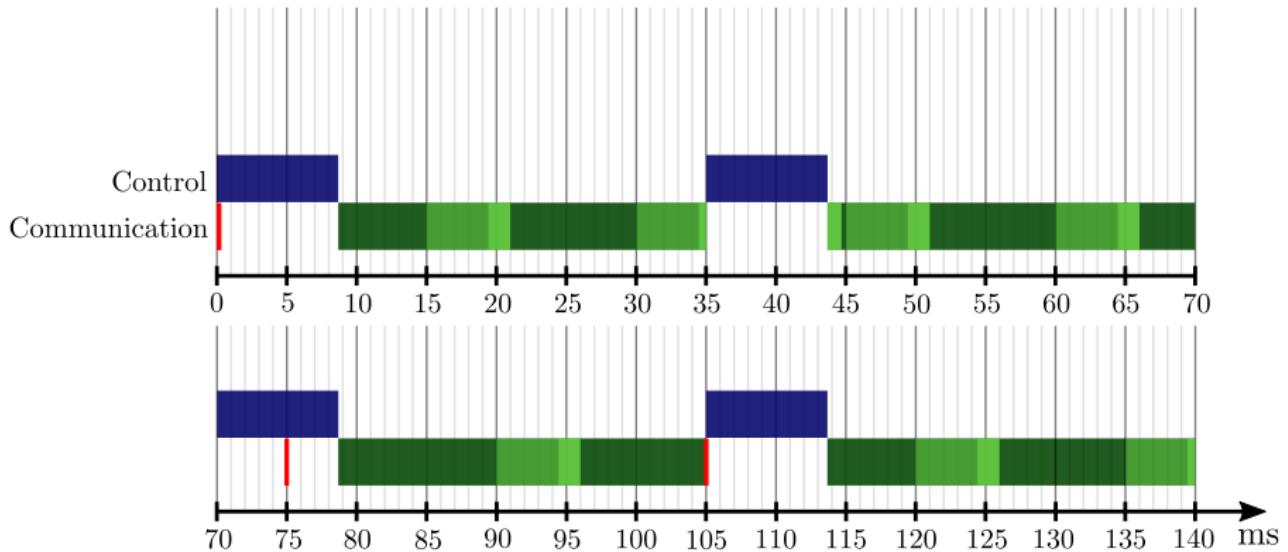
$$C_{\dot{y}_I} = 0.0038 \frac{1 + 20s}{s} \quad C_{y_I} = 0.3$$

$$C_{\dot{x}_I} = -280 \frac{s + 0.8}{s} \quad C_{z_I} = 1$$

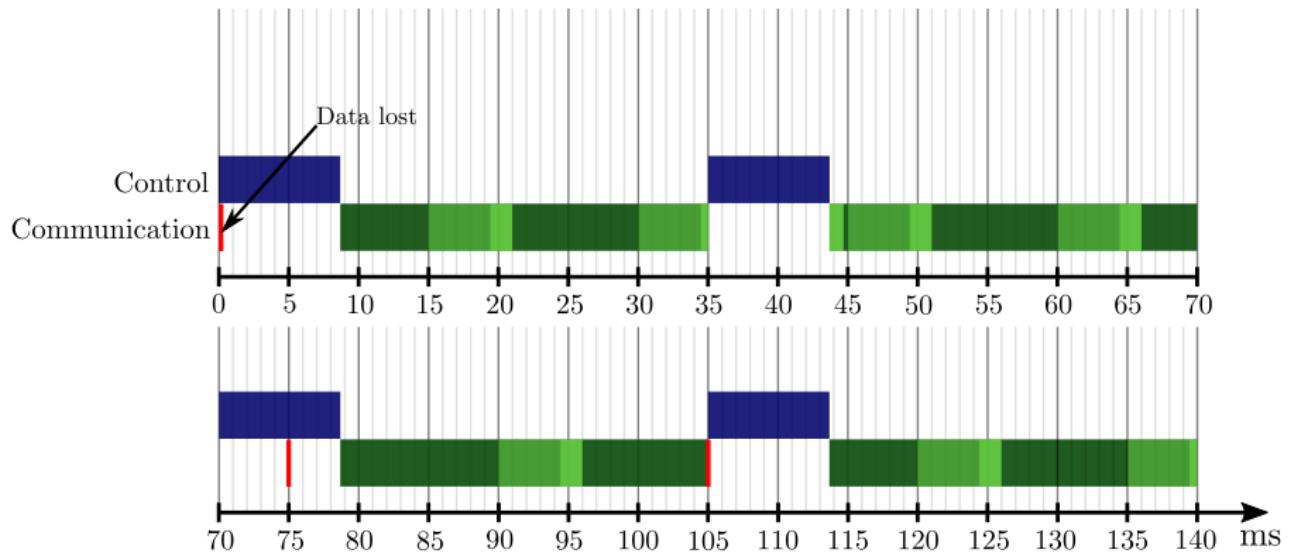
# Implementation



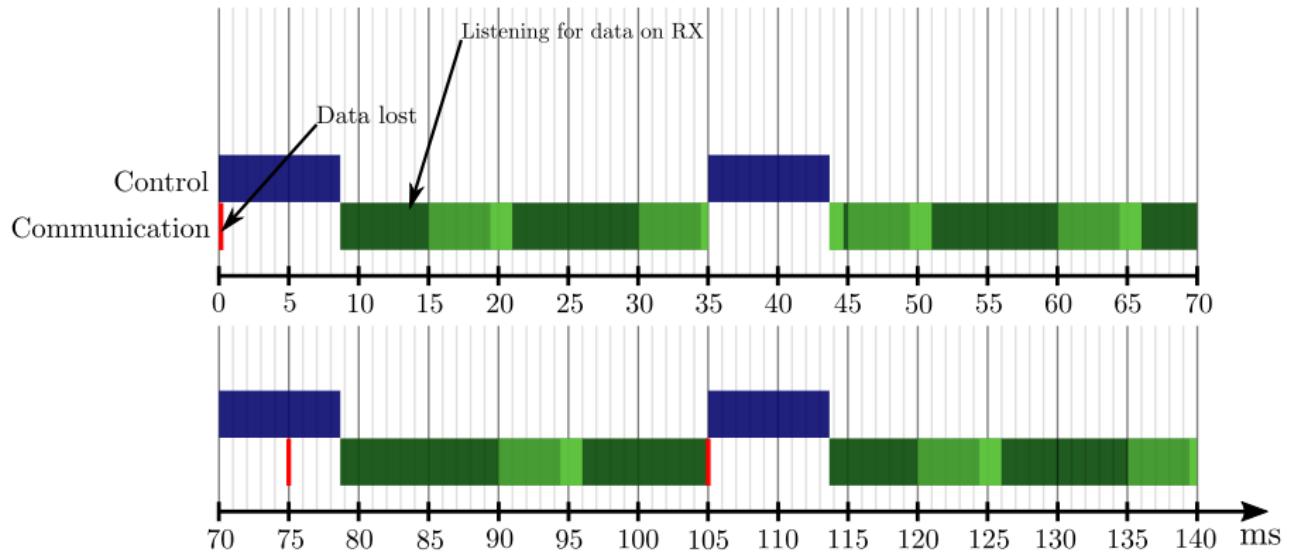
# Implementation Schedule



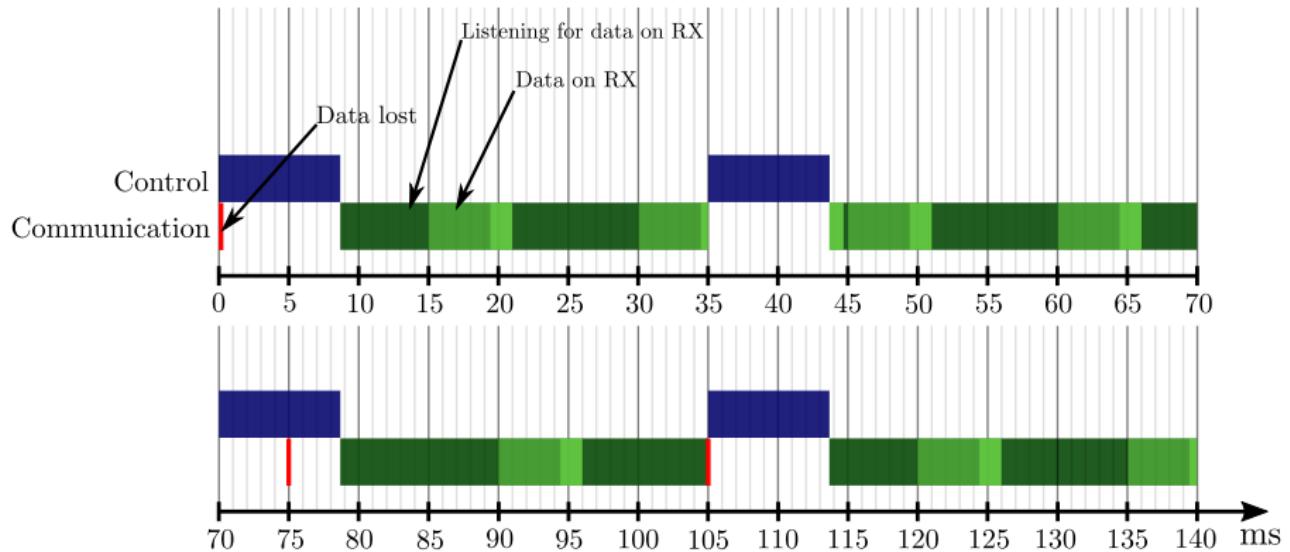
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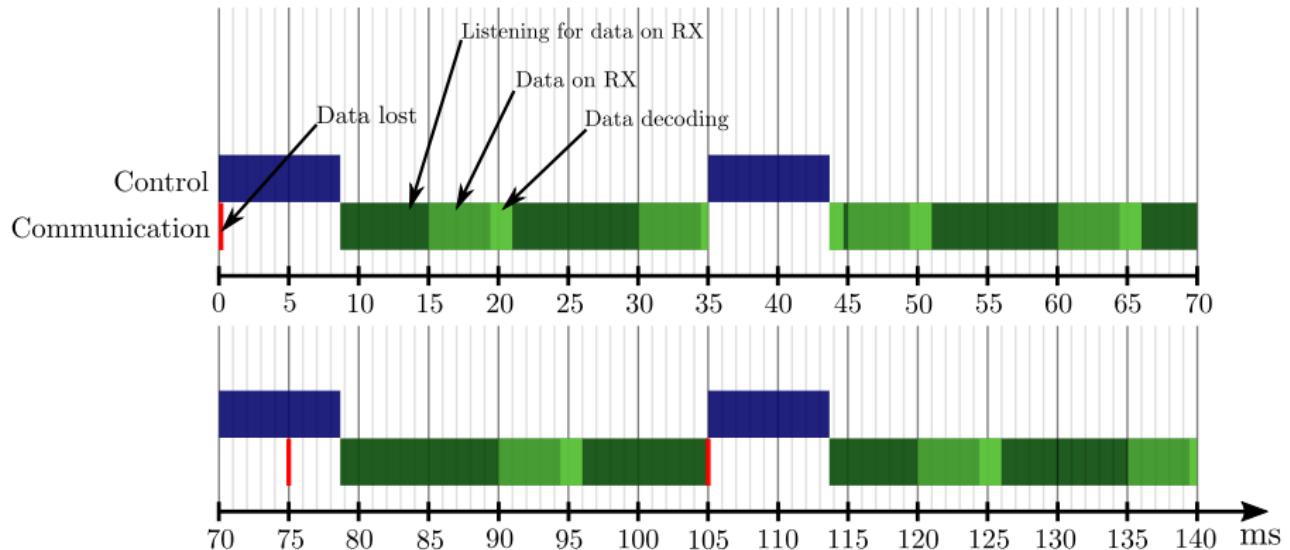
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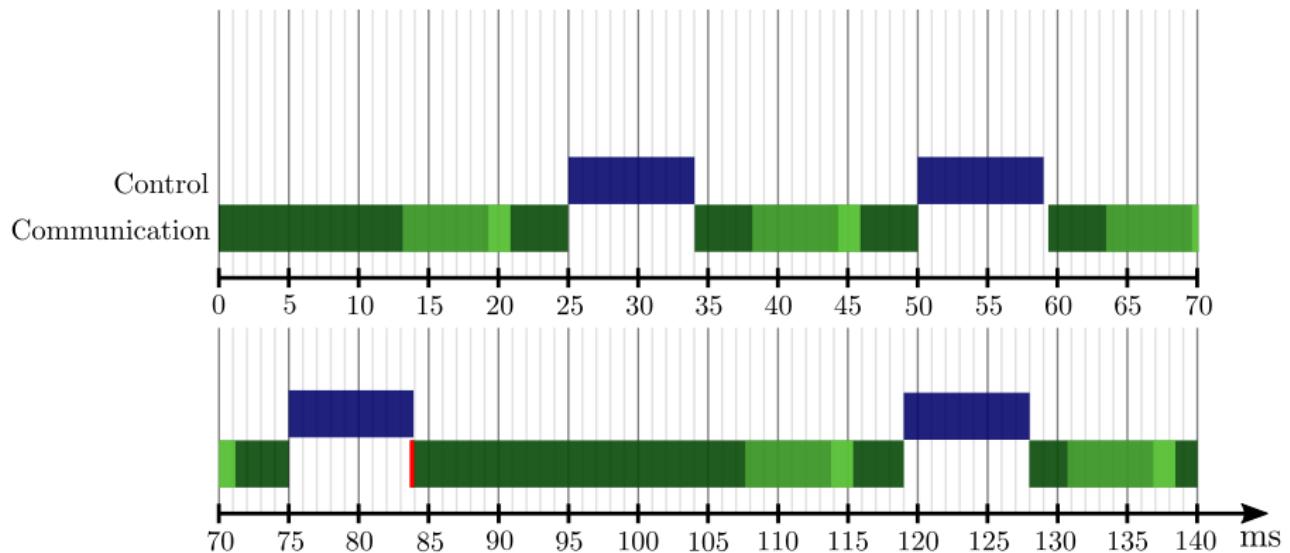
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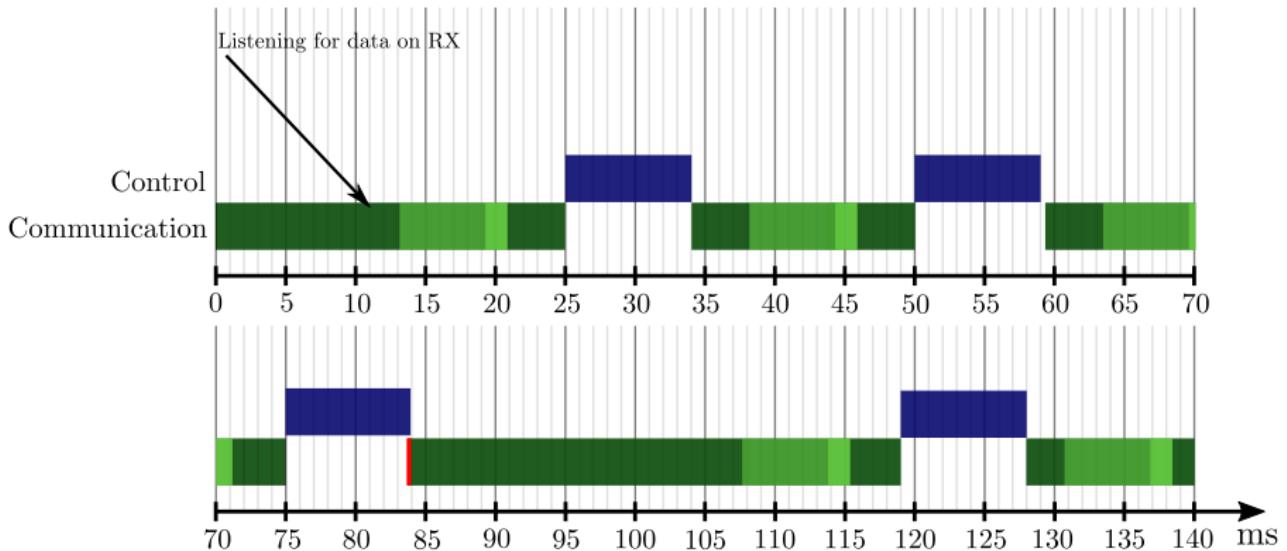
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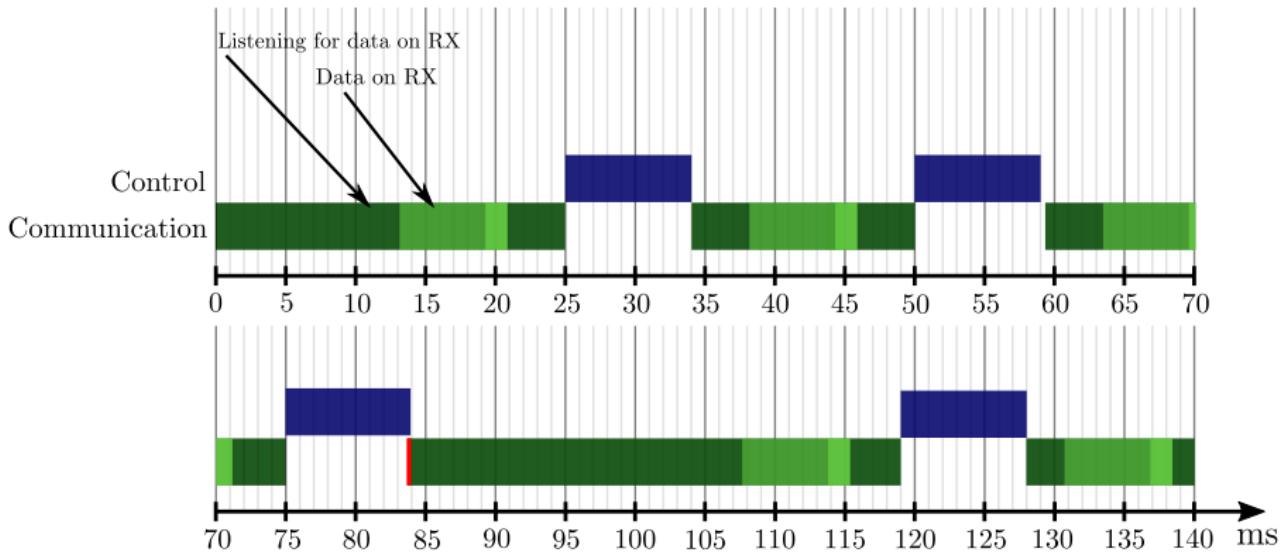
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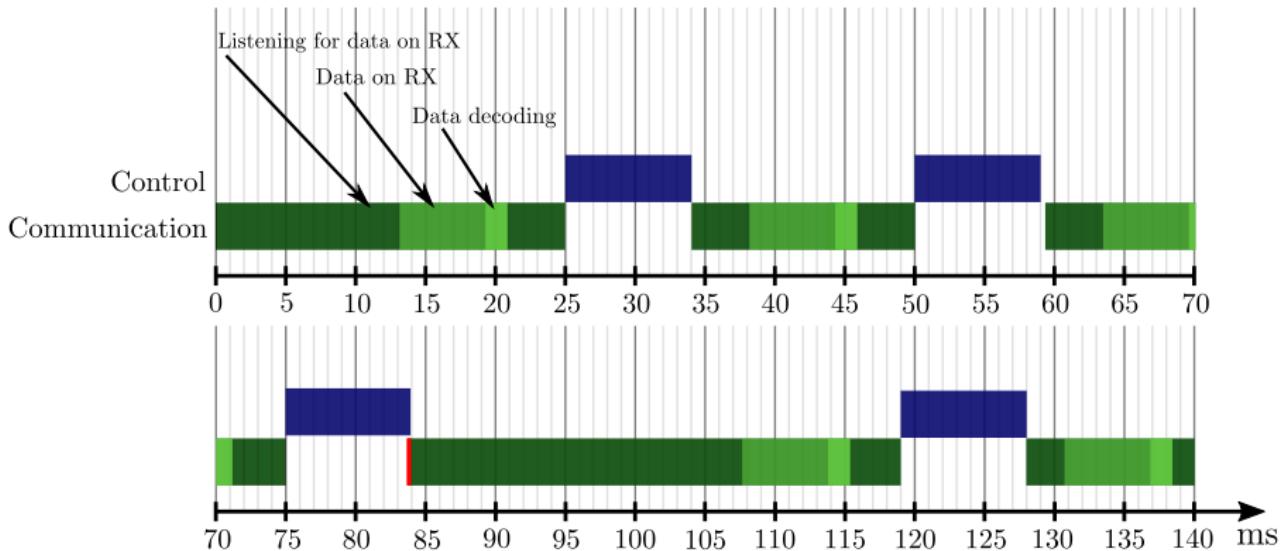
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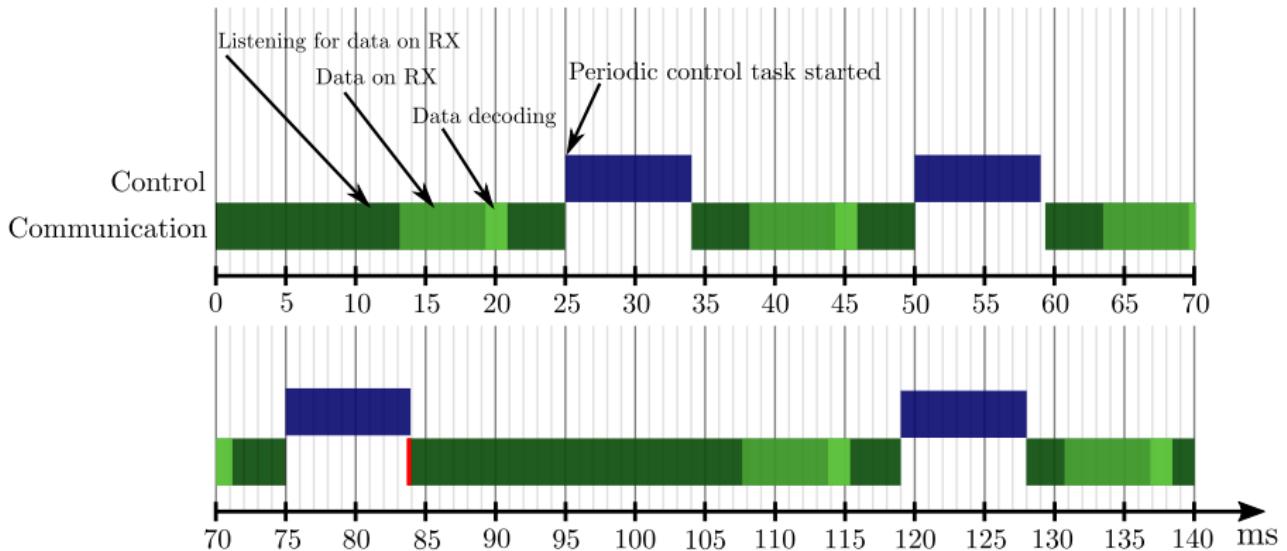
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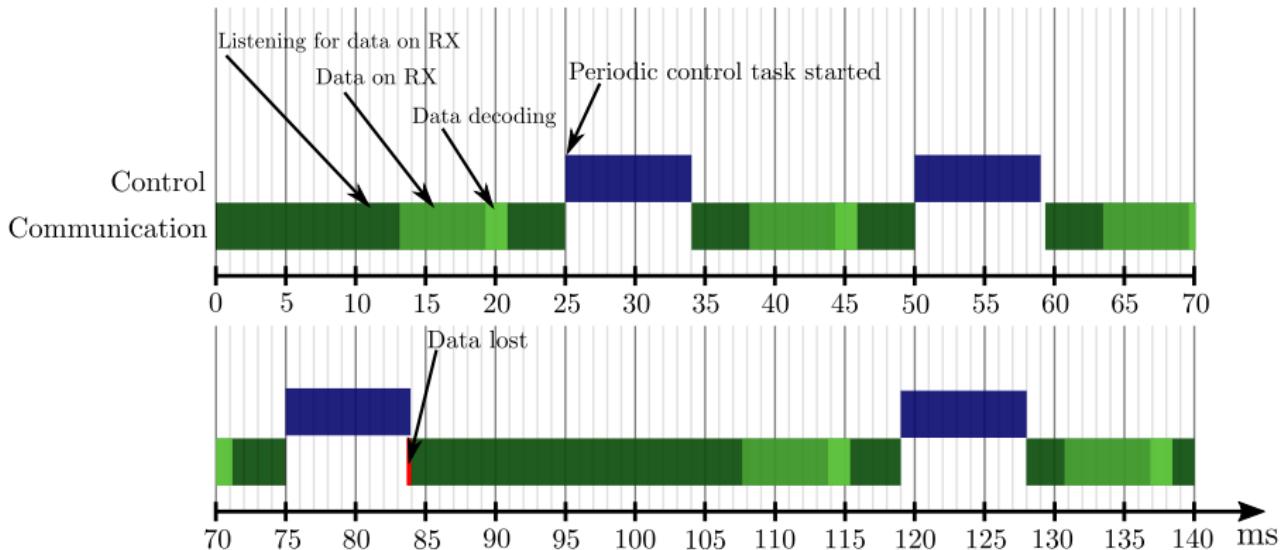
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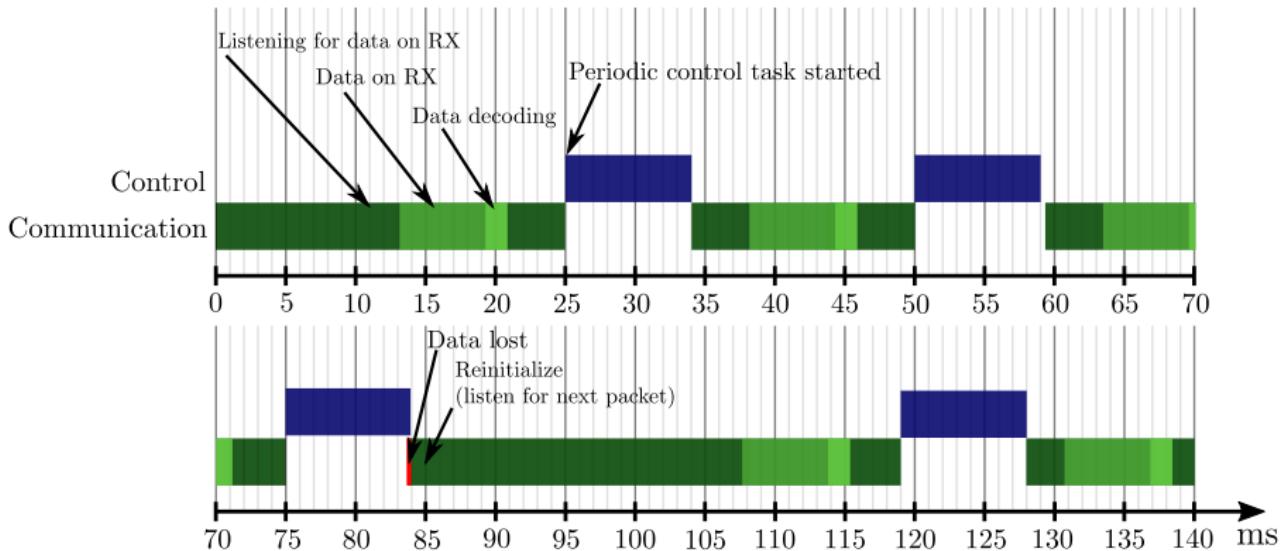
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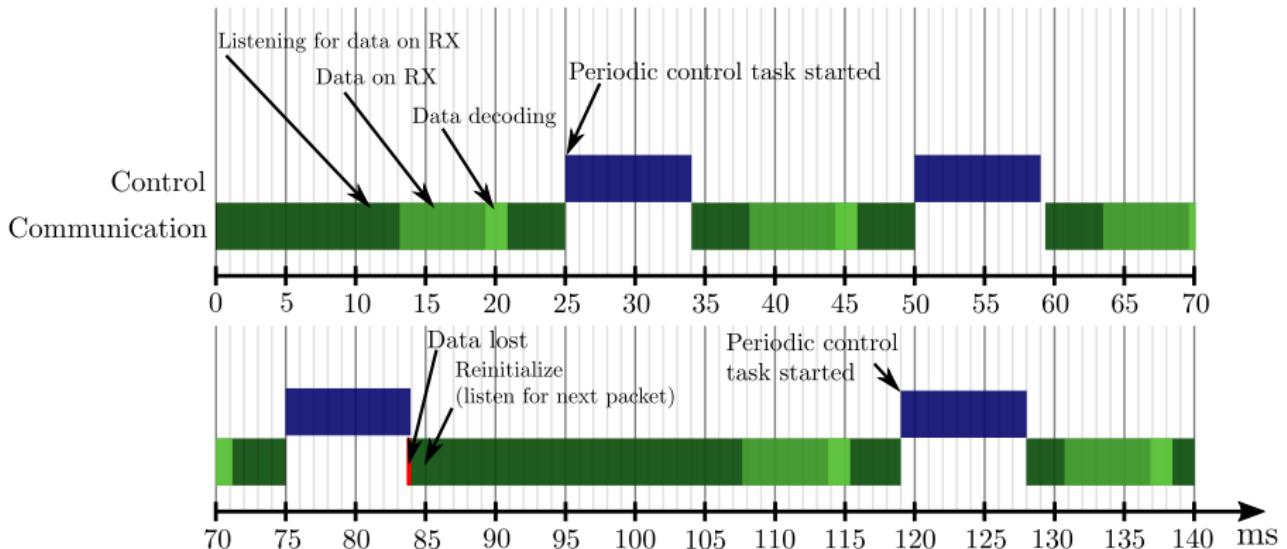
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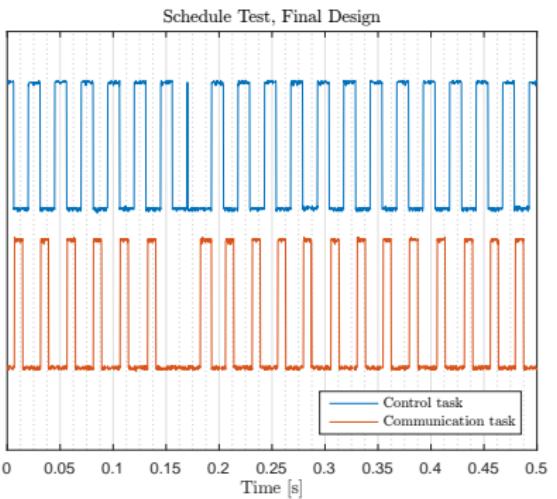
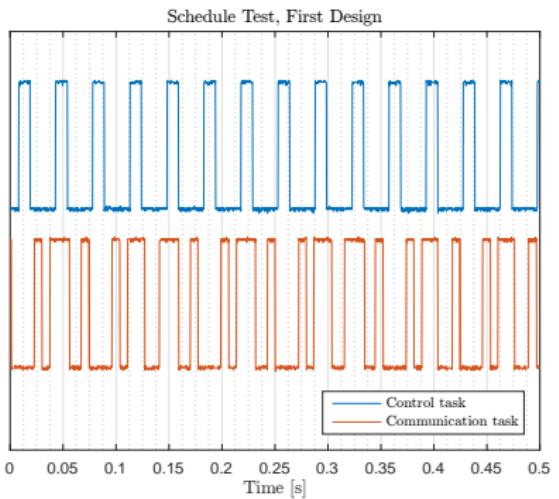
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# Implementation Schedule



29



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Network

Control Solution

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Implementation

Results

Final Statements

# Results

## Attitude Controller Simulations



# Results

Translational Controllers Simulations



# Results

## Attitude Controller Functional Tests



# Final Statements



- Limited bandwidth of controllers due to sampling.
- Control design working in simulation for attitude and translational controller.
- Implementation/Hardware issue when testing the translational controllers.
- Attitude controller tests carried out successfully.

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