

Attitude and Position Control of a Quadcopter in a Networked Distributed System

Alejandro Alonso García, Amalie V. Petersen, Andrea Victoria Tram Løvemærke,
Niels Skov Vestergaard, Noelia Villarmarzo Arruñada
School of Information and Communication Technology
Control and Automation
Aalborg University

Email: [aalons16] [apet13] [alavem13] [nveste12] [nvilla16] @student.aau.dk

Abstract—Quadcopters are becoming increasingly interesting due to the great variety of usage. A design that is able to make the quadcopter hover and move to a desired position is presented. The system's coupled behavior and instability raises a challenging control task. This task is solved by implementing a controller design, which is based upon a model that is derived by first principle modeling. This is later linearized since it is desired to use linear controllers. The system is divided into an attitude and translational control loops. These are designed by using state space and classical control methods, respectively. The quadcopter gets its attitude and position from a motion tracking system based on infrared cameras, keeping the control in a micro processor on the quadcopter. This layout constitutes a distributed system, where network issues, such as delays and packet losses, are taken into account.

I. INTRODUCTION

In the last years, the interest for quadcopters has increased due to the great possibilities they offer. Among these, the most well-known ones are surveillance, inspection of big structures and search and rescue missions in difficult environments. **droneuses**

The quadcopter constitutes a control challenge due to its unstable nature and coupled behavior. The system has six degrees of freedom, the three position coordinates and the three orientations, and there are only four actuation variables, namely the motor rotational speeds **draganflyer**

The control of a quadcopter has been addressed many times in the recent years. In Mian et al. **backstepping** the quadcopter is controlled using a back-stepping technique and non-linear controllers. Another way of solving the issue is presented in Tayebi et al. **quaternionsPD** in which the quadcopter attitude is modeled using quaternions and controlled with a PD based controller. In **MianWang** Mian and Wang model the system using its dynamic equations and use non-linear controllers to achieve a steady flight while in Mokhtari et al. **GHinf** the system is controlled by a mixture of a robust feedback linearization and a modified optimization control method.

¹ The approach presented here tries to control the attitude and translational position of the quadcopter using linear controllers and transmitting sensor data through a wireless network.

¹FiXme Note: Why we do not include specific details of our hardware or our network in the methods??

In section II, the model of the quadcopter is obtained by a first principles method. This approach yields a non-linear model that describes the attitude and translational behavior of the quadcopter. The model is then linearized around an equilibrium point, which is chosen to be in hovering position. With the linearized equations, controllers for attitude and translational behaviors are designed in section III. The attitude controller is obtained by means of a state space representation while the translational controller is designed using classical control. In the control system, the translational constitutes an outer loop and sets the reference for the attitude controller. Since the sensors are not placed in the quadcopter and the information comes from an external motion tracking system **vicon** an analysis on how the network affects the control loop is also presented, this is done in section IV. In section V, the simulations and experimental results of the designed controllers are presented. They are after discussed in section VI. Lastly, a conclusion is presented in section VII and possible future work is mentioned in section VIII. ²

II. MODEL

The quadcopter free body diagram is shown in Figure 1.

²FiXme Note: Are we to present both simulation and real results?

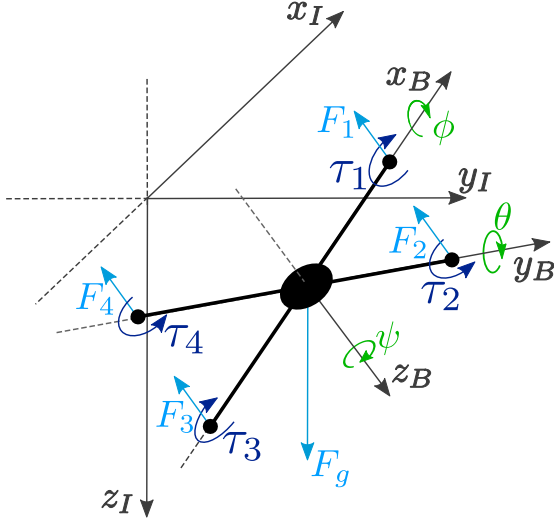


Figure 1: Forces (\mathbf{F}_i) and torques (τ_i) acting on the quadcopter and the positive references chosen for rotations and translations in both inertial and body coordinate frames.

As it is seen, the system is modeled by using two coordinate frames. The inertial frame is utilized to describe the translational movement while the body frame is attached to the quadcopter and used to characterize its attitude behavior. In the figure, also the positive references for rotational and translational movements are depicted, as well as the main forces and torques acting on the quadcopter. The positive rotations follow a right-hand fashion.

The forces generated in the propeller are readily obtained in the body coordinate frame. In order to represent them in the inertial frame a rotation matrix is used. It is built considering a 123 rotation sequence **rotationmatrix**. This means that any rotation is described as three rotations around the x_B axis first, then around the y_B axis and lastly around the z_B axis.

The dynamic model of the quadcopter is given by three sets of equations. The first describes the motor and the propeller, the second presents the attitude response of the quadcopter and the third explains how the translational variables of the system evolve.

3

A. Motor and Propeller

The four motors in the quadcopter generate a rotation in the propellers that creates the force that lifts the quadcopter. The thrust force can be modeled as proportional to the square of the motor rotational speed. The thrust coefficient for each motor is found experimentally. The units for this coefficient are $\text{Ns}^2\text{rad}^{-2}$

The rotation also generates a torque on each motor due to the aerodynamic drag. Drag torque is compensated in the quadcopter by having two of the motors turning in one

direction and the two others in the opposite direction. It is as well described as proportional to the square of the rotational speed in terms of a drag coefficient, which is also obtained experimentally. Its units are $\text{Nms}^2\text{rad}^{-2}$

The expressions for the thrust force and drag torque caused by the rotation of each propeller are

$$F = k_{th}\omega^2 \quad (1)$$

$$\tau = k_d\omega^2 \quad (2)$$

where F is the thrust force, k_{th} is the thrust coefficient, ω is the angular speed of the motor, τ is the drag torque and k_d is the drag coefficient.

These equations are used in the attitude and translational models presented below.

B. Attitude Model

The attitude model equations, which are based on Newton's Second Law for rotational movement, are as follows

$$J_x\ddot{\phi} = k_{th}(\omega_4^2 - \omega_2^2)L \quad (3)$$

$$J_y\ddot{\theta} = k_{th}(\omega_1^2 - \omega_3^2)L \quad (4)$$

$$J_z\ddot{\psi} = k_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \quad (5)$$

where J_x , J_y and J_z are the moments of inertia around the three axes of rotation, $\ddot{\phi}$, $\ddot{\theta}$ and $\ddot{\psi}$ are the angular accelerations in roll, pitch and yaw, respectively, ω_i is the rotational speed of each motor and L is the distance between the center of the quadcopter and the position of the motors.

The expressions above state how the thrust forces and the drag torques generated on the propellers affect the attitude behavior of the quadcopter.

C. Translational Model

The equations describing the response of the system along the inertial x , y and z axes are derived from Newton's Second Law of Motion. The forces that act on the system are those from the propellers and the gravitational force. These expressions are

$$m\ddot{x}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (6)$$

$$\times (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi)$$

$$m\ddot{y}_I = -k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (7)$$

$$\times (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$m\ddot{z}_I = F_g - k_{th}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (8)$$

$$\times \cos \phi \cos \theta$$

where m is the mass of the quadcopter, \ddot{x}_I , \ddot{y}_I and \ddot{z}_I are the accelerations along the inertial reference frame directions, ϕ , θ and ψ are the roll, pitch and yaw angles respectively and F_g is the gravitational force acting on the quadcopter.

It is worth mentioning that, as the thrust forces always point in the negative z_B direction, the accelerations along x_I and y_I directions are zero when pitch and roll angles are zero.

³Fixme Note: In the modeling section, what are your assumptions? What is the reason for not including model parts such as Gyroscopic torque, Air friction/drag on the quadcopter itself (even though you include it on the propellers), Aerodynamic lift, Mass placement and separation of masses, Coriolis acceleration?

D. Linearization

The model equations are linearized using the first order Taylor approximation around an equilibrium point of the system. The chosen point is the hovering position, which implies that the attitude and translational accelerations and velocities are zero. The angular position of the quadcopter is also part of the point around which the linearization is made, and it is also set to zero in the three angles.

Choosing a zero acceleration linearization point along the z_I axis yields an equilibrium rotational speed so that the necessary thrust is generated to compensate for the gravitational force. The relation is expressed as⁴

$$\bar{\omega}_i = \sqrt{\frac{mg}{4k_{th}}} \quad (9)$$

The resulting equations for the attitude model after the linearization are

$$J_x \Delta \ddot{\phi} = 2k_{th} L \bar{\omega}_4 \Delta \omega_4 - 2k_{th} L \bar{\omega}_2 \Delta \omega_2 \quad (10)$$

$$J_y \Delta \ddot{\theta} = 2k_{th} L \bar{\omega}_1 \Delta \omega_1 - 2k_{th} L \bar{\omega}_3 \Delta \omega_3 \quad (11)$$

$$J_z \Delta \ddot{\psi} = 2k_d \bar{\omega}_1 \Delta \omega_1 - 2k_d \bar{\omega}_2 \Delta \omega_2 + 2k_d \bar{\omega}_3 \Delta \omega_3 - 2k_d \bar{\omega}_4 \Delta \omega_4 \quad (12)$$

where $\Delta \ddot{\phi}$, $\Delta \ddot{\theta}$ and $\Delta \ddot{\psi}$ are the changes in rotational acceleration from the linearization point, $\bar{\omega}_i$ is the rotational speed of each motor to achieve equilibrium along the z_I axis and $\Delta \omega_i$ is the change in rotational speed of each motor from the linearization point.

Similarly, the equations of the translational model are linearized. The result is

$$m \Delta \ddot{x}_I = -k_{th} (\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \Delta \theta \quad (13)$$

$$m \Delta \ddot{y}_I = k_{th} (\bar{\omega}_1^2 + \bar{\omega}_2^2 + \bar{\omega}_3^2 + \bar{\omega}_4^2) \Delta \phi \quad (14)$$

$$m \Delta \ddot{z}_I = -2k_{th} \bar{\omega}_1 \Delta \omega_1 - 2k_{th} \bar{\omega}_2 \Delta \omega_2 - 2k_{th} \bar{\omega}_3 \Delta \omega_3 - 2k_{th} \bar{\omega}_4 \Delta \omega_4 \quad (15)$$

where $\Delta \ddot{x}_I$, $\Delta \ddot{y}_I$ and $\Delta \ddot{z}_I$ are the changes in linear acceleration from the linearization point in each direction of the inertial frame and $\Delta \phi$ and $\Delta \theta$ are the changes in roll and pitch from the linearization point, respectively.

III. CONTROL

The control of the system is divided into two control systems. One handles the attitude and the other controls the translational behavior of the quadcopter. These two are related such that the translational controller sets the references for the angles handled by the inner controller.

A. Attitude Controller

The attitude controller for the quadcopter is designed using a state space representation of the system. This helps handling the coupled angular response of the quadcopter. The chosen states for the system are the three angular positions and the three angular velocities. The input vector of the attitude system

consists of the four motor rotational speeds and the output vector consists of the three angles, roll, pitch and yaw. The state, input and output vectors are

The attitude control is based on a state feedback and an integral term in order to be able to track a given reference and reject input disturbances. As not all states are measured a reduced order observer is implemented to estimate the angular velocities of the quadcopter. Due to the separation principle, both subsystems can be designed independently. **ssReference**

Figure 2 shows how these designs are related.

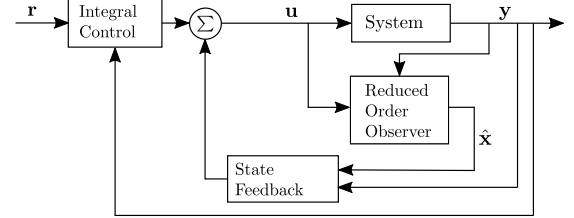


Figure 2: Control structure for the system, including the state feedback, the integral controller and the reduced order observer.

The design of the state feedback and integral control is shown in Figure 3.

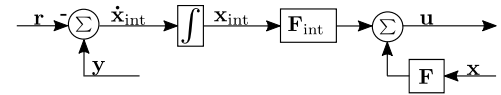


Figure 3: State feedback and integral controller in the attitude control structure.

Three states, $\mathbf{x}_{int}(t)$, are added to the already existing state vector to track the two references for ϕ and θ angles and reject input disturbances in the three angles. This leads to the extended system

$$\dot{\mathbf{x}}_e(t) = \mathbf{A}_e \mathbf{x}_e(t) + \mathbf{B}_e \mathbf{u}(t) + \begin{bmatrix} \mathbf{0} \\ -\mathbf{I} \end{bmatrix} \mathbf{r}(t) \quad (16)$$

$$\mathbf{y}(t) = \mathbf{C}_e \mathbf{x}_e \quad (17)$$

where

$$\dot{\mathbf{x}}_e(t) = \begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{x}}_{int}(t) \end{bmatrix} \quad \mathbf{A}_e = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{B}_e = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \quad \mathbf{C}_e = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix}$$

The feedback law for this design approach is given by

$$\mathbf{u}(t) = \mathbf{F} \mathbf{x}(t) + \mathbf{F}_{int} \mathbf{x}_{int}(t) . \quad (18)$$

The feedback matrix, \mathbf{F}_e is obtained using a Linear Quadratic Regulator (LQR) approach together with Bryson's rule. This makes it possible to find the optimal feedback gains while being able to prioritize the different states and set boundaries

⁴Fixme Note: Should we use the same $\bar{\omega}$?

$$J = \int_0^\infty \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} dt. \quad (19)$$

In this equation, the matrices \mathbf{Q} and \mathbf{R} must be selected as positive semi-definite and positive definite matrices that penalize, in the cost function, the error of the different states and inputs of the system.

$$Q_{ii} = \frac{1}{\text{maximum acceptable value of } [x_i^2]} \quad (20)$$

The values are selected so the controller is stable and the states converge to the equilibrium point and the references are tracked as fast as possible. This gain selection process is done using the simulation of the model, which also considers the network effects such as delay and packet loss. The values of \mathbf{Q} and \mathbf{R} are then used to find the optimal feedback matrices that minimize the cost function (19) as

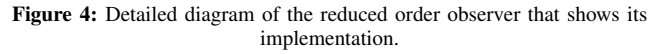
Where the matrix \mathbf{P} positive definite and its value is found using the Algebraic Riccati Equation,

Once \mathbf{F}_e is obtained, it is split into \mathbf{F} and \mathbf{F}_{int} . In this way, the controller is implemented as shown in Figure 3.

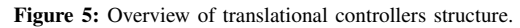
The observer is designed by finding the matrix \mathbf{L}_{obs} such that the eigenvalues of the matrix $\mathbf{A}_{22} + \mathbf{L}_{\text{obs}}\mathbf{A}_{12}$ give the desired dynamics for the estimation **ssReference**. This is done splitting the original system matrices into the submatrices

With the observer matrix, the observer equation is derived, see (24). This ensures an estimate $\hat{\mathbf{x}}_{4:6}$ which converges to $\mathbf{x}_{4:6}$ at a rate given by the chosen observer poles.

This estimation of $\mathbf{x}_{4:6}$ is implemented as shown in Figure 4.



The translational controllers are structured as cascade loops, where the velocity and position are controlled in the inner and outer loop, respectively. The relation between the controllers is presented in Figure 5.



To design the inner controllers for the velocities \dot{x} and \dot{y} , the model equations derived previously, see (6) and (7), are Laplace transformed. These are used to create a transfer function between the angles and the velocities, yielding

$$G_{\dot{y}}(s) = \frac{\dot{y}_I(s)}{\phi(s)} = \frac{k_{\text{th}}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)}{ms} \quad (26)$$

Since the plants are the same but with different signs, the controller design is carried out for \dot{x} and applied to \dot{y} afterwards with opposite sign.

⁵FiXme Note: Change to PI controllers??

The translational position loop is then designed with the same method, three times less bandwidth than the inner velocity loop. The plant of the outer loop is considered only as an integrator that transforms velocity to position. As for the inner loop, the controller of the outer loop is a proportional controller.

To be able to design the velocity loop for the z translational controller, the model equation derived previously, see (15), is Laplace transformed and written on the form of a transfer function, as

$$G_{\dot{z}} = \frac{\dot{z}_I}{\omega_{\text{sum}}} = \frac{\frac{1}{4}(-2k_{th})\bar{\omega}_{\text{sum}}}{ms} \quad (27)$$

where \dot{z}_I is the velocity in the z_I direction, ω_{sum} is the sum of the rotational speeds of the motors and $\bar{\omega}_{\text{sum}}$ is the sum of the rotational speeds in equilibrium.

Due to an integrator and a negative gain, the system's root locus moves into the right half plane as the gain increases. A proportional controller with negative gain ensures the system to be stable.

As there is not an inner control loop, input disturbances may occur such that a steady state error appears, which is not eliminated by the integrator in the plant. To remove this potential error an integrator term is added, leading to a PI-controller.

The plant for the z position controller is just an integrator and a proportional controller is utilized. This is as well designed so that the bandwidth is three times lower than the inner one.

IV. NETWORK

^{6 7} The wireless network between the sensor and the quadcopter can influence the performance of the controller. This influence can be divided into two categories: the delay, from when the sensor receives data until the data is actually used by the controller, and the loss of packages in the communication channel.

The theoretical modeling of these influences has been studied by several researchers in order to obtain a criteria for finding maximum allowable delay and package loss, while the control system remains stable **ling nirupam** However, these approaches often lead to an increased complexity as the network effect is included in the model of the system.

An alternative to account for these effects in the control system is to utilize a network simulator such as TrueTime **TrueTimeNew**

TrueTime provides the option to simulate the network and the controller together. It is thereby possible to check the effect of the delay and the package loss in the behavior and, thus, ensure that the control system is still stable.

The delay is modeled as constant in the simulation. Its value is obtained by measuring the time needed for the transmission of the data and adding the maximum time elapsed since

the data is available for the controller until it utilizes the information. In this way, the worst case scenario is considered

The packet loss, defined as a constant probability of losing a packet, is found experimentally by sending a large amount of packages and examining how many of these are available for the controller. In the experiment, it has been found that more packages are received than control loops are executed. This implies that there is always the most recent information available in the processor and thus the package loss probability can be considered to be zero.

V. RESULTS

The controllers have been simulated in MATLAB Simulink in order to generate results presented in this section. ^{8 9}

The model parameters can be seen in Table I. All of them have been obtained through tests except for the moments of inertia, which were calculated by following an analytical procedure. ¹⁰

Symbol	Value	Units
m	0.996	kg
L	0.225	m
J_x	0.01073	kg m ²
J_y	0.01073	kg m ²
J_z	0.02135	kg m ²
k_{th}	$1.32922 \cdot 10^{-5}$	Ns ² rad ⁻²
k_d	$9.39741 \cdot 10^{-7}$	Nms ² rad ⁻²
$\bar{\omega}_i$	429	rad s ⁻¹

Table I: Parameters used though the analysis and design.¹¹

The attitude controller is defined by the designed \mathbf{F} and \mathbf{F}_{int} matrices shown below and the chosen observer poles, which are $[-11, -12, -13]$.

$$\mathbf{F} = \begin{bmatrix} 0.00 & -165.15 & -223.35 & 0.00 & -44.04 & -68.52 \\ 165.15 & 0.00 & 223.35 & 44.04 & 0.00 & 68.52 \\ 0.00 & 165.15 & -223.35 & 0.00 & 44.04 & -68.52 \\ -165.15 & 0.00 & 223.35 & -44.04 & 0.00 & 68.52 \end{bmatrix}$$

$$\mathbf{F}_{\text{int}} = \begin{bmatrix} 0.00 & -220.97 & -250.00 \\ 220.97 & 0.00 & 250.00 \\ 0.00 & 220.97 & -250.00 \\ -220.97 & 0.00 & 250.00 \end{bmatrix}$$

¹² The translation velocity controllers for x and y are $C_{\dot{x}}(s) = -0.1$, $C_{\dot{y}}(s) = 0.1$ and the position ones are $C_x(s) = 0.5$, $C_y(s) = 0.5$. The PI-controller for the z translational velocity is $C_{\dot{z}}(s) = -201 \frac{s+0.8}{s}$ and the outer loop P-controller is $C_z = 0.9$. ¹³

⁸FixMe Note: split in different subsections??

⁹FixMe Note: More details on how to obtain simulations if we put them in the paper.

¹⁰FixMe Note: More details on how the parameters were obtained and a comparison of how good is the model??

¹²FixMe Note: should we have these matrices??

¹³FixMe Note: Split this into normal equations??

⁶FixMe Note: Explain more

⁷FixMe Note: More details on network, protocol, more description?

These controllers are discretized using the Tustin method with a sampling period of 35 ms, as this is the fastest available frequency in which data can be acquired from the motion tracking system and transmitted to the quadcopter.

Regarding the network, the delay is 40 ms and the package loss probability is set to zero due to the low transmission frequency used in the system. The simulation results obtained with these parameters and controllers are shown in Figure 6.

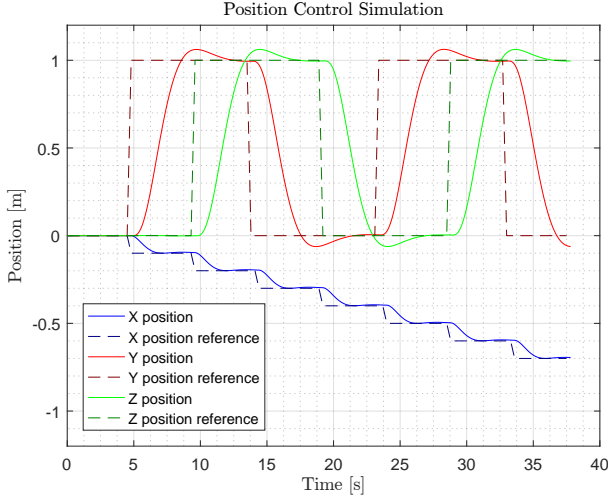


Figure 6: Position control results in the three inertial axes directions. The references given to the control system are shown with dashed lines.

The inner attitude controller results are also included and shown in Figure 7 so the performance of the attitude control can be evaluated.

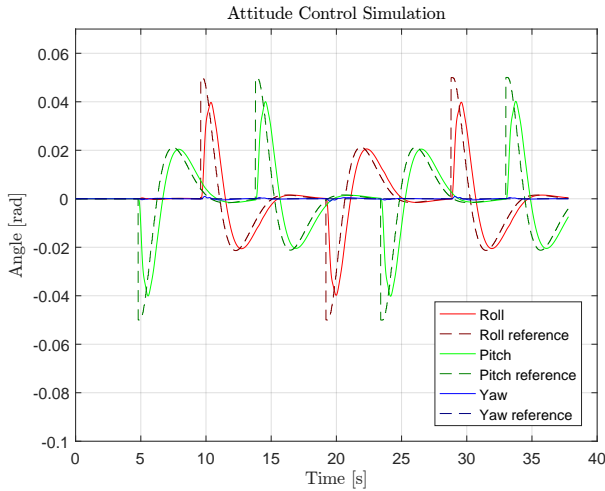


Figure 7: Attitude control results in the three angles. The references given to the attitude control system are shown with dashed lines.

VI. DISCUSSION

The results obtained in the simulations show both the attitude and position response of the quadcopter.

It is seen that the controllers achieve the desired reference even though the network delay and the sampling rate affect

the performance. The main network effect is the designed bandwidth of the controllers. This occurs due to the limited frequency in which the sensor data is obtained from the motion tracking system through the wireless connection.

It is also worth observing how the attitude controller shows a permanent error with respect to the reference. This is generated as a result of the integral controller design because it assumes a constant reference applied to it. This issue, though, does not affect the final position of the quadcopter.

VII. CONCLUSION

The behavior of a quadcopter has been modeled by first principles of physics. A linear control system has been designed in order to hover and move to a desired position. The control system has been split into an attitude and a translational controller. The former has been designed using a state space approach, including state feedback with integral control and a reduced order observer. The translational control system has been designed with a classical control approach and result in three cascade loops, including proportional and PI controllers. As the quadcopter uses an external motion tracking system to determine its position and orientation, an analysis of the issues that can arise when having a networked distributed system has been done in order to ensure the control system remains stable. The results obtained from the design show that both the attitude and the translational behavior of the quadcopter has been successfully controlled.

VIII. FUTURE WORK

The control system has been designed using a motion tracking system as an attitude sensor. In order to improve the result while keeping the same control structure, an inertial measurement unit could be installed on the quadcopter.