Time delay compensation for positive nonlinear networked control systems with bounded controls

Ling Huang
School of Automation,
Harbin University
of Science and Technology,
Heilongjiang 150080, China.
Email:mail_huangling@163.com

Cheng-Chew Lim
School of Electrical and
Electronic Engineering,
the University of Adelaide,
SA 5005, Australia
Email: cheng.lim@adelaide.edu.au

Ji Liu
School of Electrical and
Electronic Engineering,
Harbin University of
Science and Technology,
Harbin 150080, China.

***Abstract*—Delay compensation for a class of discrete-time nonlinear networked control systems with bounded controls while imposing positivity in closed-loop systems is considered in the paper. Under the state feedback and the output feedback control, compensation for the network induced delays is designed using the Lyapunov stability theory and matrix transformation with nonnegative control and bounded control. All the results are presented in linear programming (LP) forms. An example is given to show the effectiveness of the proposed methods.**

I. Introduction

Networked control systems are applicable in many diverse fields as robot teleoperation, industrial process control, system-on-chip resource control, cognitive radio networks and navigation [1]–[4]. The benefits offered by network-based systems come from ease of maintenance, decreased installation cost, use of current technology, and increased system flexibility. However, many problems still need to be solved before all of the advantages from networked control systems can be realized. Network scheme induced time delay is one of the problems, and this paper will present a delay compensation scheme to reduce the time delay effect under a closed-loop control.

Network induced delay has been investigated in a number of studies. In [5] the maximal time delay and maximal drops are investigated with the linear matrix inequalities technique. In [6], the networked predictive control based on the state observer was studied for discrete systems, where the forward and feedback channels delays were modeled as constant and random variables, and a switched system theory was used to propose condition under which the closed system is stable. In [7], an output feedback time compensation controller was designed with the forward and feedback channels delays modelled as Markovian chains, and the cone complementary linearization approach was used to calculate controllers gains. In [8], the state and output feedback time delay compensation controllers for T-S fuzzy systems were considered. By using fuzzy singular system theory and matrix transformation technique, the existence conditions for compensation controllers were given in linear matrix inequalities (LMI) forms, and the gains expressions were also presented.

Positive systems, which have nonnegative states whenever the initial conditions are nonnegative, exist in many fields [9]–[11]. Analysis and control of the systems differs from these of traditional systems, since the positivity constraint must be taken into account. Research findings of these systems are available from [12]–[19]. The stabilization of the positive discrete-time T-S fuzzy systems with multiple time delay was discussed by using state feedback and output feedback controllers in [17] and [18], and all their results were expressed in linear programming form. In [19], the exponential stability conditions with linear programming form were presented for positive continuous-time and discrete-time systems with bounded time-varying delays. Then an output feedback stabilization controller was designed with singular value decompositon theory. The H_∞ performace analysis of discrete-time and continuous-time positive systems was presented by LMIs in [20].

In this paper, the delay compensation control for a class of positive discrete-time networked control systems is considered. Our approach and the solution differ from some existing results in that the researched problem is time delay compensation for networked control system and the researched objected is positive system. Thus the novelty of the paper being that it is the first time delay compensation control is considered in positive discrete-time systems, and that some existence conditions for delay compensation controller are given in LP forms.

The paper is organized as following. The system description of a class of nonlinear systems represented by T-S fuzzy systems and some preliminaries are presented in Section II. Then for the state feedback and output feedback time delay compensation, the existence conditions for the controllers are provided in the following section. In Section IV, a numerical example is given to show the effectiveness of the proposed theory. Some closing remarks are followed in Section V.

Notation. The following notion is used in the paper. $\mathbb{R}^n$ denotes the set of vectors of size n. $\mathbb{R}^n_+$ denotes the set of positive vectors of size n. $col\{r_1, r_2, r_3\}$ means a column vector composed of r_1, r_2, r_3. $A > 0$ means that the matrix A is positive.

978-1-5090-0626-7/16/$31.00 ©2016 IEEE

II. System Description and Preliminaries

The networked predictive control system with input constraint is shown in Figure 1. The controlled plant is a nonlinear system and a T-S fuzzy system is used to model it through sector nonlinearity or local linearization [21]. Suppose that its i subsystem has the form

Rule i. IF $\theta_1(k)$ is F_i^1, $\theta_2(k)$ is F_i^2, ..., $\theta_g(k)$ is F_i^g, THEN:

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ y(k) = C_i x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector; $u(k) \in \mathbb{R}^m$ is the control input; $y(k) \in \mathbb{R}^\nu$ is the system output vector; matrices $A_i, B_i, C_i, i = 1, 2, \dots, r$ are system matrices with appropriate dimensions; $\theta_1(k), \theta_2(k), \dots, \theta_g(k)$ are premise variables; $F_i^j, i = 1, 2, \dots, r, j = 1, 2, \dots, g$ are fuzzy sets; and r is the number of IF-THEN rules.

Express the premise variables in vector form $\theta(k) = [\theta_1(k) \quad \theta_2(k) \quad \theta_g(k)]$. If a standard fuzzifier and a weighted center-average defuzzifier are used, then the fuzzy system can be inferred as follows.

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} h_i(\theta(k)) A_i x(k) + \sum_{i=1}^{r} h_i(\theta(k)) B_i u(k) \\ y(k) = \sum_{i=1}^{r} h_i(\theta(k)) C_i x(k) \end{cases} \tag{2}$$

where

$$h_i(\theta(k)) = \frac{\omega_i(\theta(k))}{\sum_{i=1}^{r} \omega_i(\theta(k))}$$

$$\omega_i(\theta(k)) = \prod_{j=1}^{g} F_i^j(\theta_j(k))$$

in which $F_i^j(\theta_j(k)$ is the grade of membership of $\theta_j(k)$ in fuzzy set F_i^j. Generally

$$\omega_i(\theta(k)) > 0$$
$$1 \geq h_i(\theta(k)) \geq 0$$

In the networked control system, The sensor samples the state of the controlled system periodically, then the value $x(k)$ and its time stamp k are sent through the network. Because of multiple devices sharing network, packet collision, network congestion, and connection interrupting, time delay for information transmission is inevitable, So time delay compensation for networked control systems is necessary and has important application value.

In the studied system, there exists communication network between the sensor and the controller, and communication network between the controller and the actuator. For compensate the network induced time delay, control prediction generator and networked delay compensation are designed. When the sampled data and its time stamp are sent to the generator with some time delay, the generator will calculate some controlled inputs for possible packet loss, and send the packet of controlled input to the compensation controller. Since the delay compensation controller is intelligent, it will judge its time through stamp and receives the latest prediction controller package, then select the appropriate control input to the controlled plant.

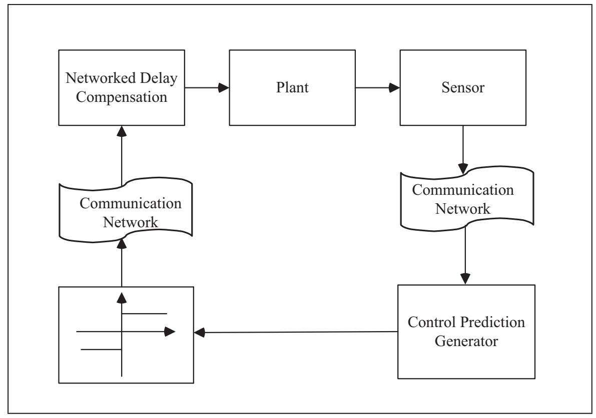

Fig. 1. Networked predictive controlled system with input constraint

The time delay from the sensor to the control prediction generator is denoted as d_k^{sc}, and the time delay from the generator to the compensation controller is denoted as d_k^{ca}. The sum of the two kinds of time delays is called the round trip delay d_k. In the paper, we introduced the following assumption.

Assumption 1. *The round trip delay is time varying and bounded, which can be described by $1 \leq d_m \leq d_k \leq d_M$, where d_m and d_M are the lower and upper bounds of the delay; in the following, we use set $\mathbb{S} \triangleq \{d_m, d_m+1, \dots, d_M\}$ to denote the range of the amplitude constraint.*

For the controlled system (2), if we use the following state feedback controller,

$$u_i^s(k) = K_i x(k), i = 1, 2, \dots, r \tag{3}$$

where $K_i, i = 1, 2, \dots, r$ are state feedback controller gains, $u_i^s(k)$ is state feedback controller for the i subsystem. Then the control prediction vector for the fuzzy subsystem can be constructed as follows.

$$u_i^s(k - d_k^{sc} + j | k - d_k^{sc}) = K_{ij} x(k - d_k^{sc}), i = 1, 2, \dots, r, j \in \mathbb{S} \tag{4}$$

where $K_{ij}, i = 1, 2, \dots, r, j \in \mathbb{S}$ are delay compensation controller gains for all the possible time delay, $u_i^s(k - d_k^{sc} + j | k - d_k^{sc})$ is state feedback controller predicting the j step compensating time delay d_k^{sc} for the i subsystem. So, a control prediction sampled signal needed to be transferred to delay compensation in the controller side is defined by

$$U_i^s(k|k-d_k^{sc}) = \begin{bmatrix} u_i^s(k-d_k^{sc}+d_m|k-d_k^{sc}) \\ u_i^s(k-d_k^{sc}+d_m+1|k-d_k^{sc}) \\ \vdots \\ u_i^s(k-d_k^{sc}+d_M|k-d_k^{sc}) \end{bmatrix} \tag{5}$$

Remark 1. *For each subsystem, there are $d_M - d_m + 1$ controllers to be generated when the prediction generator receives the data from the sensor, and only the intelligent delay compensation controller can select the best controller according to the condition of network delays.*

For the output feedback case, when the system state cannot be measured the system output variable y is used in controller.

$$u_i^o(k) = H_i y(k), i = 1, 2, \ldots, r \tag{6}$$

where $H_i, i = 1, 2, \ldots, r$ is output feedback controller gain for each T-S fuzzy subsystem. Then the control prediction vector can be constructed as follows.

$$u_i^o(k-d_k^{sc}+j|k-d_k^{sc}) = H_{ij} y(k-d_k^{sc}), i = 1, 2, \ldots, r, j \in \mathbb{S} \tag{7}$$

where $H_{ij}, i = 1, 2, \ldots, r, j \in \mathbb{S}$ are compensation controller gains in output feedback form. The control input packet in the controller can be constructed as

$$U_i^o(k|k-d_k^{sc}) = \begin{bmatrix} u_i^o(k-d_k^{sc}+d_m|k-d_k^{sc}) \\ u_i^o(k-d_k^{sc}+d_m+1|k-d_k^{sc}) \\ \vdots \\ u_i^o(k-d_k^{sc}+d_M|k-d_k^{sc}) \end{bmatrix} \tag{8}$$

For the T-S fuzzy system (2), if the same membership function is used in the controller then under the prediction control, the overall controller can be obtained as

$$u(k) = \sum_{i=1}^{r} h_i(\theta(k)) K_{id_k} x(k-d_k) \tag{9}$$

Thus the state feedback closed-loop system can be obtained as following.

$$\begin{aligned} x(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} & h_i(\theta(k))h_j(\theta(k))\{A_i x(k) \\ & + B_i K_{jd_k} x(k-d_k)\}, d_k \in \mathbb{S} \end{aligned} \tag{10}$$

Before presenting the closed-loop system of the output feedback, an assumption is introduced firstly.

Assumption 2. *The output matrices $C_i, i = 1, 2, \ldots, r$ of all the subsystems are the same, represented by C.*

Under the assumption, the closed-loop system can be obtained as

$$\begin{aligned} & x(k+1) \\ & = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\{A_i x(k) + B_i H_{jd_k} y(k-d_k)\} \\ & = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(k))h_j(\theta(k))\{A_i x(k) \\ & + B_i H_{jd_k} C y(k-d_k)\}, d_k \in \mathbb{S} \end{aligned} \tag{11}$$

For simplicity, system (10) can be rewritten as

$$x(k+1) = Ax(k) + BK_{d_k} x(k-d_k), d_k \in \mathbb{S} \tag{12}$$

where

$$\begin{aligned} A &= \sum_{i=1}^{r} h_i(\theta(k)) A_i \\ B &= \sum_{i=1}^{r} h_i(\theta(k)) B_i \\ K_{d_k} &= \sum_{i=1}^{r} h_i(\theta(k)) K_{id_k} \end{aligned} \tag{13}$$

Similarly the output feedback closed-loop system can be rewritten in the following form.

$$x(k+1) = Ax(k) + BH_{d_k} C y(k-d_k), d_k \in \mathbb{S} \tag{14}$$

where

$$H_{d_k} = \sum_{i=1}^{r} h_i(\theta(k)) H_{id_k} \tag{15}$$

Before ending this section we introduce the following preliminaries.

Definition 1. *Given any nonnegative initial state and any input function, system (1) is said to be controlled positive if the corresponding trajectory remains in the positive orthant for all k: $x(k) \in \mathbb{R}_+^n$.*

Lemma 1. *[17] System (10) is positive if and only if matrices $A_i, B_i K_{jd_k}, i, j = 1, 2, \ldots, r,\ d_k \in \mathbb{S}$ are nonnegative.*

III. Main Results

In this section, we will use matrix theory and the Lyapunov-Krasovskii function technique to analyze the stability of closed-loop systems (12) and (14).

Let $X(k) = col\{x(k) \quad x(k-1) \quad \ldots x(k-d_M)\}$. Then the state feedback closed-loop control system (12) can be augmented as

$$X(k+1) = A_{sc} X(k) \tag{16}$$

where

$$A_{sc} = \left[\begin{array}{cccc} A & 0_{n\times(d_k-1)n} & BK_{d_k} & 0_{n\times(d_M-d_k)n} \\ \hdashline \multicolumn{2}{c}{I_{d_{Mn}\times d_{Mn}}} & \multicolumn{2}{c}{0_{d_{Mn}\times n}} \end{array}\right] \tag{17}$$

Similarly the output feedback control system can be expanded as

$$X(k+1) = A_{oc}X(k) \tag{18}$$

where

$$A_{oc} = \left[\begin{array}{cccc} A & 0_{n\times(d_k-1)n} & BH_{d_k}C & 0_{n\times(d_M-d_k)n} \\ \hdashline I_{d_{Mn}\times d_{Mn}} & & 0_{d_{Mn}\times n} & \end{array}\right] \tag{19}$$

The stability conditions of (16) will be established.

A. State feedback delay compensation without input constraint controls

In this subsection, we investigate the stability condition of system (16) when the control signal is not subject to amplitude constraint. That is, given system (2), with the initial condition $0 \leq \psi(k)$ and $A_i > 0$, the problem is to find a delay compensation controller $u(k) = K_{d_k}x(k-d_k), d_k \in \mathbb{S}$ such that, under all the network delays, system (16) is positive and asymptotically stable.

It is well known that the stability of system (16) is the same as its dual system

$$X(k+1) = A_{sc}^T X(k) \tag{20}$$

So system (20) will be investigated instead of system (16). Before presenting the main result, the forms of the matrices used in the result are listed as following.

The input matrix is in form of

$$B_i = \begin{bmatrix} b_i^1 \\ b_i^2 \\ \vdots \\ b_i^n \end{bmatrix}, b_i^l \in \mathbb{R}^m, l = 1, 2, \ldots, n \tag{21}$$

The Lyapunov-Krasovskii vector is in the following form

$$\begin{aligned} \Lambda = &\quad col\{\lambda_1 \quad \lambda_2 \quad \ldots \quad \lambda_{d_M+1}\} \\ \lambda_i \in &\quad \mathbb{R}^n, i = 1, 2, \ldots, d_M + 1 \end{aligned} \tag{22}$$

$$\lambda_{d_k+1} = col\{\lambda_{d_k+1}^1 \quad \lambda_{d_k+1}^2 \quad \ldots \quad \lambda_{d_k+1}^n\} \tag{23}$$

We now state the existence condition for the compensation controller under state feedback.

Theorem 1. *Given $A_i > 0, i = 1, 2, \ldots, r$, system (16) is positive and asymptotically stable, if there exist vector $\Lambda > 0, \Lambda \in \mathbb{R}^{(d_M+1)n}$, and vector $p_{jd_k}^t \in \mathbb{R}^m, j = 1, 2, \ldots, r, t = 1, 2, \ldots, n, d_k \in \mathbb{S}$, for $i = 1, 2, \ldots, r, l = 1, 2, \ldots, m$ satisfying the following linear programmings (LPs).*

$$\begin{cases} A_i\lambda_1 + B_i \sum_{t=1}^{n} p_{jd_k}^t - \lambda_1 < 0 \\ \lambda_1 - \lambda_2 < 0 \\ \lambda_2 - \lambda_3 < 0 \\ \vdots \\ \lambda_{d_M} - \lambda_{d_M+1} < 0 \\ \Lambda > 0 \\ b_i^l p_{jd_k}^t \geq 0 \end{cases} \tag{24}$$

then the time compensation controller is

$$K_{jd_k} = \begin{bmatrix} \frac{p_{jd_k}^1}{\lambda_{d_k+1}^1} & \frac{p_{jd_k}^2}{\lambda_{d_k+1}^2} & \ldots & \frac{p_{jd_k}^n}{\lambda_{d_k+1}^n} \end{bmatrix} \quad j = 1, 2, \ldots, r, d_k \in \mathbb{S} \tag{25}$$

Proof. For system (20), we choose the Lyapunov-Krasovskii function as

$$V(X(k)) = X^T(k)\Lambda \tag{26}$$

where $\Lambda \in \mathbb{R}^{(d_M+1)n}, \Lambda > 0$.

The rate of increase of the Lyapunov-Krasovskii function is

$$\begin{aligned} \Delta V(X(k)) &= V(X(k+1)) - V(X(k)) \\ &= X^T(k+1)\Lambda - X^T(k)\Lambda \\ &= X^T(k)A_{sc}\Lambda - X^T(k)\Lambda \end{aligned} \tag{27}$$

Since (20) is a positive system and is the dual system of (12), then $X(k) > 0$. It follows that

$$A_{sc}\Lambda - \Lambda < 0 \tag{28}$$

implies $\Delta V(X(k)) < 0$, then (20) is asymptotically stable; Its dual system (16) is also asymptotically stable.

By considering the form of A_{sc} in (17) and Λ in (22), inequality (28) can be transformed to

$$\begin{aligned} A\lambda_1 + BK_{d_k}\lambda_{d_k+1} - \lambda_1 < 0 \\ \lambda_1 - \lambda_2 < 0 \\ \lambda_2 - \lambda_3 < 0 \\ \vdots \\ \lambda_{d_M} - \lambda_{d_M+1} < 0 \end{aligned} \tag{29}$$

Expressing A, B and K_{d_k} in the form of (13), the first inequality in (29) becomes

$$\sum_{i=1}^{r} h_i(\theta(k)) \sum_{j=1}^{r} h_j(\theta(k))\{A_i\lambda_1 + B_iK_{jd_k}\lambda_{d_k+1} - \lambda_1\} < 0 \quad i, j = 1, 2, \ldots, r, d_k \in \mathbb{S} \tag{30}$$

Let $K_{jd_k} = \begin{bmatrix} k_{jd_k}^1 & k_{jd_k}^2 & \ldots & k_{jd_k}^n \end{bmatrix}$. Observing the structure of λ_{d_k+1} in equation (23), then

$$K_{jd_k}\lambda_{d_k+1} = \sum_{t=1}^{n} \lambda_{d_k+1}^t k_{jd_k}^t$$

Let $p_{jd_k}^t = \lambda_{d_k+1}^t k_{jd_k}^t$. In view of the positivity of membership function, inequalities (30) can be changed into

$$A_i\lambda_1 + B_i \sum_{t=1}^{n} p_{jd_k}^t - \lambda_1 < 0, i, j = 1, 2, \ldots, r, d_k \in \mathbb{S} \tag{31}$$

To ensure that the closed-loop system (16) is positive, A, BK_{jd_k} must be nonnegative. Since

$$[BK_{jd_k}]_{lk} = b_i^l k_{jd_k}^t = b_i^l \frac{p_{jd_k}^t}{\lambda_{d_k+1}^t}, l, t = 1, 2, \ldots, n, d_k \in \mathbb{S}$$

also $\lambda_{d_k+1}^t, t=1,2,\ldots,n$ is positive, then $b_i^l p_{jd_k}^t \geq 0, l = 1,2,\ldots,m, t=1,2,\ldots,n$, which implies $B_i K_{jd_k} \geq 0$. This completes the proof. □

Remark 2. *In Theorem 1, it is clear that positivity of the closed-loop system (16) is guaranteed by the nonnegativity of matrices A and $BK_j, j \in \mathbb{S}$.*

B. State feedback delay compensation with non-symmetrically bounded controls

For time delay compensation control, we restrict the control input within non-symmetrically bounded controls, that is

$$-u_1 \leq u(k) \leq u_2 \tag{32}$$

where u_1, u_2 are two positive vectors.

The problem can be formulated as following. Given system (2) with initial condition $0 \leq \psi(k) \leq \lambda_1$, and two positive control bounds u_1, u_2, we want to find some bounded delay compensation controllers $-u_1 \leq u(k) = K_{jd_k}x(k-d_k) \leq u_2$ such that, under all the network delays, the closed-loop system (16) is positive and asymptotically stable. we can derive the following result through some deduction.

Theorem 2. *For system matrix $A_i > 0, i=1,2,\ldots,r$, and two control input bounds $u_1 > 0, u_2 > 0$, if the following linear programmings in the variables λ_{d_k}, vectors $y_{jd_k}^t, z_{jd_k}^t, d_k \in \mathbb{S}, j=1,2,\ldots,r, t=1,2,\ldots,n$, for $i=1,2,\ldots,r, l=1,2,\ldots,n$*

$$\begin{cases} A_i\lambda_1 + B_i \sum_{t=1}^{n}(p_{jd_k}^t - z_{jd_k}^t) - \lambda_1 < 0 \\ \lambda_1 - \lambda_2 < 0 \\ \lambda_2 - \lambda_3 < 0 \\ \vdots \\ \lambda_{d_M} - \lambda_{d_M+1} < 0 \\ \Lambda > 0 \\ b_i^l y_{jd_k}^t - b_i^l z_{jd_k}^t \geq 0 \\ 0 < \sum_{t=1}^{n} p_{jd_k}^t \leq u_2 \\ 0 < \sum_{t=1}^{n} z_{jd_k}^t \leq u_1 \end{cases} \tag{33}$$

is feasible, then there exist bounded time delay compensation controllers such that the closed-loop system (16) is positive and asymptotically stable under initial condition $0 \leq \psi(k) \leq \lambda_1$. Moreover, the compensation controllers can be obtained from

$$K_{jd_k} = \left[\frac{p_{jd_k}^1 - z_{jd_k}^1}{\lambda_{d_k+1}^1}, \frac{p_{jd_k}^2 - z_{jd_k}^2}{\lambda_{d_k+1}^2}, \ldots, \frac{p_{jd_k}^n - z_{jd_k}^n}{\lambda_{d_k+1}^n}\right] \quad j=1,2,\ldots,r, d_k \in \mathbb{S} \tag{34}$$

Proof. Similar to the proof of Theorem 1. $0 < \sum_{t=1}^n p_{jd_k}^t \leq u_2$ means $K_{jd_k}^+ \lambda_{d_k+1} \leq u_2$ Under the initial condition assumption, we know that $0 \leq x(k-d_k) \leq \lambda_1$. If $K_{jd_k}^+$ is nonnegative by construction, then

$$0 < u(k) = K_{jd_k}^+ x(k-d_k) \leq K_{jd_k}^+ \lambda_1 \tag{35}$$

With the relationship of $\lambda_i, i=1,2,\ldots,d_M+1$, we can extend inequality (35) to

$$u(k) \leq K_{jd_k}^+ \lambda_{d_k+1} \leq u_2$$

Similar

$$u(k) \leq K_{jd_k}^- \lambda_{d_k+1} \leq u_1$$

Then by using the fact that any matrix K_{jd_k} can be expressed as the difference of two positive matrices $K_{jd_k} = K_{jd_k}^+ - K_{jd_k}^-$, where

$$K_{jd_k}^+ = \left[\frac{p_{jd_k}^1}{\lambda_{d_k+1}^1}, \frac{p_{jd_k}^2}{\lambda_{d_k+1}^2}, \ldots, \frac{p_{jd_k}^n}{\lambda_{d_k+1}^n}\right], j=1,2,\ldots,r, d_k \in \mathbb{S}$$
$$K_{jd_k}^- = \left[\frac{z_{jd_k}^1}{\lambda_{d_k+1}^1}, \frac{z_{jd_k}^2}{\lambda_{d_k+1}^2}, \ldots, \frac{z_{jd_k}^n}{\lambda_{d_k+1}^n}\right], j=1,2,\ldots,r, d_k \in \mathbb{S}$$
$$K_{jd_k} = \left[\frac{p_{jd_k}^1 - z_{jd_k}^1}{\lambda_{d_k+1}^1}, \frac{p_{jd_k}^2 - z_{jd_k}^2}{\lambda_{d_k+1}^2}, \ldots, \frac{p_{jd_k}^n - z_{jd_k}^n}{\lambda_{d_k+1}^n}\right] \quad j=1,2,\ldots,r, d_k \in \mathbb{S}$$

Let $0 < x(k-d_k) \leq \lambda_1$, then $0 < K_{jd_k}^+ x(k-d_k) \leq K_{jd_k}^+ \lambda_1$ and $-K_{jd_k}^- \lambda_1 \leq -K_{jd_k}^- x(k-d_k) < 0$ satisfy. Summing the last two inequalities, one can obtain

$$-K_{jd_k}^- \lambda_1 \leq K_{jd_k} x(k-d_k) \leq K_{jd_k}^+ \lambda_1 \tag{36}$$

Considering $0 < \sum_{i=1}^n p_{jd_k}^i \leq u_2$, then

$$0 < K_{jd_k}^+ x(k-d_k) \leq K_{jd_k}^+ \lambda_1 \leq K_{jd_k}^+ \lambda_{d_k+1} \leq u_2 \tag{37}$$

Similarly, the following can be obtained.

$$0 > -K_{jd_k}^- x(k-d_k) \geq -K_{jd_k}^- \lambda_1 \geq -K_{jd_k}^- \lambda_{d_k+1} \geq -u_1 \tag{38}$$

Combining inequalities (36)-(38), then we can guarantee

$$-u_1 \leq u(k) \leq u_2$$

□

C. Output feedback delay compensation in positive systems

We now investigate the stability of output feedback delay compensation system (18). Similarly to state feedback control, we consider the stability of its dual system

$$X(k+1) = A_{oc}^T X(k) \tag{39}$$

Theorem 3. *For system (2) , under an output feedback delay compensation controller, the closed-loop system (18) is asymptotically stable if there exist λ_{d_k}, and vector $v_{jd_k}^t, d_k \in \mathbb{S}, j =$*

$1, 2, \ldots, r, t = 1, 2, \ldots, n$, *for* $l = 1, 2, \ldots, n, t = 1, 2, \ldots, \mu$, *such that the following inequalities*

$$\begin{cases} A_i\lambda_1 + B_i \sum_{t=1}^{\nu} v_{jd_k}^t - \lambda_1 < 0 \\ \lambda_1 - \lambda_2 < 0 \\ \lambda_2 - \lambda_3 < 0 \\ \vdots \\ \lambda_{d_M} - \lambda_{d_M+1} < 0 \\ \Lambda > 0 \\ b_i^l v_{jd_k}^t \geq 0 \end{cases} \tag{40}$$

are feasible. Then the output feedback delay compensation controller can be calculated by

$$H_{jd_k} = \begin{bmatrix} \frac{v_{jd_k}^1}{\sum_{q=1}^n c_s^q \lambda_{d_k+1}^q} & \frac{v_{jd_k}^2}{\sum_{q=1}^n c_s^q \lambda_{d_k+1}^q} & \cdots & \frac{v_{jd_k}^\nu}{\sum_{q=1}^n c_s^q \lambda_{d_k+1}^q} \end{bmatrix}$$

The controller gain matrices are in the forms of

$$H_{jd_k} = \begin{bmatrix} h_{jd_k}^1 & h_{jd_k}^2 & h_{jd_k}^\nu \end{bmatrix}, h_{jd_k}^i \in \mathbb{R}^m, i = 1, 2, \ldots, \nu \tag{41}$$

and the output matrix is in the form of

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_\nu \end{bmatrix} \tag{42}$$

$$c_s = \begin{bmatrix} c_s^1 & c_s^2 & \ldots & c_s^n \end{bmatrix}, s = 1, 2, \ldots, \nu$$

Proof. For system (39), the Lyapunov-Krasovskii function is chosen as

$$V(X(k)) = X^T(k)\Lambda$$

with $\Lambda > 0$.

The increment of the Lyapunov-Krasovskii function is

$$\begin{aligned} \Delta V(X(k)) &= X^T(k+1)\Lambda - X^T(k)\Lambda \\ &= X^T(k)A_{oc}\Lambda - X^T(k)\Lambda \end{aligned}$$

As system (18) is required to be positive, so the state of its dual system (39) $X(k) > 0$. It is easy to see that

$$A_{oc}\Lambda - \Lambda < 0 \tag{43}$$

implies $\Delta V(X(k)) < 0$. That is, system (39) is asymptotically stable, and its dual system (18) is asymptotically stable too.

By considering the special structure of A_{oc}, inequality (43) can be converted to the following inequalities.

$$\begin{cases} A\lambda_1 + BH_{d_k}C\lambda_{d_k+1} - \lambda_1 < 0 \\ \lambda_1 - \lambda_2 < 0 \\ \lambda_2 - \lambda_3 < 0 \\ \vdots \\ \lambda_{d_M} - \lambda_{d_M+1} < 0 \end{cases} \tag{44}$$

Considering the structure of A, B in (13) and H_{d_k} in (15), the first inequality in (44) can be expanded to

$$\sum_{i=1}^{r} h_i(\theta(k)) \sum_{j=1}^{r} h_j(\theta(k)) \{A_i\lambda_1 + B_i H_{jd_k} C\lambda_{d_k+1} - \lambda_1\} < 0 \tag{45}$$

Considering the positivity of $h_i(\theta(k)), i = 1, 2, \ldots, r$, inequality (45) can be simplified to

$$A_i\lambda_1 + B_i H_{jd_k} C\lambda_{d_k+1} - \lambda_1 < 0$$

Noting the structure of $H_{jd_k}, \lambda_{d_k+1}$ in (22), (41), also letting

$$v_{jd_k}^t = h_{jd_k}^t \sum_{q=1}^{n} c_s^q \lambda_{d_k+1}^q, t = 1, 2, \ldots, \nu$$

the last inequality can be transformed into the first inequality of (40).

To guarantee the positivity of the output feedback delay compensation closed-loop system, A_i and $BH_{d_k}C$ must be nonnegative. Since

$$[B_i H_{jd_k} C]_{lt} = b_i^l \frac{v_{jd_k}^t}{\sum_{q=1}^n c_s^q \lambda_{d_k+1}^q} c_s^t \tag{46}$$

and $\lambda_{d_k}^h > 0$, so

$$b_i^l v_{jd_k}^t \geq 0, l = 1, 2, \ldots, n, t = 1, 2, \ldots, \nu$$

implies $BH_{d_k}C \geq 0$. This completes the proof. □

IV. Numerical Example

To demonstrate the effectiveness of proposed method, a numerical example in [18] is used. The controlled plant comprises two subsystems, and the system matrices in the form of (1) are

$$A_1 = \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} A_2 = \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.7 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}, C_2 = C_1$$

The input matrices are chosen as

$$B_1 = \begin{bmatrix} 0.02 & 0.03 \\ 0.01 & 0.05 \end{bmatrix}, B_2 = \begin{bmatrix} 0.09 & 0.01 \\ 0.02 & 0.08 \end{bmatrix}$$

We assume that the membership functions of the controlled plant and the controller are $h_1(x_1(k)) = 1 - 0.25x_1(k) * x_1(k), h_2(x_1(k)) = 0.25x_1(k) * x_1(k)$. The system is controlled through a network, and the lower and upper bounds of the network induced time delay are 1 and 3, respectively. According to Theorem 1, the compensation controllers are

$$K_{11} = \begin{bmatrix} 0.2776 & 1.8025 \\ 1.6249 & 0.6816 \end{bmatrix}, K_{12} = \begin{bmatrix} 0.6480 & 0.5173 \\ 2.9063 & 1.7221 \end{bmatrix}$$

$$K_{13} = \begin{bmatrix} 0.4640 & 1.4041 \\ 0.6723 & 1.9467 \end{bmatrix}, K_{21} = \begin{bmatrix} 0.7339 & 0.7367 \\ 3.4935 & 1.7610 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} 0.9358 & 0.3058 \\ 1.3218 & 3.3395 \end{bmatrix}, K_{23} = \begin{bmatrix} 1.5659 & 0.5595 \\ 0.3775 & 1.4358 \end{bmatrix}$$

The initial condition is chosen as $x_0 = \begin{bmatrix} 5 & 5 \end{bmatrix}^T$. The state feedback time delay compensation results are shown in Figure 2 and Figure 3. They are shown that, after 70 steps, the states of closed-loop system tend to zero under the state compensation controller. On the contrary, the states without compensation become bigger compared with the initial states. The control inputs of the closed-loop system are given in Figure 4 and Figure 5.

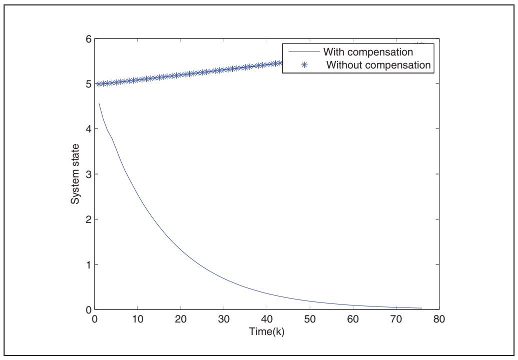

Fig. 2. State 1 of closed-loop system under state feedback without input constraints

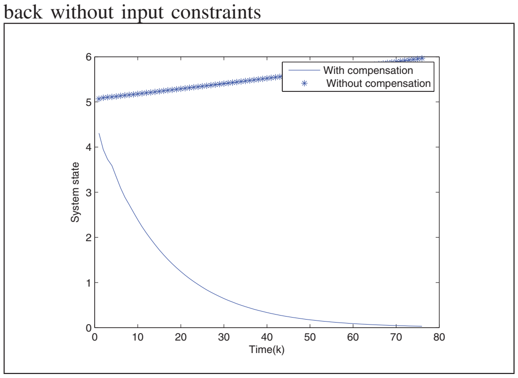

Fig. 3. State 2 of closed-loop system under state feedback without input constraints

Given two positive scalars such that the input vector is limited in the range $-0.8 < u(k) < 0.8$, using Theorem 2, the time delay controllers can be calculated.

$$K_{11} = \begin{bmatrix} 0.0201 & 0.8179 \\ 1.2819 & 1.4910 \end{bmatrix}, K_{12} = \begin{bmatrix} -0.1415 & 0.4711 \\ 1.8013 & 2.7022 \end{bmatrix}$$

$$K_{13} = \begin{bmatrix} 0.5879 & 0.1566 \\ -4.2273 & -0.4359 \end{bmatrix}, K_{21} = \begin{bmatrix} 0.2338 & 1.0411 \\ 3.0170 & 1.8606 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} 1.3703 & 0.2235 \\ 2.1611 & 1.7205 \end{bmatrix}, K_{23} = \begin{bmatrix} 1.8959 & 0.0630 \\ 0.7232 & 1.9441 \end{bmatrix}$$

The states of closed-loop systems are given in Figure 6 and Figure 7. All the states tend to zero with time, but the speed of the closed-loop system without compensation is much slower than the one of the closed-loop system with time delay compensation. The inputs of closed-loop systems are shown in Figure 8 and Figure 9, it is obviously demonstrate that they are within the restricted region.

If the output feedback is used for compensating the time delay induced by the network, then the controller can be cal-

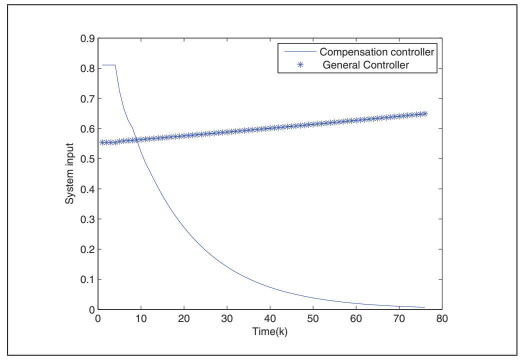

Fig. 4. Input 1 of closed-loop system under state feedback without input constraints

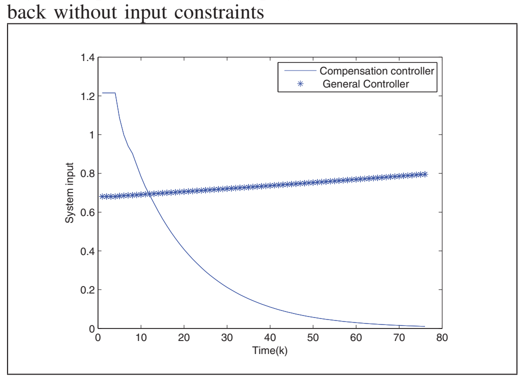

Fig. 5. Input 2 of close-loop system under state feedback without input constraints

culated by using Theorem 3, and the controller gain matrices are given in the following.

$$H_{11} = \begin{bmatrix} 0.3026 & 0.3994 \\ 3.9036 & 2.4762 \end{bmatrix}, H_{12} = \begin{bmatrix} 2.5720 & 0.2463 \\ 2.4846 & 1.8962 \end{bmatrix}$$

$$H_{13} = \begin{bmatrix} 2.9529 & -0.1955 \\ 0.8733 & 2.2606 \end{bmatrix}, H_{21} = \begin{bmatrix} 1.5650 & 0.2348 \\ 3.1230 & 1.8537 \end{bmatrix}$$

$$H_{22} = \begin{bmatrix} 1.6981 & 0.2659 \\ 4.7607 & 1.1002 \end{bmatrix}, H_{23} = \begin{bmatrix} 0.0996 & 0.3701 \\ 2.5612 & 2.2574 \end{bmatrix}$$

V. Conclusion

A time delay compensation method has been proposed to control the positive discrete-time systems. By using the Lyapunov-Krosovskii function and the matrix transformation technique, sufficient existence conditions for the delay compensation controller are presented in linear programming form. Simulation results of a numerical example show that the proposed methods are effective. Sufficient existence conditions for the delay compensation controller under input restrict are also derived in the linear programming form.

Acknowledgment

This work was supported in part by the National Natural Science Funds for Young Scholar of China under Grants 61304046 and Scientific Research Fund of Heilongjiang Provincial Education Department, China (12541118).

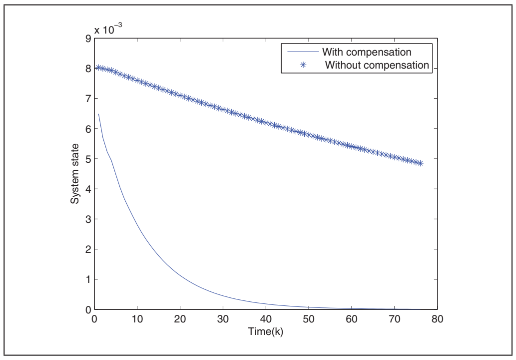

Fig. 6. State 1 of close-loop system under state feedback with input constraints

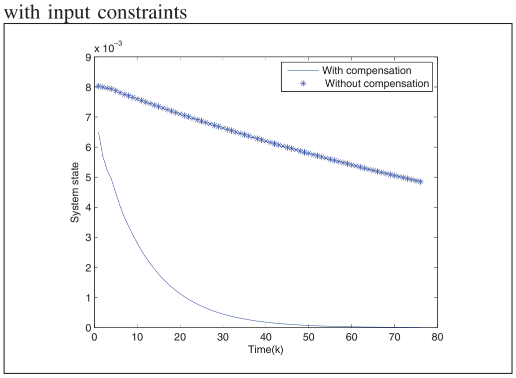

Fig. 7. State 2 of close-loop system under state feedback with input constraints

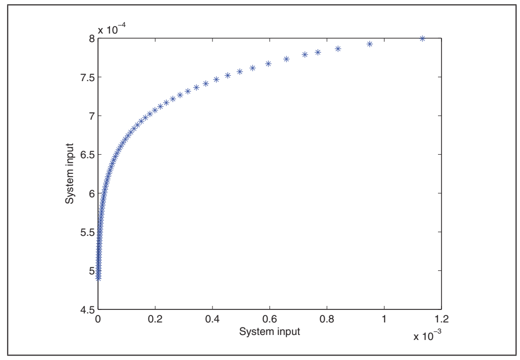

Fig. 8. Input 1 of close-loop system under state feedback with input constraints

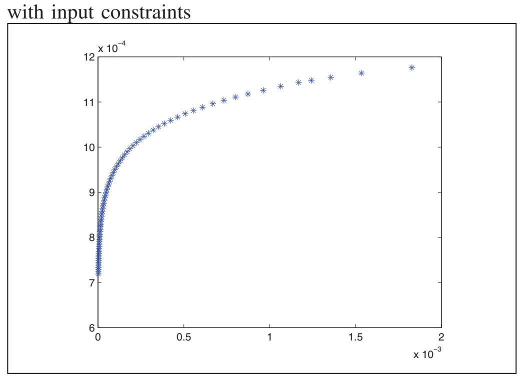

Fig. 9. Input 2 of close-loop system under state feedback with input constraints

REFERENCES

[1] B. Li, K. L. Teo, C. C. Lim, and G. R. Duan. " An optimal PID controller design for nonlinear constrained optimal control problems," *Discrete and Continuous Dynamical System - Series B*, vol. 16, pp. 1101-1118, 2011.

[2] J. Xu, and C. C. Lim. "Using transfer-resource graph for software-based verification of system-on-chip," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 27, pp. 1315-1328, 2008.

[3] F. Li, P. Shi, L. Wu, M. V. Basin, and C. C. Lim. "Quantized control design for cognitive radio networks modeled as nonlinear semi-Markovian jump systems," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 4, pp. 2330-2340, 2015.

[4] L. Zhang, H. Gao, and O. Kaynak. "Network-induced constraints in networked control systems—a survey," *IEEE Transactions on Industrial Electronics,* vol. 9, no. 1 pp. 403-416, 2013.

[5] G. R. Matłas, and B. Antonio. "Analysis of networked control systems with drops and variable delays," *Automatica*, vol. 43, pp. 2054-2059, 2007.

[6] G. Liu, Y. Xia, J. Chen, D. Rees, and W. Hu. "Networked predictive control of systems with random network delays in both forward and feedback channels ," *IEEE Transactions on Industrial Electronics,* vol. 54, no. 3, pp. 1282-1297, 2007.

[7] R. Yang, G. Liu, P. Shi, C. Thomas, and V. Basin. "Predictive output feedback control for networked control systems", *IEEE Transactions on Industrial Electronics,* vol. 61, no. 1, pp. 512-520, 2014.

[8] J. Zhang, P. Shi, and Y. Xia. "Fuzzy delay compensation control for T-S Fuzzy systems over network," *IEEE Transactions on Cybernetics*, vol. 43, no. 1, pp. 259-268, 2013.

[9] A. Berman, M. Neumann, and R. J. Stern. "Nonnegative matrices in dynamic systems," Wiley, New York, 1989.

[10] T. Kaczorek. "Positive 1D and 2D systems," Springer, 2002.

[11] L. Farina, and S. Rinaldi. "Positive Linear System: Theory and Applications," Wiley, 2000.

[12] H. Gao, J. Lam, C. Wang, and S. Xu. "Control for stability and positivity: Equivalent conditions and computation," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 52, no. 9, pp. 540-544, 2005.

[13] M. Rami, and F. Tadeo. "Controller synthesis for positive linear systems with bounded controls," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 54, no. 2, pp. 151-155, 2007.

[14] T. Tanaka, and C. Langbort. "The bounded real lemma internally positive systems and H-infinity structured static state feedback," *IEEE Transaction on Automatic Control*, vol. 56, no. 9, pp. 2218-2223, 2011.

[15] B. Shafai, R. Ghadami, and A. Oghbaee. "Constrained stabilization with maximum stability radius for linear continuous-time systems," *Proceeding of the 52nd IEEE Conference on Decision and Control*, Florence, Italy, pp. 3415-3420, 2013

[16] B. Shafai, A. Oghbaee, and T. Tanaka. "Positive stabilization with maximum stability radius for linear time-delay systems," *Proceeding of the 53rd IEEE Conference on Decision and Control*, Los Angeles, USA, pp. 1948-1953, 2014.

[17] A. Benzaouia, R. Oubah, and A. Hajjaji. "Stabilization of positive Takagi-Sugeno fuzzy discrete-time systems with multiple delays and bounded controls," *Journal of the Franklin Institute*, vol. 351, no. 7, pp. 3719-3733, 2014.

[18] A. Benzaouia, and R. Oubah. "Stability and stabilization by output feedback control of positive Takagi-Sugeno fuzzy discrete-time systems with multiple delays," *Nonlinear Analysis: Hybrid Systems*, vol. 11, pp. 154-170, 2014.

[19] S. Q. Zhu, M. Meng, and C. H. Zhang. "Exponential stability for positive systems with bounded time-varying delays and static output feedback stabilization," *Journal of the Franklin Institute*, vol. 350, pp. 617-636, 2013.

[20] Y. Ebihara , D. Peaucelle, and D. Arzelier. "LMI approach to linear positive system analysis and synthesis," *Systems & Control Letters*, vol. 63, pp. 50-56, 2014.

[21] K. Tanaka, and H. O. Wang. " Fuzzy control system design and analysis," A wiley-interscience publication, 2001.