

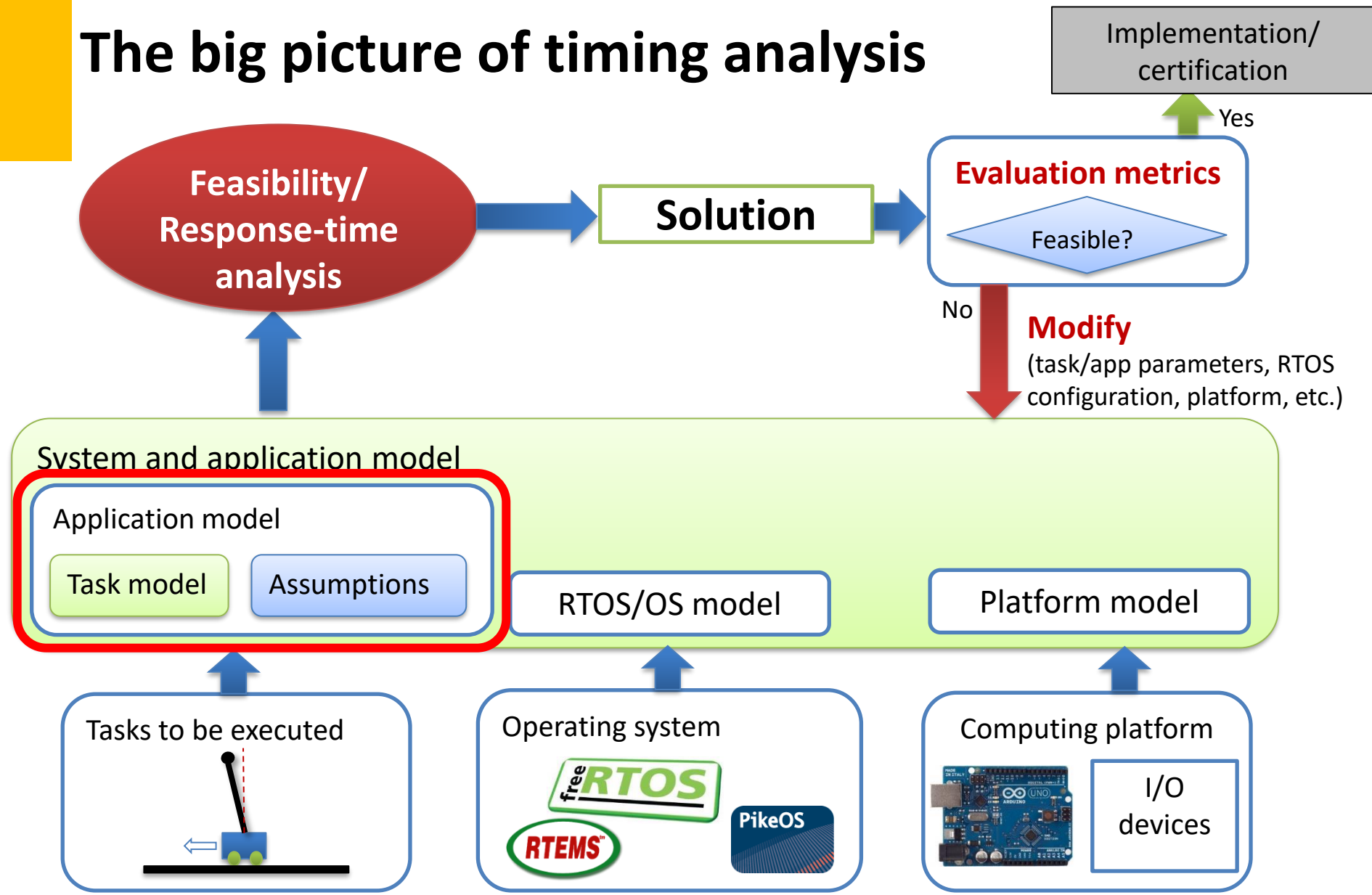
2IMN20 - Real-Time Systems

Modeling real-time systems

Geoffrey Nelissen

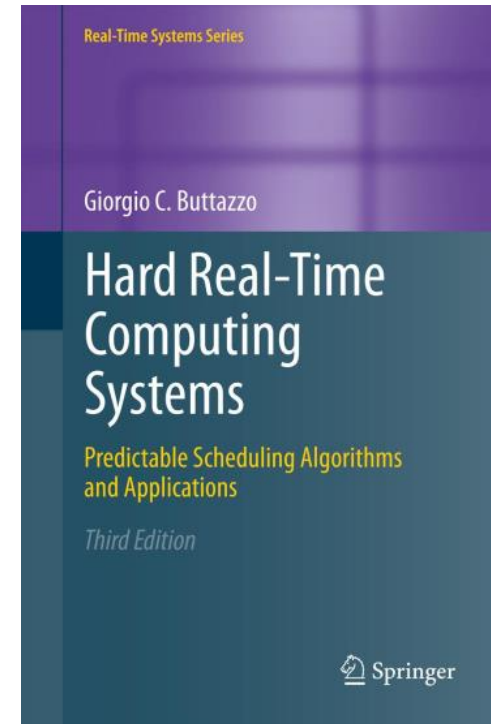
2023-2024

The big picture of timing analysis



Reading material

Buttazzo's book, chapter 1 and 2



Disclaimer:

Most slides were provided by Dr. Mitra Nasri

Some slides have been taken from [Giorgio Buttazzo](#)'s website

Recap: Task

- **Task**

- A **sequence of actions** that must be carried out to, for example, implement a functionality or respond to an event
- A task may use multiple inputs and produce multiple outputs
- It may be activated by
 - **events** (e.g., user command or an interrupt) or
 - **time** (e.g., by timer interrupts)

Question: how is this task activated (on **event** or **time**)?

Question: What is the activation model of this task (periodic, sporadic or aperiodic)?

```
While(true)
{
    mq_receive (mq, (char*)&temp, sizeof(temp));
    if (temp > 42)
        send(-1);
    else
    {
        int * array = read10Data( );
        int max = -1;
        for (int i=0; i < 10; i++)
            if (max < 0 || array[i] > max)
                max = array[i];
        send(max);
    }
    sleep (100, ms);
}
```

Workload characterization

- A task workload is characterized by three main aspects:
 - **Execution time**
 - **Activation/Arrival model**
 - **Computation model**

Workload characterization

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- ~~Execution time~~

- **Activation/Arrival model**

- **Computation model**

Arrival model of a task

- A task releases a **sequence of jobs** to be executed. Job may be released
 - **Periodically**
 - with or without **release jitter**
 - with or without **release offset** (first job released later than 0)
 - **Sporadically**
 - **Aperiodically**
- Time-triggered activation
- Event-triggered activation
- More complex arrival patterns may be modelled with **arrival curves**

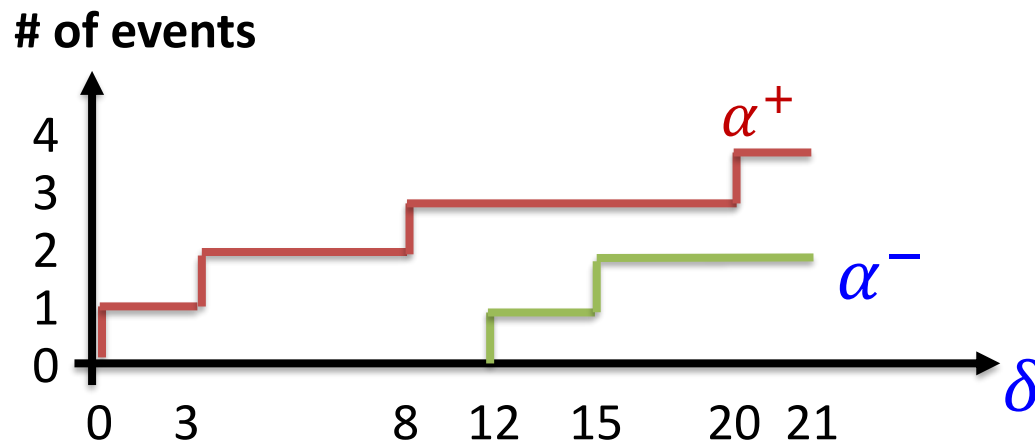
Note: many other arrival models exist. For example, **elastic tasks**, **variable-rate tasks**, **digraph tasks**, ...

Modelling complex arrival patterns with arrival curves

- An arrival curve represents the **lower bound** and **upper bound** on the **number of events in any time interval**.

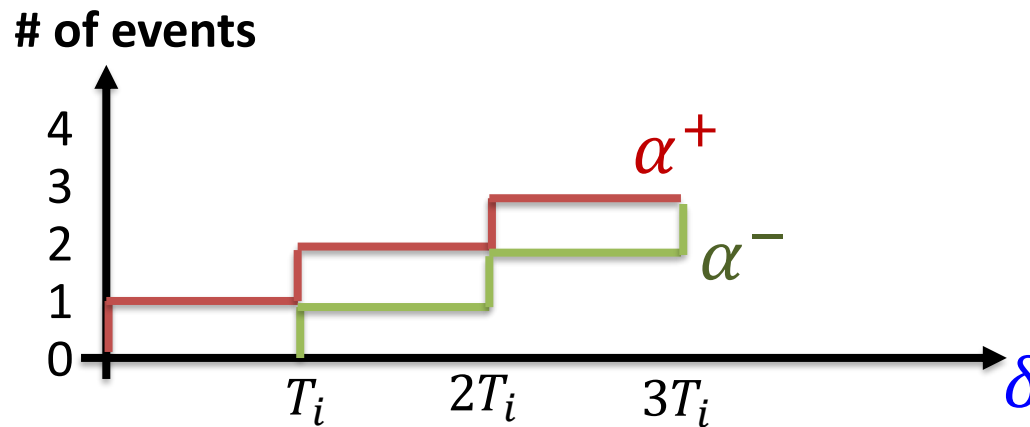
α^+ = maximum number of events
in any interval of duration δ

α^- = minimum number of events
in any interval of duration δ



Arrival curves

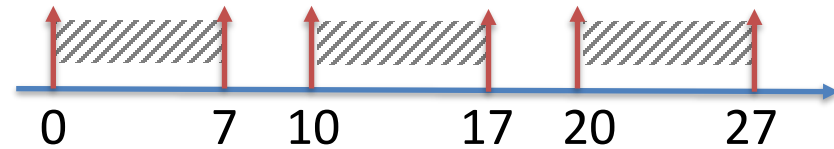
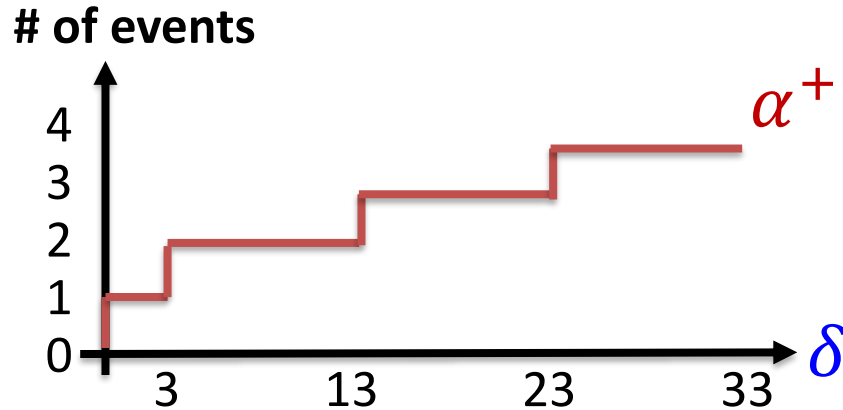
Question: How does the arrival curve of a periodic task with period T_i look like?



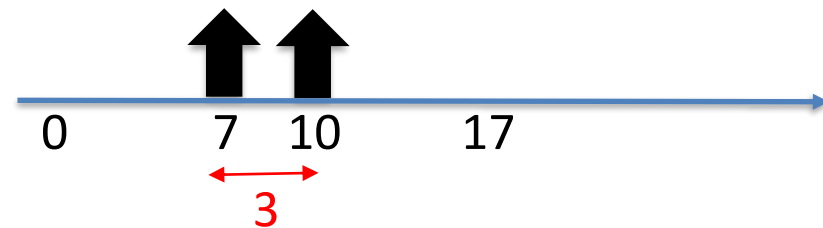
Question: Why $\alpha^- \neq \alpha^+$ even for a periodic task with no offset or release jitter?

Arrival curves

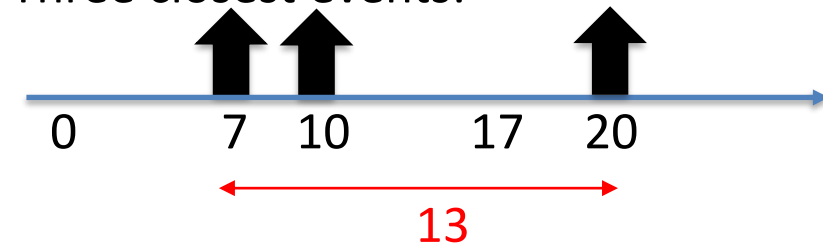
Question: How does the α^+ arrival curve of a periodic task with release jitter look like?
 Period = 10 and release jitter $\sigma = 7$



Two closest events:



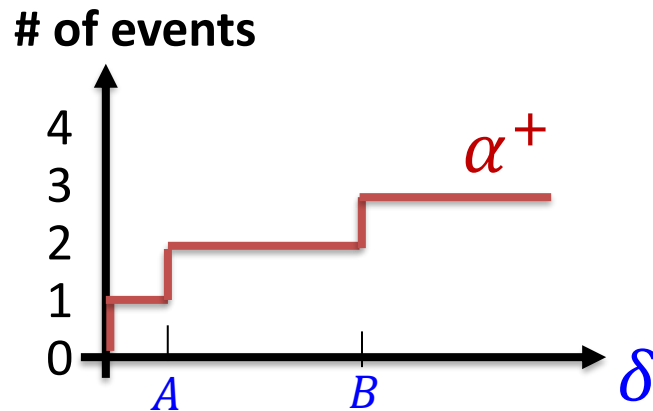
Three closest events:



At home: Derive the α^- arrival curve

Arrival curves

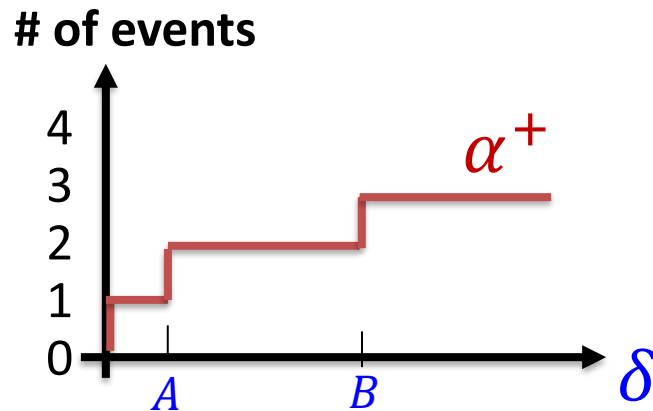
Question: How does the α^+ arrival curve of a periodic task with period T and release jitter σ look like?



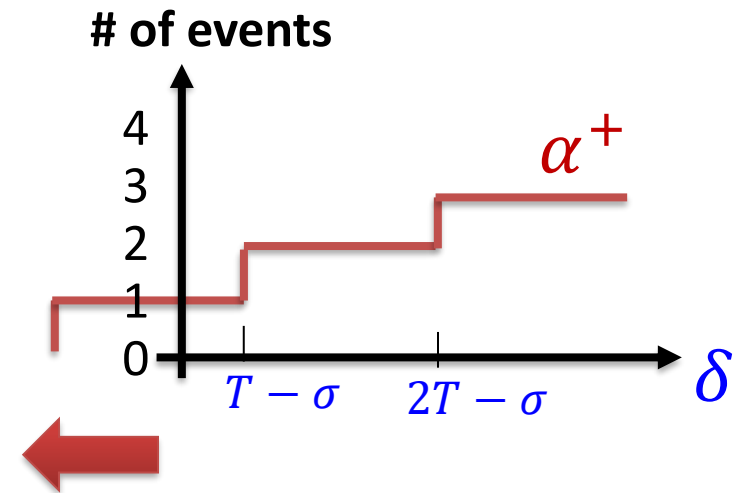
Determine A and B

Arrival curves

Question: How does the α^+ arrival curve of a periodic task with period T and release jitter σ look like?

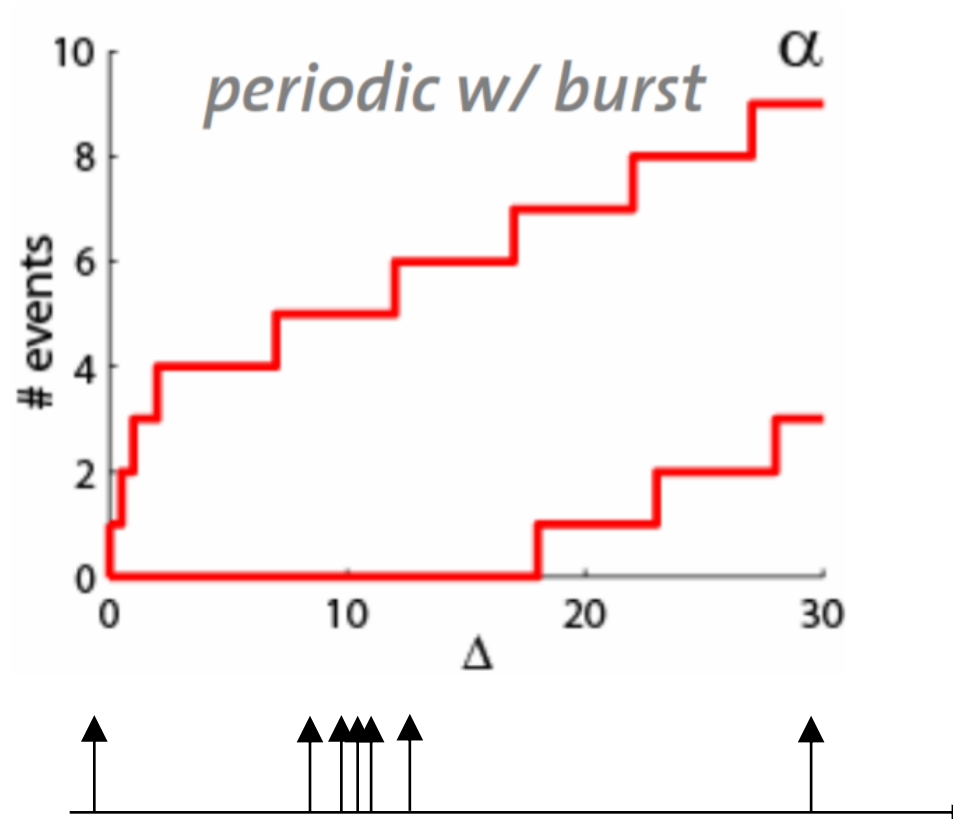


Determine A and B

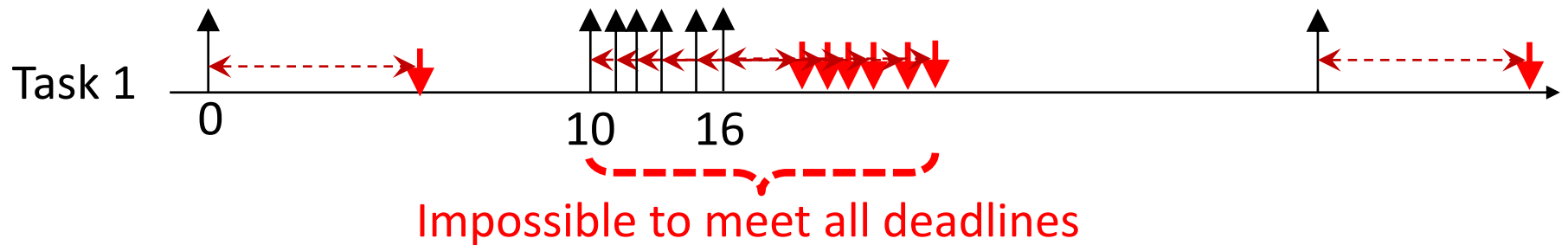


Shift α^+ of a purely periodic task by σ to the left to get α^+ for a task with release jitter σ

Other arrival curves



- Since it is hard to predict the future workload, it is hard to analyze the system to find the worst-case response time of the tasks
- With unbounded inter-arrival times, it is impossible to guarantee the timeliness of a resource-constrained system



Characterization of the deadline

- Type of deadlines
 - **Hard**
 - **Soft**
 - **Firm**
- Relation between deadline and period
 - **Implicit deadline**
 - Deadline is **equal** to the period/minimum inter-arrival time
 - **Constrained deadline**
 - Deadline is **smaller than** or equal to the period/minimum inter-arrival time
 - **Arbitrary deadline**
 - Deadline may be **smaller than, equal to, or larger than** the period/minimum inter-arrival time

Relation between deadline and period

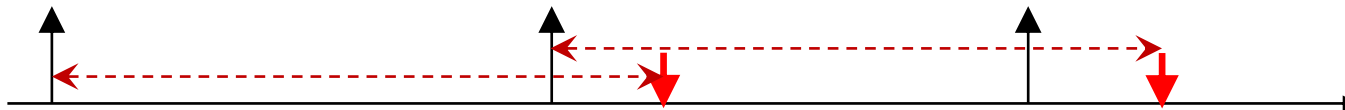
- **Implicit deadline:** deadline is equal to period



- **Constrained deadline:** deadline is smaller than or equal to period



- **Arbitrary deadline:** deadline can be smaller, equal, or larger than period



Relation between deadline and period

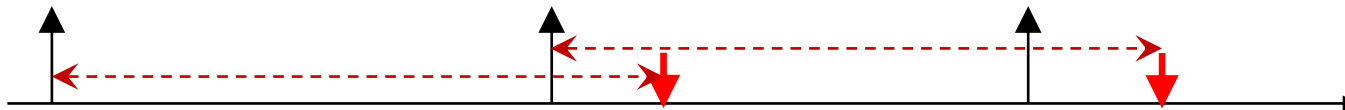
- **Implicit deadline:** deadline is equal to period



Harder to analyze

because at any time, **more than one instance of the task might be ready** for execution

- **Arbitrary deadline:** deadline can be smaller, equal, or larger than period



Notations

τ_i : the i^{th} task in the task set

C_i : the WCET of task τ_i

T_i : the period of τ_i

σ_i : the release jitter of τ_i

Φ_i : the release offset (also called phase) of τ_i

D_i : the relative deadline of τ_i

R_i : the worst-case response time of τ_i

$J_{i,j}$: the j^{th} job of task τ_i

$a_{i,j}$: arrival time of the job $J_{i,j}$

$r_{i,j}$: release time of the job $J_{i,j}$

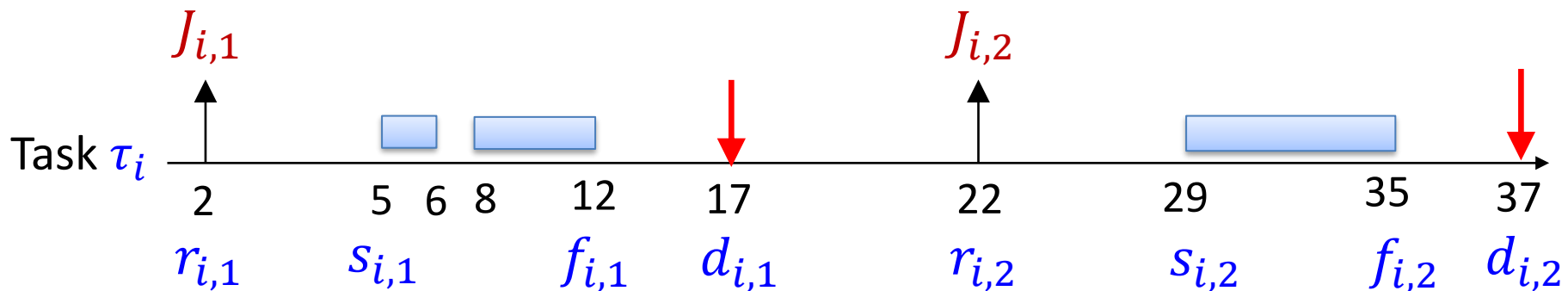
$c_{i,j}$: execution time of the job $J_{i,j}$

$d_{i,j}$: absolute deadline of the job $J_{i,j}$

$s_{i,j}$: start time of the job $J_{i,j}$

$f_{i,j}$: finish time of the job $J_{i,j}$

$R_{i,j}$: response-time of the job $J_{i,j}$



Question: What is the largest *observed* response time in this schedule?

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Question: What is the *observed* C_i in this schedule?

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Question: what is the equation to calculate $r_{i,j}$ for a given job $J_{i,j}$ of a periodic task without release jitter?

$$r_{i,j} = \Phi_i + (j - 1) \cdot T_i$$

Notations and definitions

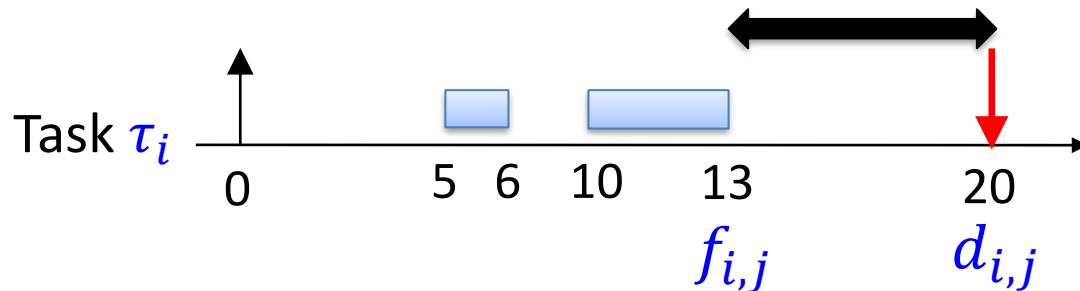
A schedule is said to be **feasible** for a real-time task τ_i if all jobs of this task **complete before their deadline**, that is, if

$$\forall j, \quad f_{i,j} \leq d_{i,j}$$

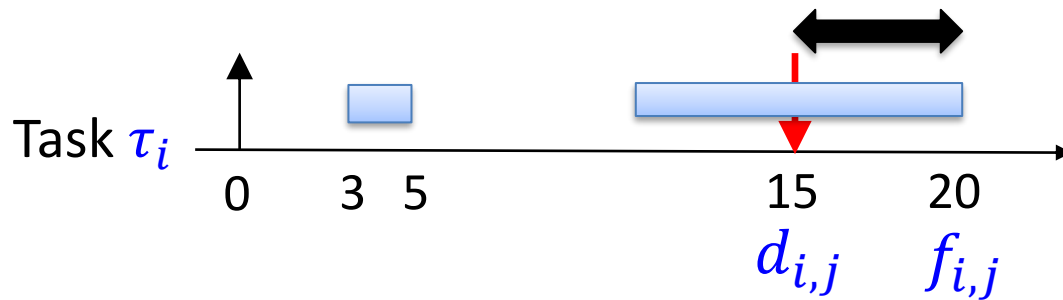
or equivalently,

$$\forall j, \quad R_{i,j} \leq D_i$$

Slack and lateness



Slack:
 $d_{i,j} - f_{i,j}$



Lateness:
 $L_{i,j} = f_{i,j} - d_{i,j}$

Tardiness: $\max(0, f_{i,j} - d_{i,j})$

Workload characterization

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- ~~Execution time~~

- ~~Activation/Arrival model~~

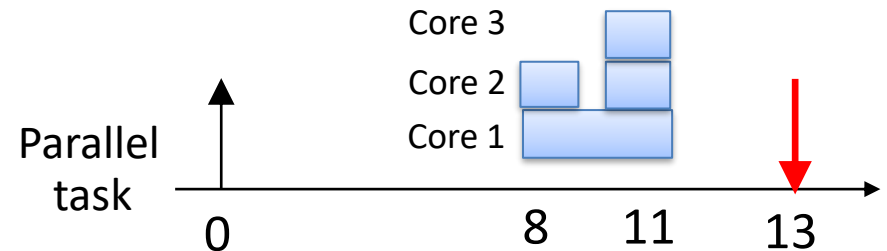
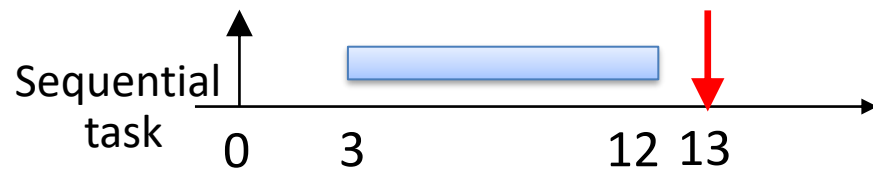
- **Computation model**

Modeling computation

Sequential

v.s.

Parallel



Sequential task



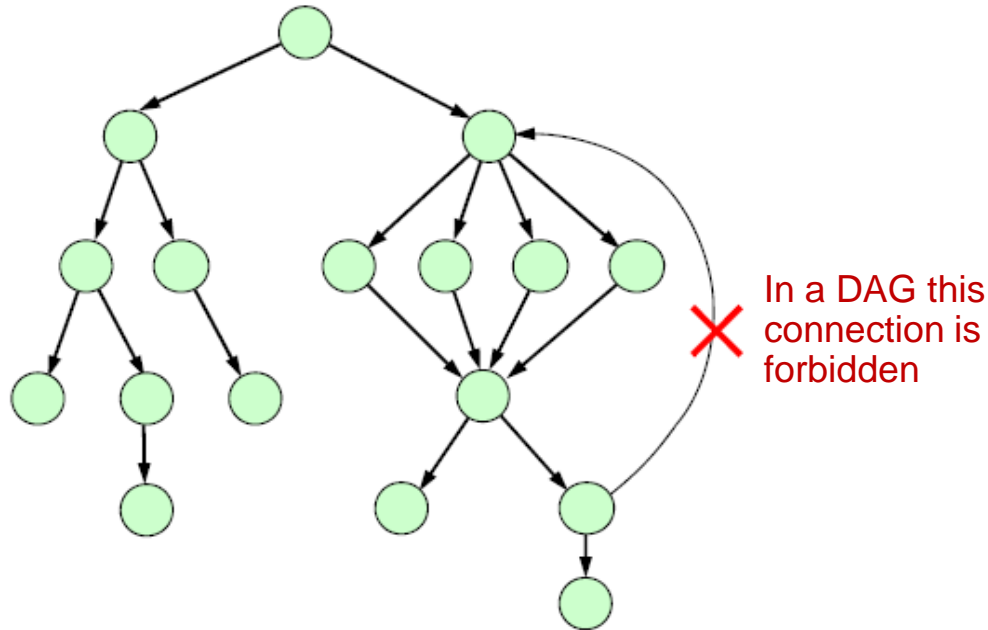
Parallel task

Parallel programming

- Existing parallel programming models/languages
 - OpenMP
 - MPI
 - Java/C++/Python/...
 - IBM's X10
 - Intel TBB
 - Sun's Fortress
 - Cray's Chapel
 - Cilk (Cilk++)
 - Codeplay's Sieve C++
 - Rapidmind Development Platform

Modeling parallel tasks

- Representing a parallel code requires more complex structures like a graph (usually a **directed-acyclic graph**, a.k.a. **DAG**):

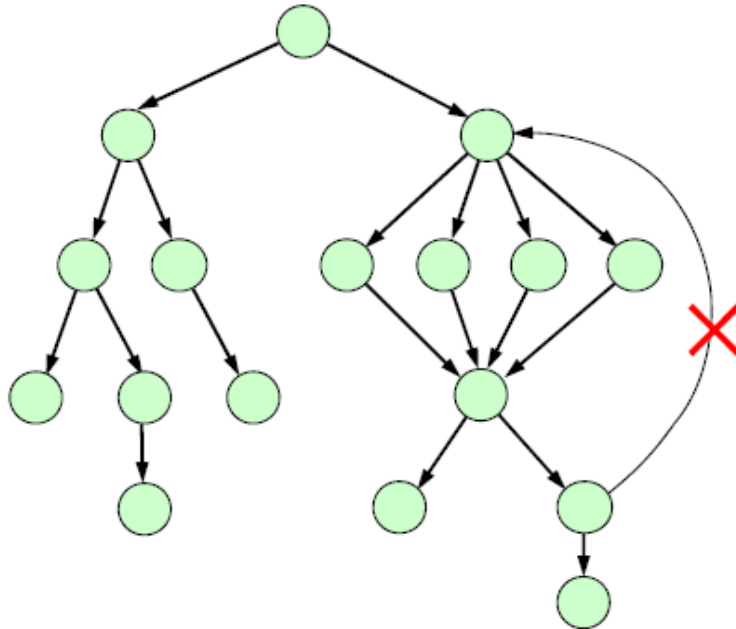


Question: What is the maximum parallelism of that task?

7

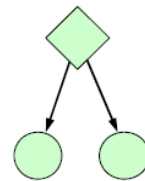
Modeling parallel tasks

- Representing a parallel code requires more complex structures like a graph (usually a **directed-acyclic graph**, a.k.a. **DAG**):

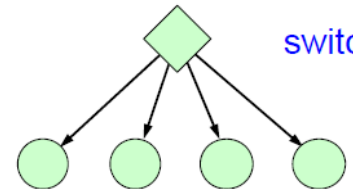


In a DAG this connection is forbidden

DAGs can be **conditioned** (e.g., by an **if-then-else** block)

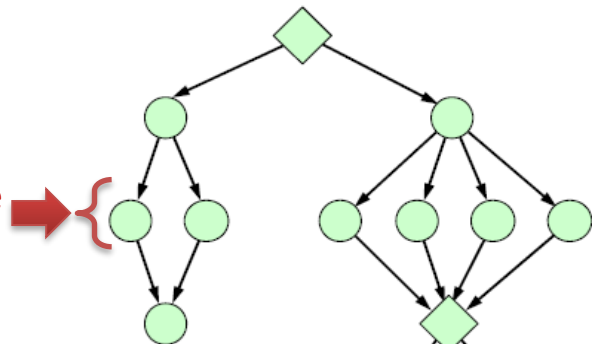


if-then

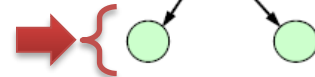


switch

Both must be executed



Only one will execute



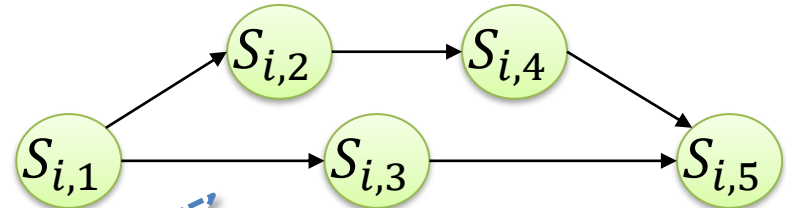
OR nodes represent conditional statements

Notations

Worst-case execution time of segment $S_{i,2}$

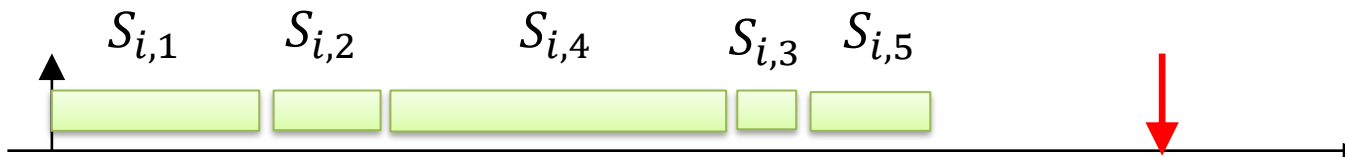
Task parameters:

$$\tau_i = (\langle C_{i,1}, C_{i,2}, \dots, C_{i,m_i} \rangle, D_i, T_i)$$



Precedence constraint between task's segments
(a.k.a. **intra-task precedence constraint**)

Single core schedule

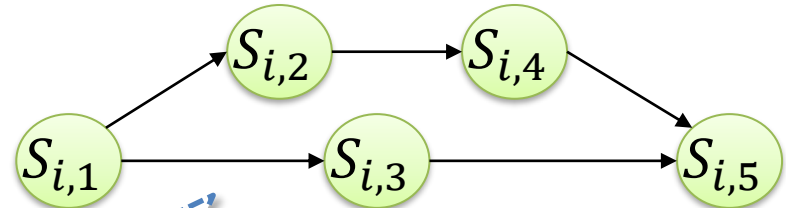


Notations

Worst-case execution time of segment $S_{i,2}$

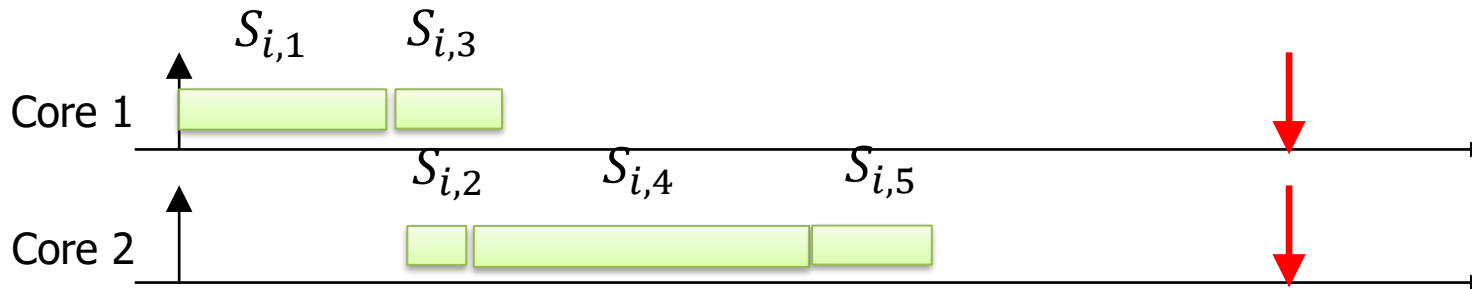
Task parameters:

$$\tau_i = (\langle C_{i,1}, C_{i,2}, \dots, C_{i,m_i} \rangle, D_i, T_i)$$



Precedence constraint between task's segments
(a.k.a. **intra-task precedence constraint**)

Multi-core (2 cores) schedule



Preemptive v.s. non-preemptive execution

- **Preemptive execution**

- The execution of a task may be forcefully preempted (stopped) by the scheduler

- **Non-preemptive execution**

- A task does not leave the processor (or resource) until it completes or suspends itself
- **Non-preemptive execution** is also called “**cooperative execution**” if the task which is running on the platform voluntarily yields the processor so that other tasks can execute

Self-suspension

Self-suspension: At some points in the code, task may suspend itself to access I/O, wait for results from a GPU or other another task, etc.

While(true)

```
{  
  int max = -1;  
  int * array = read10Data();  
  for (int i=0; i < 10; i++)  
    if (max < 0 || array[i] > max)  
      max = array[i];  
  send(max);  
  sleep (100, ms);  
}
```

Example: A **non-busy-waiting read** results in self-suspension.

In a non-busy-waiting read, the task initiates a request and then waits until it is notified (when data is ready).





Disclaimer

- The course slides came from Koen Langendoen who compiled materials found on the web and obtained from friendly colleagues together.
- Thanks to
 - Koen Langendoen
 - Akos Ledeczi
 - Gerd Gross
 - Giorgio Buttazzo
 - Reinder Bril
 - Zonghua Gu
 - and many others