2f) $\sum_{n=1}^{\infty} \frac{2^{n}+1}{2^{n+1}}$

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Hoja 1 de 2

Observemos que:

$$\frac{2^n+1}{2^{n+1}} = \frac{2^n}{2^{n+1}} + \frac{1}{2^{n+1}} = \frac{1}{2} + \frac{1}{2^{n+1}}$$

Si vemos el límite cuando $n\to\infty$:

$$\lim_{n\to\infty} \frac{1}{2} + \frac{1}{2^{n+1}} = \frac{1}{2} + 0$$

Como $\lim_{n\to\infty} a_n \neq 0$ la serie $\sum_{n=1}^{\infty} a_n$ es divergente

3d) $\sum_{n=1}^{\infty} \frac{a^n}{(n+2)(n+a)5^n}$ $a>0$.

Veamos que sucede si ~~[illegible]~~ $a<5$:

$$\frac{a^n}{(n+2)(n+a)5^n} < \frac{a^n}{5^n} = \left(\frac{a}{5}\right)^n \quad \text{ya que } (n+2).(n+a)>0$$

$$\sum_{n=1}^{\infty}\left(\frac{a}{5}\right)^n = \sum_{n=0}^{\infty}\frac{a}{5}\left(\frac{a}{5}\right)^n = \frac{a}{5}\sum_{n=0}^{\infty} r^n \quad \text{con } r=\frac{a}{5}$$

Como $r<1$, la serie geometrica converge y vale que: $\frac{a}{5}\sum_{n=0}^{\infty} r^n = \left(\frac{1}{1-r}\right)\left(\frac{a}{5}\right)$ y por criterio de comparacion, la serie $\sum_{n=1}^{\infty}\frac{a^n}{(n+2)(n+a)5^n}$ converge.

Veamos si $a>5$:

$$\lim_{n\to\infty}\frac{a^n}{(n+2).(n+a).5^n} = \lim_{n\to\infty}\frac{\left(\frac{a}{5}\right)^n}{(n+2)(n+a)} \overset{L'H}{=} \lim_{n\to\infty}\frac{\left(\frac{a}{5}\right)^n \ln\left(\frac{a}{5}\right)}{a+2n+2} \overset{L'H}{=} \lim_{n\to\infty}\frac{1}{2}\left(\frac{a}{5}\right)^n \ln^2\left(\frac{a}{5}\right) = \infty$$

Como $\lim_{n\to\infty} a_n \neq 0$ la serie $\sum_{n=1}^{\infty} a_n$ es divergente

Veamos $a=5$:

$$\frac{a^n}{(n+2)(n+5)5^n} = \frac{1}{(n+2)(n+5)} = \frac{1}{3}\,\frac{(n+5)-(n+2)}{(n+2).(n+5)} = \frac{1}{3}\left(\frac{1}{n+2} - \frac{1}{n+5}\right)$$

Luego

$$\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+3)} = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n+2} - \frac{1}{n+5} = \frac{1}{3}\left(\frac{1}{4}+\frac{1}{5}+\sum_{n=1}^{\infty} b_n - b_{n+1}\right) \quad \text{con } b_n = \frac{1}{n+2}$$

Entonces la serie es telescopica y

$$\sum_{n=1}^{\infty}\frac{1}{(n+2)(n+3)} = \frac{1}{3}\left(\frac{1}{4}+\frac{1}{5}+b_1 - \lim_{n\to\infty} b_n\right) = \frac{1}{3}\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{3} - \lim_{n\to\infty}\frac{1}{n+2}\right) = \frac{1}{3}\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{3}\right) = \frac{47}{180}.$$