

# Warehouse Optimization

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## 1 Introduction

This project is dedicated to optimizing warehouse operations, with a focus on **order batching** and **picking processes**. Efficient batching and picking are critical for reducing operational costs, minimizing travel distances, and improving overall order fulfillment performance.

This project was proposed by **Find&Order**, a company specializing in warehouse and logistics solutions.

In addition to exact mathematical models, the project explores **heuristic approaches** to generate high-quality solutions within reasonable computation times. These heuristics are particularly relevant in real-world warehouses, where the combinatorial complexity of batching and routing problems often makes exact optimization impractical.

The objective of this document is to present both the **mathematical formulations** of the batching and picking problems and the **heuristic methods** developed to address them. This provides a comprehensive view of the strategies used to improve warehouse efficiency.

## 2 Problem Description

This project addresses the optimization of warehouse order preparation operations by jointly considering *order batching* and *picking routing* problems.

### 2.1 Warehouse Environment

The warehouse is modeled as a set of storage positions where the products are located. The layout of the warehouse is represented through:

- A set of storage locations.
- An adjacency matrix describing the travel distance between every pair of locations.
- A predefined starting location and ending location for each picking tour.

A set of customer orders is provided. Each order is associated with:

- A unique identifier.
- A volume representing its physical size.
- A list of storage locations corresponding to the products that must be collected.

In this project, each order is assumed to be associated with a dedicated container, referred to as a *support*. This assumption reflects the operational framework considered and simplifies traceability and batching decisions.

During warehouse operations, a picker can transport several supports simultaneously during a single picking tour. The set of supports collected during the same picking tour is referred to as a *batch*.

## 2.2 Decision Problem

The decision problem consists of determining:

1. How to assign orders to batches.
2. The sequence in which storage locations are visited for each batch.

The objective is to minimize the total travel distance required to collect all products across all batches.

## 2.3 Batching Constraints

The batching decisions must satisfy the following operational constraints:

- Each order must be assigned to exactly one batch.
- The number of supports assigned to a batch cannot exceed a predefined maximum capacity.
- The total volume of supports assigned to a batch cannot exceed a predefined maximum volume.

## 2.4 Picking Constraints

For each batch, the picking route must satisfy the following requirements:

- Every storage location corresponding to the products of the batch must be visited.
- The picking route must start at the warehouse depot.
- The picking route must end at the designated exit location.

## 2.5 Objective Function

The cost of a solution is defined as the sum of travel distances required to complete all batch picking routes. The performance of a solution is evaluated by comparing it to a baseline solution in which each order is processed independently. In this baseline approach, each support is assigned to a single batch, and the picking route follows a predefined serpentine routing strategy, commonly referred to as an S-shape path, which corresponds to the default routing rule used in many warehouse management systems.

## 2.6 Computational Complexity

The problem addressed in this project combines two well-known combinatorial optimization problems:

- The Order Batching Problem, which is closely related to bin packing and clustering problems.
- The Picking Routing Problem, which can be formulated as a variation of the Traveling Salesman Problem (TSP).

Both subproblems are known to be NP-hard. Consequently, the integrated batching and routing problem exhibits exponential growth in the number of feasible solutions as the number of orders increases.

Due to this computational complexity, solving large-scale instances optimally becomes impractical within reasonable computation times. Therefore, heuristic and metaheuristic approaches are required to generate high-quality solutions efficiently in realistic industrial environments.

## 3 Mathematical Models

This section presents the mathematical formulations used to model the order batching and picking routing problems described in Section 2. The models are developed following a sequential optimization framework. First, orders are grouped into batches. Then, a picking route is computed for each batch.

### 3.1 Order Batching Model

#### 3.1.1 Context

The batching model aims to assign customer orders to batches while satisfying operational constraints such as batch capacity and volume limitations. The objective is to group orders that are likely to be collected together efficiently, thereby reducing the travel distance required during picking operations.

Since the exact picking route is not known at the batching stage, an order similarity measure based on shared picking locations is used as a proxy to estimate the potential travel distance reduction obtained by grouping orders.

#### 3.1.2 Sets

- $O$  : Set of customer orders, indexed by  $o$ .
- $B$  : Set of potential batches, indexed by  $b$ .

### 3.1.3 Parameters

- $v_o$  : Volume associated with order  $o$ .
- $C$  : Maximum volume capacity of a batch.
- $N$  : Maximum number of supports allowed in a batch.
- $a_{oo'}$  : Number of shared picking locations between orders  $o$  and  $o'$ .

### 3.1.4 Decision Variables

$$x_{ob} = \begin{cases} 1 & \text{if order } o \text{ is assigned to batch } b \\ 0 & \text{otherwise} \end{cases}$$

$$z_{oo'b} = \begin{cases} 1 & \text{if orders } o \text{ and } o' \text{ are assigned to the same batch } b \\ 0 & \text{otherwise} \end{cases} \quad \forall o, o' \in O \text{ with } o < o', \forall b \in B$$

The variable  $z_{oo'b}$  is introduced to linearize the quadratic term  $x_{ob} \cdot x_{o'b}$  appearing in the objective function. It ensures that  $z_{oo'b} = 1$  if and only if both orders  $o$  and  $o'$  are assigned to batch  $b$ .

### 3.1.5 Objective Function

The batching objective is to maximize the similarity between orders assigned to the same batch:

$$\max \sum_{b \in B} \sum_{\substack{o, o' \in O \\ o < o'}} a_{oo'} z_{oo'b}$$

This objective promotes grouping orders sharing common storage locations, which is expected to reduce picking travel distance.

### 3.1.6 Constraints

Each order must be assigned to exactly one batch:

$$\sum_{b \in B} x_{ob} = 1 \quad \forall o \in O$$

Batch volume constraint:

$$\sum_{o \in O} v_o x_{ob} \leq C \quad \forall b \in B$$

Batch size constraint:

$$\sum_{o \in O} x_{ob} \leq N \quad \forall b \in B$$

Linearization constraints:

$$z_{oo'b} \leq x_{ob} \quad \forall o, o' \in O, o < o', \forall b \in B$$

$$z_{oo'b} \leq x_{o'b} \quad \forall o, o' \in O, o < o', \forall b \in B$$

$$z_{oo'b} \geq x_{ob} + x_{o'b} - 1 \quad \forall o, o' \in O, o < o', \forall b \in B$$

Binary constraints:

$$x_{ob} \in \{0, 1\}, \quad \forall o \in O, \forall b \in B$$

$$z_{oo'b} \in \{0, 1\}, \quad \forall o, o' \in O, o < o', \forall b \in B$$