Handwritten Proof #2 Rod cutting problem. 313551135 林[illegible]

$$r_n = \max(p_n, r_1 + r_{n-1}, r_2 + r_{n-2}, \dots, r_{n-2} + r_2, r_{n-1} + r_1) \quad \text{— (1)}$$

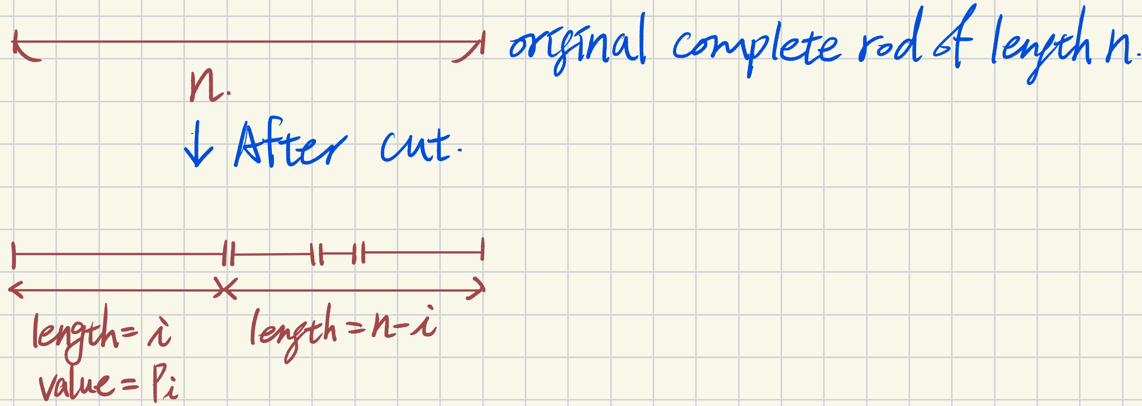
$$\Rightarrow r_n = \max_{1 \le i \le n}(p_i + r_{n-i}) \quad \text{——— (2)}$$

The first formula:

$$r_n = \max(p_n, r_1 + r_{n-1}, r_2 + r_{n-2}, \dots, r_{n-2} + r_2, r_{n-1} + r_1)$$

r_n means the largest value of the rod cut of length n is sold. The best value might be no cut, cut into 1 and $(n-1)$, etc.

Therefore, it might be cut into:



We can find that no matter what the method of cut to get the largest value is, the optimal solution must consists of a piece of length occurs after a length $1 \le i < n$ follow by a rod of length $n-i$ cut optimally. Therefore, for the left most cut rod, it's value is P_i. Instead, the right rod cut of total length $n-i$ has the largest value r_{n-i}. Thus, we either have $r_n = p_i + r_{n-i}$ or $r_n = p_n$.

Then if $r_0 = 0$, it means:

$$\begin{cases} r_n = \max\limits_{1 \le i \le n} (p_i + r_{n-i}). \\ r_n = p_n + r_0 \Rightarrow r_n = p_n, \text{ if no cut happens.} \end{cases}$$