

## Exercise 1.1

*Proof.* Let  $A, B$  and  $C$  be arbitrary sets. Let  $x$  be an element in  $(A \cap B) \cup (A \cap C)$ . By definition of the union, either  $x \in (A \cap B)$  or  $x \in (A \cap C)$ . If  $x \in (A \cap B)$ , then  $x$  is an element in both  $A$  and  $B$  by definition of the intersection. Analogously, if  $x \in (A \cap C)$ , then  $x$  is an element in both  $A$  and  $C$  by definition of the intersection. So  $x \in A$  and  $x \in (B \cup C)$ , i.e.  $x \in A \cap (B \cup C)$ , again by the definitions of the intersection and the union, which ends the proof.  $\square$

## Exercise 1.2

*Proof.* Let  $A$  and  $B$  be arbitrary sets. We want to show that if  $(A \cap B) = \emptyset$ , then  $A \setminus B = A$ . We argue by contradiction, so we assume that  $(A \setminus B) \neq A$ . By definition  $A \setminus B \subseteq A$ , so  $A \not\subseteq A \setminus B$ .....THIS IS NOT FINISHED .....  $\square$

## Exercise 1.3

*Proof.* Let  $A$  and  $B$  be arbitrary sets. We want to show that if  $A \cup B = B$ , then  $A \subseteq B$  by contrapositive. So we assume that if  $A \not\subseteq B$ , then  $A \cup B \neq B$ . Since  $A \not\subseteq B$ , there must be an  $x$  which is in  $A$  but not in  $B$ . Therefore  $A \cup B$  cannot be  $B$ .  $\square$

## Exercise 1.4

*Proof.* Let  $A = \{Lina, Leda, Alessia, Scarlet\}$ ,  $B = \{Lina, Leda, Alessia\}$  and  $C = \{Leda, Alessia, Scarlet\}$ . It's easy to see, that while  $A$  is exactly the Set  $B \cup C$ ,  $A$  is neither a subset of  $B$  or  $C$ .  $\square$