Discrete Mathematics in Computer Science

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Exercise Sheet 1

Due: Monday, October 2, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

In this exercise you should practice how to correctly write down mathematical proofs. Your proof should be formulated in such a way that it is clear and easy to follow. Write full, gramatically correct sentences; we do not accept simply lining up arguments. In the future, we also expect clearly written proofs for proof exercises but only on this sheet the focus is on the form rather than the content.

You can find examples for well-written proofs in the file `proof_examples_1.pdf` *on the course website. Since the formal definitions that we need for these proofs only will be introduced in the next chapter, we list some rules that you may use in the proof. Do not use other rules from set theory because the correctness of more complex rules first has to be proven itself.*

- $X = Y$ iff $X \subseteq Y$ and $Y \subseteq X$.
- $X \subseteq Y$ iff all elements of X are elements of Y.
- $x \in (X \cup Y)$ iff $x \in X$ or $x \in Y$.
- $x \in (X \cap Y)$ iff $x \in X$ and $x \in Y$.
- $x \in (X \setminus Y)$ iff $x \in X$ and $x \notin Y$.
- $x \notin X$ iff not $x \in X$.
- $X = \emptyset$ iff there is no $x \in X$.

Exercise 1.1 (3 marks)
Show the following statement with a *direct proof*.
For all sets A, B and C it holds that $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

Exercise 1.2 (3 marks)
Prove the following statement with a *proof by contradiction*.
For all sets A and B we have: if $(A \cap B) = \emptyset$, then $(A \setminus B) = A$.

Exercise 1.3 (2 marks)
Show the following statement with a *proof by contrapositive*.
For all sets A and B we have: if $A \cup B = B$, then $A \subseteq B$.

Exercise 1.4 (2 marks)

Refute the following statement.
For all sets A, B and C we have: if $A \subseteq (B \cup C)$, then $A \subseteq B$ or $A \subseteq C$.

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.