

Exercise 1.1

Proof. Let A, B and C be arbitrary sets. Let x be an element in $(A \cap B) \cup (A \cap C)$. By definition of the union, either $x \in (A \cap B)$ or $x \in (A \cap C)$. If $x \in (A \cap B)$, then x is an element in both A and B by definition of the intersection. Analogously, if $x \in (A \cap C)$, then x is an element in both A and C by definition of the intersection. So $x \in A$ and $x \in (B \cup C)$, i.e. $x \in A \cap (B \cup C)$, again by the definitions of the intersection and the union, which ends the proof. \square

Exercise 1.2

Proof. Let A and B be arbitrary sets. We want to show that if $(A \cap B) = \emptyset$, then $A \setminus B = A$. We argue by contradiction, so we assume that $(A \setminus B) \neq A$. By definition $A \setminus B \subseteq A$, so $A \not\subseteq A \setminus B$. To fulfill this definition, there must be an $x \in A \cup B$ \square

Exercise 1.3

Proof. Let A and B be arbitrary sets. We want to show that if $A \cup B = B$, then $A \subseteq B$ by contrapositive. So we assume that if $A \not\subseteq B$, then $A \cup B \neq B$. Since $A \not\subseteq B$, there must be an x which is in A but not in B . Therefore $A \cup B$ cannot be B . \square

Exercise 1.4

Proof. Let $A = \{\text{"Lina"}, \text{"Leda"}, \text{"Alessia"}, \text{"Scarlet"}\}$, $B = \{\text{"Lina"}, \text{"Leda"}, \text{"Alessia"}\}$ and $C = \{\text{"Leda"}, \text{"Alessia"}, \text{"Scarlet"}\}$. It's easy to see, that while A is exactly the Set $B \cup C$, A is neither a subset of B or C . \square