

# Discrete Mathematics in Computer Science

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## Exercise Sheet 7

**Due: Monday, November 13, 2023, 4pm**

**Please carefully read the exercises FAQ on ADAM!**

*Note:* Submissions that are exclusively created with  $\text{\LaTeX}$  will receive a bonus mark. Please submit only the resulting PDF file.

### **Exercise 7.1** (2 marks)

Prove or disprove the following statements:

- (a) If  $a \mid c$  and  $b \mid c$  then  $ab \mid c$ .
- (b) If  $a \mid b$  and  $a \mid b - c$  then  $a \mid c$ .

### **Exercise 7.2** (2 marks)

Show that congruence modulo  $n$  is compatible with the following operations. You may use that congruence modulo  $n$  is compatible with addition, subtraction, multiplication and translation.

- (a) scaling
- (b) exponentiation

### **Exercise 7.3** (1 mark)

Draw the digraph  $G = (\{A, B, C, D, E, F\}, \{(A, B), (C, B), (C, D), (C, F), (D, E), (E, C), (F, C)\})$  pictorially.

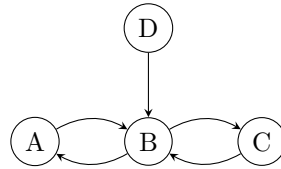
### **Exercise 7.4** (3 marks)

Specify a graph  $G = (V, E)$  with vertices  $V = \{H, I, J, K, L, M, N\}$  and the following properties. You may describe  $G$  either as a tuple or pictorially.

- (i) There is no  $x \in E$  such that  $I$  is an endpoint of  $x$ .
- (ii) Vertices  $J$  and  $M$  are adjacent.
- (iii) Vertex  $K$  has degree 5.
- (iv)  $\text{neigh}(N) = \text{neigh}(J)$
- (v)  $\text{neigh}(H) \cap \text{neigh}(J) = \{K\}$
- (vi)  $|E| = 8$

**Exercise 7.5** (2 marks)

Consider the following directed graph  $G$ :



- (a) Specify a walk from  $D$  to  $B$  that is not a path.
- (b) Specify all paths of length 2.
- (c) Specify a tour with  $v_0 = v_n = A$  that is not a cycle.
- (d) How many cycles are in the induced graph of  $G$ ?

**Submission rules:**

Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.