Exercise 1.1

Proof. Let A,B and C be arbitrary sets. Let x be an element in $(A \cap B) \cup (A \cap C)$. By definition of the union, either $x \in (A \cap B)$ or $x \in (A \cap C)$. If $x \in (A \cap B)$, then x is an element in both A and B by definition of the intersection. Analogously, if $x \in (A \cap C)$, then x is an element in both A and C by definition of the intersection. So $x \in A$ and $x \in (B \cup C)$, i.e. $x \in A \cap (B \cup C)$, again by the definitions of the intersection and the union, which ends the proof.

□

Exercise 1.2

Proof. Let A and B be arbitrary sets. We want to show that if $(A \cap B) = \emptyset$, then $A \setminus B = A$. We argue by contradiction, so we assume that $(A \setminus B) \neq A$. By definition $A \setminus B \subseteq A$, so $A \nsubseteq A \setminus B$ or else we would have an equality. Therefore there must exist an x which is element in A but not in $A \setminus B$, i.e. $x \in A \cap B$. But this means that $A \cap B$ is not empty. This is the wanted contradiction.

□

Exercise 1.3

Proof. Let A and B be arbitrary sets. We want to show that if $A \cup B = B$, then $A \subseteq B$ by contrapositive. So we assume that if $A \nsubseteq B$, then $A \cup B \neq B$.
Since $A \nsubseteq B$, there must be an x which is in A but not in B. Therefore $A \cup B$ cannot be B.

□

Exercise 1.4

Proof. Let $A = \{"Lina","Leda","Alessia","Scarlet"\}$, $B = \{"Lina","Leda","Alessia"\}$ and $C = \{"Leda","Alessia","Scarlet"\}$. It´s easy to see, that while A is exactly the Set $B \cup C$, A is neither a subset of B or C.

□