Discrete Mathematics in Computer Science

M. Helmert, G. Röger
S. Eriksson
Fall Semester 2023

University of Basel
Computer Science

Exercise Sheet 5

Due: Monday, October 30, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 5.1 (1 mark)

(a) How many equivalence relations exist over set $M = \{a, b, c\}$? Justify your answer.

(b) Consider the equivalence relation

$$\sim = \{\langle a,a\rangle, \langle b,b\rangle, \langle c,c\rangle, \langle d,d\rangle, \langle e,e\rangle, \langle c,e\rangle, \langle d,b\rangle, \langle b,d\rangle, \langle e,c\rangle\}$$

over $S = \{a, b, c, d, e\}$. Specify all equivalence classes induced by $\sim$.

Exercise 5.2 (2 marks)

Prove the following statement from the lecture:
Let $\sim$ be an equivalence relation over set S, and $E = \{[x]_\sim \mid x \in S\}$ the set of all equivalence classes.

- Every element of S is in some equivalence class in E.
- Every element of S is in at most one equivalence class in E.

Hint: For the second part, you can use an indirect proof, assuming there are $x, y, z \in S$ with $x \in [y]_\sim$, $x \in [z]_\sim$ and $[y]_\sim \neq [z]_\sim$.

Exercise 5.3 (2 marks)

Specify a partial order over $S = \{a, b, c, d\}$ with exactly two minimal elements and b being the greatest element. Justify why your relation is a partial order and specify which are the minimal elements.

Exercise 5.4 (2 marks)

Are the following statements correct? Briefly justify your answer.

(a) For all total orders R over an arbitrary set S, exactly one of xRy and yRx is true for all $x, y \in S$.

Hint: For $x = y$, the statements xRy and yRx are the same statement and thus only count as one statement.

(b) For every set S there is a strict order over S where all $x \in S$ are both minimal and maximal.

Exercise 5.5 (3 marks)

Consider the relations $A = \{\langle x, 2x\rangle \mid x \in \mathbb{N}_0\}$ and $B = \{\langle i \cdot x, x\rangle \mid i, x \in \mathbb{N}_0\}$ and $C = \{\langle 4, 2\rangle, \langle 5, 8\rangle, \langle 4, 7\rangle, \langle 8, 2\rangle, \langle 6, 4\rangle\}$ over $\mathbb{N}_0$.
Describe each of the relations described below as a set using set-builder or explicit notation.

(a) A^{-1}

(b) $B \setminus A^{-1}$

(c) $C \circ A$

(d) $A \circ (A \circ A)$

(e) A^*

(f) $A \circ B$

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.