Discrete Mathematics in Computer Science

M. Helmert, G. Röger
S. Eriksson
Fall Semester 2023

University of Basel
Computer Science

Exercise Sheet 4

Due: Monday, October 23, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 4.1 (2 marks)
Explain why the following statements are wrong.

(a) $\{2,3\} \times \emptyset = \{\langle 2,\emptyset\rangle, \langle 3,\emptyset\rangle\}$

(b) $\{\langle 1,0\rangle\} \times \{0\} = \{\langle 1,0\rangle, \langle 0,0\rangle\}$

(c) $\{\langle 1,2\rangle, \langle 2,3\rangle\} = \{\langle 2,3\rangle, \langle 2,1\rangle\}$

(d) $\{2,4\} \in \{0,1,2\} \times \{3,4,5\}$

Exercise 4.2 (2 marks)
Specify sets $A, B \subseteq \{1,\ldots,10\}$ that satisfy the given properties.

(a) $A \times B = B \times A$, $|A| = 3$, $\langle 3,5\rangle \in A \times B$

(b) $\langle\langle 1,6\rangle, 4\rangle \in (A \times A) \times B$, $|A \times B| = 6$, $|A \cup B| = 4$

Exercise 4.3 (2 marks)
Specify a non-empty relation over $S = \{a,b,c\}$ satisfying the required properties, or explain why such a relation cannot exist.

(a) Relation R_1 is neither reflexive nor irreflexive and is symmetric but not transitive.

(b) Relation R_2 is transitive, symmetric, irreflexive and contains $\langle a,b\rangle$.

Exercise 4.4 (4 marks)
Consider the following binary relation R over the natural numbers:

$$R = \{\langle i, j \cdot i\rangle \mid i,j \in \mathbb{N}_0\}.$$

Is R reflexive, irreflexive, symmetric, asymmetric, antisymmetric, transitive? Briefly justify your answer for each of the six properties.

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.