Discrete Mathematics in Computer Science

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Exercise Sheet 3
Due: Monday, October 16, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 3.1 (1 mark)
Specify sets $A, B \subseteq \{1, \ldots, 10\}$ such that $|A \cup B| = 5$, $|\mathcal{P}(A)| = 16$ and $|B| = 3$.

Exercise 3.2 (2 marks)
We consider the universe $U = \{e_0, \ldots, e_7\}$ and a bit string representation where the i-th bit (0-indexed, from left to right) represents the inclusion of e_i.

  (a) Specify the bit string representation of set $\{e_1, e_4, e_6, e_7\}$.

  (b) Which set does the bit string `01110110` represent?

  (c) Consider the section "Bitwise operators" from the following article:

    https://en.wikipedia.org/wiki/Bitwise_operation.

    Which bitwise operators compute the intersection, union and negation of sets represented as bit strings?

Exercise 3.3 (1 mark)
Refute that for all non-empty disjoint sets A and B it holds that $|A| < |A \cup B|$.

Exercise 3.4 (2 marks)
Show that $\mathbb{Z}$ is countably infinite by specifying a bijection from $\mathbb{Z}$ to $\mathbb{N}_0$. Don't forget to justify why your function is indeed a bijection.
Hint: A justification does not need to be a full formal proof, but it should be a convincing argument.

Exercise 3.5 (2 marks)
Show that $\mathbb{Q}$ is countable. You may use the fact that $\mathbb{Q}_+$ is countable.
Hint: Use the countability of $\mathbb{Q}_+$ to show the countability of $\mathbb{Q}_- = \{q \mid q \in \mathbb{Q}, q < 0\}$.

Exercise 3.6 (2 marks)
Show that the set of tarradiddles T is countable. Tarradiddles were defined in the lecture in chapter A3 (Proofs II), slide 20.

Hint: The set of tarradiddles is a subset of the set of all finite strings that consist only of symbols from the set $\Sigma = \{✈, ♥, ✿\}$.

Submission rules:
Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.