

Discrete Mathematics in Computer Science

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Exercise Sheet 8

Due: Monday, November 20, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with L^AT_EX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 8.1 (3 marks)

Are the following statements about digraphs $G = (N, A)$ true? Justify your answer.

- (a) If $(n_1, n_2) \in S_G^i$, then $(n_1, n_2) \in S_G^j$ for all $j > i \geq 1$. As a reminder, S_G^i is the i -fold composition of S_G .
- (b) Nodes n_1 and n_2 are mutually reachable iff G contains a cycle involving n_1 and n_2 .
- (c) If n_1 and n_2 are in the same strongly connected component, then they are in the same weakly connected component.

Exercise 8.2 (3 marks)

Let $G = (N, A)$ be a digraph. A *topological order* for G is a bijective function $f : N \rightarrow \{1, \dots, |N|\}$ such that $f(u) < f(v)$ for all arcs $(u, v) \in A$. Prove the following statements.

- (a) If a topological order f exists for G , then G is acyclic.
- (b) If G is acyclic, then a topological order f for G exists.

Hint: First show the lemma “each DAG contains a node v with $\text{outdeg}(v) = 0$ ”, and then show the claim by induction using the lemma.

Exercise 8.3 (3 marks)

Consider a tree $G = (V, E)$ with $|V| > 2$. How many leaves does G have if it satisfies the following property? Justify your answer.

- (a) The longest path in G has length 2.
- (b) The longest path in G has length $|V| - 1$.

Exercise 8.4 (1 mark)

Specify a graph $G = (V, E)$ with $|E| = |V| - 1$ that is not a tree.

Submission rules:

Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.