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Exercise 4.1

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$

Exercise 4.2

a) 0100 1011

b) $\{e_1, e_2, e_3, e_5, e_6\}$

c) Intersection $\rightarrow$ AND
Union $\rightarrow$ OR
Negation $\rightarrow$ NOT

Exercise 4.3

We assume $A = \mathbb{N}_0$ and $B = \mathbb{Z}_- = \{-1, -2, ...\}$. Then $|A| = |\mathbb{N}_0|$ and $|A \cup B| = |\mathbb{Z}| = |\mathbb{N}_0|$ by Exercise 3.4. So $|A| = |A \cup B|$.

Exercise 4.4

$0 \mapsto 0$
$1 \mapsto 1$
$2 \mapsto -1$
$3 \mapsto 2$
$4 \mapsto -2$
$\vdots$

In the image we only have elements from $\mathbb{Z}$ and we map to every element of $\mathbb{Z}$ just once.

Exercise 4.5

Proof. We can use the same Proof idea as in the slides for the proof that $\mathbb{Q}_+$ is countably infinite and just prepend a minus sign in front of every fraction in order to proof that $\mathbb{Q}_-$ is also countably infinite. By the theorem proven in the lecture we know that the union of $\mathbb{Q}_-$, $\mathbb{Q}_+$ and $\{0\}$ is countable. $\square$

Exercise 4.6

TBS: The set of tarradiddles T is countable.
Remark: We will use 1, 2, 3 instead of the Taradiddle symbols.

Proof. We define a first set containing just the elementary symbols: $\Sigma_1 = \{1, 2, 3\}$. Then we define a set containing all possible combinations of element tuples: $\Sigma_2 = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$, analogously also the sets Σ_3, Σ_4 and so forth. Those sets are countable since they are finite. The set $\bigcup_{i=1}^{\infty} \Sigma_i$ is countable by the lectures theorem and so is the subset T of tarradiddles. $\square$