Discrete Mathematics in Computer Science

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Exercise Sheet 2

Due: Monday, October 9, 2023, 4pm

Please carefully read the exercises FAQ on ADAM!

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 2.1 (2 marks)

Proof the following statement with mathematical induction.

$$\sum_{i=0}^{n} i = \frac{n \cdot (n+1)}{2} \text{ for all } n \in \mathbb{N}_0$$

Hint: A weak induction where in the induction hypothesis you suppose the proposition is true for $n-1$ and in the step you go from $n-1$ to n will work nicely.

Exercise 2.2 (2 marks)

We consider the set $\mathcal{B}$ of binary trees which is inductively defined as follows:

- $\square$ is a binary tree.
- If L and R are binary trees, then $\langle L, \bigcirc, R\rangle$ is a binary tree.

Given a binary tree B we define its number of leaves with

$$\begin{aligned} \mathit{leaves}(\square) &= 1 \text{ and} \\ \mathit{leaves}(\langle L, \bigcirc, R\rangle) &= \mathit{leaves}(L) + \mathit{leaves}(R), \end{aligned}$$

and its number of edges with

$$\begin{aligned} \mathit{edges}(\square) &= 0 \text{ and} \\ \mathit{edges}(\langle L, \bigcirc, R\rangle) &= \mathit{edges}(L) + \mathit{edges}(R) + 2. \end{aligned}$$

Show with structural induction that for all binary trees B it holds that $\mathit{edges}(B) = 2 \cdot \mathit{leaves}(B) - 2$.

Exercise 2.3 (2 marks)

We recursively define a set of words S as follows:

- `baa` is in S
- `c` is in S
- If x and y are in S, then so is xyx (e.g. since `c` and `baa` are in S, so is `cbaac`).
- If x is in S, then so is bxb (e.g. since `c` is in S, so is `bcb`).

Prove with structural induction that all words in S have odd length, where the length of a word is the number of characters (e.g. `baa` has length 3).

Exercise 2.4 (1 mark)

The following formalisations of the described expressions are syntactically wrong. Specify the correct syntax.

(a) The set of all odd natural numbers that are smaller than 20: $\{x \neq 2n, x < 20\}$

(b) The set of all natural numbers without 6: $\{x\} \setminus 6$

Exercise 2.5 (3 marks)

For this exercise, we consider the set of all objects to be $U = \{1, \ldots, 10\}$. In each subtask specify sets $A, B \subseteq U$ that have all the required properties.

(a) $\overline{A \cup B} = \{2, 4, 7\}$ and $A \cap B = \{1, 3\}$

(b) $A \cap B = \emptyset$ and $A \subseteq B$ and $B \setminus A = \{3, 5\}$.

(c) $A \cup B \subseteq A$ and $B \supset \{6, 8\}$.

Submission rules:

Upload a single PDF file (ending in .pdf). Put the names of all group members on top of the first page. Make sure your PDF has size A4 (fits the page size if printed on A4). There is a template that satisfies these requirements available on ADAM.