EARTH AND OCEAN SCIENCES 453

MATLAB homework assignment # 4:

Can Melting Snow Trigger Earthquakes in the Pacific Northwest?

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1 Preliminary Notes

Hydroseismicity refers to earthquakes that are triggered by dynamics associated with the hydrologic cycle. In this assignment you will investigate how the annual cycle of spring rains/melting snow on Mount Hood might influence seismicity in this region (Figure 1). In more detail, you will identify and characterize an apparent temporal relationship between seismicity and seasonal groundwater recharge. You will use this information to estimate key transport properties of the regional hydrologic system and, in turn, to interrogate some dynamical links to seismicity. Finally, you will use your analysis to assess the state of stress in the upper crust. One motivating question following from the paper by Rundle is the extent to which the crust in a tectonically-active region is maintained in a "critical" state.

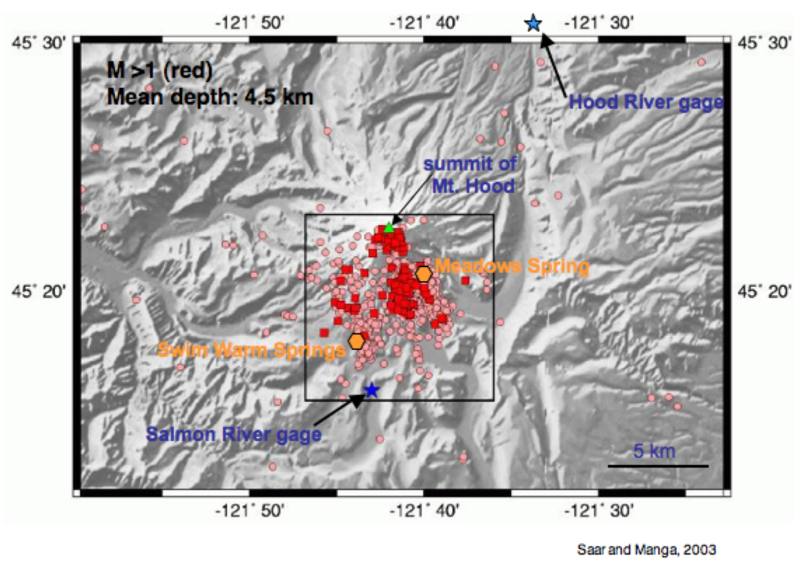

Figure 1: Earthquake Hydrology in the Oregon Cascades

1.1 Porous media flow: Darcy's law and the evolution of pore pressure

The flow of water in a porous medium is driven by differences in the fluid pressure that may be expressed as the potential

$$\phi = \rho g z + P, \tag{1}$$

where ρ is the density of water, g is gravity, z is elevation above some datum (e.g., sea level) and P is pressure. In a saturated permeable media the volume flux of water $\mathbf{q}$ is given by Darcy's law

$$\mathbf{q} = -\frac{\kappa}{\mu}\nabla\phi, \tag{2}$$

where κ is the permeability [m^2] of the medium and μ is the viscosity of water. Because the pressure P is that exerted by a column of water of height $(h - z)$, the potential in equation 1 can be written equivalently in terms of the hydraulic head

$$h = z + \frac{P}{\rho g} \tag{3}$$

which has dimensions of length and corresponds to a readily measurable quantity. The physical significance of of the hydraulic head is that it is dimensionally a potential energy per unit weight of fluid rather than a potential energy per unit mass as is implied by (1). The gradients $\nabla\phi$ and ∇h imply a force per unit mass and a force per unit weight, respectively.

In terms of (3), Darcy's law is

$$\mathbf{q} = -K\nabla h \tag{4}$$

where K is the hydraulic conductivity of the permeable medium [m s^{-1}]. The relationship between the permeability κ and the hydraulic conductivity K is

$$\kappa = \frac{\mu K}{\rho g}, \tag{5}$$

where μ is the fluid viscosity [Pa-s] (resistance to shear) and and the quantity ρg [Pa m^{-1}] is the pressure gradient driving flow through the pores. Although the permeability is dimensionally a measure of the geometry of the pore structure in a given medium it will be different for different fluids. Water is slightly compressible and the pressure effect on density is described by the equation of state

$$\rho(p) = \rho(p_0)\exp[\beta(p - p_0)] \tag{6}$$

where p_0 is some reference pressure and β is the compressibility of water $\beta = 4.5 \times 10^{-10}$ Pa^{-1}. Finally, assuming uniaxial strain in the solid matrix and a constant vertical stress, viscosity and permeability, Darcy's equation, coupled with the continuity equation,

$$\frac{\partial(\rho\mathbf{q})}{\partial t} + \nabla\cdot\mathbf{q} = 0 \qquad (7),$$

leads to a diffusion equation that governs the evolution of pore pressure:

$$\frac{\partial P}{\partial t} = D\nabla^2 P. \tag{8}$$

Here, D is the "hydraulic diffusivity" [m^2/s] and is defined as

$$D = \frac{K}{S} = \frac{g\kappa\rho}{\mu S} \quad (9),$$

where S is the standard hydrogeologic specific storage $[m^{-1}]$ and is equal to the volume of water released from the pore space per unit change in head while maintaining a constant vertical stress and no lateral strain. This physical property is analogous to the specific heat in thermal diffusion problems with which you are no doubt familiar. Dimensional analysis on equation (7) indicates that the characteristic time for changes in pressure to diffuse or relax will scale as L^2/D, where L is the characteristic length scale in the problem (such as a fault length). From some more dimensional analysis, the characteristic length scale over which pressure changes is also an intrinsic property of the saturated porous medium and will scale as $\sqrt{Dt}$.

As a final remark, equation (7) is the pressure diffusion equation used most commonly in groundwater modeling. The form of (7), however, assumes that the evolution of fluid pressure resulting from flow is uncoupled from that related to the solid matrix stress. If there is two- or three-dimensional flow or a more complicated applied stress (or resulting strain) field, an additional solid <=> fluid stress coupling must be included through an additional time derivative of the mean stress of the solid matrix (for more on this, see chapter 4 in the excellent book *Theory of Linear Poroelasticity* by Wang).

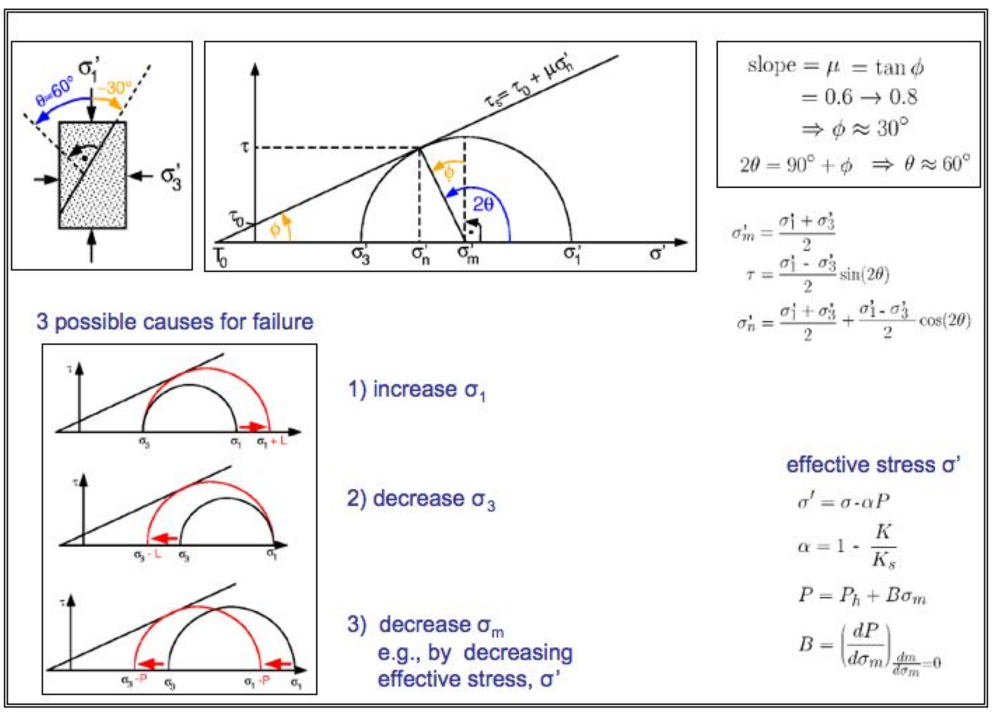

Figure 2: Mohr-Coulomb failure criterion: Some essential concepts

1.2 Causes of hydroseismicity

Some mechanisms involved in triggering hydroseismicity may be identified by combining concepts from linear poroelasticity with the Coulomb failure criterion, which is:

$$\tau_s = \tau_o + \mu\sigma_n', \tag{10}$$

where τ_s, τ_o and μ are the shear strength [Pa], cohesion [Pa] and friction coefficient [-], respectively and σ_n' is the effective normal stress across a fluid-filled fault (a positive value) (Figure 2). The effective normal stress acts to "clamp" the fault closed and, thus, governs the magnitude of the shear stress required for failure, which leads to an earthquake. Note that the effective stress tensor σ_{ij}' is modified from the regular principal stress tensor σ_{ij} to include the effects of fluid pore pressure P:

$$\sigma_{ij}' = \sigma_{ij} - \alpha P \delta_{ij}. \tag{11}$$

Examination of equation (11) indicates that if the pore-fluid $P = 0$ then this normal stress is simply the lithostatic value $\rho g z$. Here, δ_{ij} is the Kronecker-delta and the so-called Biot-Willis coefficient,

$$\alpha = 1 - K/K_s, \tag{12}$$

where K and K_s are the bulk moduli of the solid matrix as a whole and of the solid grains composing the microstructure. In terms of the principal stress components the effective normal stress is:

$$\sigma_n' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos(2\theta), \tag{13}$$

where σ_1 and σ_3 are the effective principal maximum and minimum compressive stresses and θ is the angle between σ_1' and the normal to the failure plane (Figure 1). The shear stress is

$$\tau = \left| \frac{\sigma_1' - \sigma_3'}{2}\sin(2\theta) \right| \tag{14}$$

Here, the sum $\sigma_1' + \sigma_3'$ might be viewed as the "total compressive stress" on the fault and the difference $\sigma_1' - \sigma_3'$ the effective differential stress giving rise to the shear stress that can lead to failure.

Examination of equations (10), (13) and (14) shows that an applied shear stresses in excess of the fault strength τ_o will lead to rupture expressed as motion along the fault. Moreover, such failure can be triggered by changes in either (or both!) θ and σ_{ij}'. In particular, earthquakes may be induced by 1) an increase in σ_1'; 2) a decrease in σ_3'; or 3) an equal decrease in all effective principal stresses by, for example, an increase in the fluid pore pressure P. Mechanisms 1 and 2 cause the fault to become unstable through an increase in the differential stress that leads to a shear instability. Mechanism 3 reduces the effective strength of the fault because the fluid pressure P opposes σ_n' and "unclamps" the fault. Said differently, the friction binding the two sides of the fault together is reduced because the fluid pressure forces the two sides to open parallel to the normal to the fault plane.

Temporal fluctuations in the pore-fluid pressure P denoted P' can take many forms. The regional change in response to an increase in the maximum compressive crustal stress σ_1' by, say, by adding a weight that contracts the pore space(such as a filling reservoir) is communicated instantaneously. By contrast, a local increase in the hydrostatic pore-fluid pressure by groundwater flow or by a forced injection of fluids at depth may trigger earthquakes after a time lag that is related to the diffusion of pressure from the source of the perturbation to the fault. Note that in the Mount Hood problem you will investigate below that the actual weight of the groundwater added to the crust during snow melt is trivial (there is little change in the maximum compressive stress). However, because the pore space is connected the pore fluid pressures generated by groundwater flow can be significant enough to cause earthquakes.

2 ASSIGNMENT

In this assignment you will investigate whether seasonal (i.e., periodic) groundwater recharge might drive seismicity in the Mount Hood area. Your results will be applied, in turn, to interrogate the state of stress in the upper crust. In particular, you will be able to address Rundle's conjecture that the crust is always in a "near-critical" state. In this assignment you will first do some time series analysis to identify a temporal relationship between variations in seismicity and groundwater flow. You will use the results of this analysis together with an analytical solution to the pressure diffusion equation to probe aspects and implications of the dynamics of hydroseismicity.

1. Download the files `eqmagsrawinterp.txt` and `eqmagstreamflow.txt`. The first file shows the raw daily seismic moment M_o (dynes $\times 10^{21}$) and corresponding interpolation over the period 1983-1988 beneath Mount Hood, Oregon. The time-averaged depth at which most earthquakes occur is approximately 4.5 km. These data were interpolated using a moving polynomial fit and a Gaussian smoothing algorithm. A moving polynomial interpolation scheme (rather than a simple curve fit over the whole data) is used in order to prevent introducing new frequency bands to the data while producing a continuous time series. Please plot both time series from `eqmagsrawinterp.txt` on one plot so that you can see how the histogram of seismic moment "events" has become a continuous time series. Note that the interpolation is plotted with $M_o(t) = 1$ as a baseline rather than $M_o(x) = 0$. For consistency, the same fitting/smoothing operations are applied to the groundwater data in the second file.

   The second file, `eqmagstreamflow.txt`, is the one you will analyze. The first column is time in days and the second two columns are normalized seismicity (total moment magnitude per day, which is an accurate proxy for the number of earthquakes per day) and normalized streamflow related to surface runoff (cubic meters per day). Plot the two time series normalized to their maximal values (the y-axes should be between 0 and 1). We will assume that because groundwater recharge is governed mostly by surface runoff in this region that the time series for flow rate is a reasonable approximation for the "forcing" to the hydrologic system. To get a sense of the dynamics contained in each time series you may also wish to plot the Fourier spectrum from each. A script entitled `fftbasic.m` can be downloaded from the website if you would like a brief tutorial on spectral analysis.

2. The key piece of data we would like to extract from comparison of these two time series is the time lag in days between the forcing (i.e., groundwater recharge) and the response (i.e., seismicity). To find this lag you will use a standard cross-correlation technique. For a tutorial, please download and work with `basicXcorr.m`. There are a few additional comments below as well. Alternatively, you may wish to analyze the coherence and phase of the signals. A basic setup is commented out in the same tutorial script. One advantage to this method is that you can analyze the phase shift associated with every component frequency in the time series, in

turn. A practical disadvantage is that the utility of this method depends on the accuracy of your spectral and cross-spectral estimates. Because your time series are not infinite in length this has to be done carefully. Said more directly– although a potentially better method for this problem, it will take you longer to get an answer with which you are satisfied using this technique.

Cross correlate the earthquake and groundwater runoff time series using the built-in function `xcorr.m` first. Use the "coeff" flag and plot your results for lags between 0 and 400 days (order 1 year) and over a range in correlation coefficients between 0 and 0.3. At what positive time lags are correlations maximized? Why do we care about positive lags and not negative ones? The next issue is whether to believe the results! More precisely, are the lags you identify statistically significant relative to a correlation with a signal of random phase. Do you understand your data?

To add this very important calculation, use the `xcorrc` function, which can be downloaded from the website. `xcorrc.m` calculates the statistical significance of each lag in the data by comparing the result to that which you might get if you cross-correalted one of your two signals with another signal or set of signals composed of fourier components with random phases. To implement this function use the "unbiased" flag and set the confidence limit to 0.95 (feel free to increase or decrease this value). For example:

```
[cross up lo autocor1 autocor2 lags] = xcorrc(eq,water,'unbiased',n,0.95);
```

Here, `autocor` refers to the autocorrelation functions, which are the correlation of each time series with itself. If there is periodic behavior in the time series you should see this in the autocorrelation functions when they are plotted at large lags. Try this for yourself.

Plot the correlation function `cross` as a function of positive time lags over the interval [0 400] days, as well as the upper (and lower, if you wish) confidence limits "up" (and "lo"). Vary the number of iterations `n` per lag to be 10, 20, 50, 100, 500, and 1000 (this may take a few minutes). Examine how the confidence limit changes with the number of iterations. You should see the dependence of the result on the number of iterations become small as `n` becomes large. In fact, there should be an approximate upper limit on the number of iterations required above which all you are doing is using up cpu cycles (and your patience) unnecessarily. Examine your results carefully? Are the curves indicating the uncertainties at each lag of the same form as the correlation function? Are the errors correlated? Is this good or bad?

Once you are satisfied with your data analysis, find the first and most strongly correlated positive time lag to an uncertainty of 0.1 days. You can pick this by hand first. Write a function that finds the peak in the correlation function that you want and the corresponding time lag.

3. A reasonable approximation to the "forcing" to the groundwater system resulting from periodic seasonal recharge at the surface has the form:

$$p'(t, z = 0) = p_o \cos(2\pi t/\tau_{force}), \tag{A1}$$

where the period of the forcing τ_{force} = 1yr and the reference pressure at the surface (i.e., $z = 0$) is atmospheric pressure or about 0.1 MPa. A particular solution to equation (8) with this boundary condition is (you should verify this for yourself!):

$$p'/p_o = \exp\Big(- z\sqrt{\pi/(D\tau_{force})}\Big) \cos\Big(2\pi t/\tau_{force} - z\sqrt{\pi/(D\tau_{force})}\Big) \tag{A2}$$

You should recognize the normalization of the length z with the characteristic length identified above $\sqrt{Dt}$. Note also that the periodic part of this solution has the same form as the forcing at the surface with an additional dimensionless phase lag $\left(z\sqrt{\pi/(D\tau_{force}}\right)$. In words: the input produces a response that occurs sometime later. This occurs as a result of damping in the problem and is an intrinsic property of linear differential equations characterized by linear damping (this problem is analogous to an over-damped harmonic oscillator). Diffusion acts, of course, to spread out gradients of anything– in this case pore fluid pressure. Thus, there is an exponential decay in the pressure perturbation with distance away from the surface.

Let's do a little analysis of the solution given by equation (A2):

–Find an expression for the characteristic "skin" depth at which the pressure perturbation decrease to a factor of 1/e of the original value. What is this depth and how does it compare to the depth at which earthquakes occur?

–Assuming that earthquakes occur when the pore-fluid pressure is maximized at 4.5 km depth, from inspection of equation A2 find an expression for the hydraulic diffusivity D. Find the value for D assuming also a period of 1 year (one full seasonal cycle). Is this a high value? Do a little research and justify your response.

4. On one 2D plot, use your calculated value for D and plot this solution for 5-10 depths ranging from 0 to 5 km over a period of 0 to 1000 days (your plot should be a series of curves corresponding to depth). Plot also the time lag in days determined from your data analysis as a single (vertical) line. Describe your results qualitatively. What is the effect of increasing depth on the amplitude and phase lag? When is the pressure perturbation maximized at the surface? At 2 km, say? At 4.5 km depth (i.e., where earthquakes occur)? At 10 km depth?

5. The actual dimensional pressure $P = P_h + P'$, where $P_h = \rho g z$ is the hydrostatic pressure. Make a new 2D plot in which you show (on one plot) $P(t)$ over 1000 days at depths ranging from 0 - 5 km. The plot should have $P(t)$ plotted against t at discrete depths z that you specify. To start, plot the solution at $z = 0$ and a depth increments of 0.1-0.2 km. A practical issue is how to see the effect of the perturbation P', which will be small in comparison to P_h. To amplify the structure in the solution related to P' try plotting $P = P_h + 10P'$, say. Note again the lag in time between the response at 0 and a few km depth. Do you think that failure at a given depth could ever occur at a time following the first pore-fluid pressure maximum? From equation (10) and the discussion that follows one might expect earthquakes to occur where P' is maximized and σ'_n is minimized. Is this the case at Mount Hood (see Figure 1)?

6. Find an expression and the value for the fractional increase in fluid pore pressure required for failure at 4.5 km depth assuming that failure occurs where this is maximized. What is the dimensional increase in fluid pore-pressure? How does this value compare with a typical earthquake stress-drop of order 10 MPa?

7. Do you believe your results? That is– is the increase in pore-fluid pressure at 4.5 km really big enough to do anything? One way to assess this is to ask the question: What is the magnitude of this annual increase in stress relative to the background rate at which tectonic stresses are increased as a result of Cascadia subduction? Consider again a typical earthquake stress drop of 10 MPa. Now consider that the earthquake cycle is perhaps 300-400 years. What is the average

annual rate at which the background tectonic stress is increased? Is this value comparable with the stress perturbation related to groundwater recharge? Is Rundle's conjecture that the upper crust is in a critical state of stress appropriate at Mount Hood? What do you think?

3 Notes on cross-correlation functions, uncertainties and normalization

1. The continuous average cross-correlation coefficient can be defined as the inner product of the earthquake moment time series $M_o(t)$ and the groundwater runoff time series $Q(t-\tau)$, where τ is the time lag between the two time series:

$$\varphi_{xy}(\tau) = \frac{1}{T} \int\limits_{-\infty}^{\infty} M_o(t) Q(t-\tau) \mathrm{d}\tau, \qquad (N1)$$

where T is the temporal length of $M_o(t)$ and τ is the time lag between the signals. From the form of (N1) you can see that if $M_o(t)$ contains a sinusoidal component of the same period as $Q(t)$, then $\varphi_{xy}(\tau)$ will be a periodic function with the same period as $Q(t)$. In the alternative end-member situation that $Q(t)$ and $M_o(t)$ are orthogonal, and $\varphi_{xy}(\tau)$ will vanish. However, in real physical problems there is always noise. Consequently, $\varphi_{xy}(\tau)$ will never vanish but can be characterized with a lower limit or "noise floor". This noise floor is determined by the uncertainties calculated with `xcorrc` above.

2. Confidence limits. The calculation of the confidence limits in `xcorrc` is performed in the following way. One of the two time series is assigned random phases for each intrinsic frequency band such that its autocorrelation remains unchanged. That is, the form of equation (N1) where a correlation between a time series and itself is investigated remains unchanged so that we are not introducing a new problem through this operation. A specified number of iterations of cross correlations $\varphi_{xy}(\tau)$ are done using spectral multiplication (which is very fast):

$$\varphi_{xy}(\tau) = \mathcal{F}^{-1}\{\mathcal{F}(M_o)\mathcal{F}^*(Q)\}, \qquad (N2)$$

where $\mathcal{F}$, $\mathcal{F}^{-1}$ and $\mathcal{F}^*$ are the Fourier transform, its inverse and its complex conjugate, respectively. The range of values that contain 90% of the data for a given time lag is the 90% confidence interval for that time lag.

3. Normalization. The correlation coefficients are all normalized:

$$-1 \leq \varphi_{xy}(\tau) = \frac{\varphi_{xy}(\tau)}{\sqrt{\varphi_{xx}(\tau=0)}\sqrt{\varphi_{yy}(\tau=0)}} \leq 1, \qquad (N3)$$

where the repeated subscripts indicate the autocorrelation at 0 time lag. This normalization ensures that perfect correlation, no correlation and perfect anticorrelation occur for $\varphi = 1$, 0, and -1, respectively.