① $\lim_{x \to 1} (x^5 - 3x^3 + 1)(x^2 - 2)$

$\lim_{x \to 1} (x^5 - 3x^3 + 1) \quad \lim_{x \to 1} (x^2 - 2)$

$(1)^5 - 3(1)^3 + 1 \cdot (1)^2 - 2$

$(1 - 3 + 1) \cdot (1 - 2)$

$-1 \cdot -1$

1 //

② $\lim_{x \to 16} \frac{\sqrt{x}}{x + 16}$

$\frac{\sqrt{16}}{16 + 16}$

$\frac{4}{32}$

$\frac{1}{8}$ //

③ $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$

$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \frac{\times (\sqrt{x} + 4)}{\times (\sqrt{x} + 4)}$

$\lim_{x \to 16} \frac{\cancel{x - 16}}{\cancel{(x - 16)}(\sqrt{x} + 4)}$

$\lim_{x \to 16} \frac{1}{\sqrt{x} + 4}$

$\cancel{\lim_{x \to 16}} \frac{1}{\sqrt{16} + 4}$

$\frac{1}{8}$ //

(4) $\lim_{x \to 3} \frac{x-3}{x^2-9}$

$\lim_{x \to 3} \frac{(x-3)}{(x-3)(x+3)}$

$\lim_{x \to 3} \frac{1}{(x+3)}$

$\frac{1}{6}$ //

(5) $\lim_{x \to 0} \frac{(x+5)^2 - 25}{x}$

$\lim_{x \to 0} \frac{x^2 + 10x + \cancel{25} - \cancel{25}}{x}$

$\lim_{x \to 0} \frac{\cancel{x}(x+10)}{\cancel{x}}$

$0 + 10$

10 //

(6) $\lim_{x \to 0} \frac{(x+1)^3 - 1}{x}$

$\lim_{x \to 0} \frac{x^3 + 3x^2 + 3x + \cancel{1} - \cancel{1}}{x}$

$\lim_{x \to 0} \frac{\cancel{x}(x^2 + 3x + 3)}{\cancel{x}}$

$0 + 0 + 3$

3 //

(7) $\lim_{x \to 2} \frac{x^2 - 7x + 10}{x-2}$

$\lim_{x \to 2} \frac{(x-2)(x-5)}{(x-2)}$

$2 - 5$

-3 //

⑧ $\lim\limits_{x \to \infty} x - \sqrt{x^2 + x}$

$$\lim_{x \to \infty} \frac{x - \sqrt{x^2+x}}{1} \times \frac{x + \sqrt{x^2+x}}{x + \sqrt{x^2+x}}$$

$$\lim_{x \to \infty} \frac{\cancel{x^2} - \cancel{x^2} - x}{x + \sqrt{x^2+x}}$$

$$\lim_{x \to \infty} \frac{-\cancel{x}}{\cancel{x}\left(1 + \sqrt{1 + 1/x}\right)}$$

$$\lim_{x \to \infty} \frac{-1}{1 + \sqrt{1 + 1/x}}$$

$$\frac{-1}{1 + \sqrt{1 + 1/\infty}}$$

$$\frac{-1}{1 + 1}$$

$$\frac{-1}{2} //$$

⑨ $$\lim_{x \to \infty} \frac{\sqrt{x^4 + 3x^2 + 8} - 2}{x^2 + 11x + 3}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2\left(x^2 + 3 + 8/x^2\right)} - 2}{x^2 + 11x + 3}$$

$$\lim_{x \to \infty} \frac{x\sqrt{x^2 + 3 + 8/x^2} - 2}{x^2 + 11x + 3}$$

$$\lim_{x \to \infty} \frac{\cancel{x^2}\sqrt{1 + 3/x^2 + 8/x^4} - 2/x^2}{\cancel{x^2}\left(1 + 11/x + 3/x^2\right)}$$

$$\lim_{x \to \infty} \frac{\sqrt{1 + 3/x^2 + 8/x^4} - 2/x^2}{1 + 11/x + 3/x^2}$$

$$\frac{\sqrt{1 + 1/\infty^2 + 1/\infty^4} - 2/\infty^2}{1 + 1/\infty + 1/\infty^2}$$

$$\frac{\sqrt{1 + 0 + 0} - 0}{1 + 0 + 0}$$

$$+1$$

(10) $\lim_{x \to 0} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{x}$

$\lim_{x \to 0} \dfrac{\sqrt{1+x} - \sqrt{1-x}}{x} \times \dfrac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$

$\lim_{x \to 0} \dfrac{(1+x) - (1-x)}{x \cancel{(1+\sqrt{x})} (\sqrt{1+x} + \sqrt{1-x})}$

$\lim_{x \to 0} \dfrac{\cancel{1} + x - \cancel{1} + x}{x(\sqrt{1+x} + \sqrt{1-x})}$

$\lim_{x \to 0} \dfrac{2\cancel{x}}{\cancel{x}(\sqrt{1+x} + \sqrt{1-x})}$

$\lim_{x \to 0} \dfrac{2}{(\sqrt{1+x}) + \sqrt{1-x}}$

$\dfrac{2}{\sqrt{1+0} + \sqrt{1-0}}$

$\dfrac{2}{2}$

1 //

(11) $\lim_{x \to 4} \dfrac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}}$

$\lim_{x \to 4} \dfrac{(3 - \sqrt{5+x})}{(1 - \sqrt{5-x})} \times \dfrac{(3 + \sqrt{5+x}) \times (1 + \sqrt{5-x})}{(3 + \sqrt{5+x}) \times (1 + \sqrt{5-x})}$

$\lim_{x \to 4} \dfrac{[9 - (5+x)](1 + \sqrt{5-x})}{[1 - (5-x)](3 + \sqrt{5+x})}$

$\lim_{x \to 4} \dfrac{4 - x \quad (1 + \sqrt{5-x})}{-4 + x \quad (3 + \sqrt{5+x})}$

$\lim_{x \to 4} \dfrac{-(\cancel{x-4})(1 + \sqrt{5+x})}{(\cancel{x-4})(3 + \sqrt{5+x})}$

$\lim_{x \to 4} \dfrac{-(1 + \sqrt{5-x})}{(3 + \sqrt{5+x})}$

$\dfrac{-(1 + \sqrt{5-4})}{(3 + \sqrt{5+4})} = \dfrac{-2}{6} = \dfrac{-1}{3}$ //

(12) $\lim_{x \to 1} \dfrac{x^4 - 3x^2 + 2}{x^3 - 1}$

$\lim_{x \to 1} \dfrac{(x^2 - 1)(x^2 - 2)}{(x^2 + x + 1)(x - 1)}$

$\lim_{x \to 1} \dfrac{(\cancel{x-1})(x+1)(x^2-2)}{(x^2 + x + 1)(\cancel{x-1})}$

$\dfrac{(1+1)((1)^2 - 2)}{((1)^2 + 1 + 1)}$

$\dfrac{-2}{3}$ //

(13) $\lim_{x \to \infty} \left[1 + \log x - \frac{1}{2}\log(2x^2+1)\right]$

$\lim_{x \to \infty} \left[1 + \log x - \log(2x^2+1)^{1/2}\right]$

$\lim_{x \to \infty} \left[1 + \log\left[x - \sqrt{(2x^2+1)}\right]\right]$

$\lim_{x \to \infty} \left[1 + \log\left(\dfrac{x}{\sqrt{2x^2+1}}\right)\right]$

$\lim_{x \to \infty} \left[1 + \log\left(\dfrac{x}{x\sqrt{2 + 1/x^2}}\right)\right]$

$1 + \log \dfrac{1}{\sqrt{2+0}}$

$1 + \log\left(\dfrac{1}{\sqrt{2}}\right)$

$1 + \left(\log\left[1 - \sqrt{2}\right)\right]$

$1 + \log_{10} 1 - \dfrac{1}{2}\log(2)$

$1 + 0 - \dfrac{1}{2}\log(2) = 1 - \dfrac{1}{2}\log(2) = 0.8445$ //

0.1505

$1 - \dfrac{1 \times 0.3010}{2}$

$\begin{array}{r} 1.000 \\ 0.15 \\ \hline 0.84 \end{array}$

(14) Let $\lim_{x \to 3} f(x) = 4$ and $\lim_{x \to 3} g(x) = -2$

(a) $\lim_{x \to 3} [f(x) + g(x)]$

$$\lim_{x \to 3} f(x) + \lim_{x \to 3} g(x)$$

$$4 + (-2)$$

$$2 \,//$$

(b) $\lim_{x \to 3} [f(x) - g(x)]$

$$\lim_{x \to 3} f(x) - \lim_{x \to 3} g(x)$$

$$4 - (-2)$$

$$6 \,//$$

(c) $\lim_{x \to 3} \dfrac{2f(x) - g(x)}{f(x) \cdot g(x)}$

$$\frac{\lim_{x \to 3} \left(2f(x) - g(x)\right)}{\lim_{x \to 3} f(x) \cdot g(x)}$$

$$\frac{2 \lim_{x \to 3} f(x) - \lim_{x \to 3} g(x)}{\lim_{x \to 3} f(x) \cdot \lim g(x)}$$

$$\frac{2(4) - (-2)}{4 \cdot (-2)}$$

$$\frac{8 + 2}{-8}$$

$$-\frac{\cancel{10}\,5}{\cancel{8}\,4} = -1.25 \,//$$

(15) Prove $\lim_{x \to a} \dfrac{x^n - a^n}{(x-a)} = na^{n-1}$

$$L.H.S. = \lim_{x \to a} \frac{x^n - a^n}{(x-a)}$$

$$\cancel{(x-a)\ x^{n-1}}$$

$$\lim_{x \to a} \left[\frac{\frac{d(x^n - a^n)}{dx}}{\frac{d(x-a)}{dx}} \right]$$

$$\lim_{x \to a} \frac{nx^{n-1}}{1}$$

$$\frac{na^{n-1}}{1}$$

$$na^{n-1}$$

$$= R.H.S.$$ //

(16) Prove $\lim_{x \to 0} \left(\dfrac{\sin x}{x} \right) = 1$

$$L.H.S. = \lim_{x \to 0} \left(\frac{\sin x}{x} \right)$$

$$\sin x \approx x$$

$$\lim_{x \to 0} \frac{x}{x}$$

$$\lim_{x \to 0} 1$$

$$1$$ //

(17) Prove $\lim_{x \to 0} \sin(1/x)$ does not exist.

$\lim_{x \to 0} \sin(1/x)$ we can make a new variable h so that $h = 1/x$

As $x \to 0$, $h \to \infty$ since $1/0$ is undefined. so we can say that

$$\lim_{x \to 0} \sin(1/x) = \lim_{h \to \infty} \sin(h)$$

As h get bigger, sin (h) keep fluctuating between -1 and 1, it never tends towards anything, or stop fluctuating at any point.

(18) $\lim_{x \to \pi/6} [\sin x - 4\cos 2x + \cot 3x]$

$\lim_{x \to \pi/6} [\sin x - 4 \cos 2 \times x + \cot 3 \times x]$

$\sin \pi/6 - 4 \cos(2 \times \pi/6) + \cot(3 \times \pi/6)$

$1/2 - 4 \times 1/2 + 1/\infty$

$1/2 - 2 + 0$

$1/2 - 2 + 0$

$\frac{1-4}{2}$

$-3/2$

-1.5 //

(19) $\lim_{x \to 3\pi/2} \frac{1+\sin x}{\cos x}$

$\lim_{x \to 3\pi/2} \frac{1+\sin x}{\cos x} \times \frac{(1-\sin x)}{(1-\sin x)}$

$\lim_{x \to 3\pi/2} \frac{1-\sin^2 x}{\cos x (1-\sin x)}$

$\lim_{x \to 3\pi/2} \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}(1-\sin x)}$

$\lim_{x \to 3\pi/2} \frac{\cos x}{(1-\sin x)}$

$\frac{\cos(3\times\pi/2)}{1-\sin(3\times\pi/2)} = \frac{0}{1-(-1)} = \frac{0}{2} = 0$ //

(20) $\lim_{x \to \infty} \dfrac{2x^3 - 3x^2 + 2}{x^3 - x^2 - 100x + 1}$

$\lim_{x \to \infty} \dfrac{x^3 (2 - 3/x + 2/x^3)}{x^3 (1 - 1/x - 100/x^2 + 1/x^3)}$

$\lim_{x \to \infty} \dfrac{(2 - 3/x + 2/x^3)}{(1 - 1/x - 100/x^2 + 1/x^3)}$

$\dfrac{2 - 3/\infty + 2/\infty^3}{(1 - 1/\infty - 100/\infty^2 + 1/\infty^3)}$

$\dfrac{(2 - 0 + 0)}{(1 - 0 - 0 + 0)}$

$\dfrac{2}{1}$

$\underline{\underline{2}}$

(21) $\lim_{x \to a} \sin(x)$

$\underline{\underline{\sin(a)}}$

(22) $\lim_{x \to a} \cos(x)$

$\underline{\cos(a)}$

(23) $\lim_{x \to \pi/4} \tan(x)$

$\tan(\pi/4)$

$1 //$

(24) $\lim_{x \to a} e^x$

$\underline{\underline{e^a}}$

(25) $\lim_{x \to a} \ln x$ for $a>0$

$$\ln(a)$$

(26) $\lim_{x \to a} \sin^{-1} x$ for $-1 < a < 1$

$$\lim_{x \to a} \left(\frac{d(\sin^{-1} x)}{dx} \right)$$

$$\lim_{x \to a} \frac{1}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-a^2}}$$

(27) $\lim_{x \to a} \cos^{-1}(x)$ for $-1 < a < 1$

$$\lim_{x \to a} \left(\frac{d \cos^{-1}(x)}{dx} \right)$$

$$\lim_{x \to a} \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{-1}{\sqrt{1-a^2}}$$

(28) $\lim_{x \to a} \tan^{-1}(x)$ for $-\infty < a < \infty$

$$\lim_{x \to a} \left(\frac{d \tan^{-1}(x)}{dx} \right)$$

$$\lim_{x \to a} \frac{1}{(1+x^2)}$$

$$\frac{1}{1+a^2}$$

(29) $\lim_{x \to -3\pi/2} \cot(x)$

$\lim_{x \to -3\pi/2} \frac{1}{\tan(x)}$

$\lim_{x \to -3\pi/2} \frac{\cos(x)}{\sin(x)}$

$\frac{\cos -3 \times (\pi/2)}{\sin -3 \times (\pi/2)}$

$\frac{0}{1}$

$\underline{\underline{0}}$

(30) $\lim_{x \to (\pi)^-} \cot(x)$

$\cot(\pi^-)$

$-\infty_{//}$ $-\cot(\pi)$

$-\infty_{//}$

(31) $\lim_{x \to (\pi/2)^-} \sec(x)$

$\lim_{x \to (-\pi/2)^-} \frac{-1}{\cos(x)}$

$-\frac{1}{\cos(\pi - \pi/2)}$

$\frac{-1}{-\cos(\pi/2)}$

$\underline{\underline{\infty}}$

(32) $\lim_{x \to \pi/2} \sec(x)$

$\lim_{x \to \pi/2} \frac{1}{\cos(x)}$

$\frac{1}{\cos(\pi/2)} = -\infty_{//}$

(33) $\lim_{x \to -(3\pi/2)^+} \tan(x)$

$\lim_{x \to (-3\pi/2)} \frac{\sin x}{\cos(x)}$

$\frac{\sin - (2\pi - \pi/2)}{\cos(2\pi - \pi/2)}$

$\frac{\sin(-\pi/2)}{\cos(-\pi/2)}$

$\frac{-1}{-0}$

$-\infty$ //

(34) $\lim_{x \to -(3\pi/2)^-} \tan(x)$

$\lim_{x \to (3\pi/2)^-} \frac{\sin(x)}{\cos(x)}$

$-\frac{\sin(2\pi - \pi/2)}{\cos(2\pi - \pi/2)}$

$-\left(\frac{-1}{0}\right)$

$-(-\infty)$

$+\infty$ //

(35) $\lim_{x \to 0} \sin^{-1}\left(\frac{x+1}{2}\right)$

$\lim_{x \to 0} \frac{1}{\sin\left(\frac{x+1}{2}\right)}$

$\frac{1}{\sin(1/2)}$

$\frac{1}{\sin(1/2)} = \frac{1}{\sin(0.5)} = \frac{1}{0.0087} = 2.085$ //

(36) $\lim_{x\to 0} x\cot(x)$.

$\lim_{x\to 0} x \times \frac{\cos(x)}{\sin(x)}$

$\lim_{x\to 0} \frac{x}{\sin(x)} \cdot \lim_{x\to 0} \cos(x)$

$\lim_{x\to 0} \frac{1}{\frac{\sin(x)}{(x)}} \cdot \lim_{x\to 0} \cos(x)$

$\frac{\lim_{x\to 0} 1}{\lim_{x\to 0} \frac{\sin(x)}{(x)}} \cdot \lim_{x\to 0} \cos(x)$

$\frac{1}{1} \cdot \cos(0)$

$\frac{1}{1} \cdot 1$

1 //

(37) $\lim_{x\to 0} \frac{\sin x}{\tan x}$

$\lim_{x\to 0} \frac{\sin x}{\frac{\sin x}{\cos x}}$

$\lim_{x\to 0} \frac{\sin x \cos x}{\sin x}$

$\lim_{x\to 0} \cos(x)$

$\lim_{x\to 0} \cos(x)$

$\cos(0)$

$\underline{\underline{1}}$

(38) $\lim\limits_{x \to 0} \dfrac{\tan 2x}{5x}$

$\lim\limits_{x \to 0} \dfrac{\sin(2x)}{5(x) \cdot \cos(2x)}$

$\lim\limits_{x \to 0} \dfrac{1}{\cos(2x)} \cdot \dfrac{\sin(2x)}{5x}$

$\lim\limits_{x \to 0} \dfrac{1}{\cos(2x)} \cdot \lim\limits_{x \to 0} \dfrac{\sin(2x)}{\frac{5x}{5} \times 2} \times \dfrac{2}{5}$

$\lim\limits_{x \to 0} \dfrac{1}{\cos(2x)} \cdot \lim\limits_{x \to 0} \dfrac{\sin(2x)}{2x} \times \dfrac{2}{5}$

$\dfrac{1}{1} \cdot 1 \cdot \dfrac{2}{5}$

$\underline{\underline{\dfrac{2}{5}}}$

(39) $\lim\limits_{x \to 0} \dfrac{x e^{-2x+1}}{x^2 + x}$

$\lim\limits_{x \to 0} \dfrac{\cancel{x}(e^{-2x+1})}{\cancel{x}(x+1)}$

$\cancel{x} \lim\limits_{x \to 0} \dfrac{e^{-2x+1}}{(x+1)}$

$\dfrac{e^{-2\times 0+1}}{0+1}$

$\dfrac{e^1}{1}$

$\underline{\underline{e}}$

40. $$\lim_{x \to 0} \left(\frac{1 - e^{2x}}{1 - e^{x}} \right)$$

$$\lim_{x \to 0} \frac{(1 - e^{x})(1 + e^{x})}{(1 - e^{x})}$$

$$\lim_{x \to 0} (1 + e^{x})$$

$$1 + e^{0}$$

$$1 + 1$$

$$\underline{\underline{2}}$$