Assignment 3 - Program Structures & Algorithms Fall 2021

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Task

- Implement height-weighted quick union with path compression
- Experiments to find out how many operations required to connect all sites

Conclusion

$$Number\ of\ pairs\ (m) \approx \frac{1}{2} N \ln (N)$$

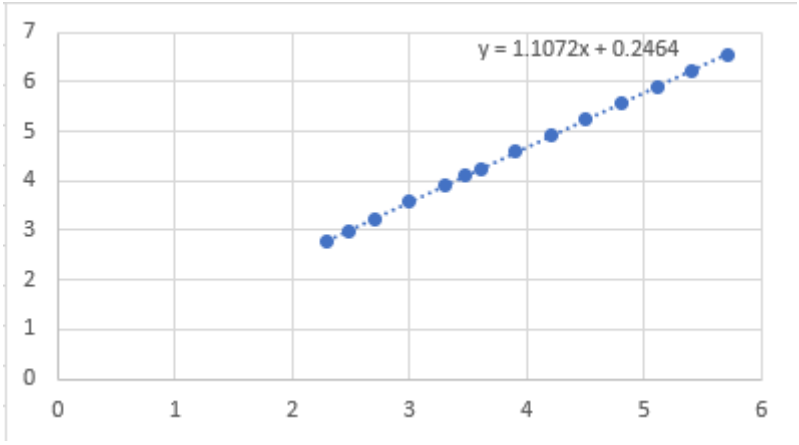
Evidence

We selected seven different `N` values: 200, 300, 500, 1000, 2000, ..., 1024000, 2048000.

And for each `N` value, we called `count()` method 1000 times and calculate the mean value.
Results as below:

number of sites (N)	number of pairs (m)	log10(N)	log10(m)	1/2*N*ln(N)
200	588.36	2.301029996	2.769643139	529.8317367
300	936.7	2.477121255	2.97160052	855.5673712
500	1692.4	2.698970004	3.228503017	1553.652025
1000	3738.9	3	3.57274385	3453.877639
2000	8181.7	3.301029996	3.912843551	7600.90246
3000	12842	3.477121255	4.108632666	12009.55135
4000	17830	3.602059991	4.251151343	16588.09928
8000	38346	3.903089987	4.583720068	35948.78728
16000	82452	4.204119983	4.916201195	77442.75201
32000	174290	4.505149978	5.24127247	165975.8589
64000	371460	4.806179974	5.569912054	354132.4276
128000	790736	5.10720997	5.898031511	752626.2747
256000	1666575	5.408239965	6.221824863	1593975.389
512000	3522392	5.709269961	6.546837686	3365396.455
1024000	7404624	6.010299957	6.86950301	7085684.267

And we plot the N-m data series to log-log graph (in Excel):

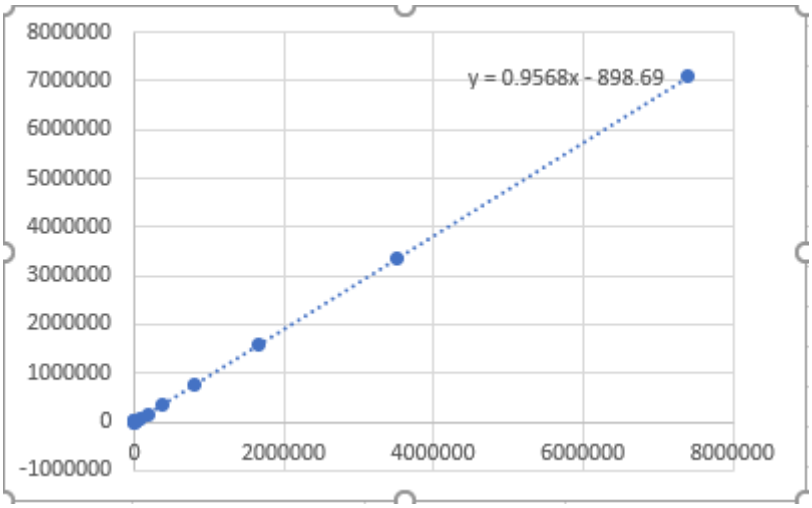


The regression line's expression has coefficient as 1.1, which seems like the original relation between `N` and `m` is not linear. (and as we have quite a lot experiment data with large `N` values, this doesn't look like an allowable error)

So the guess is to have one more term combined with linear term, like `N*(another term)`, and this term's growth rate needs to be something closer to `N^0.1`. Here I chose natural log as the extra term. (Why not binary? well the graph doesn't show any property that "something can be split into two things", and as an edge can connect to any other edge, it might be more appropriate to use natural log)

I calculated `N*ln(N)`, which seems quite large. So I divided it by two, and the results are shown in previous data table.

We can plot it against `m`:



Looks pretty close to a 1:1 relation. In fact, if we get rid of those data with small `N` values, the coefficient will get closer to 1.

We can use this `1/2*N*ln(N)` as an approximation of the relation between `N` and `m`. It suggests that, on average, every site in the graph will need to connect to at least `ln(N)` other sites so that the graph will only have one connected component.

Code

- `UF_HWQUPC` - implement union-find structure
- `UF_Experiment` - UF client file to run experiments. The main function defines a list of N values to run experiments. It might take long time to run it.

Unit tests