

Series solution of Differential equations and Special Functions.

POWER SERIES METHOP

Consider à homogenous linear second order

D. E. mith nariable coefficients

Po(x)d2y + P,(x) dy + P,(x)y = 0 - (1)

dx2 dx

dry + P1(x) dy + P2(x) y=0
dry Po(x) dx Po(x)

Let $f_1(x) = p(x)$ and $p_2(x) = q(x)$ $p_0(x)$ $p_0(x)$

then, d2y + P(v)dy + 960)y -0 -2

EQ is called normal form or canonical form of eq. (1)

The power series solution of eq 2 about a point x = x0 depends on the following definitions.

Ordinary point / Regular point:

A point x=xo is called an ordinary point
of eq2 if P(x) and P(x) are both
analytic Cie, differentiable at x=xo

If Po(n) \$0 at n= no, then no is



an ordinary point

Singular point

point n= no is called a singular point of eq. 2) if either P(x) or Q(x) or ueth one not analytic at xo.

If Po(x)=0 at x=20, then xo is a singular point

Regular singular point.

singular point is called a regular singular point of 200 if (x-x0) P(x) and $(x-x0)^2$ g(x) both are analytic (or differentiable) at x=x0

 $\lim_{x \to \infty} (x-x_0) P(x) = \text{ finite value}$

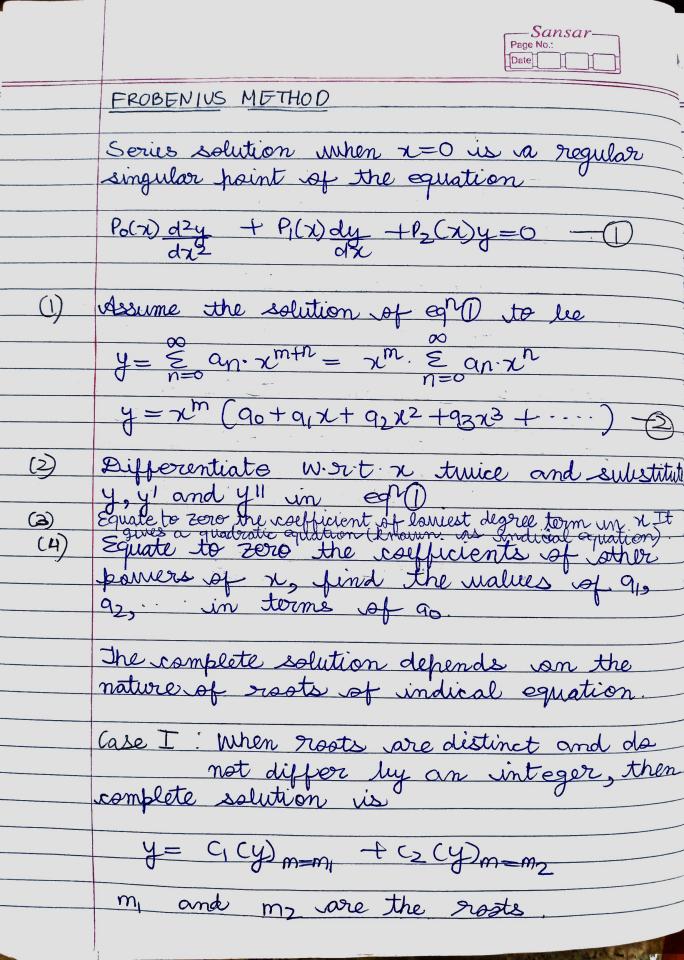
 $\lim_{x\to\infty} (x-xo)^2 g(x) = \text{finite value}$

Doregular Singular Point

or both one not analytic at x=x0



Series solution about on Ordinary Point Let the power series solution of an PO(x) y" + P((x))y + P2(x) y = 0 about an ordinary point no be given as $y = \frac{8}{5} a_n (n-n_0)^n = a_0 + a_1 (n-n_0) + a_2 (n-n_0)^2 + a_1 (n-n_0) + a_2 (n-n_0)^2$ The coefficients a, a2, a3, ... are obtained as follows: 1) Let $y = \sum_{n=0}^{\infty} a_n(x-x_0)^n$ be series solution
of given equation.
2) Differentiate y' wiret a twice to get y, y', y'' then substitute y, y', y'' in ear 03) Shift the summation index to obtain a common power of x in each term. Equate the coefficients of narious poneers of x to zero to obtain 9,92, ... in terms of 90. 5) Substitute 91, 92, ..., in aprox to obtain the raph solution of given equation.





(ase II: When roots are repeated, then the complete solution is

$$y = q(y)_{m=m_1}$$
 to $(\frac{\partial y}{\partial m})_{m=m_1}$

my is the root.

case III: When roots are distinct and differ by an integer, making a coefficient of y infinite.

Let my b m2 be the roots (such that m1<m2). If some of co-efficients of y- series become infinite when m=my, we modify the form of y by replacing as by bo (m-mi). Then the complete solution is

$$y = q(y)_{m=m_2} + c_2 (y)_{m=m_1}$$