Computability

* Given a Turing Machine T, does T eventually enter every one of its non-halting states if it begins with blank tape
  + UNDECIDABLE
  + We reduce the problem of a Turing machine accepting the empty string which is known to be undecidable
  + We construct T’ which has all the states of T plus the state of q,
  + T’ has all the symbols of T plus additional one $.
  + Any move that causes T to halt, T’ instead moves to the start of T and places $ on the tape
  + $ causes T’ to cycle through its non halting states ending with q
  + If T accepts the empty string, then some move causes T to halt, T’ enters all its non-halting states
  + If T does not accept the empty string, then since T never executes a halting move, T’ will never enter state q and does not enter its non-halting states
* Given Turing Machine T, does it accept the empty string in an even number of moves
  + UNDECIDABLE
  + Reduce the halting problem to this machine.
  + Given a TM T, we construct T’ which copies T but also keeps track of whether it has made an even number or odd number of moves
  + Also different between T and T’ is that if T halts after an odd number of moves, then T makes an additional move before halting.
  + T’ accepts the same strings as T, but halts after an even number of moves
  + T halts iff T’ halts after an even number of moves
* Given Turing Machine T and string w, does T loop forever on input w?
  + UNDECIDABLE
  + We reduce the problem of whether a Turing Machine halts over a given string
  + We construct T’ that accepts the same language but never crashes
  + For any input string w, T fails to accept w iff T’ loops forever on w.
  + The problem of halting over a given string is therefore reducible to an undecidable one
* Given a TM T, does T halt within 10 moves on every string
  + DECIDABLE
  + Within 10 moves, a Turing Machine cannot move its head any further right than square 10
  + We can look at the first 10 moves for every possible string of length 10 or less
  + After this examination, we know whether T halts within 10 moves