Computability

Countability - If a set has the same cardinality as a subest of the natural numbers N then the set is countable

If |A| = |N| then set A is countably infinite or denumerable

If a set is not countable then it is uncountable

Limits of Turing Machines

Uncomputable Problems

Any program can be regarded as function which takes input,does some manipulation and produces output

Universal Turing machines

Input is a specification for Turing machine M and string W

Output is the output from M on W

U((M,w)) - M(w)

Halting Problem

Given Turing Machine M and string W will M halt when run on W

If there is a turing machine T that solves such a problem then this is defined as follows:

H((m,w)) = Y if M halts on input w

N, otherwise

Theorm No turing Machine can solve the halting problem

Proof Contradiction

We define a Turing Machine C which takes the encoding of another Turing Machine M as its input and behaves as follows

Runs H on input (M,M)

Loops forever if M halts on input M

Halts otherwise

Reducibility

In order to prove that other problems are undecidable, rather than perform more diagonalisation proofs, we show that these can be reduced to another problem which is known to be undecidable

Proof

Suppose M’ decided the language {L(M): accepts E}

Given M and string W we create a new machine Mw that operates as follows on empty input

Write w on the tape

Simulate execution of M

Now apply M’ to Mw to solve the original halting problem

Thus M’ must not exist