HW #6

Problem I

$$\vec{a} = \sum_{i=1}^{k} c_i \vec{v}_i$$

Is $\vec{w}$ a linear combination of $Aug(\vec{x}^{(1)}), \ldots, Aug(\vec{x}^{(n)})$

initial $\vec{w} = \vec{0}$

Prediction rule: $H(\vec{x}^{(i)}) = Aug(\vec{x}) \cdot \vec{w}$

$$\vec{w}^{(t)} = \vec{w}^{(t-1)} - \alpha \begin{cases} Aug(\vec{x}^{(i)}), & \vec{w}^{(t-1)} \cdot Aug(\vec{x}^{(i)}) \geq 0 \\ -Aug(\vec{x}^{(i)}), & \vec{w}^{(t-1)} \cdot Aug(\vec{x}^{(i)}) < 0 \end{cases}$$

Answer: $\vec{w}$ is not a linear combination of $Aug(\vec{x}^{(1)}), \ldots, Aug(\vec{x}^{(n)})$. Even though $\vec{w}$ can be expressed as a linear combination of $Aug(\vec{x}^{(i)})$, as seen in the update rule above, $\vec{w}$ is only updated when a point is misclassified. This implies $\vec{w}$ is only a linear combination of the $Aug(\vec{x}^{(i)})$ when $\vec{x}^{(i)}$ is misclassified, not for all $\vec{x}^{(i)}$.

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Problem 2

$$\vec{w}^{(t)} = \vec{w}^{(t-1)} - \alpha \begin{cases} \text{Aug}(\vec{x}^{(i)}), & \vec{w}^{(t-1)} \cdot \text{Aug}(\vec{x}^{(i)}) \geq 0 \\ -\text{Aug}(\vec{x}^{(i)}), & \vec{w}^{(t-1)} \cdot \text{Aug}(\vec{x}^{(i)}) < 0 \end{cases}$$

$n = a + b$ updates

a on data points with label $+1$

b on data points with label -1

Since we know the correct labels on the misclassified data, then we know the sign of α for each update so:

a times α will be positive since previously the point was classified negative the update rule will change the sign to positive.

$\vec{w}^{(1)} = \vec{0}$ and $w_0 = 0$, w_0 will add 1 each time a data point with label $+1$ is misclassified so,

$$w_0 = 0 + 1(a)$$

The same is true for data points with label -1, but now α will be changed from positive to negative since it was misclassified before. This occurs b times so:

$$w_0 = 0 - 1(b)$$

combining these two scenarios we get:

$$\boxed{w_0 = a - b}$$

☆ We only add 1 since $\vec{w} \cdot \text{Aug}(\vec{x}^{(i)})$ just adds 1 to w_0 in $\vec{w}$.

Problem 3

$\vec{w} = (-12, 3, 4)^T$

a) Decision boundary is denoted by: $Aug(\vec{x}) \cdot \vec{w}$

So expanding that out we get:

$$w_0 + w_1 x_1 + w_2 x_2$$

Solve for x_2 as the vertical axis:

$$\Rightarrow w_2 x_2 = -w_0 - w_1 x_1$$

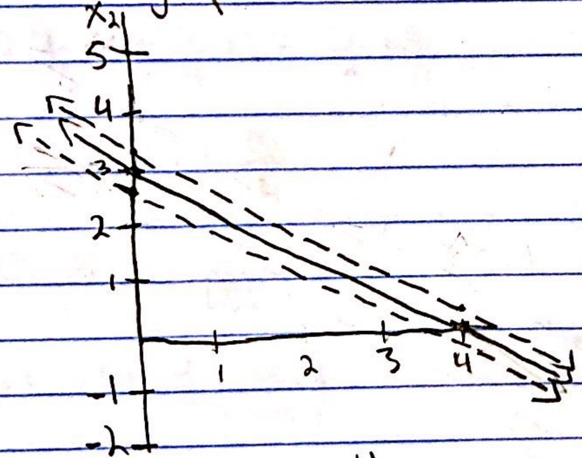
$$\Rightarrow x_2 = \frac{-w_0}{w_2} - \frac{w_1 x_1}{w_2}$$

Plugging $\vec{w} = (-12, 3, 4)^T$ into equation:

$$x_2 = \frac{12}{4} - \frac{3}{4} x_1$$

$$x_2 = 3 - \frac{3}{4} x_1$$

On a graph that looks like:



b) We can solve the same equation $Aug(\vec{x}) \cdot \vec{w}$ now with 1 and -1 on the other side:

$$w_0 + w_1 x_1 + w_2 x_2 = c$$

$$\Rightarrow w_2 x_2 = -w_0 - w_1 x_1 + c$$

$$\Rightarrow x_2 = \frac{-w_0}{w_2} - \frac{w_1 x_1}{w_2} + \frac{c}{w_2}$$

where $c = 1 \Rightarrow x_2 = \frac{12}{4} - 3/4\, x_1 + 1/4 = 13/4 - 3/4\, x_1$

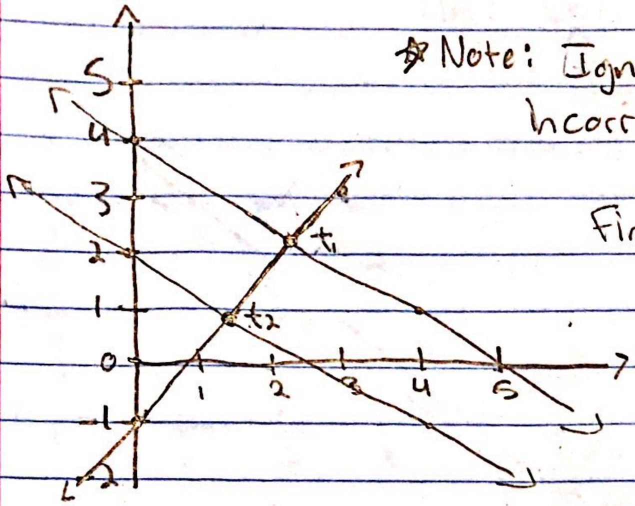
where $c = -1 \Rightarrow x_2 = 11/4 - 3/4\, x_1$

* Lines drawn on graph above as ----

c) $X_2 = \frac{13}{4} - 3/4 X_1$

$X_2 = 11/4 - 3/4 X_1$

Choose arbitrary perpendicular line: $X_2 = -1 + \frac{4}{3} X_1$



* Note: Ignore graph it was drawn incorrectly.

Finding intersection points t_1, t_2

t_1:

$$-1 + \frac{4}{3} X_1 = \frac{13}{4} - \frac{3}{4} X_1$$

$$\Rightarrow \frac{4}{3} X_1 + \frac{3}{4} X_1 = \frac{13}{4} + \frac{4}{4}$$

$$\Rightarrow \frac{16}{12} X_1 + \frac{9}{12} X_1 = 17/4$$

$$\Rightarrow \frac{25}{12} X_1 = \frac{17}{4}$$

$$\Rightarrow X_1 = \frac{204}{100}$$

$$X_2 = -1 + \frac{4}{3}\left(\frac{204}{100}\right)$$

$$= \frac{-300}{300} + \frac{816}{300} = \frac{516}{300}$$

$$t_1 = \left(\frac{204}{100}, \frac{172}{100}\right)$$

t_2: $-1 + \frac{4}{3} X_1 = \frac{11}{4} - \frac{3}{4} X_1$

$$\Rightarrow \frac{4}{3} X_1 + \frac{3}{4} X_1 = \frac{11}{4} + \frac{4}{4}$$

$$\Rightarrow \frac{25}{12} X_1 = \frac{15}{4}$$

$$\Rightarrow X_1 = \frac{180}{100}$$

$$X_2 = -1 + \frac{4}{3}\left(\frac{180}{100}\right)$$

$$= \frac{-300}{300} + \frac{720}{300} = \frac{420}{300} = \frac{140}{100}$$

$$t_2 = \left(\frac{180}{100}, \frac{140}{100}\right)$$

Now find distance between t_1 and t_2

$$d = \sqrt{\left(\frac{140}{100} - \frac{172}{100}\right)^2 + \left(\frac{180}{100} - \frac{204}{100}\right)^2} = \sqrt{\left(-\frac{32}{100}\right)^2 + \left(\frac{-24}{100}\right)^2}$$

$$= \boxed{\frac{2}{5}}$$

Answer: $\frac{2}{5}$ is the minimum distance between the two margin lines.

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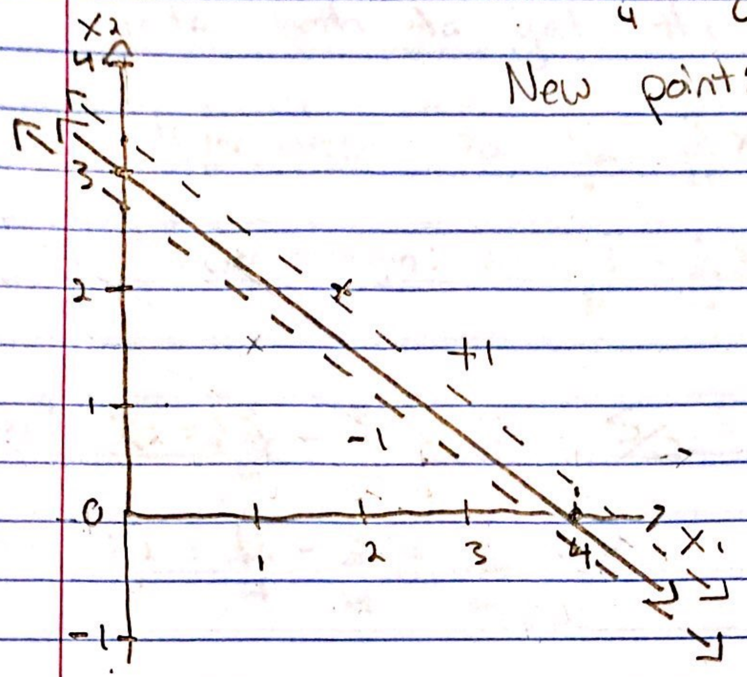
d) decision boundary: $X_2 = 3 - \frac{3}{4}x_1$

margin lines: $X_2 = \frac{13}{4} - \frac{3}{4}x_1$ for +1

$X_2 = \frac{11}{4} - \frac{3}{4}x_1$ for −1

New point: (2, 2)



★ Check if (2,2) is on the upper margin line:

$$2 = \frac{13}{4} - \frac{3}{4} \cdot 2 \Rightarrow 2 = \frac{13}{4} - \frac{6}{4} \Rightarrow 2 \neq \frac{9}{4}$$

(2,2) does not fall on margin line. Is it above or below?

new point: $X_2 = 2$ for upper margin: $X_2 = \frac{13}{4} - \frac{6}{4} = \frac{7}{4}$

Since $2 > \frac{7}{4}$ then the point falls above upper margin.

Answer: The new point lies above the upper margin so we classify it as +1.

e)

We know that there must be a support vector on both margin lines so we simply plug $x_1 = 1$ into both to get the two vectors.

Upper margin: $x_2 = \frac{13}{4} - \frac{3}{4}x_1$

Lower margin: $x_2 = \frac{11}{4} - \frac{3}{4}x_1$

Upper: $x_2 = \frac{13}{4} - \frac{3}{4} = \frac{10}{4} = 5/2 = 2.5$

Lower: $x_2 = \frac{11}{4} - \frac{3}{4} = \frac{8}{4} = 2$

Answer: So the two support vectors are: $(1, 2.5)$ and $(1, 2)$

Problem 4

words: that, is, a, big, bug, no, seriously, really, everybody, run

dictionary = [a, big, bug, everybody, is, no, really, run, seriously, that]

s1 = that is a big bug

$\vec{V}_1 = (1, 1, 1, 0, 1, 0, 0, 0, 0, 1)^T$

s2 = no seriously that is a really big bug

$\vec{V}_2 = (1, 1, 1, 0, 1, 1, 1, 0, 1, 1)^T$

s3 = everybody run

$\vec{V}_3 = (0, 0, 0, 1, 0, 0, 0, 1, 0, 0)^T$

Answers