MAT157: Analysis I — Tutorial 19

Topics: Convergence Tests.

Question 1. Determine the convergence of each of the following series

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \log(n)^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \prod_{k=1}^{n} \frac{2k-1}{3k-1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}+n^5+157}{n^2+e^n}$

Question 2. Let (a_n) be a sequence of non-zero terms for which $\left|\frac{a_{n+2}}{a_n}\right| \to \frac{1}{157}$. Can we conclude that $\sum_n |a_n|$ converges? Provide a proof or a counterexample. How, if at all, does the result change if we assume $\left|\frac{a_{n+p}}{a_n}\right| \to \frac{1}{157}$ for an arbitrary $p \in \mathbb{N}$?

Question 3. Prove that the series $\displaystyle\sum_{n=4}^{\infty} \frac{1}{\log(\log(n))^{\log(n)}}$ converges. *Hint:* Use the integral test a couple of times. Note that $\frac{e^x}{\log(x)^x}$ is decreasing for $x \geq 4$.

Question 4. Let (a_n) be a sequence such that $\sum_n a_n$ converges absolutely. If $q(x) \in \mathbb{R}[x]$ is a polynomial such that $q(0) = 0$, prove that $\sum_n q(a_n)$ converges absolutely.