MAT157: Analysis I — Tutorial 22

Topics: Power and Taylor series

Question 1. Let (a_n) be a sequence in $\mathbb{R}$ for which all but finitely many of the terms are nonzero and define

$$R_1 = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| \quad \text{and} \quad R_2 = \lim_{n\to\infty} |a_n|^{1/n}$$

(a) If $R_1 \in [0, \infty]$, prove that $\sum_n a_n(x-a)^n$ has radius of convergence R_1.

(b) If $R_2 \in [0, \infty]$, determine the radius of convergence of $\sum_n a_n(x-a)^n$ as a function of R_2.

Question 2. Determine the radius and interval of convergence for each of the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{n^2+2}(x+1)^n$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{7}{3\sqrt{n+1}-2}(x-1)^n$.

Question 3. Find a Taylor series representation for each of the following functions. Be sure to specify the radius of convergence.

(a) $\displaystyle\frac{2x^2}{1+x^3}$.

(b) $\displaystyle\frac{x^p}{(1-x)^{p+1}}$ for $p \in \mathbb{N}$.

(c) $\log(157 - x)$.

Question 4. Let $f : \mathbb{R} \to \mathbb{R}$ be a smooth function. If f is odd, determine an expression for the Taylor series expansion of f centered at $x = 0$. What if f is even?