例2.

求序列极限

$$\lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{\sum_{i=1}^{n} n^i}$$

Solution.

对原式变形有

$$\begin{aligned}
\lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{\sum_{i=1}^{n} n^i} &= \lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{\dfrac{n^{n+1}-1}{n-1}} \\
&= \lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{n^n}\cdot\lim_{n\to\infty}\frac{n^{n+1}-n^n}{n^{n+1}-1} \\
&= \lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{n^n}\cdot\lim_{n\to\infty}\frac{1-\dfrac{1}{n}}{1-\dfrac{1}{n^{n+1}}} \\
&= \lim_{n\to\infty}\frac{\sum_{i=1}^{n} i^n}{n^n}
\end{aligned}$$