Linear Algebra Done Right 6A

1. 证明:如果$v_1, \cdots, v_m \in V$,那么

$$\sum_{j=1}^{m}\sum_{k=1}^{m}\langle v_j, v_k\rangle \geqslant 0$$

Proof.

我们有

$$\begin{aligned}||v_1 + \cdots + v_m||^2 &= \sum_{j=1}^{m}||v_j||^2 + 2\sum_{j,k\in\{1,\cdots,m\},j\neq k}\langle v_j, v_k\rangle \\ &= \sum_{j=1}^{m}\sum_{k=1}^{m}\langle v_j, v_k\rangle\end{aligned}$$

于是

$$\sum_{j=1}^{m}\sum_{k=1}^{m}\langle v_j, v_k\rangle = ||v_1 + \cdots + v_m||^2 \geqslant 0$$