

1.1 (1) 已阅读

(2) 对于 $s = f(t) = at + b$ 的模型而言 $a = \frac{\overline{st} - \bar{s}\bar{t}}{\bar{t}^2 - \bar{t}^2}$ $b = \frac{\bar{t}^2 \bar{s} - \bar{t} \bar{st}}{\bar{t}^2 - \bar{t}^2}$

由 $\langle (s - \langle s \rangle)(t - \langle t \rangle) \rangle = r\sigma^2$ 可得 $\langle st - \langle s \rangle t - t \langle s \rangle + \langle s \rangle \langle t \rangle \rangle = r\sigma^2$

而 $\langle s \rangle = \langle t \rangle = \mu$ 于是 $\langle st \rangle - \mu \langle s + t \rangle + \mu^2 = r\sigma^2$ 即 $\bar{st} = \langle st \rangle = r\sigma^2 + \mu^2$

由 $\langle (t - \langle t \rangle)^2 \rangle = \sigma^2$ 可得 $\langle t^2 \rangle - 2\langle t \rangle^2 + \langle t \rangle^2 = \sigma^2$ 即 $\bar{t}^2 = \sigma^2 + \mu^2$

于是 $a = \frac{r\sigma^2 + \mu^2 - \mu^2}{\sigma^2 + \mu^2 - \mu^2} = r$ $b = \frac{(\sigma^2 + \mu^2)\mu - \mu(r\sigma^2 + \mu^2)}{\sigma^2 + \mu^2 - \mu^2} = \frac{\mu(1-r)\sigma^2}{\sigma^2} = \mu(1-r)$

于是回归结果为 $s = f(t) = rt + \mu(1-r)$

平均而言一样高 父亲更高

(3) 考虑 s 与 t 的对称性, 不难有 $t = g(s) = rs + \mu(1-r)$

平均而言一样高 儿子更高

1.2 设 A_n 为连续抛 n 次正面朝上, B 为取出秋青钱.

$P(A_3) = P(A_3|B)P(B) + P(A_3|\bar{B})P(\bar{B}) = 1 \cdot \frac{1}{8} + \frac{1}{8} \cdot \frac{1}{2} = \frac{9}{16}$

于是 $P(B|A_3) = \frac{P(A_3|B)P(B)}{P(A_3)} = \frac{\frac{1}{8}}{\frac{9}{16}} = \frac{8}{9}$

同理 $P(B|A_4) = \frac{\frac{1}{16}}{\frac{17}{32}} = \frac{16}{17}$

第四次反面朝上, 是秋青钱的概率为 0.