P429-430.

5(a) $\|\vec{x}-\tilde{\vec{x}}\|_\infty = \max\{8.571\times10^{-4}, 6.667\times10^{-4}\} = 8.571\times10^{-4}$.

$\|A\tilde{\vec{x}}-\vec{b}\|_\infty = \max\{2.063\times10^{-4}, 1.190\times10^{-4}\} = 2.063\times10^{-4}$.

7. $A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ $B=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$ $\|A\|_\infty = 1$ $\|B\|_\infty = 1$ $\|AB\|_\infty = 2 > \|A\|_\infty \cdot \|B\|_\infty$.

13. $\|A\| = \max_{\|x\|=1}\|Ax\| \le \|A\|\cdot\|x\|$ ① $\max_{\|x\|=1}\|A\vec{x}\| \ge 0$. ② $\max_{\|x\|=1}\|\alpha A\vec{x}\| = |\alpha|\,\|A\|$.

∴ The matrix norm have this form. ③ $\|A+B\| = \max_{\|x\|=1}\|(A+B)\vec{x}\| = \max_{\|x\|=1}\|A\vec{x}\| + \max_{\|x\|=1}\|B\vec{x}\| = \|A\|+\|B\|$ ④ $\|AB\vec{x}\| \le \|A\|\cdot\|B\vec{x}\| = \|A\|\cdot\|B\|$.

∴ $\|AB\| = \max_{\|x\|=1}\|AB\vec{x}\| \le \|A\|+\|B\|$

$(\|A\vec{x}\| \le \|\vec{x}\|\max\|A\vec{x}\| = \|A\|\|\vec{x}\|)$

P436

3. (a) $\begin{vmatrix}2-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} = (2-\lambda)^2 - 1 = 0$ $\lambda = 1$或3 $\rho(A) = 3 > 1$.

(b) $-\lambda(1-\lambda)-1=0$. $\lambda = \frac{1\pm\sqrt5}{2}$ $\rho(A) = \frac{1+\sqrt5}{2} > 1$

(c) $\lambda^2 - \frac14 = 0$ $\lambda = \pm\frac12$ $\rho(A) = \frac12 < 1$

(d) $\begin{vmatrix}1-\lambda & 1\\ -2 & -2-\lambda\end{vmatrix} = (\lambda-1)(2+\lambda)+2=0$ ∴ $\lambda = 0$或-1 $\rho(A) = 1 = 1$

(e) $\begin{vmatrix}2-\lambda & 1 & 0\\ 1 & 2-\lambda & 0\\ 0 & 0 & 3-\lambda\end{vmatrix} = (3-\lambda)((2-\lambda)^2-1) = 0$ $\lambda = 3$或3或1. $\rho(A) = 3 > 1$.

(f) $(-1-\lambda)(3-\lambda)(7-\lambda) = 0$ $\lambda_1 = -1$ $\lambda_2 = 3$ $\lambda_3 = 7$ $\rho(A) = 7 > 1$

(g) $\lambda_1 = \lambda_2 = 1$ $\lambda_3 = 5$ $\rho(A) = 5 > 1$.

(h) $\begin{vmatrix}3-\lambda & 2 & -1\\ 1 & -2-\lambda & 3\\ 2 & 0 & 4-\lambda\end{vmatrix} = (\lambda-3)(\lambda-4)(-2-\lambda) = 0$ ∴ $\lambda_1 = 3$ $\lambda_2 = 4$ $\lambda_3 = -2$

$\rho(A) = 4 > 1$

$\therefore$ (C) is convergent.

P453.

13. $T_w = (D-wL)^{-1}\{(1-w)D + wU\}$

$\|T_w\| = |D-wL|^{-1} \cdot |(1-w)D + wU|$

$= |D|^{-1} \cdot (1-w)^n |D|$

$= (1-w)^n.$

$= |\prod_{i=1}^{n} \lambda_i| \leq |\prod_{i=1}^{n} \max_{1 \leq k \leq n} \{\lambda_k\}|$

$\therefore |1-w| \leq |\max\{\lambda\}| = \rho(T_w)$

$\therefore \rho(T_w) \geq |w-1|.$