a. $\left|\frac{\sinh}{h}-1\right| = \left|\frac{h-\frac{h^3}{6}-h+O(h^5)}{h}\right| = \left|-\frac{h^2}{6}+O(h^4)\right| \le O(h^2)$

b. $\left|\frac{1-\cosh}{h}-0\right| \le \left|\frac{1-(1-\frac{h^2}{2})}{h}\right| = \frac{1}{2}h \le O(h)$.

c. $\left|\frac{\sinh-h\cosh}{h}-0\right| \le \left|\frac{h-h(1-\frac{h^2}{2})}{h}\right| = \frac{1}{2}h^3 \le O(h^2)$

d. $\left|\frac{1-e^h}{h}-1\right| = \left|\frac{1-(1h+\frac{h^2}{2}+O(h^3))-h}{h}\right| = \frac{1}{2}h+O(h^2) \le O(h)$.

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13. $x^3-x-1=0$ The root is 1.324717796.

with $[1,2]$, $n=14$, $P_{14}=1.32477$ is 10^{-4} to the solution.

15. $\forall N>0$, $\exists n>N$

$\exists p>0$ $|a_{n+p}-a_n| = \left|\frac{1}{n+1}+\cdots+\frac{1}{n+p}\right|$.

$\cancel{若}$ $p=n$, $|a_{n+p}-a_n| = \frac{n}{2n} = \frac{1}{2}$ $\therefore \{P_n\}$ doesn't converge

P64.

3. (a). $g(x)=\frac{20x+\frac{21}{x^2}}{21}$ $g(1)=1.95$ $g(2)=2.15$ $g(3)\approx 2.97$

$\therefore 2<x<3$ $g'(x) = \frac{20}{21}-\frac{2}{x^3} < 1$ It's descending.

(b) $1<x<20$ $g'(x) = \frac{2}{3}-\frac{14}{x^3} < 1$ $\therefore$ It's converge

(c) $g(1)=0$ $\therefore g(x)$ doesn't converge.

(d) $1<x<5$ $|g'(x)| = \left|-\frac{\sqrt{21}}{5}\cdot\frac{1}{x^{\frac{3}{2}}}\right| < 1$. $\therefore$ It's converge.

$\therefore$ (a)(b)(d) is OK, and the speed: $b>d>a$

19. (a) $g(x)=\frac{1}{2}x+\frac{1}{x}$.

$x=f(x) = x-\frac{1}{2}x-\frac{1}{x} = \frac{1}{2}x-\frac{1}{x}$.

$f(\sqrt{2})=0$ $\because f'(x)=\frac{1}{2}+\frac{1}{x^2}>0$ $\therefore x>g(x)$.

$b>x_0>\sqrt{2}=a$ $\therefore a\le g(a)<g(b)\le b$.

$g'(x)\le\frac{1}{2}$ $\therefore \exists p\in[a,b]$, $p=g(p)$

(b) $g'(x)=\frac{1}{2}-\frac{1}{x^2}$ 若 $0<x<\sqrt{2}$, $g'(x)<0$.

$\therefore g(x)\ge\sqrt{2}$. $\therefore x_1=g(x)\ge\sqrt{2}$

(c) If $x_0\in(0,\sqrt{2})$. $\because x_1>\sqrt{2}$, if $x_0>\sqrt{2}$, x_n converge to $\sqrt{2}$ (a)(b).

$\therefore$ it converge to $\sqrt{2}$.

If $x_0>\sqrt{2}$, then X_n coverge to $\sqrt{2}$ (a).