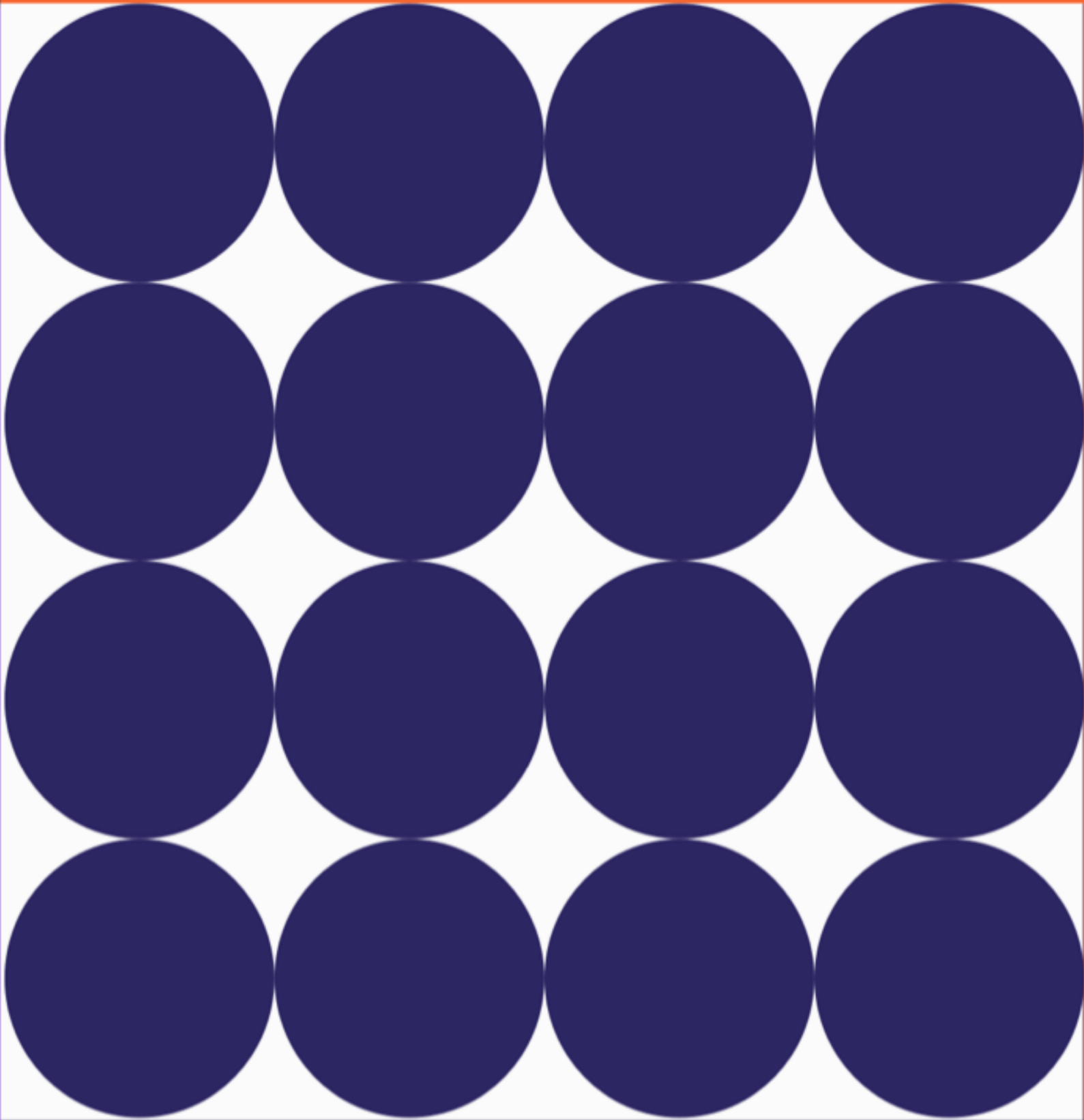


NO. _____

Title:

ADS

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Creative notes

By reading we enrich the mind;
by writing we polish it.

by Ihmd

§7 Divide and Conquer

1. Closest Points

$$T(N) = a T(N/b) + f(N)$$

↓
divide

↓
conquer

↓
combine

距离最短问题：寻找一堆点中的最短距离

正常解法： $\frac{N(N-1)}{2} \Rightarrow T = O(N^2)$

分治法：

- ① 按X轴坐标排序，从中间划分
- ② 找到左部分和右部分两点间最小距离 δ
- ③ 从中间划分处向左右走 δ 距离，此区间内的所有点做讨论
- ④ 对区间内每个点，找Y轴方向 δ 距离内的点，做计算距离。
- ⑤ 计算三个最小距离的最小值。

\Rightarrow 分治法 $\left\{ \begin{array}{l} \text{Substitution method} \\ \text{Recursion-tree method} \\ \text{Master method} \end{array} \right.$

2. Substitution method - 归纳法.

Example: $T(N) = 2T(\lfloor N/2 \rfloor) + N$

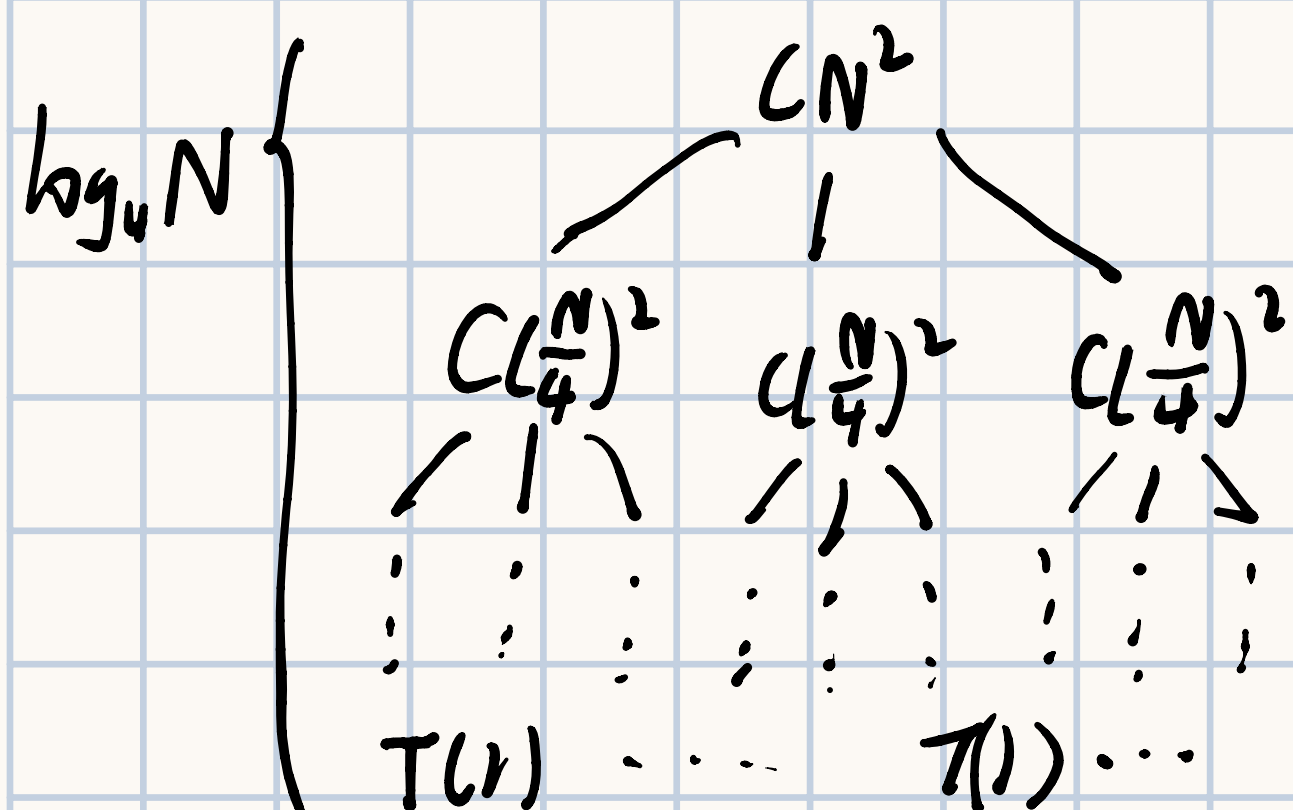
Guess: $T(N) = O(N \log N)$

假设 $n < N$ 时都成立，则 $T(N) = 2T(\lfloor N/2 \rfloor) + N \leq 2c(\lfloor N/2 \rfloor) \log \lfloor N/2 \rfloor + N$
 $\leq cN \log N - (c \geq 1)$

但是此方法若 guess 不正确会有错误答案。

3. Recursion-tree method

$$T(N) = 3T(N/4) + \Theta(N^2)$$



$$T(N) = \sum_{i=0}^{\log_4 N-1} \left(\frac{3}{4}\right)^i CN^2 + \Theta(N^{\log_4 3})$$

4 Master method

(1) Form 1

考虑 $T(n) = aT(\frac{n}{b}) + f(n)$, $T(1) = O(1)$

$$a \cdot f(\frac{n}{b}) = f(n) \Rightarrow T(n) = \Theta(f(n) \cdot \log n)$$

$$a \cdot f(\frac{n}{b}) < f(n) \Rightarrow T(n) = \Theta(f(n))$$

$$a \cdot f(\frac{n}{b}) > f(n) \Rightarrow T(n) = \Theta(n^{\log_b a})$$

(2) Form 2

考虑 $T(n) = aT(\frac{n}{b}) + f(n)$, $T(1) = O(1)$, $\varepsilon > 0$

$$f(n) = O(n^{\log_b a - \varepsilon}) \Rightarrow T(n) = \Theta(f(n) \cdot \log n)$$

$$f(n) > n^{\log_b a + \varepsilon} \Rightarrow T(n) = \Theta(f(n))$$

$$f(n) < n^{\log_b a - \varepsilon} \Rightarrow T(n) = \Theta(n^{\log_b a})$$

(3) Form 3

考虑 $T(n) = aT(\frac{n}{b}) + n^d$, $T(1) = O(1)$

$$a = b^d \Rightarrow T(n) = \Theta(f(n) \cdot \log n)$$

$$a < b^d \Rightarrow T(n) = \Theta(f(n))$$

$$a > b^d \Rightarrow T(n) = \Theta(n^{\log_b a})$$

