

- 解:

(1) 第一次:  $a + \beta$

第二次:  $\gamma^2(a + \beta)$

...

$$\therefore P_0 = \sum_{i=1}^{\infty} P_i = \sum_{i=1}^{\infty} \gamma^{2i}(a + \beta) = \frac{1 - \gamma^{2n}}{1 - \gamma^2} (a + \beta) = \frac{a + \beta}{1 - \gamma^2}$$

$$(2) P(A) = a + \beta \cdot a + \gamma \cdot b$$

$$(3) a = \gamma + \beta \cdot P_0$$

$$b = 1 - P_0$$

$$P(\text{新}) = a + \beta a + \gamma b$$

$$= a + \beta \left( \gamma + \beta \cdot \frac{a + \beta}{1 - \gamma^2} \right) + \gamma \cdot \left( 1 - \frac{a + \beta}{1 - \gamma^2} \right)$$

$$= a + \frac{\gamma^2}{1 + \gamma} + \frac{\beta^2 + \gamma\beta}{1 + \gamma} = a + \frac{\gamma^2 + \beta^2 + \gamma\beta + \gamma^2\beta}{1 + \gamma}$$

$$= a + \frac{2\gamma^2 + \gamma + \beta^2}{1 + \gamma}$$

$$P(\text{旧}) = \frac{1}{1 + \gamma}$$

从公平性角度考虑, 为使比赛更加公平, 应给每队参赛者尽可能多且相同的客观条件, 让他们展示实力(晋级)

旧赛制一锤定音的方式显然具有更高的偶然性,  $\therefore$  新赛制更合理

二. 解:

$$(1) E(X_n) = (1 - p)^n + [1 - (1 - p)^n] E(Y_n)$$

$$(2) E(X_n) = (1 - p)^n + (1 - (1 - p)^n) \cdot (1 - p)^{\frac{n-1}{2}} (2 + E(Y_{\frac{n-1}{2}})) + (1 - (1 - p)^n) (1 - (1 - p)^{\frac{n-1}{2}}) \cdot (2 + E(Y_{\frac{n-1}{2}}))$$

$$\text{利用第(1)问对 } E(X_n) \text{ 替换得上式右边} = (1 - p)^n + [1 - (1 - p)^n] E(Y_n)$$

