P280. #1.

(a) $\hat{y}_{n+1} = y_n + 0.5(te^{3t} - 2y_n)$

$y_{n+1} = y_n + \frac{h}{2}[te^{3t} - 2y_n + (t+h)e^{3(t+h)} - 2\hat{y}_{n+1}]$

n	x_n	y_n	$\hat{y}_{n+1}$	y_{n+1}	(True value
0	0.0	0	0	0.56021	0
1	0.5	0.56021	1.12042	5.30149	0.28362
2	1.0	5.30149	10.04277	2.65074	3.21916

P281. #10.

(a) $y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = y_n'$

$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$

$k_3 = f(t + \frac{h}{2}, y_n + \frac{h}{2}k_2)$

$k_4 = f(t+h, y_n + hk_3)$.

t	y_n	True value
0	0	0
0.5	0.2970	0.2836
1.0	3.3143	3.21916

P281. #13.

Midpoint method:

$k_1 = -w_i + hi + 1$

$k_2 = (\frac{h}{2} - 1)w_i - \frac{h^2}{2}i + hi + 1$

$y^{(1)}_{n+1} = y_n + hk_2 = (\frac{h^2}{2} - h + 1)y_n + (h^2 - \frac{h^3}{2})i + h$

Heun's method:

$k_1 = -y_n + hi + 1$

$K_2 = (\frac{2}{3}h - 1)y_n - \frac{2}{3}h^2 i + hi + 1$

$y^{(2)}_{n+1} = (\frac{h^2}{2} - h + 1)y_n + i(h^2 - \frac{h^3}{2}) + h$

Modified Euler method:

$k_1 = -y_n + hi + 1$.

$k_2 = (\frac{h}{2} - 1)y_n - \frac{h^2}{2}i + hi + 1$

$y^{(3)}_{n+1} = (\frac{h^2}{2} - h + 1)y_n + [h^2 + (-\frac{h^3}{2})]i + h$.

$\therefore y^{(1)}_{n+1} = y^{(2)}_{n+1} = y^{(3)}_{n+1}$

P271. #5.

(a). $y'(t) = \frac{2}{t}y + t^2 e^t$

$y''(t) = \frac{2}{t^2}y + (4t+t^2)e^t$.

$y_{n+1}(t) = y_n + h\left(\frac{2}{t}y + t^2e^t\right) + \frac{h^2}{2}\left(\frac{2}{t^2}y + 4te^t + t^2e^t\right)$

$= y_n + \frac{0.2}{t}y_n + 0.105t^2e^t + \frac{0.01}{t^2}y + 0.02te^t$.

n	t	w_n	y_n
0	1.0	0	0
1	1.1	0.33978	0.34592
2	1.2	0.85214	0.86664
3	1.3	1.58177	1.60722
4	1.4	2.58100	2.62036
5	1.5	3.91098	3.96767
6	1.6	5.64308	5.72096
7	1.7	7.86038	7.96387
8	1.8	10.65951	10.79362
9	1.9	14.15268	14.32308
10	2.0	18.46999	18.68310.

(b). ~~Newton~~ Newton Interpolating Method.

$\hat{y}(1.04) = 0.11778$ $\hat{y}(1.55) = 4.72187$ $\hat{y}(1.97) = 17.07973$

$y(1.04) = 0.11999$ $y(1.55) = 4.78864$ $y(1.97) = 17.27930$.

P301. #10.

$$\begin{cases} f_{i-1} = y_i' - h y_i'' + \frac{h^2}{2} y_i''' + O(h^3) \\ f_{i-2} = y_i' - 2h y_i'' + 2h^2 y_i''' + O(h^3) \end{cases}$$

$$y_{i+1} = y_i + h y_i' + \frac{h^2}{2} y_i'' + \frac{h^3}{6} y_i''' + O(h^4)$$

compare the coefficient of the formula,

$$\begin{cases} a+b+c=1 \\ -b-2c = \frac{1}{2} \\ \frac{b}{2}+2c = \frac{1}{6} \end{cases} \quad \therefore \begin{cases} a = \frac{23}{12} \\ b = -\frac{4}{3} \\ c = \frac{5}{12}. \end{cases}$$