

P397

7. (a) step 2: $n-1$ step 3-5: $\sum_{i=2}^n (\sum_{j=i}^n 2) + \sum_{j=i+1}^n \sum_{k=i+1}^n 2 = \frac{1}{3}n^3 - \frac{1}{2}n + 2$

step 6: $n-1$ $\therefore A = 2n-2 + \frac{1}{3}n^3 - \frac{1}{2}n + 2 = \frac{1}{3}n^3 - \frac{1}{3}n$

step 3-5: $\sum_{i=2}^n (\sum_{k=i}^n 1 + \sum_{j=i+1}^n \sum_{k=i+1}^n 2) = \frac{n^3}{3} - \frac{n^2}{2} - \frac{5}{6}n + 1$

step 6: $n-1$ $\therefore B = \frac{n^3}{3} - \frac{n^2}{2} + \frac{1}{6}n$

(b) $y_i = \frac{b_i}{L_{ii}} = b_i$

$y_i = \frac{1}{L_{ii}} [b_i - \sum_{j=1}^{i-1} L_{ij} y_j]$

c. multiplications/divisions: $\sum_{i=2}^n \sum_{j=1}^{i-1} 1 = \frac{1}{2}n^2 - \frac{1}{2}n$

additions/subtractions: $\sum_{i=2}^n [\sum_{j=1}^{i-1} 1 - 1 + 1] = \frac{1}{2}n^2 - \frac{1}{2}n$

(c) multiplications/divisions additions/subtractions

factoring $A=LU$ $\frac{1}{3}n^3 - \frac{1}{2}n$ $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n$

$Ly=b$ $\frac{1}{2}n^2 - \frac{1}{2}n$ $\frac{1}{2}n^2 - \frac{1}{2}n$

$Ux=y$ $\frac{1}{2}n^2 + \frac{1}{2}n$ $\frac{1}{2}n^2 - \frac{1}{2}n$

Total $\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$ $\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$

(d) multiplications/divisions additions/subtractions

factoring $A=LU$ $\frac{1}{3}n^3 - \frac{1}{2}n$ $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n$

$Ly^{(k)} = b^{(k)}$ $(\frac{1}{2}n^2 - \frac{1}{2}n)m$ $(\frac{1}{2}n^2 - \frac{1}{2}n)m$

$Ux^{(k)} = y^{(k)}$ $(\frac{1}{2}n^2 + \frac{1}{2}n)m$ $(\frac{1}{2}n^2 - \frac{1}{2}n)m$

$\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$ $\frac{1}{3}n^3 + (m - \frac{1}{2})n^2 - (m - \frac{1}{6})n$