

P280. #1.

$$(a) \hat{y}_{n+1} = y_n + 0.5(t e^{3t} - 2y_n)$$

$$[y_{n+1} = y_n + \frac{h}{2} [t e^{3t} - 2y_n + (t+h) e^{3(t+h)} - 2\hat{y}_{n+1}]]$$

n	x _n	y _n	\hat{y}_{n+1}	y _{n+1}	(True value)
0	0.0	0	0	0.56021	
1	0.5	0.56021	1.12042	5.30149	0.28362
2	1.0	5.30149	10.04277	2.65074	3.21910

P281. #10.

$$(a) y_{n+1} = y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = y'_n$$

$$k_2 = f(t + \frac{h}{2}, y_n + \frac{h}{2} k_1)$$

$$k_3 = f(t + \frac{h}{2}, y_n + \frac{h}{2} k_2)$$

$$k_4 = f(t + h, y_n + h k_3)$$

t, y, True value

0.0 0.0 0.2836

0.5 0.2970 0.2836

1.0 3.3143 3.2191

Heun's method:

$$k_1 = -y_n + h i + 1$$

$$k_2 = (\frac{2}{3}h - 1)y_n - \frac{2}{3}h^2 i + h i + 1$$

$$y_{n+1}^{(2)} = (\frac{h^2}{2} - h + 1)y_n + (h^2 - \frac{h^3}{2})i + h$$

Modified Euler method:

$$k_1 = -y_n + h i + 1$$

$$k_2 = (\frac{h}{2} - 1)y_n - \frac{h^2}{2}i + h i + 1$$

$$y_{n+1}^{(3)} = (\frac{h^2}{2} - h + 1)y_n + [h^2 + (-\frac{h^3}{2})]i + h$$

$$\therefore y_{n+1}^{(1)} = y_{n+1}^{(2)} = y_{n+1}^{(3)}$$

P281. #13.

Midpoint method:

$$k_1 = -w_i + h i + 1$$

$$k_2 = (\frac{h}{2} - 1)w_i - \frac{h^2}{2}i + h i + 1$$

$$y_{n+1}^{(1)} = y_n + h k_2 = (\frac{h^2}{2} - h + 1)y_n + (h^2 - \frac{h^3}{2})i + h$$

Modified Euler method:

$$k_1 = -y_n + h i + 1$$

$$k_2 = (\frac{h}{2} - 1)y_n - \frac{h^2}{2}i + h i + 1$$

$$y_{n+1}^{(3)} = (\frac{h^2}{2} - h + 1)y_n + [h^2 + (-\frac{h^3}{2})]i + h$$

$$\therefore y_{n+1}^{(1)} = y_{n+1}^{(2)} = y_{n+1}^{(3)}$$

P323. #5.

(a) n	t	w_t	y_t
0	0	0	0
1	0.1	0.000009	0.000009
2	0.2	0.000154	0.000154
3	0.3	0.000834	0.000834
4	0.4	0.002833	0.002832
5	0.5	0.007431	0.007430
6	0.6	0.016565	0.016563
7	0.7	0.033003	0.032998
8	0.8	0.060570	0.060562
9	0.9	0.104417	0.104405
10	1.0	0.171347	0.171329

(b) n	t	w_t	y_t
0	1	1	1
1	1.1	0.990178	0.990179
2	1.2	0.961524	0.961526
3	1.3	0.915455	0.915457
4	1.4	0.853637	0.853640
5	1.5	0.777968	0.777972
6	1.6	0.690562	0.690567
7	1.7	0.593732	0.593738
8	1.8	0.489979	0.489986
9	1.9	0.381980	0.381988
10	2.0	0.272581	0.272589

(c) n	t	w_t	y_t
0	0	1	1
5	1.0	3.731863	3.731704
10	2.0	11.314626	11.314529
15	3.0	34.045482	34.045172

(d) n	t	w_t	y_t
0	1	0	0
2	1.2	0.272738	0.272738
4	1.4	0.746488	0.746488
6	1.6	1.516251	1.516265
8	1.8	2.684005	2.684015
10	2.0	4.361573	4.361578

P334. #7.

$$\lambda^2 + 4\lambda - 5 = 0 \quad \therefore \lambda_1 = -5, \quad \lambda_2 = 1. \quad |\lambda_1| > 1 \quad \therefore \text{unstable.}$$