Class and Branch: TE – AI & DS
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EXPERIMENT NO.: 08

AIM: Study and Implementation of Bayes Rule.

OBJECTIVES: From this experiment, it will be able to:

- Understand Bayes Rule and its application in AI.

THEORY:

What is Bayes' Theorem?

Bayes' Theorem is used to determine the conditional probability of an event. It was named after an English statistician, Thomas Bayes who discovered this formula in 1763. Bayes Theorem is a very important theorem in mathematics, that laid the foundation of a unique statistical inference approach called the Bayes' inference. It is used to find the probability of an event, based on prior knowledge of conditions that might be related to that event.

Bayes theorem (also known as the Bayes Rule or Bayes Law) is used to determine the conditional probability of event A when event B has already occurred.

The general statement of Bayes' theorem is "The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A and the probability of A divided by the probability of event B." i.e.

$$P(A|B) = P(B|A)\ P(A) / P(B)$$

Where,

- P(A) and P(B) are the probabilities of events A and B also P(B) is never equal to zero.
- P(A|B) is the probability of event A when event B happens
- P(B|A) is the probability of event B when A happens

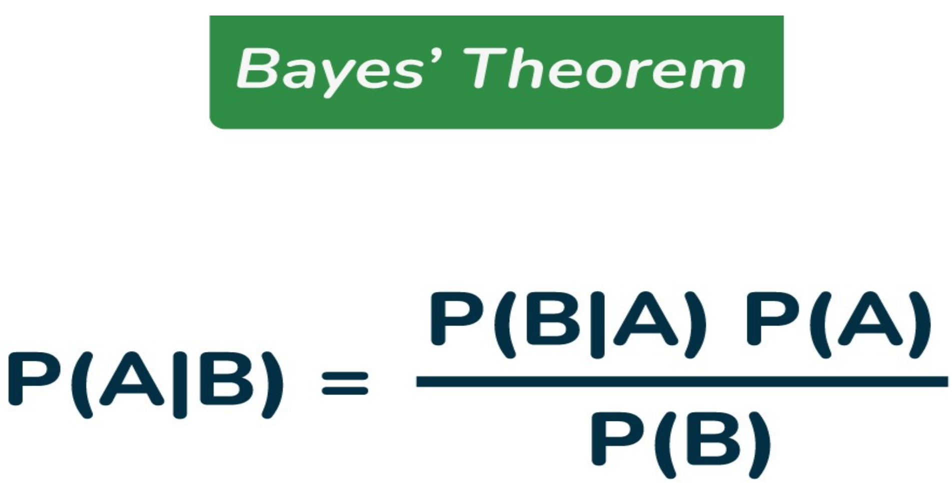

Fig: Bayes' Theorem

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Bayes Theorem Derivation

The proof of Bayes' Theorem is given as, according to the conditional probability formula,

$P(Ei|A) = P(Ei \cap A) / P(A)$…..(i)

Then, by using the multiplication rule of probability, we get

$P(Ei \cap A) = P(Ei)P(A|Ei)$……(ii)

Now, by the total probability theorem,

$P(A) = \sum P(Ek)P(A|Ek)$…..(iii)

Substituting the value of $P(Ei \cap A)$ and $P(A)$ from eq (ii) and eq(iii) in eq(i) we get,

$P(Ei|A) = P(Ei)P(A|Ei) / \sum P(Ek)P(A|Ek)$

Bayes' theorem is also known as the formula for the probability of "causes". As we know, the Ei 's are a partition of the sample space S, and at any given time only one of the events Ei occurs. Thus, we conclude that the Bayes' theorem formula gives the probability of a particular Ei, given the event A has occurred.

Applications Of Baye's Theorem

1. Medical Diagnosis: Determines the probability of a disease given a positive test result, factoring in test accuracy and disease prevalence.
2. Spam Filtering: Classifies emails as spam or legitimate by calculating the likelihood of certain words appearing in spam emails.
3. Machine Learning: Used in Naive Bayes classifiers for predicting the category of data, such as in text classification and sentiment analysis.
4. Weather Forecasting: Updates predictions of weather events (like rain) based on new meteorological data.
5. Financial Modeling: Assesses and updates the likelihood of market movements using new economic or stock data.
6. Genetics: Estimates the probability of an individual having a genetic trait or disorder based on family history and genetic data.
7. Forensic Science: Evaluates the probability of a suspect's guilt by combining evidence like DNA with background information.
8. A/B Testing: Guides decision-making in marketing by updating the probability that one variant (e.g., an ad) performs better than another.
9. Speech Recognition: Predicts word sequences in speech by calculating the probability of a particular word given the observed acoustic signals.
10. Reliability Engineering: Updates the likelihood of a system or machine failing based on past performance and operating conditions.

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EXAMPLES:

Example 1: A person has undertaken a job. The probabilities of completion of the job on time with and without rain are 0.44 and 0.95 respectively. If the probability that it will rain is 0.45, then determine the probability that the job will be completed on time.

Ans: Let E1 be the event that the mining job will be completed on time and E2 be the event that it rains. We have,

$P(A) = 0.45$,

P (no rain) $= P(B) = 1 - P(A) = 1 - 0.45 = 0.55$

By multiplication law of probability,

$P(E1) = 0.44$, and $P(E2) = 0.95$

Since, events A and B form partitions of the sample space S, by total probability theorem, we have

$P(E) = P(A)\,P(E1) + P(B)\,P(E2)$

$\Rightarrow P(E) = 0.45 \times 0.44 + 0.55 \times 0.95$

$\Rightarrow P(E) = 0.198 + 0.5225 = 0.7205$

So, the probability that the job will be completed on time is 0.7205

Example 2: There are three urns containing 3 white and 2 black balls; 2 white and 3 black balls; 1 black and 4 white balls respectively. There is an equal probability of each urn being chosen. One ball is equal probability chosen at random. what is the probability that a white ball is drawn?

Ans: Let E1, E2, and E3 be the events of choosing the first, second, and third urn respectively. Then,

$P(E1) = P(E2) = P(E3) = 1/3$

Let E be the event that a white ball is drawn. Then,

$P(E/E1) = 3/5$, $P(E/E2) = 2/5$, $P(E/E3) = 4/5$

By theorem of total probability, we have

$P(E) = P(E/E1) \,.\, P(E1) + P(E/E2) \,.\, P(E2) + P(E/E3) \,.\, P(E3)$

$\Rightarrow P(E) = (3/5 \times 1/3) + (2/5 \times 1/3) + (4/5 \times 1/3)$

$\Rightarrow P(E) = 9/15 = 3/5 = 0.6$

Therefore, the probability that a white ball is drawn is 0.6

Example 3: A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. find the probability of the lost card being a heart.

Ans: Let E1, E2, E3, and E4 be the events of losing a card of hearts, clubs, spades, and diamonds respectively.

Then P(E1) = P(E2) = P(E3) = P(E4) = 13/52 = 1/4.

Let E be the event of drawing 2 hearts from the remaining 51 cards. Then,

P(E|E1) = probability of drawing 2 hearts, given that a card of hearts is missing

⇒ P(E|E1) = 12C2 / 51C2 = (12 × 11)/2! × 2!/(51 × 50) = 22/425

P(E|E2) = probability of drawing 2 clubs ,given that a card of clubs is missing

⇒ P(E|E2) = 13C2 / 51C2 = (13 × 12)/2! × 2!/(51 × 50) = 26/425

P(E|E3) = probability of drawing 2 spades ,given that a card of hearts is missing

⇒ P(E|E3) = 13C2 / 51C2 = 26/425

P(E|E4) = probability of drawing 2 diamonds ,given that a card of diamonds is missing

⇒ P(E|E4) = 13C2 / 51C2 = 26/425

Therefore,

P(E1|E) = probability of the lost card is being a heart, given the 2 hearts are drawn from the remaining 51 cards

⇒ P(E1|E) = P(E1) . P(E|E1)/P(E1). P(E|E1) + P(E2). P(E|E2) + P(E3) . P(E|E3) + P(E4) . P(E|E4)

⇒ P(E1|E) = (1/4 × 22/425) / {(1/4 × 22/425) + (1/4 × 26/425) + (1/4 × 26/425) + (1/4 × 26/425)}

⇒ P(E1|E) = 22/100 = 0.22

Hence, the required probability is 0.22.

Example 4: Suppose 15 men out of 300 men and 25 women out of 1000 are good orators. An orator is chosen at random. Find the probability that a male person is selected.

Ans: Given,

Total Men = 300

Total Women = 1000

Good Orators among Men = 15

Good Orators among Women = 25

Total number of good orators = 15 (from men) + 25 (from women) = 40

Probability of selecting a male orator:

P (Male Orator) = Numbers of male orators / total no of orators = 15/40 = 0.375

Therefore, the probability that a male person is selected is 0.375

Example 5: A man is known to speak the lies 1 out of 4 times. He throws a die and reports that it is a six. Find the probability that is actually a six.

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Ans: In a throw of a die, let

E1 = event of getting a six,

E2 = event of not getting a six and

E = event that the man reports that it is a six.

Then, P(E1) = 1/6, and P(E2) = (1 – 1/6) = 5/6

P(E|E1) = probability that the man reports that six occurs when six has actually occurred

⇒ P(E|E1) = probability that the man speaks the truth

⇒ P(E|E1) = 3/4

P(E|E2) = probability that the man reports that six occurs when six has not actually occurred

⇒ P(E|E2) = probability that the man does not speak the truth

⇒ P(E|E2) = (1 – 3/4) = 1/4

Probability of getting a six ,given that the man reports it to be six

P(E1|E) = P(E|E1) × P(E1)/P(E|E1) × P(E1) + P(E|E2) × P(E2)   [by Bayes' theorem]

⇒ P(E1|E) = (3/4 × 1/6)/{(3/4 × 1/6) + (1/4 × 5/6)}

⇒ P(E1|E) = (1/8 × 3) = 3/8 = 0.375

Hence the probability required is 0.375

RESULTS

The experiment was designed to study and implement Bayes' Rule, a fundamental concept in probability theory, to update prior beliefs with new evidence. Bayes' Rule provides a way to revise initial estimates based on new information, which is useful in fields like medicine, machine learning, and decision-making.

The theoretical result shows that prior probability (existing knowledge) plays a critical role in the final outcome. Without considering this, one might overestimate the likelihood of an event (e.g., having a disease based on a test result).

CONCLUSION

The experiment confirms that Bayes' Rule is highly useful in situations where decisions or predictions need to be updated with new evidence. It highlights the practical importance of understanding conditional probability, especially in real-world applications like medical diagnosis, spam filtering, and reliability engineering. The result demonstrates the robustness of the theorem in integrating prior knowledge with observed data for better-informed decisions.

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