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Alcuin's Sequence

The integer sequence 1, 0, 1, 1, 2, 1, 3, 2, 4, 3, 5, 4, 7, 5, 8, 7, 10, 8, 12, 10, 14, 12, 16, 14, 19, 16, 21, 19, ... (OEIS A005044) given by the coefficients of the Maclaurin series for

$$\frac{1}{(1-x^2)(1-x^3)(1-x^4)} = 1 + x^2 + x^3 + 2x^4 + x^5 + \ldots. \tag{1}$$

A binary plot of the first few terms in the sequence is illustrated above.

Closed forms include

$$a_n = \frac{1}{288}\left\{107 + 54n + 6n^2 + (-1)^n\left[81 + 18n + 64\cos\left(\tfrac{1}{3}\pi n\right)\right] + 36\left[\cos\left(\tfrac{1}{2}\pi n\right) - \sin\left(\tfrac{1}{2}\pi n\right)\right]\right\} \tag{2}$$

$$= \frac{1}{288}\left\{107 + 54n + 6n^2 + (-1)^n\left[81 + 18n + 64\cos\left(\tfrac{1}{3}\pi n\right)\right]\right. \tag{3}$$

$$\left. + 36(-1)^{\lfloor (n+1)/2\rfloor}\right, \tag{4}$$

where $\lfloor x \rfloor$ is the floor function.

The number of different triangles which have integral sides and perimeter n is given by

$$T(n) = P_3(n) - \sum_{1\le j\le \lfloor n/2\rfloor} P_2(j) \tag{5}$$

$$= \left\lfloor \frac{n^2}{12}\right\rfloor - \left\lfloor \frac{n}{4}\right\rfloor\left\lfloor \frac{n+2}{4}\right\rfloor \tag{6}$$

$$= \begin{cases} \left\lfloor \frac{n^2}{48}\right\rfloor & \text{for } n \text{ even} \\ \left\lfloor \frac{(n+3)^2}{48}\right\rfloor & \text{for } n \text{ odd,} \end{cases} \tag{7}$$

where $P_2(n)$ and $P_3(n)$ are partition functions, with $P_k(n)$ giving the number of ways of writing n as a sum of k terms, $\lfloor x \rceil$ is the nearest integer function, and $\lfloor x \rfloor$ is the floor function (Jordan *et al.* 1979, Andrews 1979, Honsberger 1985). Strangely enough, $T(n)$ for $n = 3$, 4, ... is precisely Alcuin's sequence.

SEE ALSO:
Integer Triangle, Partition Function P, Triangle Dissection

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