Math Help - Very tricky and interesting modular question

July 9th 2010, 01:29 PM #1

elemental
Junior Member
Joined: Jul 2010
From: NJ
Posts: 68

Very tricky and interesting modular question

This is a very tricky question:
Find the last three digits of
2008^2007^2006^2005^...^1

If you're having trouble thinking about how the exponents are arranged, just imagine every consecutive number a power of the previous.

Good luck, I know the answer, and will tell you if you get it!

July 9th 2010, 01:57 PM #2

Also sprach Zarathustra

MHF Contributor

Joined:	Dec 2009
From:	Russia
Posts:	1,506
Thanks:	1

> Originally Posted by **elemental**
>
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James Bond +1 ?

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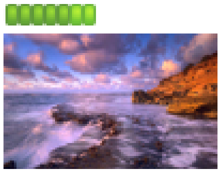
July 9th 2010, 03:07 PM #3

undefined

MHF Contributor

MHF EXPERT



Joined:	Mar 2010
From:	Chicago
Posts:	2,340
Awards:	1

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Spoiler: Show

Preamble: I think there's a more elegant way to write down the same type of reasoning, but this is the first that came to mind.

Combination of Euler's theorem and Chinese remainder theorem.

$$2008^{2007^{2006^{\cdots^{1}}}} \equiv 8^{2007^{2006^{\cdots^{1}}}} \equiv x \pmod{1000}$$

Solve

$$8^{2007^{2006^{\cdots^{1}}}} \equiv a \pmod{2^3}$$

$$8^{2007^{2006^{\cdots^{1}}}} \equiv b \pmod{5^3}$$

and then combine with CRT.

Note that $8^{2007^{2006^{\cdots^{1}}}} \equiv 0 \pmod{2^3}$

Solve $8^{2007^{2006^{\cdots^{1}}}} \equiv b \pmod{5^3}$

$$\gcd(8, 5^3) = 1$$

Use Euler's theorem

$$\varphi(5^3) = 100$$

Reduce to finding

$$2007^{2006^{2005^{\cdots^{1}}}} \equiv 7^{2006^{2005^{\cdots^{1}}}} \equiv c \pmod{100}$$

$$\gcd(7, 100) = 1$$

Use Euler's theorem

$$\varphi(100) = 40 = 2^3 \cdot 5$$

$$2006^{2005^{2004^{\cdots^{1}}}} \equiv 6^{2005^{2004^{\cdots^{1}}}} \equiv d \pmod{40}$$

$$\gcd(6, 40) = 2$$

Solve

$$6^{2005^{2004^{\cdots^{1}}}} \equiv e \pmod{2^3}$$

$$6^{2005^{2004^{\cdots^{1}}}} \equiv f \pmod{5}$$

Note that $6^{2005^{2004^{\cdots^{1}}}} \equiv 0 \pmod{2^3}$

Note that $6^{2005^{2004^{\cdots^{1}}}} \equiv 1^{2005^{2004^{\cdots^{1}}}} \equiv 1 \pmod{5}$

Now work backwards...

CRT: $2006^{2005^{2004^{\cdots^{1}}}} \equiv 16 \pmod{40}$

Euler: $2007^{2006^{2005^{\cdots^{1}}}} \equiv 7^{16} \equiv 1 \pmod{100}$

Euler: $2008^{2007^{2006^{\cdots^{1}}}} \equiv 8^{1} \equiv 8 \pmod{5^3}$

CRT: $2008^{2007^{2006^{\cdots^{1}}}} \equiv 8 \pmod{1000}$

So the answer is 008

Just like Also sprach Zarathustra said!

July 9th 2010, 03:13 PM #4

melese

Member

Joined: Jun 2010

From: Israel

Posts: 148

Ignore

> Originally Posted by **elemental**
>
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Assuming you meant $2008^{2007^{2006^{2005^{2004\dots^{1}}}}}$, call it P and let $a=2007^{2006^{2005^{2004\dots^{1}}}}$ and $b=2006^{2005^{2004\dots^{1}}}$.

I start with the fact that $2008\equiv 3\equiv -2(mod\ 5)$.

$b=2k$ for some integer k, so
$a=2007^b=2007^{2k}\equiv 3^{2k}\equiv 9^k\equiv 1(mod\ 4)$.
$a=4t+1$ for some integer t, so
$(-2)^a=(-2)^{4t+1}=16^t(-2)\equiv -2(mod\ 5)$

Now, $P=2008^a\equiv(-2)^a\equiv(-2)(mod\ 5)$. Also, obviously P is even, hence $P\equiv -2(mod\ 2)$. These two congruences give $P\equiv -2\equiv 8(mod\ 10)$.

The last digit of $2008^{2007^{2006^{2005^{2004\dots^{1}}}}}$ is 8.

Last edited by melese; July 9th 2010 at 03:49 PM. ***Reason:*** *RTQ (read the question)*

July 9th 2010, 06:06 PM #5

elemental

Junior Member

Joined: Jul 2010
From: NJ
Posts: 68

Well done guys, 008 is indeed correct.

July 9th 2010, 08:03 PM #6

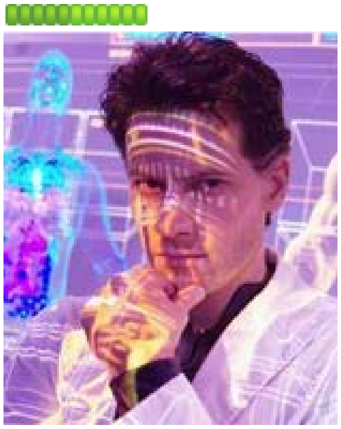
mr fantastic

Flow Master

MHF 2010 Best Statistician

MHF 2010 Funniest Member

MHF MOD



Joined: Dec 2007
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Posts: 16,948
Thanks: 5

Challenge questions belong in the Challenege Questions subforum (read the rules at that subforum before posting in it).

July 9th 2010, 08:41 PM #7

simplependulum

Super Member

MHF EXPERT

Joined: Jan 2009
Posts: 715

We wish to find out the remainder when $2006^{2005^{\cdots}}$ is divided by 40

Let x be $2006^{2005^{\cdots}}$

then

$x \equiv 1 \bmod 5$ and

$x \equiv 0 \bmod 8$ so

$\boxed{x \equiv 16 \bmod 40}$

Then consider 7^x in modulo 100

it is $7^{40k+16} \equiv 7^{16} \bmod 100$

Let $y = 7^{16}$ we have

$y \equiv (-1)^{16} \equiv 1 \bmod 4$ and

$y = 7^{16} = (49)^8 \equiv (-1)^8 \equiv 1 \bmod 25$ so

$y \equiv 1 \bmod 100$

Then 8^y in modulo 125

Since $(8, 125) = 1$ we have $8^{100} \equiv 1$

Therefore , $8^y = 8^{100k+1} \equiv 8 \bmod 125$

Together with $8^y \equiv 0 \bmod 8$ we obtain $8^y \equiv 8 \bmod 1000$

Actually , the answer is exactly $8 \equiv 8^y \equiv (2008)^y \bmod 1000$

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