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# Alcuin's Sequence



The integer sequence 1, 0, 1, 1, 2, 1, 3, 2, 4, 3, 5, 4, 7, 5, 8, 7, 10, 8, 12, 10, 14, 12, 16, 14, 19, 16, 21, 19, ... (OEIS A005044) given by the coefficients of the Maclaurin series for

$$\frac{1}{(1-x^2)(1-x^3)(1-x^4)} = 1 + x^2 + x^3 + 2x^4 + x^5 + \dots$$
 (1)

A binary plot of the first few terms in the sequence is illustrated above.

Closed forms include

$$a_{n} = \frac{1}{288} \left\{ 107 + 54 \, n + 6 \, n^{2} + (-1)^{n} \left[ 81 + 18 \, n + 64 \cos \left( \frac{1}{3} \, \pi \, n \right) \right] + 36 \left[ \cos \left( \frac{1}{2} \, \pi \, n \right) - \sin \left( \frac{1}{2} \, \pi \, n \right) \right] \right\}$$

$$= \frac{1}{288} \left\{ 107 + 54 \, n + 6 \, n^{2} + (-1)^{n} \left[ 81 + 18 \, n + 64 \cos \left( \frac{1}{3} \, \pi \, n \right) \right] \right\}$$

$$+ 36 \left[ (-1)^{\lfloor (n+1)/2 \rfloor} \right],$$
(4)

where [x] is the floor function.

The number of different triangles which have integral sides and perimeter n is given by

$$T(n) = P_3(n) - \sum_{1 \le j \le \lfloor n/2 \rfloor} P_2(j)$$

$$= \left\lceil \frac{n^2}{12} \right\rceil - \left\lfloor \frac{n}{4} \right\rfloor \left\lfloor \frac{n+2}{4} \right\rfloor$$

$$= \left\lceil \left\lceil \frac{n^2}{48} \right\rceil \quad \text{for } n \text{ even} \right\rceil$$

$$\left\lceil \left( \frac{(n+3)^2}{48} \right\rceil \quad \text{for } n \text{ odd,}$$

$$(5)$$

where  $P_2(n)$  and  $P_3(n)$  are partition functions, with  $P_k(n)$  giving the number of ways of writing n as a sum of kterms, [x] is the nearest integer function, and [x] is the floor function (Jordan et al. 1979, Andrews 1979, Honsberger 1985). Strangely enough, T(n) for n = 3, 4, ... is precisely Alcuin's sequence.

### SEE ALSO:

Integer Triangle, Partition Function P, Triangle Dissection

Andrews, G. "A Note on Partitions and Triangles with Integer Sides." Amer. Math. Monthly 86, 477, 1979.

Honsberger, R. Mathematical Gems III. Washington, DC: Math. Assoc. Amer., pp. 39-47, 1985.

Jordan, J. H.; Walch, R.; and Wisner, R. J. "Triangles with Integer Sides." Amer. Math. Monthly 86, 686-689, 1979.

Sloane, N. J. A. Sequence A005044/M0146 in "The On-Line Encyclopedia of Integer Sequences."

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= foci of hyperbola with semiaxes

invert colors of Apatasaurus

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