Machine Learning
LABORATORY: LSTM Homework

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Objectives:

- The goal of this assignment is to deepen your understanding of the Long Short-Term Memory (LSTM) architecture by implementing a manual LSTM cell from scratch.
- You will apply your manual LSTM to the MNIST digit classification task by treating each 28×28 image as a sequence of 28 time steps.
- Through this assignment, you will:
  - o Understand how the LSTM gates (forget, input, and output) interact to update the hidden and cell states.
  - o Implement the LSTM forward pass manually based on the given mathematical formulas.
  - o Train a deep learning model using PyTorch.
  - o Evaluate the model's performance on unseen data.
  - o Tune hyperparameters to improve accuracy.
- This assignment directly connects theoretical concepts (LSTM equations) with practical implementation for real-world applications.

Part 1. Instruction

In this assignment, you will implement a manual Long Short-Term Memory (LSTM) cell for sequence classification using PyTorch, without using any high-level RNN modules (no nn.LSTM, no optim.SGD, etc.).

You will manually implement:

- A step-by-step update of the hidden state and cell state based on the LSTM equations.
- A simple output layer to classify handwritten digits (0-9) from the MNIST dataset.
- Training using manual forward computation for each time step.

The general LSTM computations for each time step are as follows:

$$i_t = \sigma\left(W_i h_{t-1} + U_i x_t + b_i\right)$$

$$f_t = \sigma\left(W_f h_{t-1} + U_f x_t + b_f\right)$$

$$o_t = \sigma\left(W_o h_{t-1} + U_o x_t + b_o\right)$$

$$\tilde{c}_t = tanh\left(W h_{t-1} + U x_t + b\right)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t$$

$$h_t = o_t \odot tanh\left(c_t\right)$$

After the final time step, you apply an output layer:

$$logits = W_{out} h_t + b_{out}$$

You must implement the full forward computation manually for each time step. In addition to completing the forward pass and classification:

- Hyperparameter Tuning: You are required to adjust the hyperparameters (e.g., learning rate, batch size, number of hidden units, number of epochs, optimizer) to improve the final test accuracy as much as possible.
- Testing Loop: You must fill in the testing loop to calculate and print the overall accuracy on the MNIST test dataset (10,000 images).
- Visualization: You must visualize 10 example images from the test set (ideally showing digits 0–9 if possible).

In your pdf report, you must display:

```
input_size = 28
hidden_size = 32
num_layers = 1             #
num_classes = 10
batch_size = 128
learning_rate = 0.00006
num_epochs = 2
```

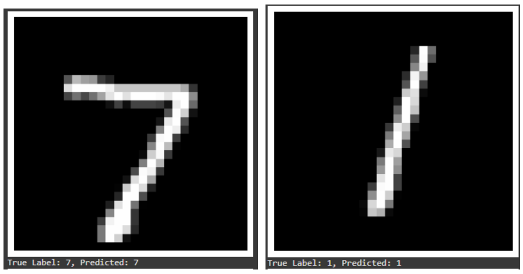
(a) Hyperparameter

```
Epoch [1/2], Step [100/469], Loss: 2.3036
Epoch [1/2], Step [200/469], Loss: 2.2966
Epoch [1/2], Step [300/469], Loss: 2.2974
Epoch [1/2], Step [400/469], Loss: 2.2918
Epoch [2/2], Step [100/469], Loss: 2.2489
Epoch [2/2], Step [200/469], Loss: 2.1501
Epoch [2/2], Step [300/469], Loss: 2.1722
Epoch [2/2], Step [400/469], Loss: 2.0087
```

(b) Training Loss record

```
Test Accuracy: 28.48%
```

(c) Test Accuracy



(d) Prediction results

<table>
<tr><th colspan="2">Part 2. Code Template</th></tr>
<tr><th>Step</th><th>Procedure</th></tr>
<tr><td>1</td><td><pre>
==
Assignment: Manual LSTM Cell for MNIST Digit Classification
==

import torch
import torch.nn as nn
import torch.optim as optim
import torchvision
import torchvision.transforms as transforms
import matplotlib.pyplot as plt
import numpy as np
import os

==
Hyperparameters - you may change the parameter to get the better accuracy
==

input_size = 28
</pre></td></tr>
</table>

	hidden_size = 32 num_layers = 1 num_classes = 10 batch_size = 128 learning_rate = 0.00006 num_epochs = 2
2	# == # Load the MNIST Dataset # == train_dataset = torchvision.datasets.MNIST(root='./data', train=True, transform=transforms.ToTensor(), download=True) test_dataset = torchvision.datasets.MNIST(root='./data', train=False, transform=transforms.ToTensor(), download=True) train_loader = torch.utils.data.DataLoader(dataset=train_dataset, batch_size=batch_size, shuffle=True) test_loader = torch.utils.data.DataLoader(dataset=test_dataset, batch_size=batch_size, shuffle=False)
3	# == # TODO 1 : Build Manual LSTM Cell # == class ManualLSTMCell(nn.Module): def __init__(self, input_size, hidden_size): super(ManualLSTMCell, self).__init__() # TODO: Define weight matrices for # - Forget gate (Weight f) # - Input gate (Weight i) # - Output gate (Weight o) # - Candidate cell (Weight c) def forward(self, x, h_prev, c_prev): # TODO: # 1. Concatenate input x and previous hidden state h_prev # 2. Calculate forget gate f_t # 3. Calculate input gate i_t # 4. Calculate candidate cell state c_tilde # 5. Update cell state c_t

	`` # 6. Calculate output gate o_t # 7. Update hidden state h_t # HINT: use torch.sigmoid and torch.tanh return h_t, c_t # Full LSTM network class ManualLSTMClassifier(nn.Module): def __init__(self, input_size, hidden_size, num_classes): super(ManualLSTMClassifier, self).__init__() # TODO: Create ManualLSTMCell # TODO: Create fully connected layer def forward(self, x): # TODO: # 1. Initialize h_t and c_t to zeros # 2. Unroll through the sequence (for each time step) # 3. Update h_t and c_t at each time step # 4. Pass last h_t into fully connected layer return out ```
4	``` # == # Training and Testing - #You are allowed to change the optimizer # == #Define model, criterion, optimizer device = torch.device('cuda' if torch.cuda.is_available() else 'cpu') model = ManualLSTMClassifier(input_size, hidden_size, num_classes).to(device) criterion = nn.CrossEntropyLoss() optimizer = optim.Adam(model.parameters(), lr=learning_rate) # Training loop for epoch in range(num_epochs): for i, (images, labels) in enumerate(train_loader): images = images.reshape(-1, 28, 28).to(device) labels = labels.to(device) outputs = model(images) loss = criterion(outputs, labels) optimizer.zero_grad() loss.backward() optimizer.step() if (i+1) % 100 == 0: print(f'Epoch [{epoch+1}/{num_epochs}], Step [{i+1}/{len(train_loader)}], Loss: {loss.item():.4f}') ```
5	``` # TODO 2: Testing loop to print the accuracy print(f'Test Accuracy: {100 * correct / total:.2f}%') # == # TODO 3: Visualization prediction # Print the accuracy from test_data ```

```
# Show 10 example images including true label and prediction
# ==========================================================

# (imshow)
```

Grading Assignment & Submission (70% Max)

Implementation:

1. (30%) Manual LSTM Cell: Correctly build a manual LSTM cell based on the provided LSTM equations.
2. (15%) Training and Hyperparameter Tuning: Successfully train the model and fine-tune hyperparameters to improve the final test accuracy; **the achieved test accuracy will determine the points awarded in this section.**
3. (5%) Testing Loop: Correctly implement the testing loop to calculate and print the test accuracy over the full 10,000 test images.
4. (5%) Visualization: Display 10 example test images, clearly showing both the true labels and the predicted labels.

Question:

5. (5%) Explain briefly the role of the forget gate, input gate, output gate, and candidate cell in an LSTM.
6. (5%) Describe what hyperparameters you tuned and how they affected your model's final accuracy.
7. (5%) Between a simple RNN and an LSTM, which one is better for sequence learning tasks? Explain your reasoning, and discuss in which situations LSTM is more useful and in which situations a simple RNN might still be sufficient.

Submission :

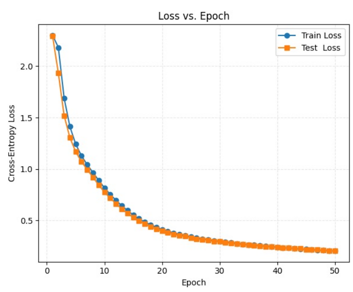
1. Report: Provide your screenshots of your results in the last pages of this PDF File.
2. Code: Submit your complete Python script in either .py or .ipynb format.
3. Upload both your report and code to the E3 system (**Labs7 Homework**). Name your files correctly:
   a. Report: StudentID_Lab7_Homework.pdf
   b. Code: StudentID_Lab7_Homework.py or StudentID_Lab7_Homework.ipynb
4. Deadline: Sunday 21:00 PM
5. Plagiarism is **strictly prohibited**. Submitting copied work from other students will result in penalties.

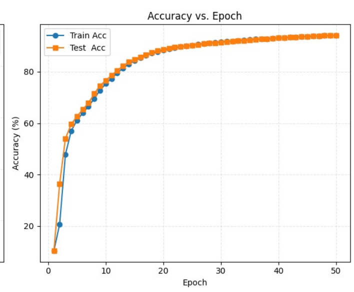
Results and Discussion:

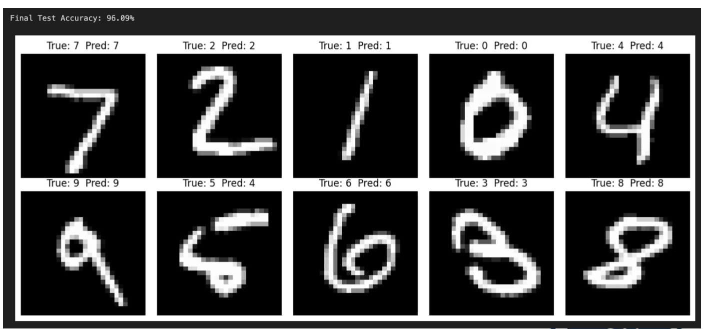
Baseline

```
input_size = 28
hidden_size = 32
num_layers = 1
num_classes = 10
batch_size = 128
learning_rate = 0.00006
num_epochs = 50
```

```
Epoch [1/50], Step [100/469], Loss: 2.3095
Epoch [1/50], Step [200/469], Loss: 2.2907
Epoch [1/50], Step [300/469], Loss: 2.2956
Epoch [1/50], Step [400/469], Loss: 2.2848
>>> Epoch [1/50]  Train Loss: 2.3011 | Train Acc: 10.32%  || Test Loss: 2.2917 | Test Acc: 10.27%
Epoch [2/50], Step [100/469], Loss: 2.2944
Epoch [2/50], Step [200/469], Loss: 2.2523
Epoch [2/50], Step [300/469], Loss: 2.1601
Epoch [2/50], Step [400/469], Loss: 2.0573
>>> Epoch [2/50]  Train Loss: 2.1820 | Train Acc: 20.71%  || Test Loss: 1.9319 | Test Acc: 36.46%
Epoch [3/50], Step [100/469], Loss: 1.7662
Epoch [3/50], Step [200/469], Loss: 1.7138
Epoch [3/50], Step [300/469], Loss: 1.6212
Epoch [3/50], Step [400/469], Loss: 1.5028
>>> Epoch [3/50]  Train Loss: 1.6895 | Train Acc: 47.80%  || Test Loss: 1.5194 | Test Acc: 53.99%
Epoch [4/50], Step [100/469], Loss: 1.5216
Epoch [4/50], Step [200/469], Loss: 1.3944
Epoch [4/50], Step [300/469], Loss: 1.4687
Epoch [4/50], Step [400/469], Loss: 1.2942
>>> Epoch [4/50]  Train Loss: 1.4129 | Train Acc: 56.91%  || Test Loss: 1.3056 | Test Acc: 59.56%
Epoch [5/50], Step [100/469], Loss: 1.3624
Epoch [5/50], Step [200/469], Loss: 1.2881
Epoch [5/50], Step [300/469], Loss: 1.2593
Epoch [5/50], Step [400/469], Loss: 1.0927
>>> Epoch [5/50]  Train Loss: 1.2435 | Train Acc: 61.08%  || Test Loss: 1.1692 | Test Acc: 62.66%
...
Epoch [50/50], Step [200/469], Loss: 0.2002
Epoch [50/50], Step [300/469], Loss: 0.2763
Epoch [50/50], Step [400/469], Loss: 0.1312
>>> Epoch [50/50]  Train Loss: 0.2031 | Train Acc: 94.20%  || Test Loss: 0.2053 | Test Acc: 94.20%
Output is truncated. View as a scrollable element or open in a text editor. Adjust cell output settings...
```



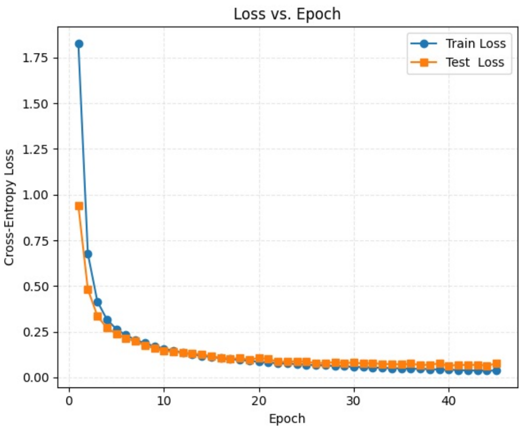


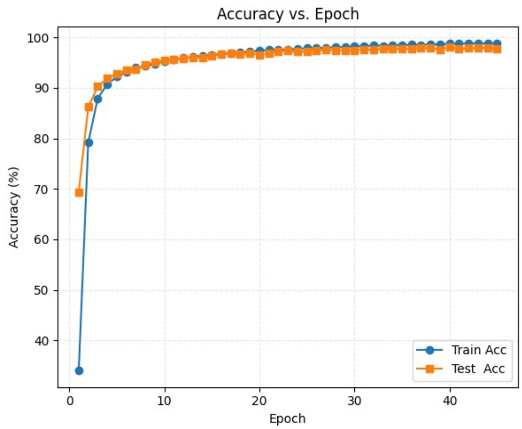


Fine tuning

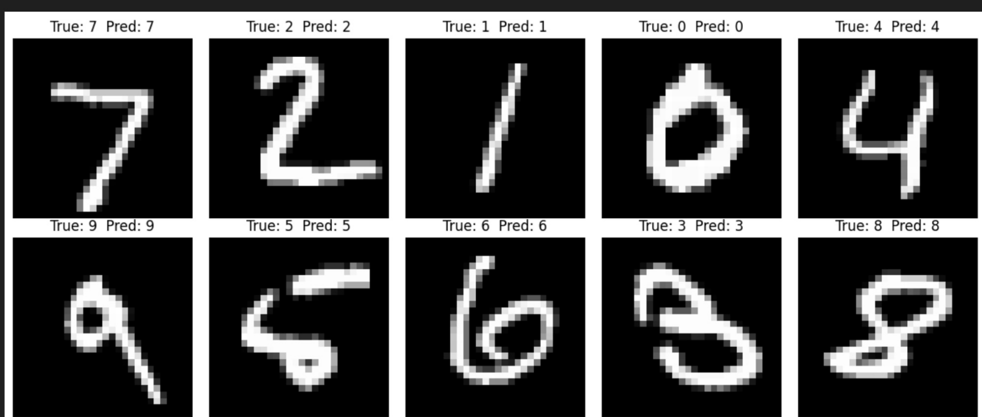
```
input_size = 28
hidden_size = 128
num_layers = 2
num_classes = 10
batch_size = 128
learning_rate = 0.00006
num_epochs = 50
```

```
Epoch [1/50], Step [100/469], Loss: 2.2994
Epoch [1/50], Step [200/469], Loss: 2.1549
Epoch [1/50], Step [300/469], Loss: 1.8010
Epoch [1/50], Step [400/469], Loss: 1.1729
>>> Epoch [1/50]  Train Loss: 1.8285 | Train Acc: 33.97%  || Test Loss: 0.9395 | Test Acc: 69.37%
Epoch [2/50], Step [100/469], Loss: 0.8601
Epoch [2/50], Step [200/469], Loss: 0.8276
Epoch [2/50], Step [300/469], Loss: 0.6787
Epoch [2/50], Step [400/469], Loss: 0.5600
>>> Epoch [2/50]  Train Loss: 0.6769 | Train Acc: 79.19%  || Test Loss: 0.4822 | Test Acc: 86.22%
Epoch [3/50], Step [100/469], Loss: 0.5810
Epoch [3/50], Step [200/469], Loss: 0.5668
Epoch [3/50], Step [300/469], Loss: 0.4923
Epoch [3/50], Step [400/469], Loss: 0.3672
>>> Epoch [3/50]  Train Loss: 0.4159 | Train Acc: 87.95%  || Test Loss: 0.3365 | Test Acc: 90.28%
Epoch [4/50], Step [100/469], Loss: 0.3568
Epoch [4/50], Step [200/469], Loss: 0.3450
Epoch [4/50], Step [300/469], Loss: 0.2427
Epoch [4/50], Step [400/469], Loss: 0.2649
>>> Epoch [4/50]  Train Loss: 0.3174 | Train Acc: 90.72%  || Test Loss: 0.2703 | Test Acc: 91.88%
Epoch [5/50], Step [100/469], Loss: 0.2451
Epoch [5/50], Step [200/469], Loss: 0.3107
Epoch [5/50], Step [300/469], Loss: 0.3994
Epoch [5/50], Step [400/469], Loss: 0.2722
>>> Epoch [5/50]  Train Loss: 0.2641 | Train Acc: 92.23%  || Test Loss: 0.2396 | Test Acc: 92.83%
...
Epoch [45/50], Step [400/469], Loss: 0.0866
>>> Epoch [45/50]  Train Loss: 0.0368 | Train Acc: 98.88%  || Test Loss: 0.0756 | Test Acc: 97.81%
Early stopping counter: 5/5
Early stopping triggered after epoch 45
```






```
Final Test Accuracy: 99.22%
```



A5:

Forget gate: Decides what information from the previous cell state should be discarded.

Input gate: Controls how much new information is added to the cell state.

Candidate cell: Represents potential new content to be added to the cell state.

Output gate: Determines what information from the cell state is passed to the hidden state.

A6:

Hidden size: Larger size captures more features but may overfit.

Batch size: Larger batches speed up training.

Number of layers: More layers improve performance but increase complexity.

Epochs: Larger epochs have better results but overfitting should be avoided. (Use early stop to avoid)

7. LSTM is better for most sequence tasks because it handles long-term dependencies and avoids vanishing gradients.

Use LSTM for: language modeling, speech recognition, financial or weather forecasting.

Use simple RNN when: sequences are short, resources are limited, or for quick prototyping.