Machine Learning
LABORATORY: Backpropagation In Class

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Objectives:

- Understand the **core concept of backpropagation** as used in training neural networks.
- Simulate and visualize **forward-mode and reverse-mode automatic differentiation** to trace how gradients are propagated.
- Interpret how gradient values are calculated during backprop through a computational graph.

Part 1. Instruction

- In this assignment, please **train a logistic classifier** to recognize whether an MNIST digit image is the target digit (e.g., "Is it a 3?") or not. ***(Last week)***
- You will integrate a backpropagation autodiff mechanism into the SGD training loop to compute gradients used for weight updates.
- Integrate an **autodiff module** that traces:
  - **Primal values** through the forward pass (e.g., intermediate variables like $v_3 = x_1 x_2$).
  - **Forward-mod**e using the chain rule from inputs to output.
  - **Reverse-mode** representing the backpropagation path from output to inputs.
- You will complete the code template provided in the in-class assignment.
- Use only NumPy for all computations. Do not use libraries like scikit-learn or PyTorch.
- Evaluate your results and answer the questions.

Part 2. Code Template

Step	Procedure

1	<pre># ========== Load Dataset ==========
	def load_images(filename):
	    with open(filename, 'rb') as f:
	        _, num, rows, cols = struct.unpack(">IIII", f.read(16))
	       data = np.frombuffer(f.read(), dtype=np.uint8).reshape((num, rows * cols))
	        return data.astype(np.float32) / 255.0
	
	def load_labels(filename):
	    with open(filename, 'rb') as f:
	        _, num = struct.unpack(">II", f.read(8))
	        return np.frombuffer(f.read(), dtype=np.uint8)</pre>

2	<pre># ========== 1. Sigmoid Function ==========
	def sigmoid(z):
	    # TODO: Implement sigmoid function (optional)</pre>

```
    pass

def sigmoid_derivative(z):
    pass

TODO: Implement Backprop Autodiff
# ====== 3. Forward and Reverse Autodiff Trace ========
def trace_autodiff_example(x1, x2):
    # Primal

    # Forward tangent

    # Reverse adjoint

    return table

TODO: Implement SGD (use your codes last week), then use the backprop
inside
# ========== 2. SGD: Algorithm 7.1 ==========
def your_sgd_logistic(X, y, eta, max_iters):

      for i in range(max_iters):

        if i == 0:
            trace = trace_autodiff_example(_,_)
    return w, trace
```

3

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# ==========Show Misclassified Samples ==========
def show_misclassified(X, y_true, y_pred, max_show=10):
    mis_idx = np.where(y_true != y_pred)[0][:max_show]
    if len(mis_idx) == 0:
        print("No misclassifications!")
        return
    plt.figure(figsize=(10, 2))
    for i, idx in enumerate(mis_idx):
        plt.subplot(1, len(mis_idx), i + 1)
        plt.imshow(X[idx, 1:].reshape(28, 28), cmap='gray')
        plt.axis('off')
        plt.title(f"T:{y_true[idx]}\nP:{y_pred[idx]}")
    plt.suptitle("Misclassified Samples")
    plt.show()
```

```
# ========== Plot Trace Graph ==========
def plot_autodiff_traces(trace_df):
    variables = trace_df['Variable']
    primal = trace_df['Primal (v)'].astype(float)
    forward = pd.to_numeric(trace_df['Forward Tangent (ẋ)'],
errors='coerce')
    reverse = pd.to_numeric(trace_df['Reverse Adjoint (v̄)'],
```

```
errors='coerce')

    fig, ax = plt.subplots(3, 1, figsize=(10, 8), sharex=True)

    ax[0].bar(variables, primal, color='skyblue')
    ax[0].set_ylabel("Primal (v)")
    ax[0].set_title("Primal Values")

    ax[1].bar(variables, forward, color='lightgreen')
    ax[1].set_ylabel("Forward Tangent (ẋ)")
    ax[1].set_title("Forward-Mode Autodiff")

    ax[2].bar(variables, reverse, color='salmon')
    ax[2].set_ylabel("Reverse Adjoint (v¯)")
    ax[2].set_title("Reverse-Mode Autodiff")
    ax[2].set_xlabel("Variables")

    plt.tight_layout()
    plt.show()
```

4

```
# ========== 3. Main ==========
def main():
    # === Load MNIST Data ===
    X_train = load_images("train-images.idx3-ubyte__")
    y_train = load_labels("train-labels.idx1-ubyte__")
    X_test = load_images("t10k-images.idx3-ubyte__")
    y_test = load_labels("t10k-labels.idx1-ubyte__")

    # === Binary Classification ===
    TARGET_DIGIT = 3 # TODO: Fill in (0 to 9) based on your student
number
    y_train_bin = np.where(y_train == TARGET_DIGIT, 1, 0)
    y_test_bin = np.where(y_test == TARGET_DIGIT, 1, 0)

    # === Add Bias ===
    X_train = np.hstack([np.ones((X_train.shape[0], 1)), X_train])
    X_test = np.hstack([np.ones((X_test.shape[0], 1)), X_test])

    # === Train ===
    # w, autodiff_trace =

    # === Predict ===
    # pred_probs =
    # preds =

    # === Accuracy ===
    # acc = np.mean(preds == y_test_bin)
    # print(f"\nTest Accuracy (is {TARGET_DIGIT} or not): {acc:.4f}")
```

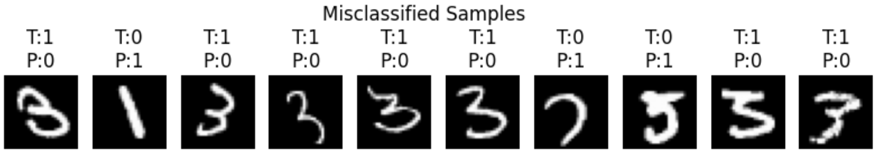
```python
    # === Show Misclassifications ===
    # show_misclassified(X_test, y_test_bin, preds)

    # === Visualize Autodiff Trace ===
    # print("\nAutodiff Trace Table (sample features):")
    # print(autodiff_trace)
    # plot_autodiff_traces(autodiff_trace)

if __name__ == "__main__":
    main()
```

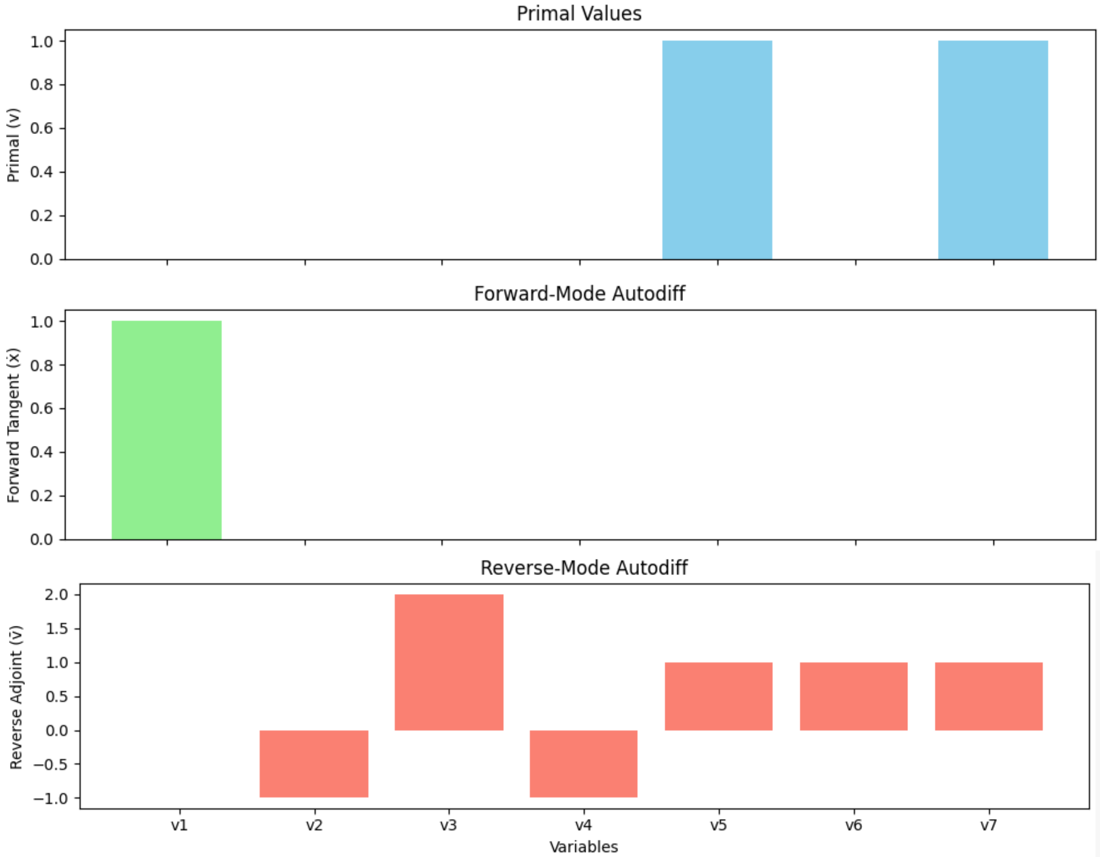
5

```
#Example Output:
Test Accuracy (is 3 or not): 0.9774
```



Autodiff Trace Table (sample features):

	Variable	Primal (v)	Forward Tangent ($\dot{x}$)	Reverse Adjoint ($\bar{v}$)
0	v1	0.0	1.0	0.0
1	v2	0.0	0.0	-1.0
2	v3	0.0	0.0	2.0
3	v4	0.0	0.0	-1.0
4	v5	1.0	0.0	1.0
5	v6	0.0	0.0	1.0
6	v7	1.0	0.0	1.0



Grading Assignment & Submission (30% Max)

Implementation:

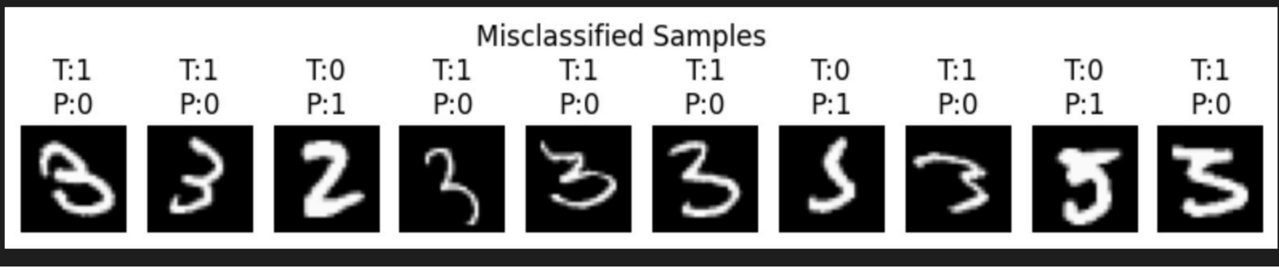
1. (10%) Implement the backpropagation autodiff
2. (10%) The model runs successfully without errors, use the provided MNIST dataset, and output the primal values, forward and reverse mode autodiff
3. (5%) Set the class of binary classification to the last digit of your student ID. (e.g., if your ID ends in 7, use the class '7'). Displays the result as shown as the "Example Output" in the last pages of this document.
4. (5%) Briefly discuss your results. For example, explain what the graph represents and why you obtained those results.

Submission:

1. Report: Provide your screenshots of your results including the discussion in the last pages of this PDF File.
2. Code: Submit your complete Python script in either .py or .ipynb format.
3. Upload both your report and code to the E3 system (**Labs5 In Class Assignment**). Name your files correctly:
    a. Report: StudentID_Lab5_InClass.pdf
    b. Code: StudentID_Lab5_ InClass.py or StudentID_Lab5_InClass.ipynb
4. Deadline: 16:20 PM
5. Plagiarism is **strictly prohibited**. Submitting copied work from other students will result in penalties.

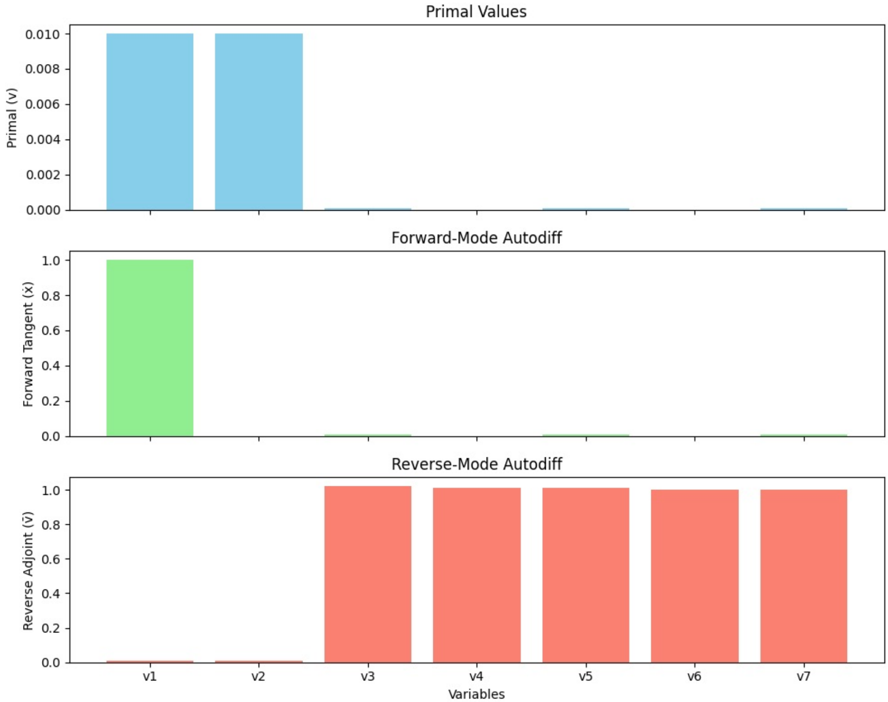
Results and Discussion:

```
Test Accuracy (is 3 or not): 0.9777
```



Autodiff Trace Table (sample features):

	Variable	Primal (v)	Forward Tangent (ẋ)	Reverse Adjoint (v̄)
0	v1	0.010000	1.000000	0.010302
1	v2	0.010000	0.000000	0.010302
2	v3	0.000100	0.010000	1.020100
3	v4	0.000001	0.000100	1.010000
4	v5	0.000101	0.010100	1.010000
5	v6	0.000001	0.000202	1.000000
6	v7	0.000102	0.010302	1.000000




```
# ====== 3. Forward and Reverse Autodiff Trace ========
def trace_autodiff_example(x1, x2):
    # Primal
    v1 = x1
    v2 = x2
    v3 = v1 * v2
    v4 = v3 * v2
    v5 = v4 + v3
    v6 = v5 * v1
    v7 = v6 + v5

    primal = [v1, v2, v3, v4, v5, v6, v7]

    # Forward tangent
    ft_v1 = 1        # derivative of v1 w.r.t v1
    ft_v2 = 0        # derivative of v2 w.r.t v1
    ft_v3 = ft_v1 * v2 + v1 * ft_v2
    ft_v4 = ft_v3 * v2 + v3 * ft_v2
    ft_v5 = ft_v4 + ft_v3
    ft_v6 = ft_v5 * v1 + v5 * ft_v1
    ft_v7 = ft_v6 + ft_v5

    forward_tangent = [ft_v1, ft_v2, ft_v3, ft_v4, ft_v5, ft_v6, ft_v7]
```

```
    # Reverse adjoint
    ra = [0] * 7
    ra[6] = 1                          # dv7/dv7
    ra[5] = ra[6] * 1                  # dv7/dv6 = 1
    ra[4] = ra[6] * 1 + ra[5] * v1     # dv7/dv5 + dv6/dv5
    ra[3] = ra[4] * 1                  # dv5/dv4
    ra[2] = ra[4] * 1 + ra[3] * v2     # dv5/dv3 + dv4/dv3
    ra[1] = ra[3] * v3 + ra[2] * v1    # dv4/dv2*v3 + dv3/dv2*v1
    ra[0] = ra[2] * v2 + ra[5] * v5    # dv3/dv1*v2 + dv6/dv1*v5

    reverse_adjoint = ra

    table = pd.DataFrame({
        'Variable': ['v1', 'v2', 'v3', 'v4', 'v5', 'v6', 'v7'],
        'Primal (v)': primal,
        'Forward Tangent (ẋ)': forward_tangent,
        'Reverse Adjoint (v̄)': reverse_adjoint
    })

    return table
```

```
def your_sgd_logistic(X, y, eta, max_iters):
    w = np.zeros(X.shape[1])
    trace = None  # initialization of trace
    for iter in range(max_iters):
        for i in range(X.shape[0]):
            xi, yi = X[i], y[i]
            pred = sigmoid(np.dot(w, xi))
            gradient = (pred - yi) * xi

            if iter == 0 and i == 0:
                trace = trace_autodiff_example(xi[1]+1e-2, w[1]+1e-2)

            w -= eta * gradient

    return w, trace
```

- Primal valuesAutodiffated from small, non-zero initial inputs (x1, x2) resulting in small but visible bars.
- Forward-mode Autodiff: Starts at input variable v1 (set as 1), showing how sensitivity propagates forward.
- Reverse-mode Autodiff: Shows gradients propagating backward from the final output (v7), indicating each variable's contribution to the output.