Modeling an adiabatic quantum computer (AQC).

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Plan

1. Motivation
2. Computation: spins and qubits
3. Simulation: my laptop becomes AQC

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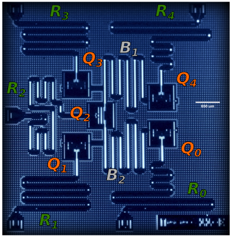
Why quantum computation?

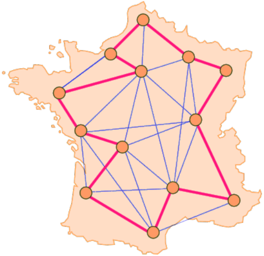
- new space for exploration:
  D-Wave company, IBM Quantum Experiance,
  IonQ, Rigetti, Quantum Inspire (Delft)
- Google achieved “quantum advantage” $\rightarrow$ extreme effort to find a practical application
- crossroad of physics and computer science



What to solve?

typically combinatorial optimization problems
$\longrightarrow$ complexity classes of solutions

- traveling salesman problem
- knapsack problem
- factorize $M = pq$ (by minimizing $E = (M - pq)^2$)



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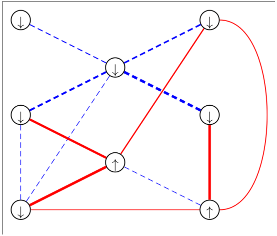
Spin-glass problem

- optimization problem related to physics of magnetism
- N spins, which can take only values ± 1 (or $\uparrow, \downarrow$)
- energy of the system is our cost function
  (increases for a pair of misplaced objects)

$$E(s_1, \ldots, s_N) = -\sum_{ij} J_{ij} s_i s_j - \sum_i h_i s_i$$



- competing links connecting spins: F or AF

Qubit – elementary building block

- states of the form: $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ (computational basis)
- important state: $|\rightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$
  random results along z, oriented along x

Spin operators:

along z-axis: $s_z|\uparrow\rangle = |\uparrow\rangle$, $s_z|\downarrow\rangle = -|\downarrow\rangle$
along x-axis: $s_x|\uparrow\rangle = |\downarrow\rangle$, $s_x|\downarrow\rangle = |\uparrow\rangle$
simple check: what is $s_x|\rightarrow\rangle =$?

Exponential curse or opportunity?

- N-qubit system is represented by 2^N-component vector
- probablitiy amplitudes do really exist (entanglement)
- make quantum system “explore” all combinatorial possibilities at once $\longrightarrow$ quantum speed up
- our focus on a particular approach AQC; different then circuit QC, but ”in theory” equivalently powerful (universal)

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Abstract engineering

TARGET: rewritten form of the spin-glass problem

$$H_1 = -\sum_{ij} J_{ij} s_i^z s_j^z - \sum_i h_i s_i^z$$

the corresponding ground state is $|\mathrm{GS}\rangle_1 = |s_1, s_2, \ldots\rangle$ where $\{s_i\}$ minimizes the target function $E(s_1, s_2, \ldots, s_N)$.

FLUCTUATIONS: most democratic state as the groundstate of

$$H_0 = -\sum_i s_i^x$$

with $|\mathrm{GS}\rangle_0 = |\rightarrow_1, \rightarrow_2, \rightarrow_3\rangle$
$= \frac{1}{\sqrt{2}^3}\left(|\uparrow,\uparrow,\uparrow\rangle + |\uparrow,\uparrow,\downarrow\rangle + |\uparrow,\downarrow,\uparrow\rangle + |\uparrow,\downarrow,\downarrow\rangle + \ldots\right)$
for $N = 3$ as an example.

Parametric dependence

Transversal field Ising model

$$H(\lambda) = (1 - \lambda)H_0 + \lambda H_1$$

where we control the “experimental” knob with $\lambda \in [0, 1]$.

Evolve slowly: from $\lambda = 0$ untill $\lambda = 1$ and read the solution!

Worst case scenario:
when ΔE close to zero, but we have to avoid exciting the system (energy gap $\Delta E(\lambda) = E_1(\lambda) - E_0(\lambda)$ for lowest eigenenergies of $H(\lambda)$).

How quickly can we run AQC?

Landau-Zener formula

$$v(\lambda) \propto (\Delta E(\lambda))^2$$

guaranties that speed is low enough.

We define
running time of adiabatic quantum computation

$$T_{\mathrm{AQC}} = \int_0^1 \frac{d\lambda}{v(\lambda)} = \int_0^1 \frac{d\lambda}{[\Delta E(\lambda)]^2}.$$

- study the scaling of T_{AQC} with N

Final remarks

- we can solve the optimization problem for $T_{\mathrm{AQC}} \leq P(N)$ (**BQP** class)
- few analytical results exist eg. database (unstructured) search $T_{\mathrm{AQC}} \propto \sqrt{2^N}$ (Grover algorithm for circuit QC)
- AQC idea useful as a sub-routine e.g. in QC studies of molecules (new VQE, QAOA approaches)
- amazing link with quantum phase transitions (QPT): $\Delta E \to 0$ for $N \to \infty$ at QPT

Our reading material may provide you with some more detailed explanation:
J. Rodriguez-Laguna and S. N. Santalla, Am. J. Phys. **86**, 360 (2018).