LAB I
Simulations of the standard map

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Plan

1. Time dependence

2. Phase space portrait

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2. Phase space portrait

Our task 1

We need to gain some intuition about the standard map.

Calculate and plot x_n and p_n as a function of the kick number n for a single trajectory following the dynamics of the map. Calculate x_n modulo 2π as it represents the angle.

Next show a two–panel plot, each panel for one trajectory (both x_n, p_n). One trajectory starts with the initial conditions $(x_0, p_o) = (3, 1.9)$, another with $(x_0, p_o) = (3, 1.8999)$, $K = 1.2$, let's plot $n < 50$.

Improve the plot by (obligatory) adding: axis labels, title, grid and legend.

what is your intuition: is the trajectory chaotic?

Our task 1 – Hints

Matplotlib plot uses arrays of coordinates as input, one may work with NumPy arrays or Python native lists.

Remember the following Matplotlib commands to adjust your plot:

```
plt.xlabel('...'); plt.ylabel('...')
plt.title('...', fontsize=..)
plt.grid(True);
plt.xlim(0,1) plt.ylim(-1,1)

plt.subplot(2,1,1)
# first plot
plt.plot(..., label='something'); plt.legend(loc='best')

plt.subplot(2,1,2)
# second plot
...

plt.savefig('trajectories.png')
```

Our task 2

Test: plot a single trajectory in the phase space (x_n, p_n) for $n < 1000$ kicks. To get a compact plot take momentum p_n modulo 2π (map is periodic).

Show: the phase space portrait filled with 100 trajectories initiated with random initial conditions. Mark each trajectory with a different (random) color. Prepare plots for $K = 1.2, 2.1, 5.5$.

Hints

```
r, g, b = np.random.random(3)     # tuple
                                  # of random colors

x0, p0 = np.random.random(2)*2*np.pi  # random initial poin

plt.plot(x, p, '.', color=(r,g,b) ) # rough pixel marks or
plt.plot(x, p, ',', color=(r,g,b))  # very detailed
                                    # pixel plot
```

Our task: extra

Implement the map for a kicked top.

A simplified version of the dynamical equations is given in https://arxiv.org/abs/1806.06184 Eq. (13). Reproduce the phase space plots Fig. 1, illustrate transition from integrable, through mixed phase space to (mostly) chaotic dynamics.