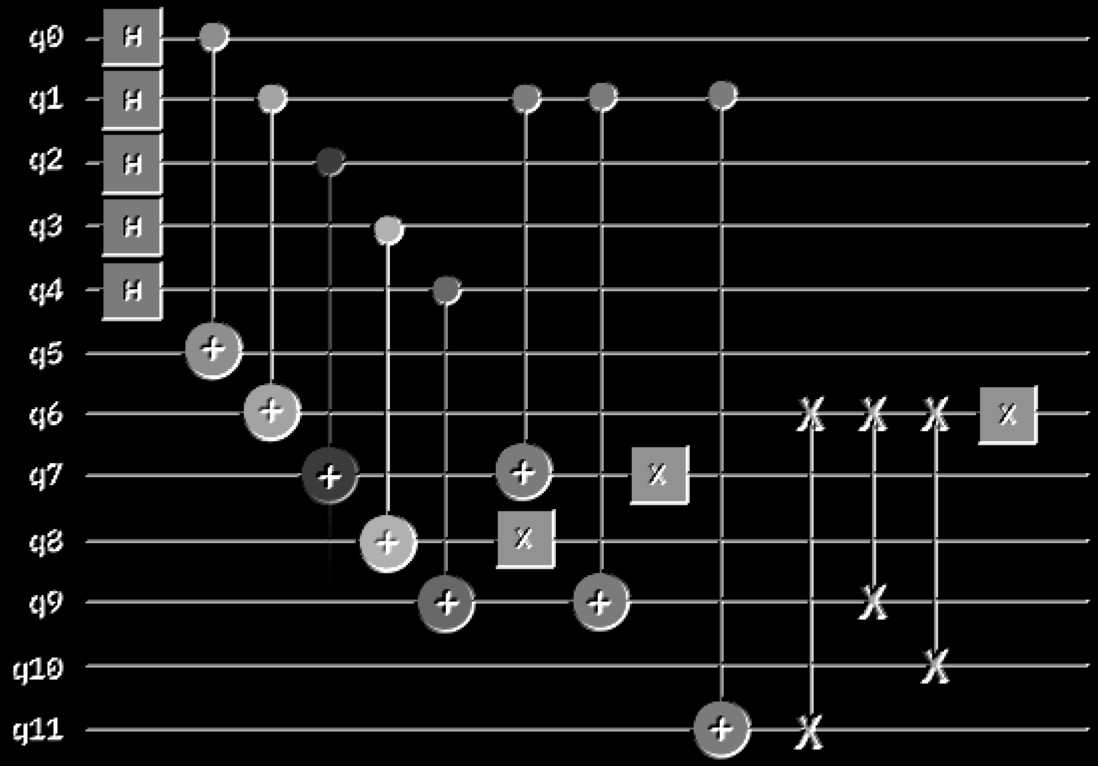
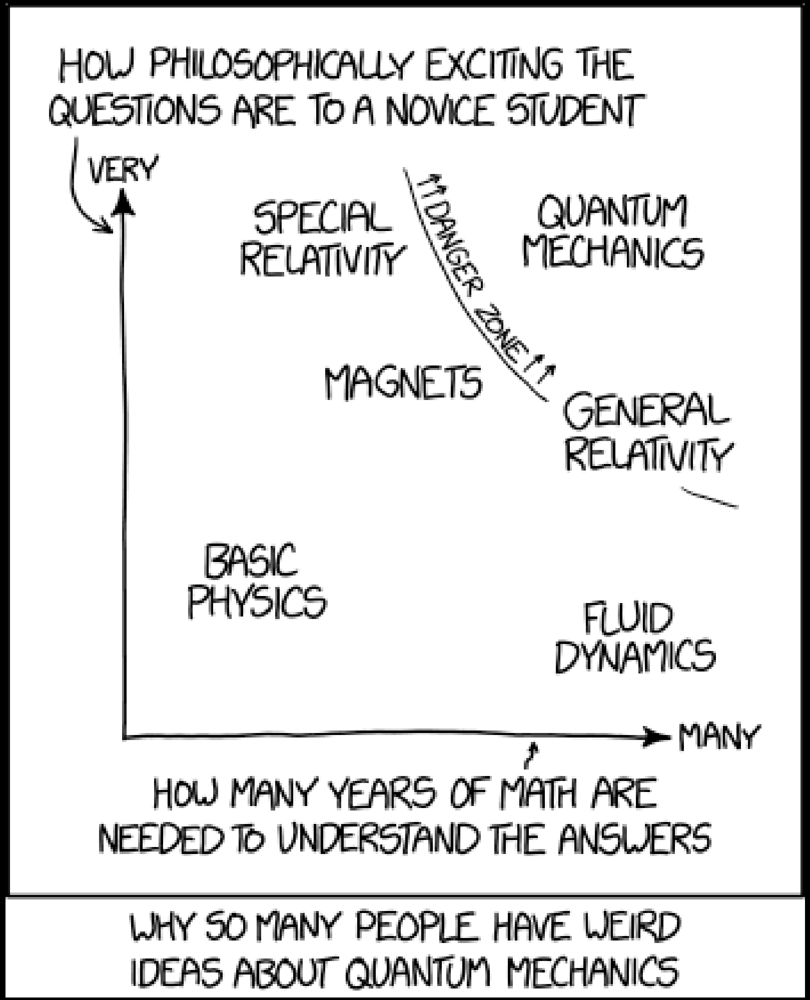
Computer modeling of physical phenomena

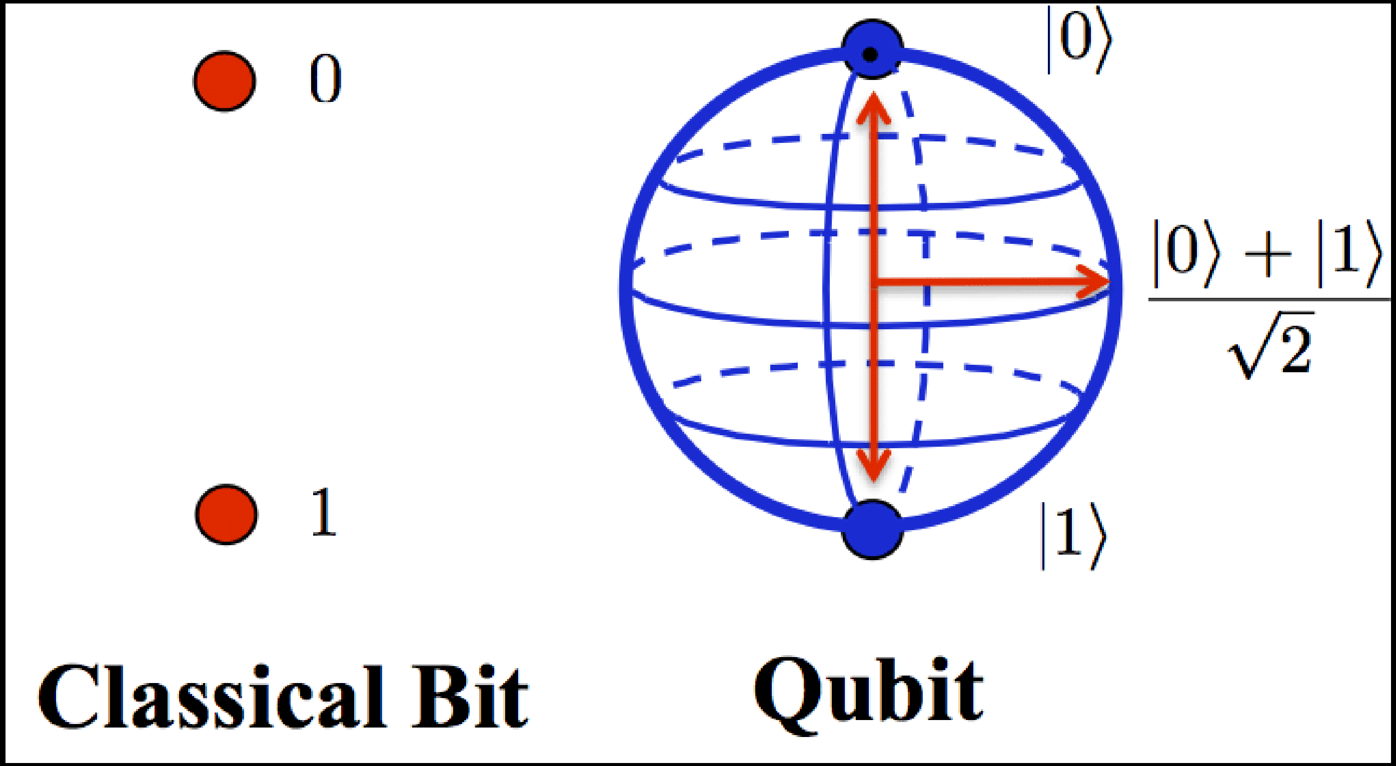


Lecture XII: Quantum Circuits

Crash course on QM



Classical bits vs quantum bits



Qubit is a basic unit of quantum information.

Qubit

basis vectors

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Psi = \alpha\,|0\rangle + \beta\,|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

Measurement

initial state

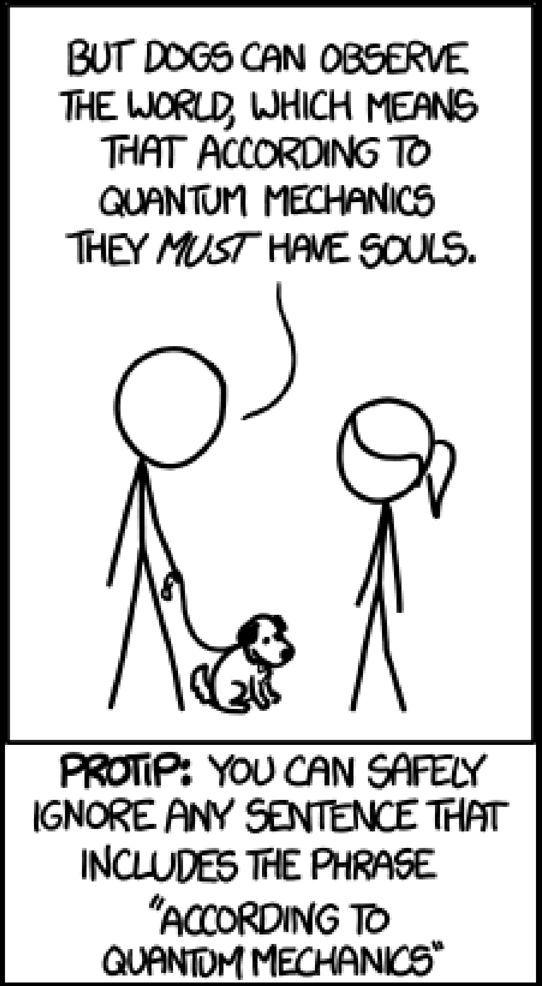
$$\Psi = \alpha\,|0\rangle + \beta\,|1\rangle$$

measurement probability

$|\alpha|^2$ $\quad$ $|\beta|^2$

system state after measurement

$|0\rangle$ $\quad$ $|1\rangle$



Quantum register

basis vectors

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Psi = \alpha\,|00\rangle + \beta\,|01\rangle + \\ + \gamma\,|10\rangle + \delta\,|11\rangle$$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

Entanglement

We consider two arbitrary quantum systems, A and B, first in state $|\psi\rangle_A$ and second in $|\phi\rangle_B$. The state of a composite system is then $|\psi\rangle_A \otimes |\phi\rangle_B$. In general, the state can be of form

$$|\psi\rangle_{AB} = \sum_{i,j} c_{i,j} |i\rangle_A \otimes |j\rangle_B$$

$|i\rangle_A$, $|j\rangle_B$ - basis vectors of each system

If we can separate $|\psi\rangle_{AB}$ into $|\psi\rangle_A = \sum_i c_i^A |i\rangle_A$ $|\phi\rangle_B = \sum_j c_j^B |j\rangle_B$ with $c_{i,j} = c_i^A c_j^B$, the state is called separable. Otherwise, it is **entangled**.

Entanglement (2)

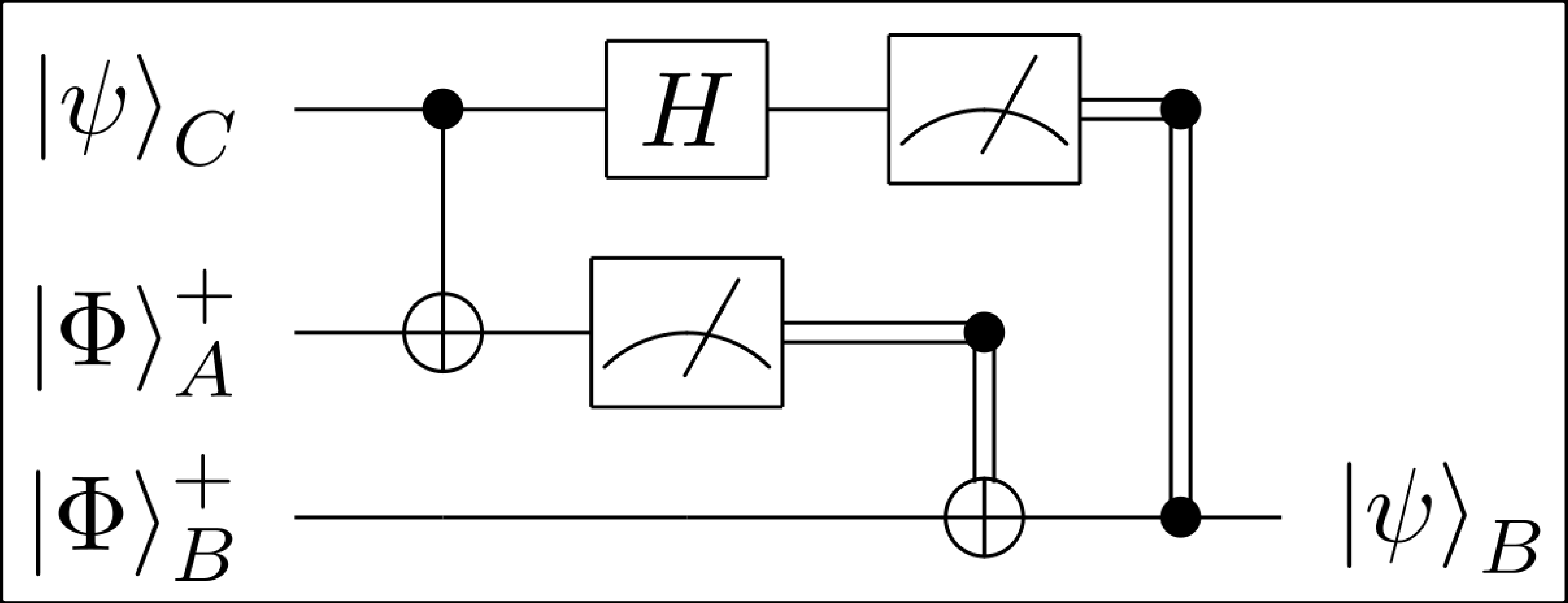
Let's consider an entangled state:

$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

Now suppose an observer in system A measures $|0\rangle_A$ - the composite system collapses then to state $|0\rangle_A \otimes |1\rangle_B$, so an observer in system B measuring his state in the same basis will always measure $|1\rangle_B$.

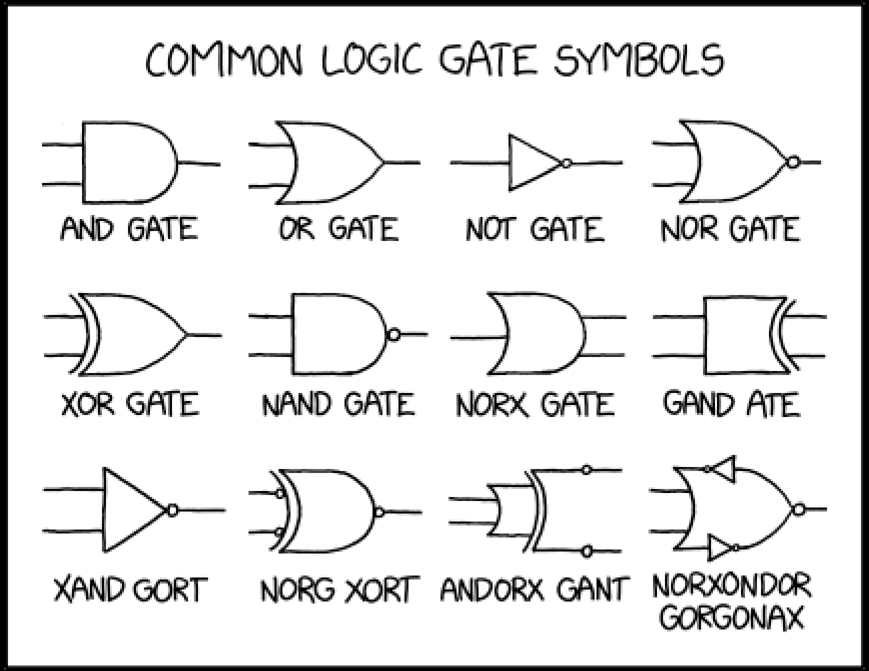
For entangled states, the measurement results are **correlated**!

Quantum Circuits



Collections of quantum gates interconnected by quantum wires capable of doing quantum computation…

Classic gates vs quantum gates



Basic gates

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{P}(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$\hat{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Multi-qubit gates

$$\widehat{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\widehat{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Prisoner’s Dilemma recap

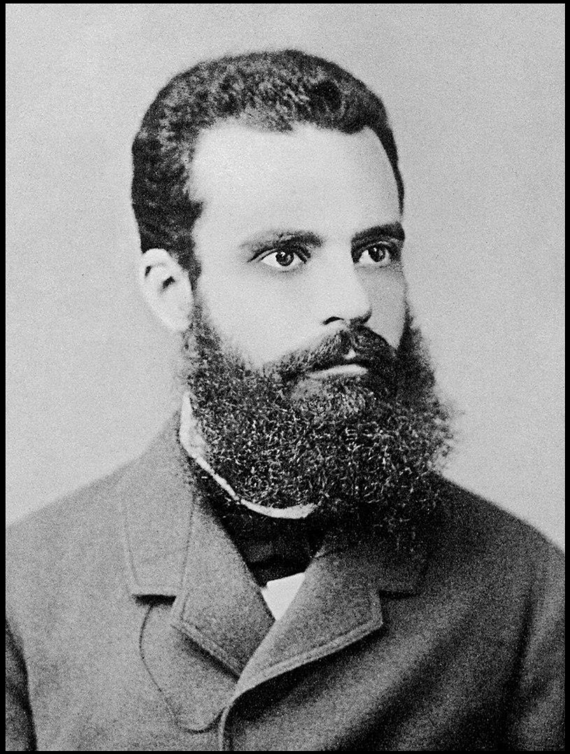
Prisoner’s Dilemma recap

Bonnie \ Clyde	cooperate	defect
cooperate	(3, 3) reward for mutual cooperation	(0, 5) sucker’s payoff and temptation to defect
defect	(5, 0) temptation to defect and sucker’s payoff	(1, 1) punishment for mutual defection

Nash equilibrium

Pareto efficiency

A situation when given an initial state, no set of strategies is available which would make one player better off without making another worse off.



Prisoner's Dilemma recap

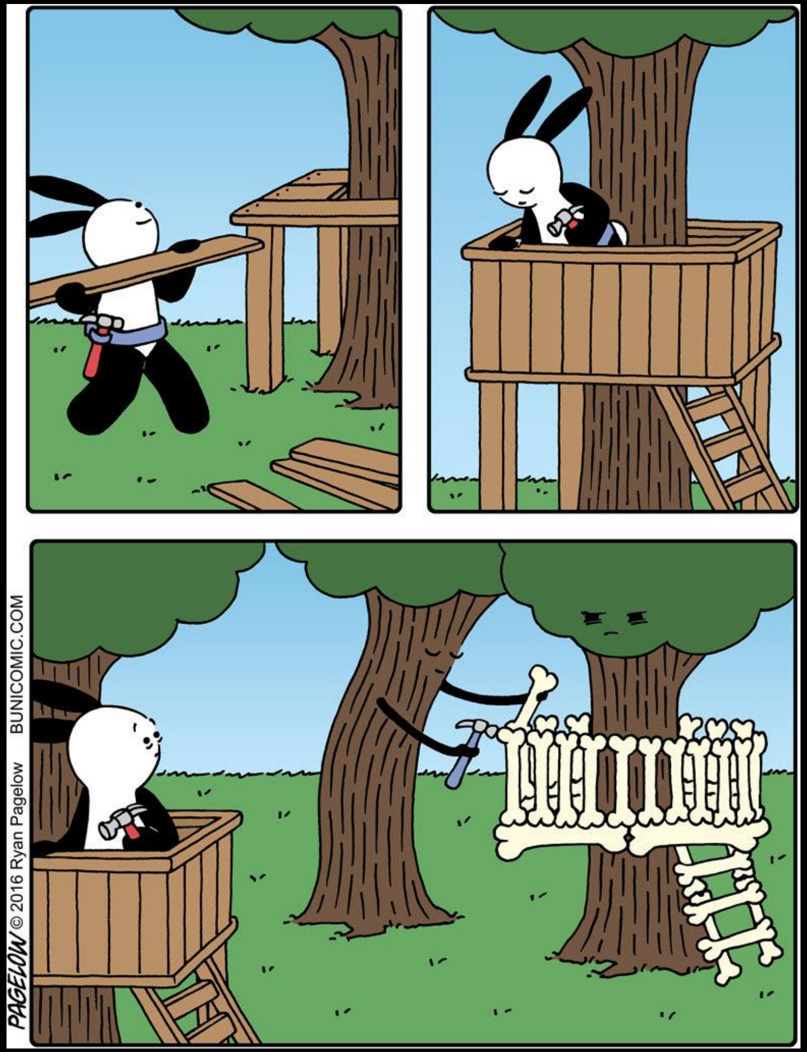
Bonnie \ Clyde	cooperate	defect
cooperate	(3, 3) reward for mutual cooperation	(0, 5) sucker's payoff and temptation to defect
defect	(5, 0) temptation to defect and sucker's payoff	(1, 1) punishment for mutual defection

Nash equilibrium is not Pareto efficient

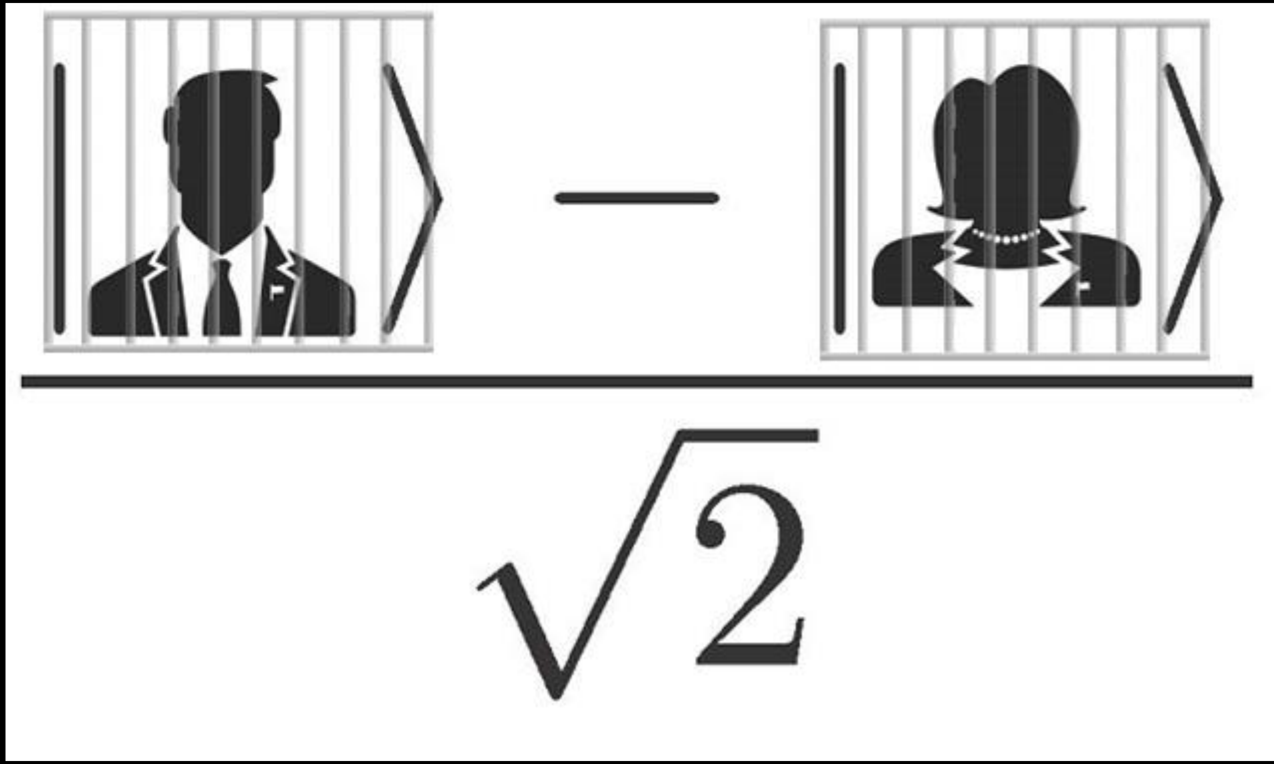
Solution: Iterated PD…

In Iterated PD, memory-based strategies do on average much better than (D, D).

- Tit-for-Tat
- Pavlov



…or Quantum PD!



QPD Intro

Bonnie and Clyde now encounter their dilemma in a quantum setup. They both start in state $|C\rangle$ and they can decide, what unitary operation they want to perform on their state. Then they measure their final states in basis $\{|C\rangle, |D\rangle\}$. Their payoff corresponds to the probability of measuring each outcome.

basis vectors

$|C\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - cooperate

$|D\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ - defect

payoffs

$$\$_1 = r\, P_{CC} + t\, P_{DC} + s\, P_{CD} + p\, P_{DD}$$

$$\$_2 = r\, P_{CC} + t\, P_{CD} + s\, P_{DC} + p\, P_{DD}$$

QPD Entanglement

In the basic scheme, Bonnie and Clyde use gates corresponding to classical decisions:

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \widehat{D} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

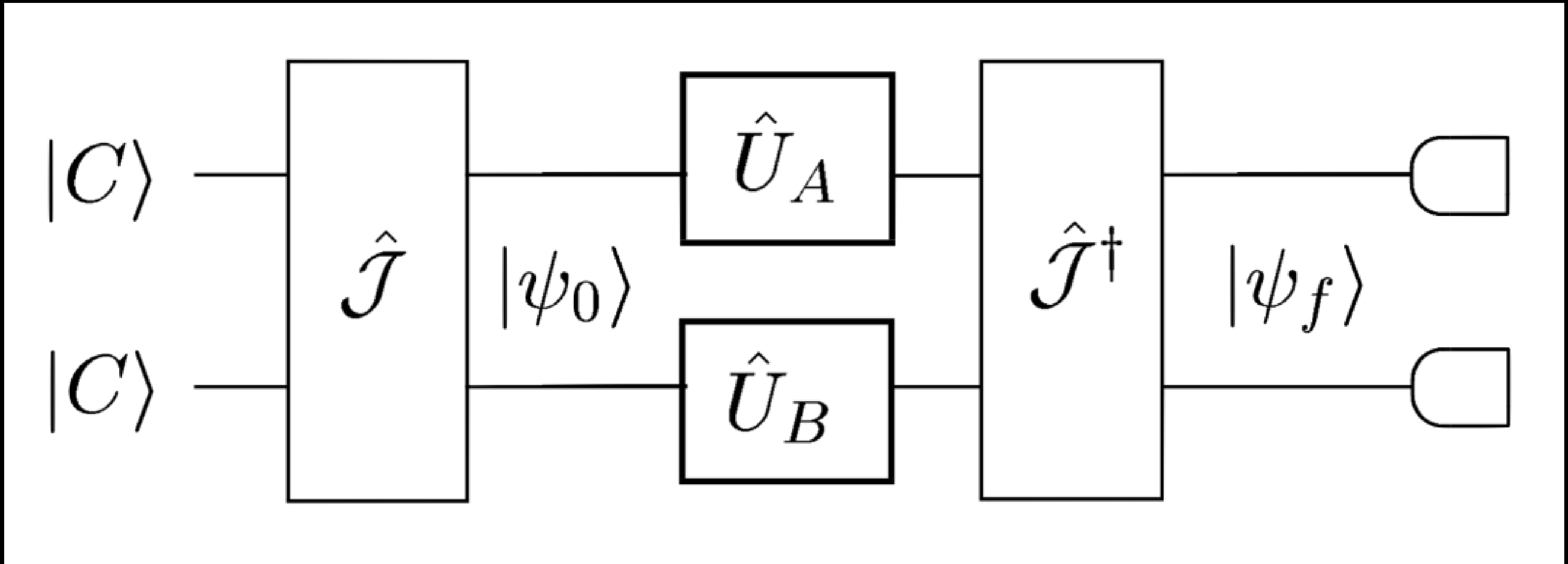
cooperate defect

Then it's like the classical version: $\widehat{D} \times \widehat{D}$ is the Nash equilibrium.

It gets interesting when the initial state is entangled – we prepare it using entanglement gate $\hat{J}$, defined as:

$$\hat{J} = \exp(-i\,\gamma\,\widehat{D} \otimes \widehat{D}\,/\,2)$$

QPD Circuit


$|C\rangle$
$|C\rangle$
$\hat{\mathcal{J}}$
$|\psi_0\rangle$
$\hat{U}_A$
$\hat{U}_B$
$\hat{\mathcal{J}}^\dagger$
$|\psi_f\rangle$

QPD Entanglement (2)

New strategies emerge! In particular, for large enough entanglement, Nash equilibrium is obtained by using strategy

$$\hat{Q} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

What is even better, this strategy is also Pareto-efficient!

Dilemma resolved!

Quantum computing…