Ans-8- Observed data $\rightarrow$

$$z_1, z_2, \ldots \quad z_{100}$$

definition of autoregressive model, with memory M -

$$\hat{z}_{t+1} = \theta_t z_t + \ldots + \theta_M z_{t-M+1}$$

$$t = M, M+1, \ldots, 100$$

(a) This can be presented as least squares as follows -

$$\min_{\theta \in \mathbb{R}^M} \|A\theta - b\|_2^2 \quad \text{with } \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix} \in \mathbb{R}^{M \times 1}$$

(b)

$$A = \begin{bmatrix} z_M & z_{M-1} & \cdots & z_1 \\ z_{M+1} & z_M & \cdots & z_2 \\ \vdots & \vdots & \ddots & \vdots \\ z_{99} & z_{98} & \cdots & z_{100-M} \end{bmatrix} \in \mathbb{R}^{(100-M) \times M} \qquad b = \begin{bmatrix} z_{M+1} \\ z_{M+2} \\ \vdots \\ z_{100} \end{bmatrix} \in \mathbb{R}^{(100-M) \times 1}$$

(c) The diagonal entries of A are all same, equal to z_M

(until $\min(100-M, M)$) $\rightarrow$ special structure of A.

(d) The rank of A = rank(A).

Memory = M.

in general

$\text{rank}(A) \leq M$

(since rank can't exceed no of cols)

For A to be full column rank,

$\text{rank}(A) = M$ (no. of columns in A = M)

no of rows $\geq$ no of columns

$$100 - M \geq M$$

$\boxed{M \leq 50}$ for A to have full column rank.

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