Ans - 4 - Consider $A \in \mathbb{R}^{m\times n}$, $A = \begin{bmatrix} | & | & & | \\ a_1 & a_2 & \cdots & a_n \\ | & | & & | \end{bmatrix}$, $a_i \in \mathbb{R}^m$

$b \in \mathbb{R}^m$; $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$, $x \in \mathbb{R}^n = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$Ax = b$

Ax = linear combination of columns of A

i.e. $Ax = x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} + \cdots + x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}$

For existence of x

$b \in \text{colspace}(A)$

$= \text{span}\{a_1, a_2, \ldots, a_n\}$

since $b = x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + \cdots + x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}$.

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For uniqueness of x

all a_i should be linearly independent

$\Rightarrow$ the span $\{a_1, a_2, \ldots, a_n\}$ has $\{a_1, a_2, \ldots, a_n\}$ as its basis

$\Rightarrow$ $\{a_1, a_2, \ldots, a_n\}$ form a basis for colspace (A).

then unique x exists, s.t. $Ax = b$.

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