**Q.1)** Consider standard unit vectors $e_1, e_2, e_3$ in $\mathbb{R}^3$. What is the trace of matrix of linear transformation which acts on $e_1, e_2, e_3$ as follows:

$$e_1 \longmapsto \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \; ; \; e_2 \longmapsto \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \; ; \; e_3 \longmapsto \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

**Q.2)** Let $a = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct rotators $Q_1$ and $Q_2$ such that $Q_2^T Q_1^T a = \begin{bmatrix} * \\ 0 \\ 0 \end{bmatrix}$.

Compute $Q = Q_2^T Q_1^T$.

Further, compute $L_{21}$ and $L_{31}$, lower triangular matrices such that

$$L_{31} L_{21} a = \begin{bmatrix} * \\ 0 \\ 0 \end{bmatrix}$$

Compute $L = L_{31} L_{21}$.

(Clearly show all the steps and final answers).

**Q.3)** Let $A \in \mathbb{R}^{3\times 2}$ be a matrix with full column rank. Let $\eta \in \mathbb{R}^{3\times 1}$ be a nonzero vector such that $\eta^T A = 0$. Then prove that for any $b \in \mathbb{R}^{3\times 1}$ such that $\eta^T b = 0$, the system of equations $Ax = b$ always has a unique solution.

**Q.4)** Let $\mathcal{L} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \ x_1 + x_2 + x_3 = 0 \right\} \subseteq \mathbb{R}^3$.

i) Prove that $\mathcal{L}$ is a subspace of $\mathbb{R}^3$.

ii) Construct a reflector $Q$ which reflects every vector of $\mathbb{R}^3$ through $\mathcal{L}$.

**Q. 5)** For $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$, check whether the following transformations are linear ?? Justify your answer in each case.

(i)
$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longmapsto \begin{bmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ x_2 \\ \frac{x_2+x_3}{2} \\ \vdots \\ \frac{x_{n-1}+x_n}{2} \\ x_n \end{bmatrix}$$

(ii)
$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \longmapsto \sum_{i=1}^{n} |x_i|$$

(iii)
$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longmapsto \begin{bmatrix} \max\{x_1, 0\} \\ \max\{x_2, 0\} \\ \vdots \\ \max\{x_n, 0\} \end{bmatrix}$$