Q.1) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. What is the distance (in the Frobenius norm) of the nearest singular matrix to A ??

Q.2) Let $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 1 & -1 \end{bmatrix}$ be the given matrix.

Let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in \mathbb{R}^2$ be such that

$$u = \underset{\|u\|_2 = 1}{\operatorname{argmax}} \|Au\|_2 .$$

Let $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \in \mathbb{R}^3$ be such that $v = Au$.

What is the value of $v_1 + v_2 + v_3$?

Q.3) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Let $\lambda_1, \lambda_2, \lambda_3$ be eigenvalues of A. What is the value of $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1\lambda_2\lambda_3$??

Q.4) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Let $\hat{x}$ be the least squares solution of $Ax = b$.

Compute $2\|\hat{x}\|_\infty - \|x\|_1$.

Q.5) Let $u, v \in \mathbb{R}^3$ be unit vectors such that $u \neq -v$.

$$A = uv^T + vu^T + uu^T + vv^T$$

What is the rank of matrix A ??

Q.6) Let $A = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & 1/3 \\ 2/3 & 0 & 1/3 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Consider the sequence $v, Av, A^2v, A^3v, \ldots$

Let $\lim_{n \to \infty} A^n v = u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$.

What is the value of $u_1 + u_2 + u_3$?