Ans 5. Given A is invertible, then columns of A are linearly independent & (if $b \in colspan(A)$) b can be expressed uniquely as $Ax = b$.

$$\therefore \boxed{x = A^{-1}b}$$

$A \in \mathbb{R}^{n \times n}$
$b \in \mathbb{R}^{n}$

computation of A^{-1} is difficult and takes large number of operations (even with LU, QR etc. decompositions)

But with A as orthogonal

then $a_i^T a_j = 0$ $(\forall i \neq j)$ and

$$A^T A = I$$

$$\Rightarrow A^T = A^{-1}$$

Hence to obtain A^{-1}, we just need to find A^T which then leads to fewer computation while finding x.

$$x = A^{-1} b \Rightarrow \boxed{x = A^T b}$$

The advantage of A being orthogonal, is that we can explicitly calculate A^{-1}, and computation to find x reduces significantly.

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