Ans-6 - Any matrix X, which satisfies $XA = I$, for a given matrix A is called the left inverse of A. (7)

Left inverse exists when the columns of A are independent. Since A in both (a) & (b) below satisfy above condition, thus left inverse exists.

(a) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}_{5\times 1}$ $\quad X = \begin{bmatrix} 2 & 0 & 0 & -1 & 0 \end{bmatrix}^T$

X is a left inverse of A.

The characterization of all such X would be

$$X = \begin{bmatrix} a_1 & a_2 & a_3 & (1-a_1) & a_4 \end{bmatrix}^T$$

$$X^T A = [1] = I_{1\times 1}$$

(b) $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix}_{3\times 2}$ $\quad X = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & -1/2 & 0 \end{bmatrix}_{2\times 3}$

$$XA = I$$

Let $X = \begin{bmatrix} p & q & r \\ a & b & c \end{bmatrix}$

$$XA = \begin{bmatrix} 2p + 3r & -2q + 3r \\ 2a + 3c & -2b + 3c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a = -\frac{3c}{2}$ — (1) $\qquad \frac{3}{2} r = q$ — (2)

$2p = 1 - 3r \qquad 3c - 1 = 2b$

$p = \frac{1-3r}{2}$ — (3) $\qquad b = \frac{3c-1}{2}$ — (4)

$$X = \begin{bmatrix} \frac{1-3r}{2} & \frac{3r}{2} & r \\ -\frac{3c}{2} & \frac{3c-1}{2} & c \end{bmatrix}$$

is a general characterization of left inverse of A.