Ans-5 - Consider the matrices $A \in \mathbb{R}^{p\times q}$, $B \in \mathbb{R}^{q\times n}$, $C \in \mathbb{R}^{n\times t}$

Commutativity

$AB = BA$ is not possible iff $n \neq p$

Now consider $A \in \mathbb{R}^{p\times q}$ $B \in \mathbb{R}^{q\times p}$

order$(AB) = p\times p$ order$(BA) = q\times q$

$AB = BA$ is not possible iff $p \neq q$

Now consider $A \in \mathbb{R}^{p\times p}$ $B \in \mathbb{R}^{p\times p}$ $A = [a_{ij}]_{p\times p}$

$$AB = \left[\sum_{k=1}^{p} a_{ik} b_{kj}\right]$$ $B = [b_{ij}]_{p\times p}$

$$BA = \left[\sum_{k=1}^{p} b_{ik} a_{kj}\right]$$

Now $\sum_{k=1}^{p} a_{ik} b_{kj} \neq \sum_{k=1}^{p} b_{ik} a_{kj}$ (generally)

thus $[AB]_{ij} \neq [BA]_{ij}$

Hence not commutative.

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Associativity

$$(AB)C = \left[\sum_{k=1}^{q} a_{ik} b_{kj}\right] \cdot C.$$

$$= \left[\sum_{m=1}^{n}\sum_{k=1}^{q} a_{ik} b_{km} c_{mj}\right] \leftarrow ij^{th} \text{ element of } (AB)C$$

$$A(BC) = A \cdot \left[\sum_{m=1}^{n} b_{im} \cdot c_{mj}\right]$$

$$= \left[\sum_{m=1}^{n}\sum_{k=1}^{q} a_{ik} b_{km} c_{mj}\right] \leftarrow ij^{th} \text{ element of } A(BC)$$

since $[A(BC)]_{ij} = [(AB)C]_{ij}$

Thus matrix multiplication is associative

computations required in $(AB)C = pqn + pnt$

$= pn(q+t) = pn(q+t)$

computations required in $A(BC) = pqt + qnt$

$= qt(p+n)$

for $(AB)C$ to be faster than $A(BC)$

we need $\boxed{pn(q+t) < qt(p+n)}$.

we can have both p and n as small as possible i.e. A and C are ~~tall~~ wide matrices