Indian Institute of Technology Kharagpur
Department of Computer Science and Engineering

Test 1

CS41001: Theory of Computation — 7th of September, 2021
Duration = 90 minutes — Total Marks = 50

Answer all questions. State all assumptions you make. Keep your answers concise.
Write on paper, then scan and submit a single pdf file.

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1. Consider the languge $\{(\mathcal{M}, x, p) | \mathcal{M}$ on input x visits state p during the computation$\}$. (Here, $p \in Q$ with Q being the set of states of $\mathcal{M}$ and $x \in \Sigma^*$ where Σ is the input alphabet of the Turing machine $\mathcal{M}$.) Is this language decidable? Justify. $\boxed{10}$

2. For a set $A \subseteq \Sigma^*$, define $A^{\mathbf{R}} = \{w^{\mathbf{R}} \mid w \in A\}$ where $w^{\mathbf{R}}$ denotes w reversed. Is it decidable for a given TM $\mathcal{M}$ whether $L(\mathcal{M}) = L(\mathcal{M})^{\mathbf{R}}$? Justify your answer. $\boxed{10}$

3. Show that given a CFG G, it is undecidable whether
   (a) $L(G) = L(G)L(G)$. (For a set A, $AA = \{xy \mid x, y \in A\}$, where xy denotes concatenation of x and y). $\boxed{4}$
   (b) G is ambiguous. (A grammar G is ambiguous if there exists a string in $L(G)$ with two different derivations in G.)
   Hint: Use PCP. $\boxed{6}$

4. Are the following variants of PCP decidable? Justify.
   (a) PCP over unary alphabet 1. That is, given two lists $A = \{w_1, w_2, \ldots, w_k\}$, $B = \{x_1, x_2, \ldots, x_k\}$ of strings from 1^+, does there exist a sequence $i_1, i_2, \ldots, i_r$ of integers from $\{1, 2, \ldots, k\}$ such that $w_{i_1} w_{i_2} \cdots w_{i_r} = x_{i_1} x_{i_2} \cdots x_{i_r}$? $\boxed{5}$
   (b) Given two lists $A = \{w_1, w_2, \ldots, w_k\}$, $B = \{x_1, x_2, \ldots, x_k\}$ of strings from Σ^+, the problem is to determine whether there exist two sequences $i_1, i_2, \ldots, i_r$ and $j_1, j_2, \ldots, j_s$ such that $w_{i_1} w_{i_2} \cdots w_{i_r} = x_{j_1} x_{j_2} \cdots x_{j_s}$. $\boxed{5}$

5. A Turing machine $\mathcal{M}$ is minimal if it has the fewest states among all TMs that accept $L(\mathcal{M})$. Prove that there does not exist an infinite *r.e.* set of minimal TMs.
   Hint: Roger's fixed point theorem. $\boxed{10}$