भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology
Hyderabad

Quiz 3 EE 1390

Introduction to AI-ML

February 21, 2019
Duration: 30 min
Max. Marks 10

- Step by step solution has to be provided for all questions.

1. The multivariate Gaussian distribution is defined as

$$p_{\mathbf{x}}(x_1, \dots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1} (\mathbf{x}-\mu)\right\} \tag{1}$$

where μ is the mean vector, $\Sigma = E\left[(\mathbf{x}-\mu)(\mathbf{x}-\mu)^T\right]$ is the covariance matrix and $|\Sigma|$ is the determinant of Σ.

2. Show that

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)} \times \left\{\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{\left(y-\mu_y\right)^2}{\sigma_y^2} - \frac{2\rho(x-\mu_x)\left(y-\mu_y\right)}{\sigma_x\sigma_y}\right\}\right] \tag{2}$$

where

$$\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \Sigma = \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \tag{3}$$

3. Let

$$\mathbf{s}_0 = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{4}$$

$$\mathbf{s}_1 = \begin{pmatrix} 0 \\ a \end{pmatrix} \tag{5}$$

If

$$\mathbf{x} = \mathbf{s} + \mathbf{n} \tag{6}$$

where $\mathbf{s} \in \{\mathbf{s}_0, \mathbf{s}_1\}$ and $\mathbf{n} \sim \mathcal{N}\left(0, \sigma^2\mathbf{I}\right)$. Find

$$p_{\mathbf{x}|\mathbf{s}_0}(\mathbf{x}) \tag{7}$$

$$p_{\mathbf{x}|\mathbf{s}_1}(\mathbf{x}) \tag{8}$$

4. How will you decide between $\mathbf{s}_0$ and $\mathbf{s}_1$ if you have $\mathbf{x}$