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Q1 If ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are satisfiable then

$\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4 = T$ for some values of x_1, x_2, x_4

$$\Rightarrow (x_1 \vee x_2 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee x_4) \wedge (\neg x_2)$$

$$\equiv (x_1 \vee x_2 \vee \neg x_4) \wedge (\neg x_1 \vee x_2 \vee \neg x_4) \wedge (\underbrace{(x_2 \wedge \neg x_2)}_{\Downarrow \atop F} \vee (x_4 \wedge \neg x_2))$$

(By Distributive Law)

$$\Rightarrow (x_1 \vee (x_2 \vee \neg x_4)) \wedge (\neg x_1 \vee (x_2 \vee \neg x_4)) \wedge (x_4 \wedge \neg x_2)$$

$$\Rightarrow \underbrace{(x_1 \wedge \neg x_1)}_{\Downarrow \atop F} \vee (x_2 \vee \neg x_4) \wedge (x_4 \wedge \neg x_2)$$

$$\Rightarrow (x_2 \vee \neg x_4) \wedge (x_4 \wedge \neg x_2) \equiv (x_2 \vee \neg x_4) \wedge (\neg (x_2 \vee \neg x_4))$$

$$\equiv F$$

$$\therefore \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4 \equiv F \not\equiv T$$

Hence proved by contradiction
The set of clauses are not satisfiable

Q2 i) Let $X \vee \phi_1$, $\neg X \vee \phi_2$ be satisfiable

i.e $(X \vee \phi_1) \wedge (\neg X \vee \phi_2) \equiv \text{True}$ — ①

We know that either X or $\neg X$ is true.

So, without loss of generality

Let X be true

$\therefore$ Eq ① reduces to ϕ_2

$\therefore$ ϕ_2 must be true

Similarly let X be false

Now we can see that ϕ_1 must be true

$\therefore$ It can be clearly be seen that in both the case $\phi_1 \vee \phi_2$ is satisfiable

Now let $\phi_1 \vee \phi_2$ be satisfiable i.e $\phi_1 \vee \phi_2 \equiv T$ — ②

So to show that $X \vee \phi_1$, $\neg X \vee \phi_2$ are satisfiable given Eq ②, we will show that there always exists a value of X such that

$(X \vee \phi_1) \wedge (\neg X \vee \phi_2) \equiv T$

So

ϕ_1	ϕ_2	$\phi_1 \vee \phi_2$	$(x \vee \phi_1) \wedge (\neg x \vee \phi_2)$
T	F	T	T if x = False
F	T	T	T if x = T
T	T	T	T

$\therefore$ ~~we can~~ So $x \vee \phi_1$, $\neg x \vee \phi_2$ is satisfiable

when $\phi_1 \vee \phi_2$ is satisfiable

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Q 2

ii) Let $X = X_1$

$\phi_1 = \neg X_2 \vee X_3 \vee X_4$

$\phi_2 = \neg X_2 \vee \neg X_3$

$\therefore (X_1 \vee \neg X_2 \vee X_3 \vee X_4) \wedge (\neg X_1 \vee \neg X_2 \vee \neg X_3)$

$\equiv (X \vee \phi_1) \wedge (\neg X \vee \phi_2) \quad — ①$

For ① to be satisfiable, sufficient condition is

$\boxed{\phi_1 \vee \phi_2 \equiv T}$

i.e $(\neg X_2 \vee X_3 \vee X_4) \vee (\neg X_2 \vee \neg X_3) \equiv T$

$\neg X_2 \vee X_4 \equiv T$

$\therefore$ Solutions are

X_1, X_3	X_2	X_4
T, F	T	T
T, F	F	T
T, F	F	F

Q3 Let pMin, pMax be min and max probabilities of C

A	B	C	$(A \vee B) \to C$	Probability
1	1	1	1	x_1
1	1	0	0	x_2
1	0	1	1	x_3
1	0	0	0	x_4
0	1	1	1	x_5
0	1	0	0	x_6
0	0	1	1	x_7
0	0	0	1	x_8

$P(C=True) = x_1 + x_3 + x_5 + x_7$

$\therefore$ pMin = Min(P(C=True))

pMax = Max(P(C=True))

Solutions

① For Min : [0.7, 0, 0, 0.1, 0, 0, 0, 0.2]

$\Rightarrow$ pMin = 0.7 + 0 + 0 = 0.7

② For Max : [0.4, 0.1, 0.3, 0, 0.2, 0, 0, 0]

$\Rightarrow$ pMax = 0.4 + 0.3 + 0.2 = 0.9

Q4 Given any probabilistic reasoning problem which can be constructed into following LP problem

$$\min f^T x \text{ such that } Ax \leq b,$$

we can show that $x_i \geq 0$ and $x_i \leq 1 \ \forall i$

where x_i is probability

Proof: In any LP formulation of probabilistic reasoning, we can represent it as

$$\begin{bmatrix} A \\ -A \end{bmatrix} p \leq \begin{bmatrix} b \\ -b \end{bmatrix}$$

For solving this LP problem, we constraint matrix by adding the following

① $\sum_i p_i \leq 1$ and $\sum_i p_i \geq 1 \Rightarrow \sum_i p_i = 1 \ \forall i$

② $-\sum_i p_i \leq 0 \ \forall i \Rightarrow \sum_i p_i \geq 0$

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$\therefore$ From ① & ② we have

$$\sum_{i=1}^{n} p_i = 1 \Rightarrow p_i = 1 - \sum_{j \neq i}^{n} p_j \quad -③$$

$$\text{Max } p_i = \text{Max}\left(1 - \sum_{j \neq i}^{n} p_j\right)$$

$$\text{Max } p_i = 1 - \min \sum_{j \neq i}^{n} p_j$$

Now since ~~all~~ $p_j \geq 0$ ~~and~~ $\forall j$

$$\min p_j \geq 0 \Rightarrow \min \sum_{i \neq j}^{n} p_j \geq 0$$

$$\therefore \text{Max } p_i \leq 1$$

$$\therefore \text{Max } p_i \leq 1 \text{ and } \text{Min } p_i \geq 0 \quad \forall i$$

$$p_i \in [0, 1], \text{ i.e } 0 \leq p_i \leq 1$$

$\therefore$ Hence LP formulation of probabilistic reasoning never leads to absurd conclusions. related to probability.