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Quiz 2 EE 1390

Introduction to AI-ML

February 21, 2019
Duration: 30 min
Max. Marks 10

- Step by step solution has to be provided for all questions.

1. Find the equation of the plane P containing the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \tag{1}$$

2. Show that the vector

$$\mathbf{y} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

lies outside P.

3. Find the point $\mathbf{w} \in P$ closest to $\mathbf{y}$.

4. Show that

$$\|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 = \|\mathbf{y}\|^2 - \mathbf{w}^T\mathbf{X}^T\mathbf{y} \tag{3}$$
$$- \mathbf{y}^T A\mathbf{w} + \mathbf{w}^T\mathbf{X}^T\mathbf{X}\mathbf{w} \tag{4}$$

5. Assuming 2×2 matrices and 2×1 vectors, show that

$$\frac{\partial}{\partial \mathbf{w}}\mathbf{w}^T\mathbf{X}^T\mathbf{y} = \frac{\partial}{\partial \mathbf{w}}\mathbf{y}^T\mathbf{X}\mathbf{w} = \mathbf{y}^T\mathbf{X} \tag{5}$$

6. Show that

$$\frac{\partial}{\partial \mathbf{w}}\mathbf{w}^T\mathbf{X}^T\mathbf{X}\mathbf{w} = 2\mathbf{w}^T\left(\mathbf{X}^T\mathbf{X}\right) \tag{6}$$

7. Show that

$$\hat{\mathbf{w}} = \min_{\mathbf{w}} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 \tag{7}$$
$$= \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{y} \tag{8}$$

8. Let

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 & \mathbf{x}_2 \end{pmatrix}. \tag{9}$$

from (1). Verify (7).