JEE problem in python

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The question

A hyperbola passes through the point

$$\vec{P} = (\sqrt{2}, \sqrt{3}) \tag{1}$$

and has foci at $(\pm 2, 0)$. Find the equation of the tangent to this hyperbola at $\vec{P}$.

Approach

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We also know that,

$$b^2 = a^2 * (e^2 - 1)$$

Approach

Each point is represented as an numpy array.

A hyperbola of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

in matrix form is represented as:

$$x^T * V * x = c$$

which is same as:

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1/a^2 & 0 \\ 0 & -1/b^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

Approach

The hyperbola is:

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1/a^2 & 0 \\ 0 & -1/b^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

And the point form of tangent at point $P(x_1, y_1)$ for the above hyperbola is:

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1/a^2 & 0 \\ 0 & -1/b^2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$

Graphical Verification