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Quiz 1 EE 1390

Introduction to AI-ML

February 18, 2019
Duration: 45 min
Max. Marks 10

- Step by step solution has to be provided for all questions.

1) Let $\mathbf{R}$ be the reflection of $\mathbf{P}$ about the line
$$L: \quad \mathbf{n}^T\mathbf{x} = c \tag{1}$$
If $PQ \perp L$, show that
$$2\mathbf{Q} = \mathbf{P} + \mathbf{R} \tag{2}$$
$$\mathbf{n}^T\mathbf{Q} = c \tag{3}$$
$$\mathbf{m}^T\mathbf{R} = \mathbf{m}^T\mathbf{P} \tag{4}$$
where $\mathbf{m}$ is the direction vector of L.

2) Show that
$$\begin{pmatrix}\mathbf{m} & \mathbf{n}\end{pmatrix}^T \mathbf{R} = \begin{pmatrix}\mathbf{m} & -\mathbf{n}\end{pmatrix}^T \mathbf{P} + \begin{pmatrix}0\\2c\end{pmatrix} \tag{5}$$

3) Letting
$$\mathbf{V} = \begin{pmatrix}\mathbf{m} & \mathbf{n}\end{pmatrix} \tag{6}$$
with the condition that $\mathbf{m}, \mathbf{n}$ are orthonormal, show that
$$\mathbf{V}^T\mathbf{V} = \mathbf{I} = \mathbf{V}\mathbf{V}^T \tag{7}$$

4) Show that
$$\begin{pmatrix}\mathbf{m} & -\mathbf{n}\end{pmatrix} = \begin{pmatrix}\mathbf{m} & \mathbf{n}\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \tag{8}$$

5) Show that
$$\mathbf{R} = \mathbf{V}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{V}^T\mathbf{P} + 2c\mathbf{n} \tag{9}$$

6) Show that
$$\frac{\mathbf{R}}{2} = \frac{\mathbf{m}\mathbf{m}^T - \mathbf{n}\mathbf{n}^T}{\mathbf{m}^T\mathbf{m} + \mathbf{n}^T\mathbf{n}}\mathbf{P} + c\mathbf{n} \tag{10}$$
when $\mathbf{m}, \mathbf{n}$ are orthogonal.

7) Find the equation of the circle, which is the mirror image of the circle
$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}2 & 0\end{pmatrix}\mathbf{x} = 0 \tag{11}$$
in the line
$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 3. \tag{12}$$

8) For the previous problem, find
$$\mathbf{A} = \mathbf{V}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\mathbf{V}^T \tag{13}$$

9) Let λ be such that
$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \tag{14}$$
Find λ and $\mathbf{x}$.

10) Comment.