

难题集锦 A

1. If j and k are even integers and $j < k$, which of the following equals the number of even integers that are greater than j and less than k ?

- A. $(k-j-2)/2$
- B. $(k-j-1)/2$
- C. $(k-j)/2$
- D. $k-j$
- E. $k-j-1$

2. $m=10^{32} + 2$, when m is divided by 11, the remainder is r .

Quantity A r

Quantity B 3

3. In a single line of people waiting to purchase tickets for a movie, there are currently 10 people behind Shandra. If 3 of the people who are currently in line ahead of Shandra purchase tickets and leave the line, and no one else leaves the line, there will be 8 people ahead of Shandra in line. How many people are in the line currently?

4. A and B are independent events, and the probability that both events occur is $1/2$. Which of the following could be the probability that event A occurs? Indicate all such probabilities.

- A. 0
- B. $1/4$
- C. $1/2$
- D. $3/4$
- E. 1

5. How many positive integers less than 10,000 are such that the product of their digits is 210?

- A: 24
- B: 30
- C: 48
- D: 54
- E: 72

6. From a group of 8 people, it is possible to create exactly 56 different k -person committees. Which of the following could be the value of k ? Indicate all such values.

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5
- F. 6
- G. 7

7. Yesterday's closing prices of 2,420 different stocks listed on a certain stock exchange were all different from today's closing prices. The number of stocks that closed at a higher price today than yesterday was 20 percent greater than the number that closed at a lower price. How many of the stocks closed at a higher price today than yesterday?

- A. 484
- B. 726
- C. 1,100
- D. 1,320
- E. 1,694

8. Of the students in a school, 20 percent are in the science club and 30 percent are in the band. If 25 percent of the students in the school are in the band but are not in the science club, what percent of the students who are in the science club are not in the band?

- A. 5%
- B. 20%
- C. 25%
- D. 60%
- E. 75%

9. The company at which Mark is employed has 80 employees, each of whom has a different salary. Mark's salary of \$43,700 is the second-highest salary in the first quartile of the 80 salaries. If the company were to hire 8 new employees at salaries that are less than the lowest of the 80 salaries, what would Mark's salary be with respect to the quartiles of the 88 salaries at the company, assuming no other changes in the salaries?

- A. The fourth-highest salary in the first quartile
- B. The highest salary in the first quartile
- C. The second-lowest salary in the second quartile
- D. The third-lowest salary in the second quartile
- E. The fifth-lowest salary in the second quartile

10. x is a positive integer. When x is divided by 2, 4, 6, 8, the remainder is 1.

- Quantity A x
Quantity B 24

11. Which of the following could be the sum of three consecutive integers?

- A. 29
- B. 46
- C. 57
- D. 92
- E. 100

12. $x^a y^b z^c$ equals the product of 154 and 56, $z > y > x$ and $a > b > c$, then what is the value of $a^x b^y c^z$?

- A. 1024
- B. 2048
- C. 8624
- D. 22528
- E. cannot be determined by the condition given

13. If $1 \leq n \leq 100$, and $\frac{n+7}{2}$ is a multiple of 4 but not a multiple of 3, then which of the following could be true?

Indicate all possible values.

- A. n is even
- B. n is odd
- C. n is prime
- D. n is a multiple of 3
- E. n is a multiple of 4

14. f, g, h are consecutive prime numbers such that $f < g < h$

Quantity A	Quantity B
$f + g + h$	$3g$

15. k is a positive integer and 225 and 216 are both divisors of k . If $k = 2^a 3^b 5^c$, where a, b and c are positive integers, what is the least possible value of $a + b + c$?

- A. 4
- B. 5
- C. 6
- D. 7
- E. 8

16. x is an integer greater than 3.

Quantity A	the number of even factors of $2x$
Quantity B	the number of odd factors of $3x$

17. In how many ways can 16 different gifts be divided among four children such that each child receives exactly four gifts?

- A. 16^4
- B. $(4!)^4$
- C. $16! / (4!)^4$
- D. $16! / 4!$
- E. 4^{16}

18. If $8^{n+1} + 8^n = 36$, then $n =$

- A. $1/3$
- B. $1/2$
- C. $3/5$
- D. $2/3$
- E. $4/5$

19. If 55 percent of a group of people have brown hair and 80 percent of the same group do not have red hair, what's the fraction of those who do not have brown hair have red hair?

- A. $1/4$
- B. $4/11$
- C. $4/9$
- D. $5/9$
- E. $4/5$

20. Last year Melania had a total of \$20000 invested in two mutual funds, Capital Growth Fund and Venture Index Fund. At the end of the year, she analyzed her investments and found that her earnings on her shares of Capital Growth Fund were three times half of her earnings on her investment in Venture Index Fund. If she earned a total of \$1250 on her investments in the two funds, and had three times as much money invested in Capital Growth Fund as in Venture Index Fund, what percent interest did Melania earn on her investment in Venture Index Fund?

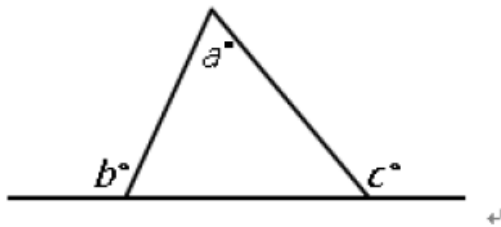
- A. 0.075
- B. 0.01
- C. 7.5
- D. 10
- E. 500

21. The function f is defined by $f\left(\frac{x+2}{2}\right) = 3x^2 - x + 5$

Quantity A: $f(4)$

Quantity B: 75

22.



Quantity A: $180 + a$

Quantity B: $b + c$

23. The average of three different positive integers is 6.

Quantity A The product of the three integers

Quantity B 25

24. List K consists of 16 positive numbers. List M is obtained from list K by multiplying each number in list K by -1

Quantity A The standard deviation of K

Quantity B The standard deviation of M

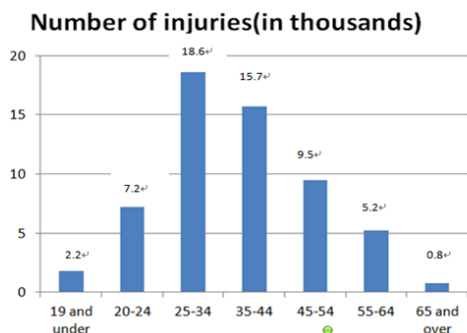
25-26 are based on the graph given.

25. How many of the age-groups each accounted for more than 15 percent of the total number of occupational injuries in State X in 1998?

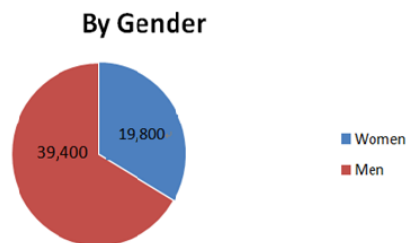
A. One B. Two C. Three D. Four E. Five

NUMBER OF OCCUPATIONAL INJURIES IN STATE X, 1998¹

By Age-Group



By Gender²



26. In 1998, if one-half of the occupational injuries in the combined 34-and under age-groups were incurred by men, what was the number of occupational injuries incurred by men in the combined 35-and-over age-groups?

A. 33,500 B. 31,900 C. 30,500 D. 25,400 E. 21,700

27. For the 55-64 age-group in 1998, the average (arithmetic mean) number of work-hours lost per occupational injury was 48.5. If a workweek is 40 work-hours, which of the following is closest to the total number of workweeks lost due to occupational injuries in the 55-64 age-group in 1998?

A. 4500 B. 5200 C. 5500 D. 5900 E. 6300

28. In a graduating class of 236 students, 142 took algebra and 121 took chemistry. What is the greatest possible number of students that could have taken both algebra and chemistry?

29. P, Q, and R are three points in a plane, and R does not lie on line PQ. Which of the following is true about the set of all points in the plane that are the same distance from all three points?

A. It contains no points.
 B. It contains one point.
 C. It contains two points.
 D. It is a line.
 E. It is a circle.

30. The random variable X is normally distributed. The values 650 and 850 are at the 60th and 90th percentiles of the distribution of X, respectively.

Quantity A The value at the 75th percentile of the distribution of X

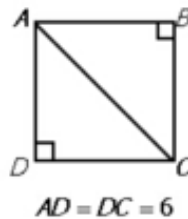
Quantity B 750

31. In a probability experiment, G and H are independent events. The probability that G will occur is r , and the probability that H will occur is s , where both r and s are greater than 0.
 Quantity A the probability that either G will occur or H will occur, but not both
 Quantity B $r+s-rs$

32. Quantity A The number of primes that are divisible by 9
 Quantity B The number of primes that are divisible by 19

33. What is the sum of all possible solutions of the equation $|x+4|^2 - 10|x+4| + 24 = 0$
 A. -16
 B. -14
 C. -12
 D. -8
 E. -6

34.
 Quantity A AB
 Quantity B BC



35. If k is the greatest positive integer such that 3^k is a divisor of $15!$ then $k =$
 A. 3
 B. 4
 C. 5
 D. 6
 E. 7

36. Sid intended to type a seven-digit number, but the two “3” he meant to type did not appear. What appeared instead was the five-digit number 52115. How many different seven-digit numbers could Sid have meant to type?
 A. 10
 B. 16
 C. 21
 D. 24
 E. 27

37. How many integers between 1 and 10^{21} are such that the sum of their digits is 2?
 A. 190
 B. 210
 C. 211
 D. 230
 E. 231

38. At a certain university, 60% of the professors are women, and 70% of the professors are tenured. If 90% of the professors are women, tenured, or both, then what percent of the men are tenured?

- A. 25
- B. 37.5
- C. 50
- D. 62.5
- E. 75

39. A popular website requires users to create a password consisting of digits only. If no digit may be repeated and each password must be at least 9 digits long, how many passwords are possible?

- A. $9! + 10!$
- B. $2 \times 10!$
- C. $9! \times 10!$
- D. $10!$
- E. $20!$

40. W, X, Y and Z each represent a different number. If the sum of each column is shown beneath that column, and the sum of each row is shown beside that row, then $n =$

- A. 6
- B. 7
- C. 8
- D. 9
- E. 10

<i>X</i>	<i>W</i>	<i>X</i>	<i>Z</i>	3
<i>W</i>	<i>Y</i>	<i>X</i>	<i>W</i>	6
<i>Z</i>	<i>Y</i>	<i>X</i>	<i>X</i>	9
<i>Z</i>	<i>Z</i>	<i>W</i>	<i>Y</i>	0
-2	7	<i>n</i>	5	

【参考答案】 1-5 A, C, 22, CDE, D 6-10 CE, D, E, E, D 11-15 C, E, BCD, D, E
 16-20 DDDCD 21-25 ACACC 26-30 D, E, 121, B, B 31-35 B, B, D, D, D
 36-40 C, E, E, B, C

难题集锦 B

- Find the remainder when $3192^{2107} - 3159^{2107}$ is divided by 11.
- Compute the sum of the numerator and the denominator of the simplest fraction of the repeating decimal $0.63636363\dots$
- When the positive integer n is divided by 7, the quotient is q and the remainder is 4. When $2n$ is divided by 7, the remainder is 1 and the quotient in terms of q , is ().
A. $q/2$
B. $q/2 + 1$
C. $2q$
D. $2q + 1$
E. $2q + 2$
- | | |
|--------------------------------|----------|
| Column A | Column B |
| $(77*78*79*80*81*82*83)^{1/7}$ | 80 |
- If the sum of two positive integers is 24 and the difference of their squares is 48, what is the product of the two integers?
A. 108
B. 119
C. 128
D. 135
E. 143
- If the sum of four consecutive even integers is 340, then the average of the two larger integers is ().
A. 86
B. 86.5
C. 87
D. 87.5
E. 88
- A certain holiday is always on the fourth Tuesday of Month X. If Month X has 30 days, on how many different dates of Month X can the holiday fall?

8. How many integers between 360 and 630 are there such that they have odd number of divisors?
- A. 3
B. 4
C. 5
D. 6
E. 7
9. When 20 is divided by the positive integer k , the remainder is $k-2$, which of the following is a possible value of k ?
- A. 8
B. 9
C. 10
D. 11
E. 12
10. Suppose n is a two-digit positive integer with units digit 5 and tens digit u . Now, if $E = (n^2 - 25)/100$, then express E in terms of u .
- A. $u+1$
B. u^2+1
C. u^2-u
D. u^2+u
E. u^2+u+1

【参考答案】 1. 0 2. 18 3. D 4. B 5. E 6. C 7. 7 8. E 9. D 10. D

难题集锦 C

1. If n is a positive integer, then n^+ denotes a number such that $n < n^+ < n+1$.
- | | |
|------------|----------|
| Column A | Column B |
| $20^+/4^+$ | 5^+ |
2. x and n are positive integers, such that $7^x = 10^n - 1$. What is the 99th smallest possible value of n ?
3. Suppose a, b, c are different integers, and the repeating decimal $0.\overline{abc} = m/n$, where $0 < m < n < 100$, then
- | | |
|------------|------------|
| Quantity A | Quantity B |
| n | 39 |
4. A survey shows 28% of the respondents like swimming, 30% like basketball and 42% like football. 8% like both swimming and basketball, 10% like both basketball and football, and 5% like both swimming and football. If 3% respondents like all of them, then how many of respondents like none of them?
- A. 14%
B. 15%
C. 17%
D. 18%
E. 20%

5. Let S be the set of all positive integers n such that n^2 is a multiple of both 24 and 108. Which of the following integers are divisor of every integer n is S ? Indicate all such integers.

- A. 12
- B. 24
- C. 36
- D. 72

6. Suppose a, b, c, d, e are selected randomly from the set $\{1, 2, 3, 4, 5\}$ and they can repeat. Find the probability that $a*b*c*d+e$ is odd.

- A. $12/25$
- B. $27/125$
- C. $243/3125$
- D. $1632/3125$
- E. $1794/3125$

7. k is a non-zero real number, such that $k^2x^2+x-1=0$. In which of the following intervals the equation must have a zero?

- A. $(-3, -2)$
- B. $(-2, -1)$
- C. $(-1, 0)$
- D. $(0, 1)$
- E. $(1, 2)$

8. The quadratic function $f(x) = -k^2x^2 - mkx - 1/16$, where m, k are constants, does not have any intersection with x -axis. In which of the following interval of m the statement given above does not necessarily hold true?

- A. $(-9/16, -7/16)$
- B. $(-7/16, -5/16)$
- C. $(-1/16, 1/16)$
- D. $(1/16, 3/16)$
- E. $(3/16, 5/16)$

9. How many non-repeating 6-digit integers can be formed using 2, 3, 4, 5, 6, 7 with the first digit no larger than 5 and can be divided by 334?

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

10. Suppose there are 20 monkeys and they all have some number of chestnuts. If in average they have 88 chestnuts per monkey and only one of them has less than 60. If every monkey has an integer number of chestnuts and they all have different number of nuts. Then how many chestnuts does the monkey who has tenth most chestnuts have at least if given none of them has more than 100 chestnuts?

- A. 88
- B. 89
- C. 90
- D. 91
- E. 92

【参考答案】 1. D 2. 594 3. B 4. E 5. A、C 6. E 7. D 8. A 9. A 10. B