(№1)

$$\vec a \times \vec b = (a_2b_3 - a_3b_2)\vec i + (a_3b_1 - a_1b_3)\vec j + (a_1b_2 - a_2b_1)\vec k$$

$$a'_i = \alpha_{in} a_n$$

$$b'_i = \alpha_{im} b_m$$

$$\vec a' \times \vec b' = \begin{vmatrix} \vec i' & \vec j' & \vec k' \\ \alpha_{1n} & \alpha_{2n} & \alpha_{3n} \\ \alpha_{1m} & \alpha_{2m} & \alpha_{3m} \end{vmatrix} a_n b_m = (\alpha_{2n}\alpha_{3m} - \alpha_{3n}\alpha_{2m}) a_n b_m \vec i' +$$

$$+ (\alpha_{3n}\alpha_{1m} - \alpha_{1n}\alpha_{3m}) a_n b_m \vec j' + (\alpha_{1n}\alpha_{2m} - \alpha_{2n}\alpha_{1m}) a_n b_m \vec k'$$

$$\vec a \times \vec b = \vec c$$

$$\vec a' \times \vec b' = \vec c'$$

$$c'_1 = (\alpha_{2n}\cdot\alpha_{3m} - \alpha_{3n}\alpha_{2m}) a_n b_m$$

$$c_1 = a_2 b_3 - a_3 b_2$$

$$c'_1 = \alpha_{2n}\alpha_{3m} a_n b_m - \alpha_{3n}\alpha_{2m} a_n b_n = \alpha_{2n}\alpha_{3m} a_n b_m - \alpha_{3m}\alpha_{2n} a_m b_n =$$

$$= \underbrace{\alpha_{2n}\alpha_{3m}}_{\alpha_{1k}} \underbrace{(a_n b_m - a_m b_n)}_{c_k} = \alpha_{1k} c_k$$

Аналогично c'_2, c'_3. C изм. по векторному закону =>

=> c_k – вектор.

№2

1) $\{c\}_i = c_i = e_{ijk} a_j b_k$

$\{c\}_k = c_k = e_{ijk} a_i b_j$

$e_{ijk} a_j b_k = e_{jik} a_i b_k = e_{kij} a_i b_j$ - так как индексы немые

$e_{kij} = e_{ijk} \Rightarrow e_{ijk} a_j b_k = e_{ijk} a_i b_j$

2) $\{\nabla f\}_i = \frac{\partial f}{\partial x_i}$ $\qquad \nabla \times \nabla f = 0$

$\nabla \times \nabla f = e_{ijk} \frac{\partial f}{\partial x_i} = 0$, т.к. e_{ijk} - антисимметричный тензор, ∇f - вектор (всегда симметричный тензор)

3) $\{r\}_i = x_i$ $\qquad \nabla \times \vec{r} = 0$

$\nabla \times \vec{r} = e_{ijk} x_i = \vec{q}$

$q_1 = e_{123} x_1 + e_{132} x_1 = x_1 - x_1$ - аналогично остальные компонента равны 0 $\Rightarrow \nabla \times \vec{r} = 0$

4)

4) $\{\nabla f^2\}_i = \frac{\partial f^2}{\partial x_i} = 2\frac{\partial f}{\partial x_i} f$

$\{\nabla f^\alpha\}_i = \frac{\partial}{\partial x_i}(f^\alpha) = \alpha f^{\alpha-1}\frac{\partial f}{\partial x_i}$

5) $\vec{r}^{\,2} \equiv r^2 = x_i x_i$

$\{\nabla r^2\}_i = \frac{\partial}{\partial x_i}(x_i x_i)^2 = \cancel{2x_i} \frac{\partial x_i}{\partial x_i}x_i + \frac{\partial x_i}{\partial x_i}x_i = 2x_i$

$\nabla(r^2)$

$\alpha = \frac{1}{2}$: $\{\nabla r\} = \frac{1}{2} r^{\frac{1}{2}-1}\frac{\partial r}{\partial x_i} = \frac{1}{2}\frac{1}{\sqrt{x_i}}$

№3

1) $e_{ijl} e_{ijm} = 2\delta_{lm}$

$e_{ijk} e_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km} \Rightarrow e_{ijl} e_{ijm} = \delta_{jj}\delta_{lm} - \delta_{jm}\delta_{lj} =$
$= 3\delta_{lm} - \delta_{lm} = 2\delta_{lm}$

2) $\nabla \times (\vec{a} \times \vec{b}) = a(\nabla \cdot b) - (\nabla \cdot a) b + (b \cdot \nabla) a - (a \cdot \nabla) b$

$\{\nabla \times (\vec{a} \times \vec{b})\}_i = e_{ijk} \dfrac{\partial (\vec{a} \times \vec{b})_k}{\partial x_j} = e_{ijk} e_{klm} \dfrac{\partial (a_l b_m)}{\partial x_j} \ominus$

$e_{ijk} e_{klm} = e_{ijk} e_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

$\ominus (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) \dfrac{\partial (a_l b_m)}{\partial x_j} = \dfrac{\partial (a_i b_j)}{\partial x_j} - \dfrac{\partial (a_j b_i)}{\partial x_j} =$

$= a_i \underbrace{\dfrac{\partial b_j}{\partial x_j}}_{\nabla \cdot b} + b_j \dfrac{\partial a_i}{\partial x_j} - a_j \dfrac{\partial b_i}{\partial x_j} - b_i \underbrace{\dfrac{\partial a_j}{\partial x_j}}_{\nabla \cdot a} = a_i (\nabla \cdot b) - b_i (\nabla \cdot a) +$

$+ b_j \dfrac{\partial a_i}{\partial x_j} - a_j \dfrac{\partial b_i}{\partial x_j} = a_i(\nabla \cdot b) - b_i(\nabla \cdot a) + \dfrac{\partial a_i}{\partial \vec{b}} - \dfrac{\partial b_i}{\partial \vec{a}} =$

$= \left| \dfrac{\partial}{\partial \vec{a}} = (\vec{a} \cdot \nabla),\ \dfrac{\partial}{\partial \vec{b}} = (\vec{b}, \nabla) \right| = a_i(\nabla \cdot b) - b_i(\nabla \cdot a) +$

$+ a_i(\vec{b} \cdot \nabla) - b_i(\vec{a} \cdot \nabla)$

3) $(\vec{a} \cdot \nabla) \cdot \vec{r} = \vec{a}$

$$\left\{(\vec{a} \cdot \nabla)\vec{r}\right\}_i = a_i \cdot \frac{\partial}{\partial x_i} x_i = a_i$$

4) $\nabla \cdot (\vec{a} \times \nabla) \vec{r} = \nabla \times \vec{a}$

$$\vec{a} \times \nabla = e_{ijk} \frac{\partial a_j}{\partial x_k}$$

$$\nabla (\vec{a} \times \nabla) = \frac{\partial}{\partial x_i} \left(e_{ijk} \frac{\partial a_j}{\partial x_k} \right)_i$$