Frames of reference

Say you are moving on a bus, and bus suddenly stops, you are thrown forward.

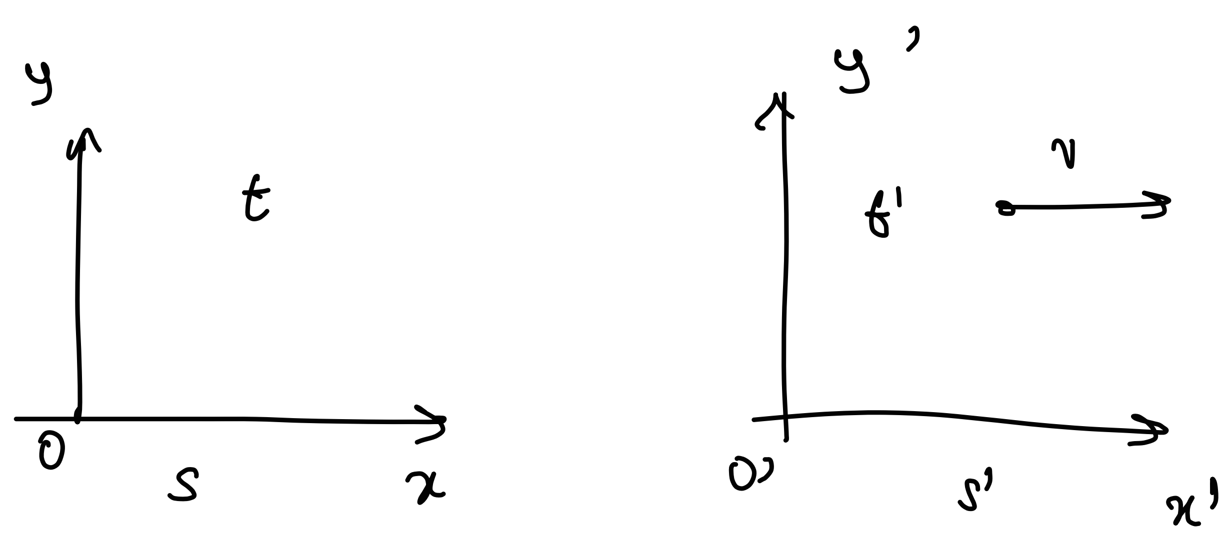
An obj. at rest should remain at rest when $F=0$ (w.r.t bus frame)

but you got thrown forward??

This is thus a non-inertial frame where Newton's law 1st law is not valid.

We add something called pseudo-force in such situations to make Newton's laws valid

More on inertial frames -



If S' moves with uniform velocity w.r.t S, then S' is also inertial

When O crosses O', we say by definition that

$\Rightarrow t=0$
$\Rightarrow t'=0$ | in that frame

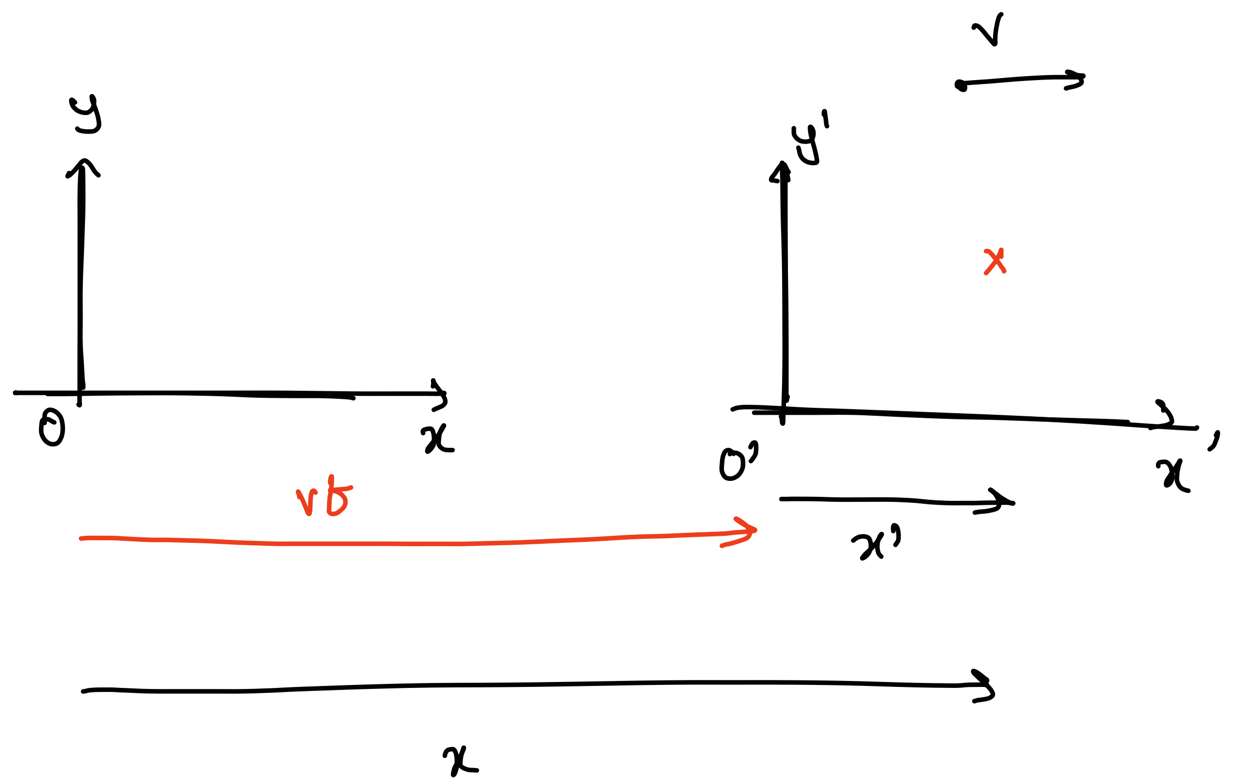
for any event

you need to say where and when | w.r.t

(x,y,z) (t) | S frame

(x',y',z') (t') | w.r.t S' frame

frame transformation



$x = vt + x'$ | $y = y'$

$x' = x - vt$

If v is small $t = t'$

If we know x', t'

$x = vt + x' \iff x = vt' + x' \quad (\because t = t')$

Gallileon transformations

velocity of frame invariant with time

$v_x = \frac{dx}{dt}$

$v_y = \frac{dy}{dt}$

$v_z = \frac{dz}{dt}$

$v_x' = \frac{dx'}{dt'} = \frac{d}{dt'}(x - vt) = \frac{dx}{dt'} - v\frac{dt}{dt'}$

$= v_x - v$

$v_y' = \frac{dy'}{dt'} = \frac{dy}{dt} = v_y$

$v_z' = \frac{dz'}{dt'} = \frac{dz}{dt} = v_z$

$v_x' = v_x - v$

$v_y' = v$

$v_z' = v_z$

$a_x' = \dfrac{dv_x'}{dt'} - \underbrace{\dfrac{dv}{dt}}_{=0}$ (∵ const. velocity of frame)

$= \dfrac{d(v_x - v)}{dt} = \dfrac{dv_x}{dt} = a_x$

acceleration is same!!

Acceleration of an object in any inertial frame is the same,
(in a Gallilean transformation) → when velocity is small

S ⇒ बल = द्रव्यमान × त्वरण

$F = m \times a$

$\quad\quad \parallel \quad \parallel$

S' ⇒ $F' = m' \times a'$

same same

$F = F'$! always

Since most of the dynamics is explainable by this laws it follows that the event is same in all frames.

ex: I throw chalk and it hits my feet in s frame.

chalk hits my feet acc. to s' frame also.

WHATEVER OUTCOMES YOU GET IN ONE FRAME

IF YOU REPEAT IN ANOTHER FRAM, YOU GET SAME

OUTCOME!

Principle of relativity.

Non-invariance of Maxwell's equations under Gallilian Transformation

Maxwell's equations-

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

If you re-arrange -

$$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2\vec{B} = \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

3×10^8 m/s

in any frame

But Gallilian transformation claims

velocity isn't same in all inertial frames.

$\therefore$ Maxwell's equations according to Gallilian transformation is false.

which claim $c = 3\times10^8$ despite frame iff all u laws are sme

OR ANOTHER WAY TO THINK OF THIS IS IF GALLILEON TRANSFORMATION IS TRUE ⇒ MAXWELL'S EQ ARE TRUE ONLY FOR ONE FRAME, IF YOU CHANGE YOUR FROM THE EQ AND HENCE THE OUTCOMES WILL BE DIFFERENT

Michelson - Morley Experiment

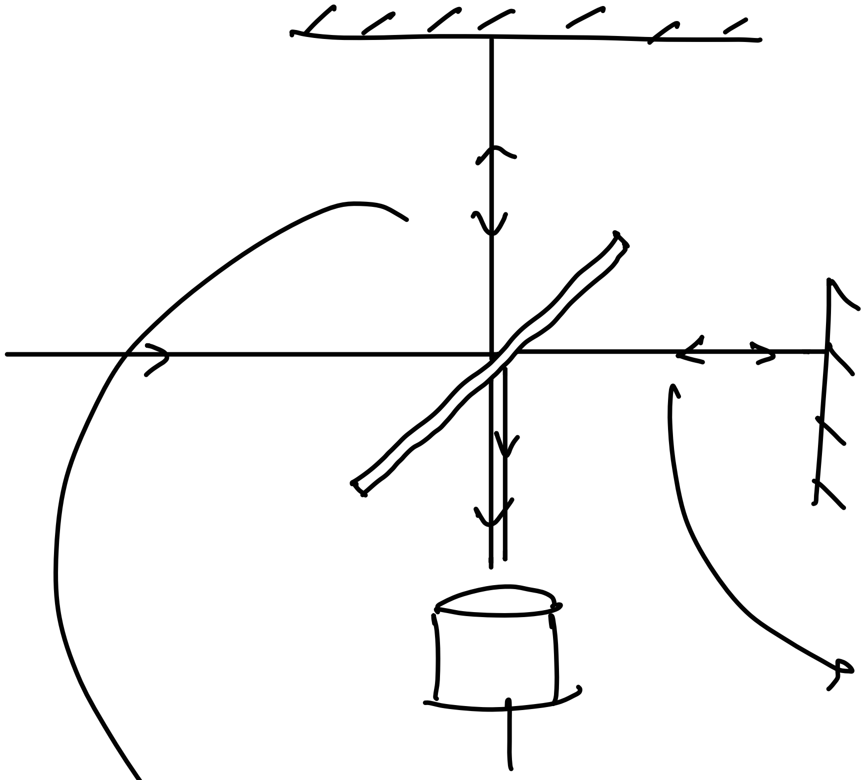
In their time, they used to believe in a material called aether in which light propogated.

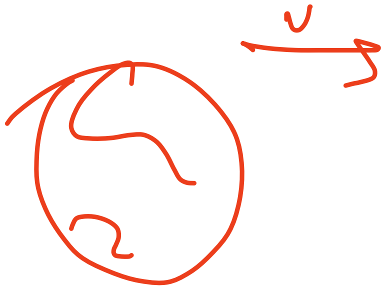
They also believed that this aether was an absolute frame w.r.t which all frames moved.

It was then believed that $c = 3\times10^8$ only in that frame, Maxwell's equation was true only in that frame.

Objective of Michelson - Morley experiment

Aether के फ्रेम में धरती का वेग निकालना।





if Earth moves this way

this should take a diff time

than this.

But nope, it wasn't the case

It was exactly the same — c always remains c from every frame.

So Maxwell's eq. are saved ∵ they always predict c.

So Einstein's postulates—

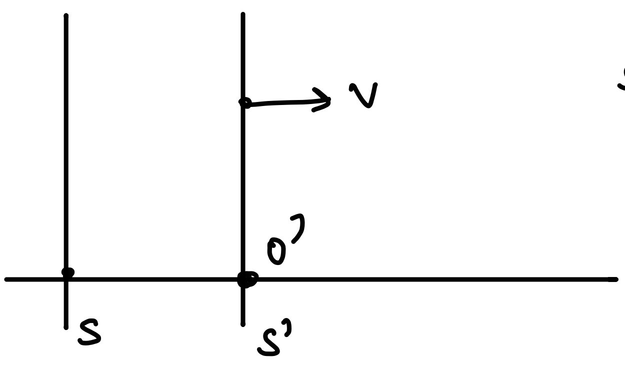
1) Speed of light is c in all medium

2) Principle of relativity is true $\longrightarrow$ we just need to figure out how for Newtonian mechanics - it anyway works for Maxwell's eq.

With these two postulates in mind, let's see how frame transformations might get affected.

$x' = ax + bt$, $y' = y$, $z' = z$

$t' = dx + ft$



For this event:

S: t,
says $(x = vt)$ $(y = 0)$ $(z = 0)$

S': $x' = 0$ $y = 0$ $z = 0$
says t'

so $ax+bt=0$

$\downarrow$

$a(vt)+bt=0$

$\underbrace{t}_{\neq 0}\underbrace{(av+b)}_{=0} = 0$ (∵ its true for all t) | $av+b=0$

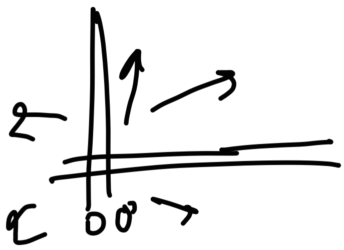
| $b=-av$

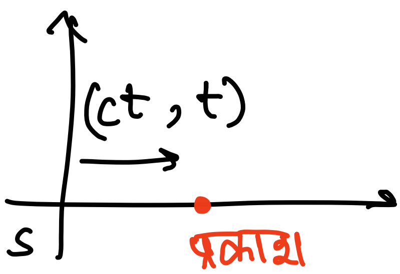
$\therefore\ x' = ax-(av)t$

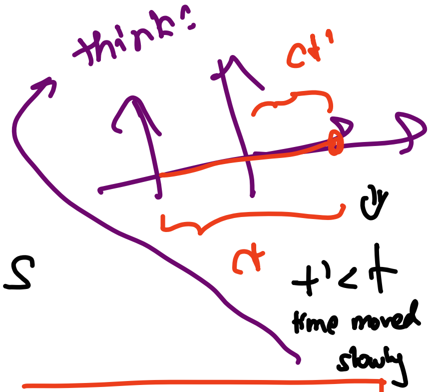
$= a(x-vt)$

मान लो कि O'-O' crossing के साथ प्रकाश की एक तरंग दूस O/O' से सभी ओर फैलती है।

अगर S - फ्रेम से देखे :







Event 1: $(ct, 0, 0, t)$ as seen by S

as seen by S'? $x' = a(ct-vt) = \boxed{at(c-v) = ct'}$

$t' = ?$ $\quad dx+ft$

$$d(ct) + ft \Rightarrow t(dc+f)$$

$$x' = at(c-v) = ct'$$

$$t' = t(dc+f)$$

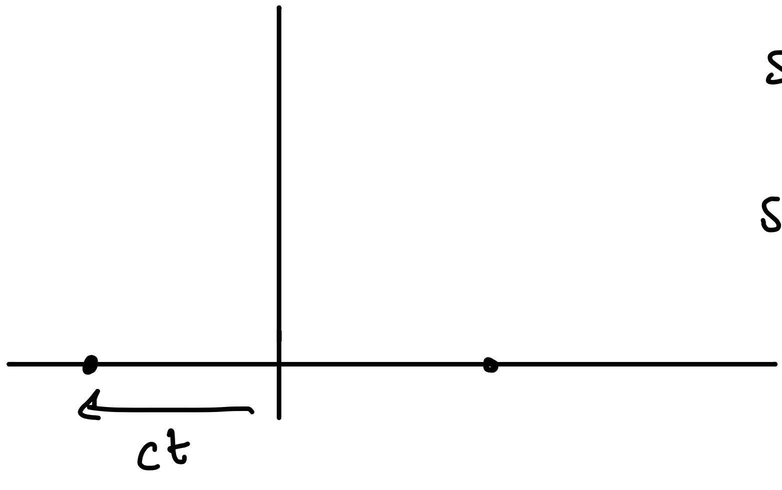
$$ct' = a(c-v)\left(\frac{t'}{dc+f}\right)$$

$$\Rightarrow c = \frac{a(c-v)}{dc+f}$$

$$\Rightarrow dc^2 + cf = ac - av$$

$$\Rightarrow dc^2 + cf = a(c-v) \longrightarrow (1)$$

Event 2 :



$S:$ $x = -ct$, $y=0$, $z=0$, t

$S':$ $x' = a(-ct - vt)$, $y'=0$, $z'=0$

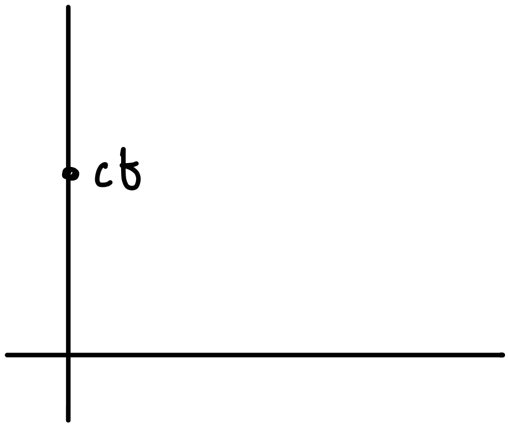
$-ct'$

$$t' = d(-ct) + ft$$

$$= (-dc+f)t$$

$$c = \frac{a(c+v)}{-dc+f} \Rightarrow -dc^2 + fc = a(c+v) \rightarrow (2)$$

Event 3;



$$S: \quad x=0 \quad y=ct \quad z=0 \quad t=t$$

$$S': \quad x' = a(0-vt) \quad y' = ct \quad z=0$$

$$t' = dx + ft$$

$$= ft$$

Now the relativity postulate-wala guy:

distance from O' in S' coords

$$d^2 = x'^2 + y'^2 + z'^2 = a^2v^2t^2 + c^2t^2 = (ct')^2$$

$$c^2t'^2 = c^2f^2t^2 = a^2v^2t^2 + ct^2$$

$$= a^2v^2 + c^2 = c^2f^2 \rightarrow (3)$$

so, the 3 equations -

$$dc^2 + fc = a(c-v)$$

$$-dc^2 + fc = a(c+v)$$

$$a^2v^2 + c^2 = f^2c^2$$

$$2fc = 2ac \quad (1+2)$$

$$f = a$$

$f=a$ in 3

$$a^2v^2 + c^2 = a^2c^2$$

$$c^2 = a^2(c^2-v^2)$$

$$a^2 = \frac{c^2}{c^2-v^2} = \frac{1}{1-v^2/c^2}$$

$$\Rightarrow a = \frac{1}{\sqrt{1-v^2/c^2}}$$

using a, f, c

$$d = \frac{-v^2/c^2}{\sqrt{1-v^2/c^2}}$$

$$\therefore x' = \frac{x-vt}{\sqrt{1-v^2/c^2}}$$

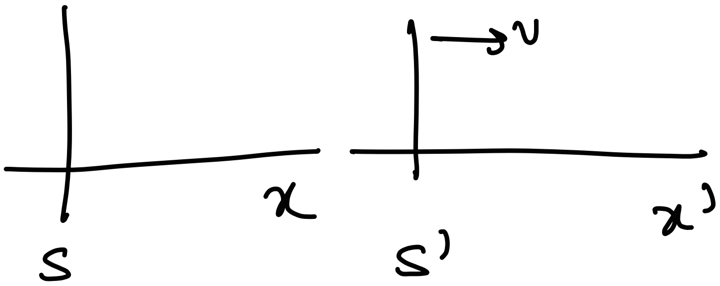
$$y' = y \qquad z' = z$$

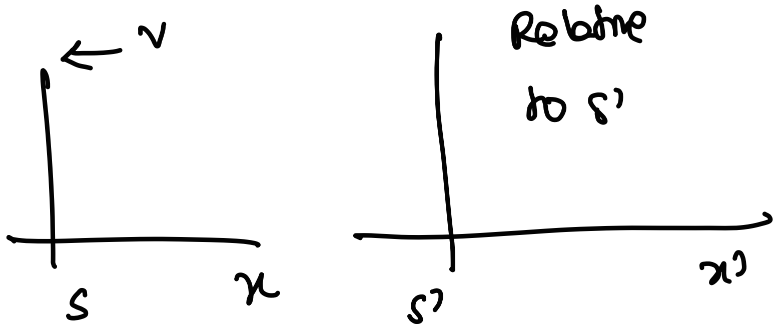
$$t' = \delta x + ft = \frac{\frac{-v}{c^2}x}{\sqrt{1 - v^2/c^2}} + \frac{t}{\sqrt{1 - v^2/c^2}}$$

$\therefore$ Lorentz Transform –

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad y = y' \qquad z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$





$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z$$

$$t = \frac{t' + \frac{x'v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

New quantity

$$x^2 + y^2 + z^2 - c^2t^2$$

$$x'^2 + y'^2 + z'^2 - c^2t_1'^2$$

We can show from Lorentz transform that these are equivalent ie they are invariant

$$\left[\frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}\right]^2 + y^2 + z^2 - c^2\left[\frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right]^2$$

$$\frac{x^2+v^2t^2-2xvt+c^2t^2-\left(\frac{x^2v^2}{c^4}-\frac{2xvt}{c^2}\right)c^2}{1-\frac{v^2}{c^2}}+y^2+z^2$$

$$\Rightarrow \frac{x^2-\cancel{2xvt}+v^2t^2+\cancel{2xvt}-x^2v^2/c^2+c^2t^2}{1-\frac{v^2}{c^2}}+y^2+z^2$$

$$\Rightarrow \frac{x^2\left(1-\frac{v^2}{c^2}\right)+t^2\left(v^2-c^2\right)}{1-\frac{v^2}{c^2}}+y^2+z^2$$

$$\Rightarrow \frac{x^2\left(1-\frac{v^2}{c^2}\right)-c^2t^2\left(1-\frac{v^2}{c^2}\right)}{1-v^2/c^2}+y^2+z^2$$

$$\Rightarrow x^2+y^2+z^2-c^2t^2 \;!$$

Yay!

Consider 2 events

Event 1 : (x_1, y_1, z_1, t_1) (x_1', y_1', z_1', t_1')

Event 2 : (x_2, y_2, z_2, t_2) (x_2', y_2', z_2', t_2')

Now, consider the quantity:

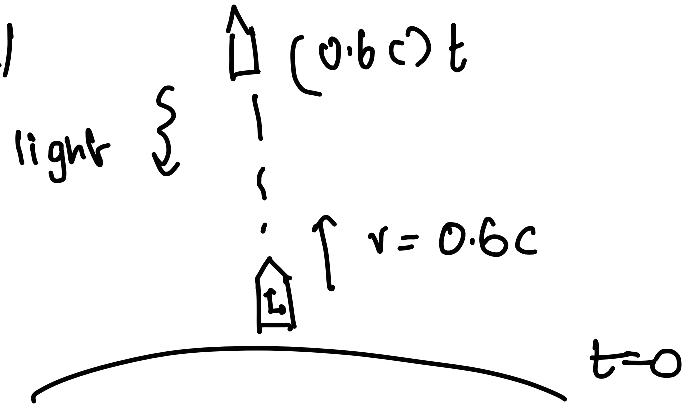
$$(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2 - c^2(t_2-t_1)^2$$

This qty. also apparently =

$$(x_2'-x_1')^2 + (y_2'-y_1')^2 + (z_2'-z_1')^2 - c^2(t_2'-t_1')^2$$

Also called: Interval btw 2 events : invariant w.r.t Lorentz transformation.

Q/



S': after 1 hour in S' they send signal

S: when did signal come?

Event : light signal being sent

S' : $t' = 1\text{hour}$ $x' = 0$

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$t = \frac{t' + (v/c^2)x'}{\sqrt{1 - v^2/c^2}} = \frac{1 + 0}{\sqrt{1 - v^2/c^2}} = \frac{1}{\gamma}$$

$$= \frac{1}{\sqrt{1 - 0.36}}$$

$$= 1.25 \text{ hrs}$$

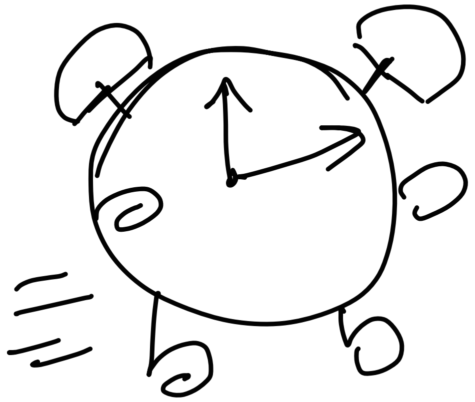
Gallileon Transformation

$$x' = x - vt$$

Lorentz transformation

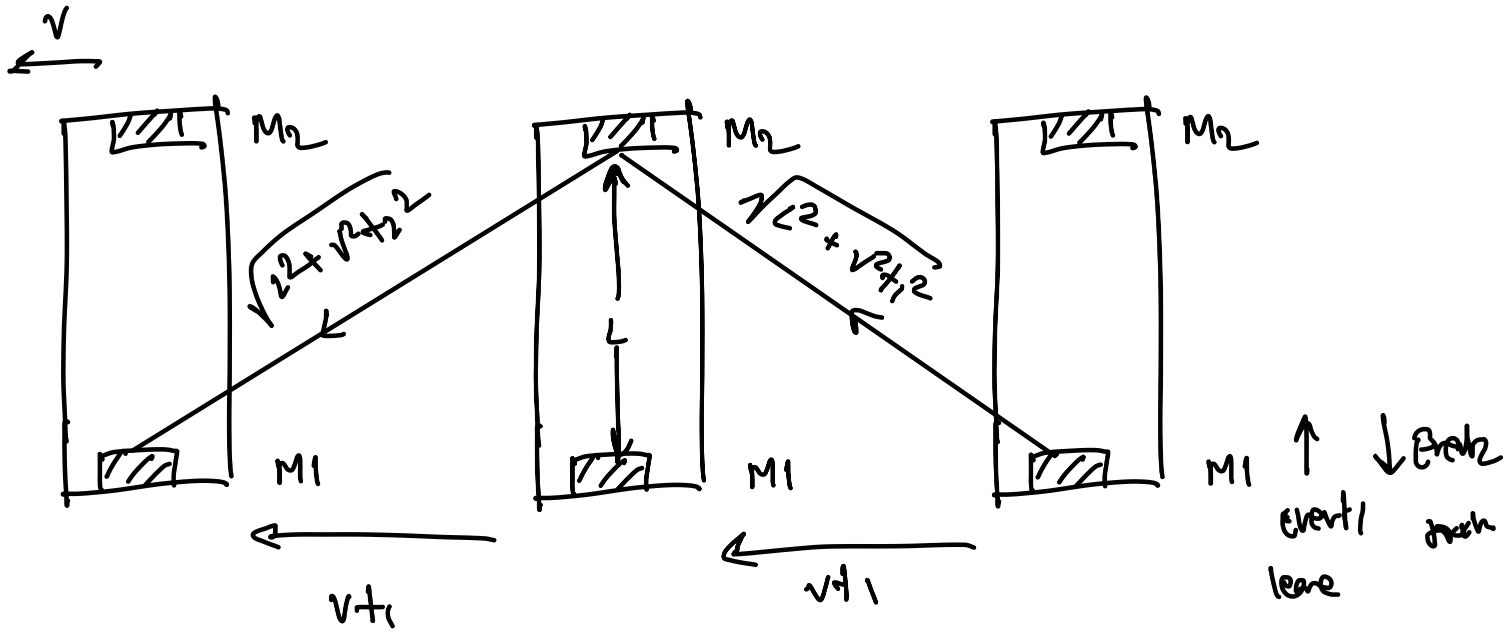
$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

Time Dilation



faster you move through space

the slower you move through time



S' frame

In S frame : $2t_1$

S' frame : $\dfrac{2\times\sqrt{L^2+v^2t_1^2}}{c}$

$$\cancel{2}t_1 = \frac{\cancel{2}\times\sqrt{L^2+v^2t_1^2}}{c}$$

$$\Rightarrow ct_1 = \sqrt{L^2+v^2t_1^2}$$

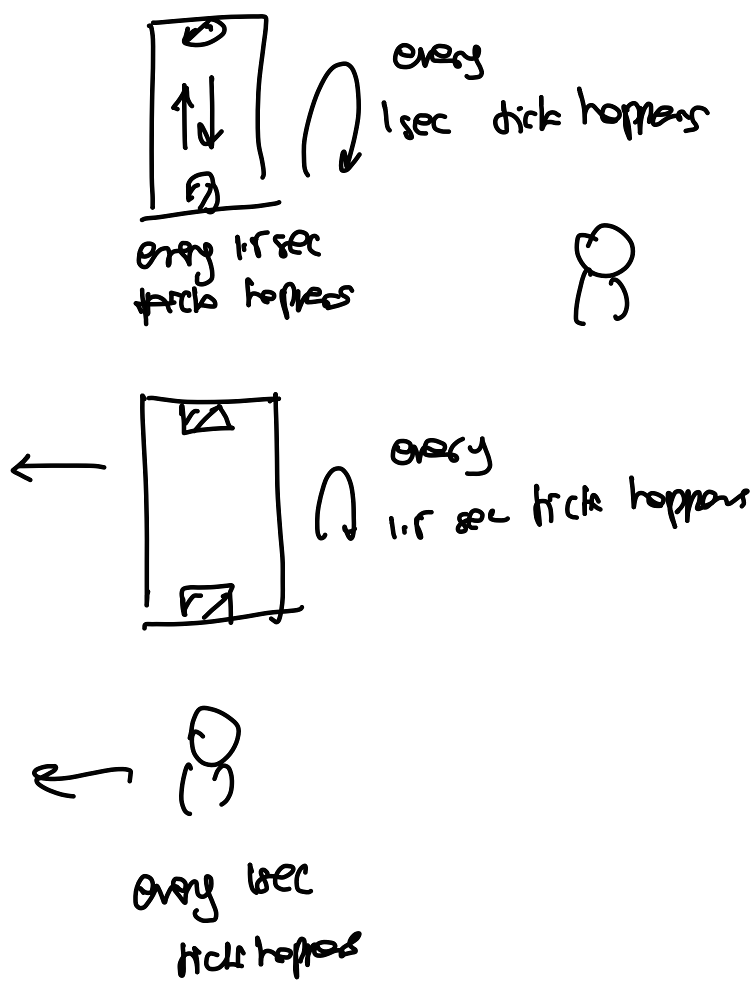
$$\Rightarrow c^2(t_1)^2 = L^2+v^2t_1^2$$

$$\Rightarrow t_1^2 = \frac{L^2}{c^2-v^2}$$

$$\Rightarrow t_1 = \frac{L}{\sqrt{c^2-v^2}}$$

$$2t_1 = \frac{2L}{c\sqrt{1-v^2/c^2}} = \qquad \Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time b/w 2 events. depends on which frame you see it from



So, to clarify:

When can you use $\Delta t' = \dfrac{\Delta t}{\sqrt{1 - v^2/c^2}}$? $\longrightarrow$ ⓐ

when in S frame they happen at same place $\Delta x = 0$

but $\Delta x'$ i.e in S' frame the two events aren't at the same place

\# What if even in x-frame $\Delta x \neq 0$

Consider

$$E_1 = (x_1, y_1, z_1)\, t_1 \qquad (x_1', y_1', z_1')\, t_1'$$

$$E_2 = (x_2, y_1, z_1)\, t_2 \qquad (x_2', y_1', z_1')\, t_2'$$

$$\underbrace{\hphantom{E_2 = (x_2, y_1, z_1)\, t_2}}_{S} \qquad \underbrace{\hphantom{(x_2', y_1', z_1')\, t_2'}}_{S'}$$

If $\Delta t' = t_2' - t_1'$ | Then relation ⓐ, will not hold

$\Delta t = t_2 - t_1$ | Check it out with Lorentz transform

$$t_2' - t_1' = \left(\frac{t_2 - \frac{x_2 v}{c^2}}{\sqrt{1 - v^2/c^2}} - \frac{t_1 - \frac{x_1 v}{c^2}}{\sqrt{1 - v^2/c^2}} \right)$$

$$= \left(\frac{(t_2-t_1) - \frac{v}{c^2}(x_2-x_1)}{\sqrt{1-\frac{v^2}{c^2}}} \right)$$ Is not zero

$$\neq \frac{\Delta t}{\sqrt{1-v^2/c^2}}$$

(Bruh. I never realised how directly you could get it)

✱ Simultaneity of relativity (समकालीनता की आपेक्षिकता)

When 2 events happen at same time, how are they related w.r.t each frame?

S: $E_1\ (x_1, y, z, t)$

$E_2\ (x_2, y, z, t)$

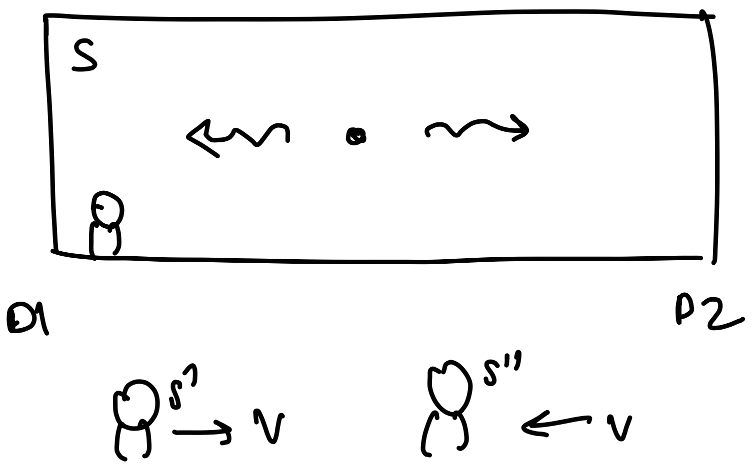
S': $$t_1' = \frac{t_1 - \frac{x_1 v}{c^2}}{\sqrt{1-v^2/c^2}}$$

$$t_2' = \frac{t_2 - \frac{x_2 v}{c^2}}{\sqrt{1-v^2/c^2}}$$

clearly: $t_1' \neq t_2'$

So, in S' frame these events are not simultaneous

Consider the following thought experiment -

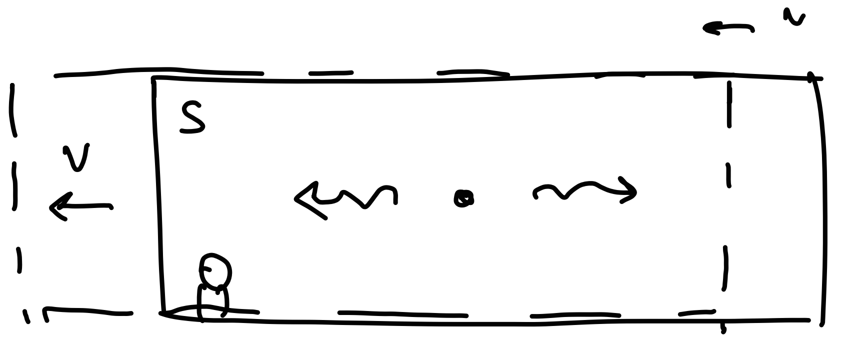


D1: D1 door opens

D2: D2 door opens

In S frame; D1, D2 are simultaneous

In S' frame;



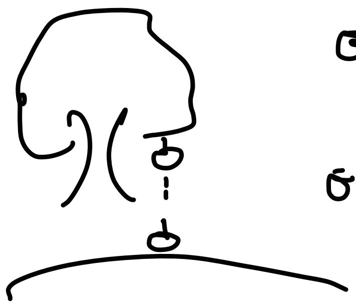
 S' $\because$ D2 is closer, it will open before D1

 S'' $\because$ D1 is closer, it will open before D2

Even the order can be reversed based on which frame you are looking from!!

This idea brings up some really mind-boggling thoughts -

On Earth frame -



E_1: Apple breaks from tree

E_2: Apple falls on ground

Can there be frame in which E_2 happens before E_1?

Let us see -

Consider if E_1, E_2 happen at same place in S

then

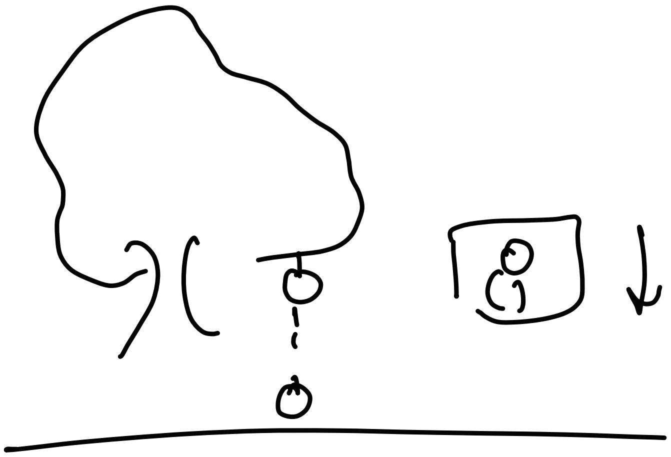
$$\Delta t' = \frac{\Delta t}{\gamma} \equiv t_2' - t_1' = \frac{t_2 - t_1}{\gamma}$$

i.e if $t_2 - t_1 > 0 \Rightarrow t_2 > t_1$ or E_2 after E_1 acc. to S frame.

$\Rightarrow t_2' - t_1' > 0 \Rightarrow t_2' > t_1'$ or E_2 after E_1 acc. to S' frame as well!

So if any 2 events happen at the same place in any inertial frame — the time b/w those two events might vary from one frame to another but their order IS preserved.

OK let's review our apple :



If you consider the frame that moves with the apple;

E_1 : $x_1' = 0$ $y_1' = 0$ $z' = 0$

E_2 : $x_1' = 0$ $y_1' = 0$ $z' = 0$

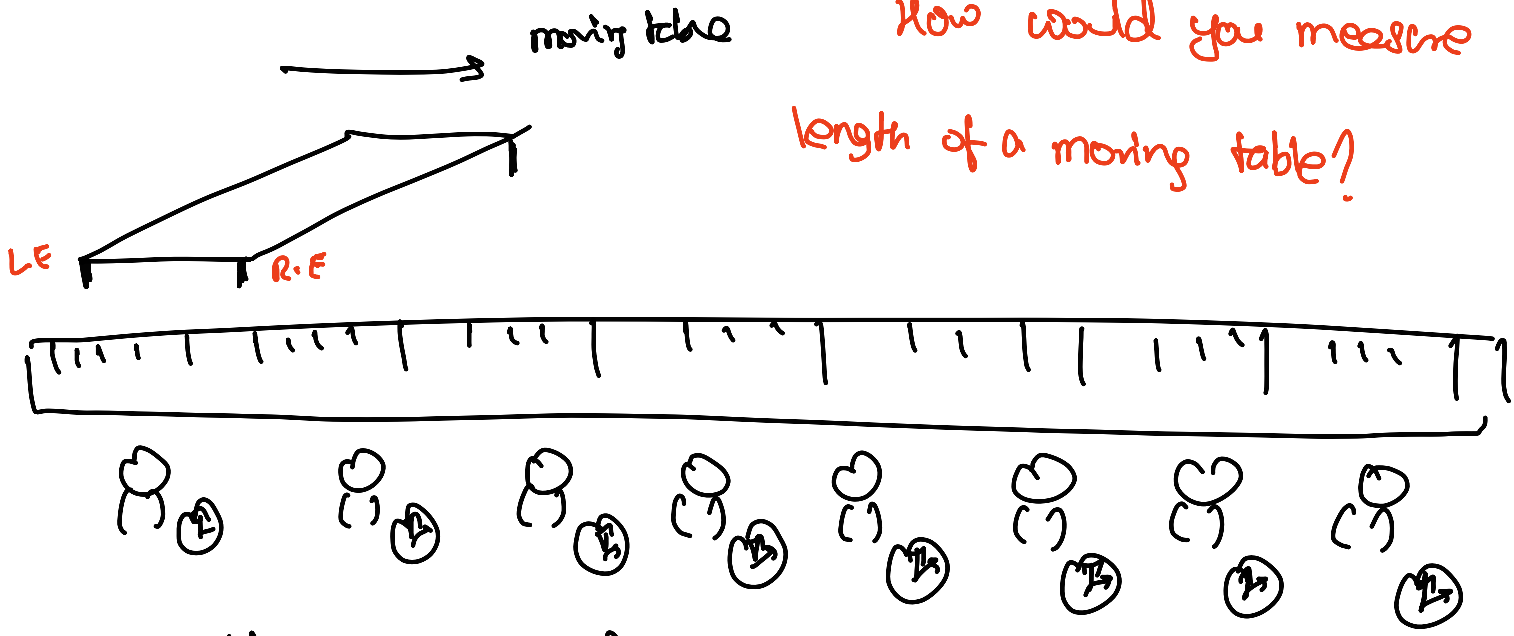
i.e they happen in the same place!!

Thus its causality has to be maintained!, in every frame apple breaks first!

PRINCIPLE OF CAUSALITY

I am not writing notes for synchronisation of clocks 'cuz I think its not there

* Length contraction

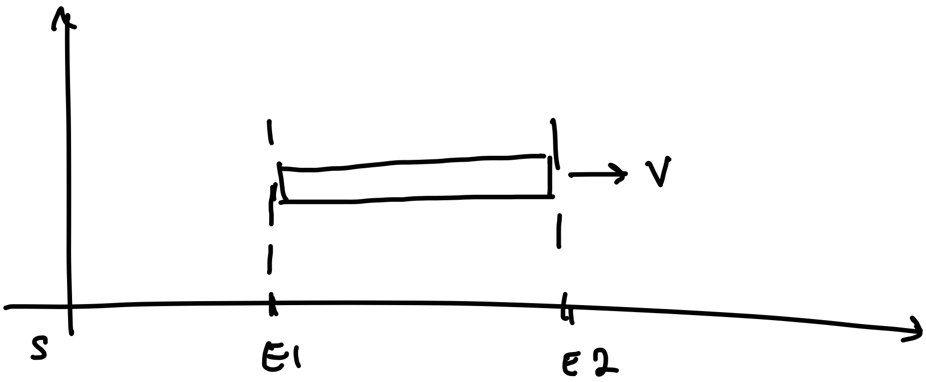


How would you measure length of a moving table?

How would you measure?

You will tell at exactly time 't'; the guy who is in front of L.E measure x-co-ordinate and R.E guy measure x-co-ordinate

$\therefore L = x_2 - x_1$: you'll end up finding that L is different from when it would be stationary



E1: x_1, t

E2: x_2, t

E1: x_1', t_1'

E2: x_2', t_2'

S में लंबाई $= x_2 - x_1$

S' में लंबाई $= x_2' - x_1'$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \qquad x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

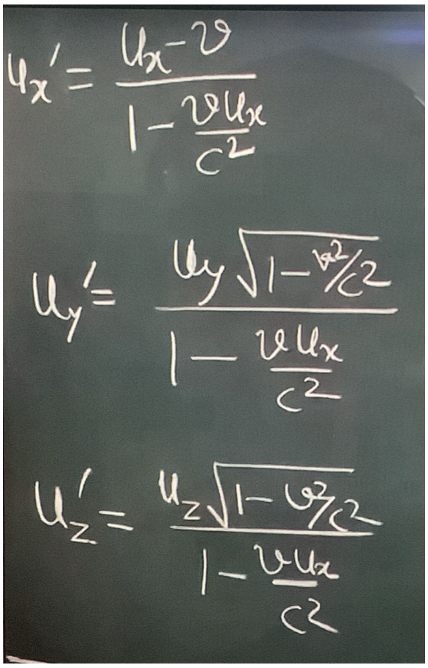
$$t_1' = \frac{t - \frac{vx_1}{c^2}}{\sqrt{1 - v^2/c^2}} \qquad t_2' = \frac{t - \frac{vx_2}{c^2}}{\sqrt{1 - v^2/c^2}}$$

It's chill even if you measure at different times since, the rod is stationary anyway in s' frame.

However, remember that if 'L' is moving, both the ends must be measured at the same time.

$$L_{rest} = \frac{L_{moving}}{\gamma} \Rightarrow L_{moving} = (L_{rest}) \times \underset{<1}{\gamma}$$

$\therefore$ it's smaller



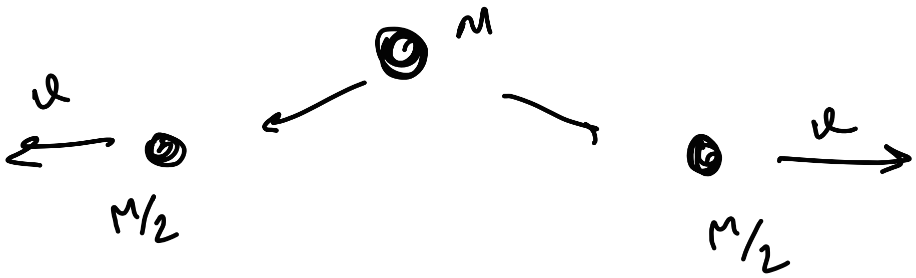
$$u_x = \frac{x_2 - x_1}{t_2 - t_1} \qquad u_x' = \frac{x_2' - x_1'}{t_2' - t_1'}$$

$$u_y = \frac{y_2 - y_1}{t_2 - t_1} \qquad u_y' = \frac{y_2 - y_1}{t_2' - t_1'}$$

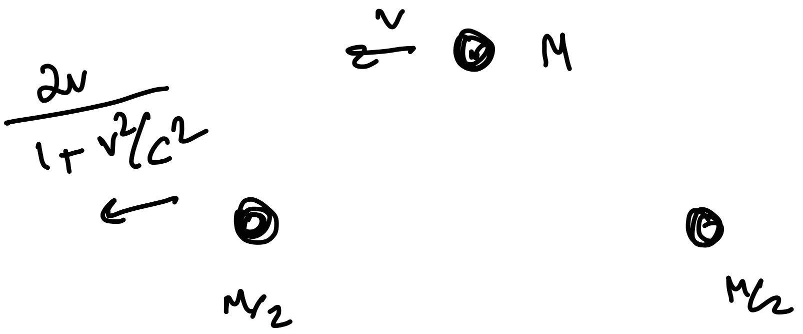
$$u_z = \frac{z_2 - z_1}{t_2 - t_1} \qquad u_z' = \frac{z_2 - z_1}{t_2' - t_1'}$$

Do Lorentz Transform to get this

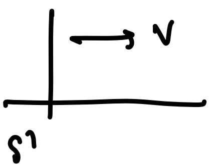
Relativistic Linear Momentum



In S frame



In S' frame



Initial momentum : $-Mv$

Final momentum :

$$-\frac{M}{2} \times \frac{2v}{1+\frac{v^2}{c^2}}$$

It's not same!

But by 2nd postulate,

all inertial frames have same laws of physics, so conservation must hold!

Let's thus assume $p = mv\,f(v)$; where as

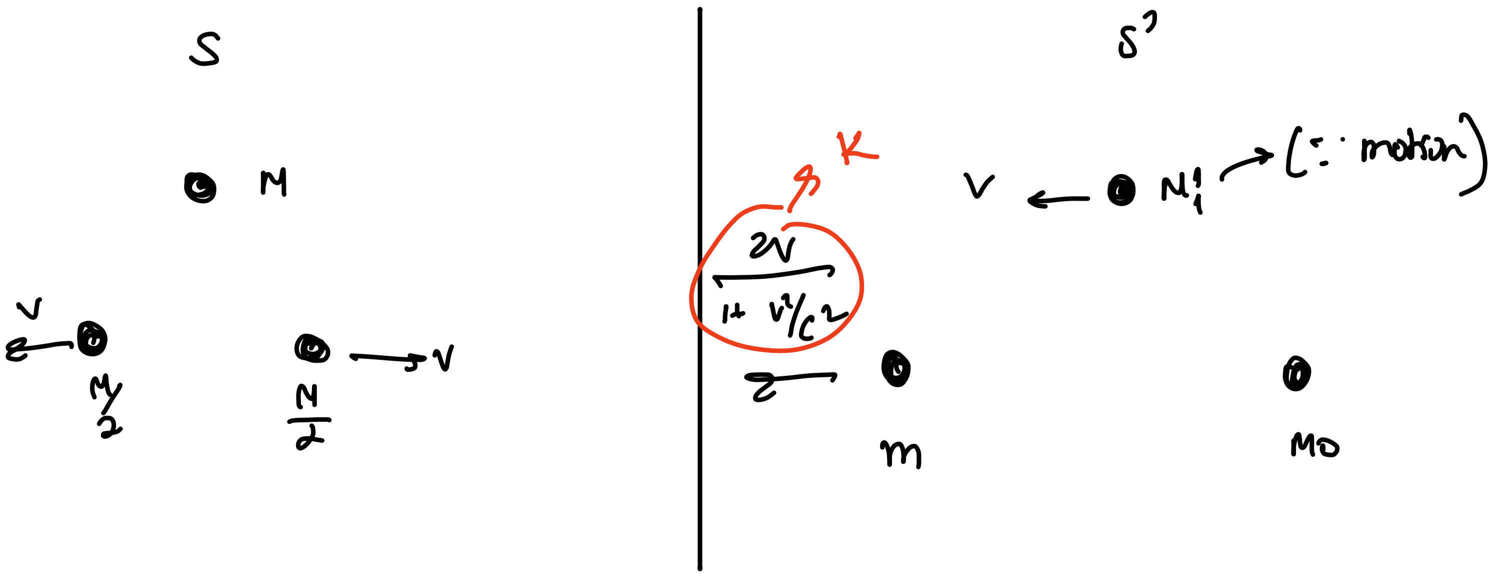
$v \to 0 \quad f(v) \to 1$

($\because$ we know normally $p = mv$)

$\boxed{m \times f(v)} \longrightarrow$ Relativistic mass

$\boxed{m_0 \times f(v)} \longrightarrow$ m_0 : rest mass

* Let's redo our situation in S'



① Mass conservation

$$M' = m + m_0 \longrightarrow ①$$

②

$$-M'v = -m \times \frac{2v}{1+\frac{v^2}{c^2}} \Rightarrow M' = \frac{2m}{1+v^2/c^2}$$

Combining ①, ②

$$m + m_0 = \frac{2m}{1 + v^2/c^2}$$

$$\Rightarrow \quad m_0 = m\left[\frac{2}{1 + v^2/c^2} - 1\right]$$

$$\Rightarrow \quad m_0 = m\,\frac{\left(1 - \frac{v^2}{c^2}\right)}{1 + \frac{v^2}{c^2}} = m \times \sqrt{1 - \frac{k^2}{c^2}}$$

(You do the maths, you get this)

$$\Rightarrow \quad \underbrace{m}_{\text{mass of particle in } S' \text{ frame.}} = \frac{m_0}{\sqrt{1 - \frac{k^2}{c^2}}}$$

velocity of particle in S' frame

$$\vec{p} = m \cdot \vec{v} = \left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \right) \times v$$

Relativistic momentum

गतिज ऊर्जा ; $K = \frac{1}{2}mv^2$ (for low velocity?)

$$dK = F dx = \frac{dp}{dt} \times dx = \frac{d(mv)}{dt} dx$$

if $v <<< c$

$$= d(mv)\, v$$

$$= m \times dv \times v \Rightarrow \frac{mv^2}{2}$$

At relativistic speeds,

$$dK = F dx = \frac{dp}{dt} dx$$

$$= \frac{d}{dt} \left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}} \right) dx$$

$$= m_0\, d \left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} \right) v$$

$$= m_0 v \times d\left(\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$= m_0 v \times \left[v\, d\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) + \frac{1}{\sqrt{1-v^2/c^2}}\, dv \right]$$

MILLION STOPS LATER

$$dk = m_0 \frac{v\,dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

$$K = \int_0^k dk = m_0 \int_0^v \frac{v\,dv}{(1-v^2/c^2)^{3/2}}$$

- MATHS happens again -

$$K = m_0 c^2 \left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]$$

$$\therefore\ K = m_0 c^2 \left[\frac{1}{\sqrt{1-v^2/c^2}} - 1\right]$$; For small v

$$= m_0c^2\left[\cancel{1}+\frac{1}{2}\frac{v^2}{c^2}\cancel{-1}\right] = \frac{1}{2}m_0v^2$$

(which is expected)

$$* \quad K = m_0c^2\left[\frac{1}{\sqrt{1-v^2/c^2}}-1\right]$$

Anyway, look at this

$$K = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} - m_0c^2 = \underbrace{mc^2 - m_0c^2}$$

at $v=0$ | $m_0c^2 - m_0c^2 = 0$

$v\uparrow$ | $mc^2\uparrow - m_0c^2 = 0\uparrow$

Rest mass energy

$$\Rightarrow \underbrace{K}_{\text{Kinetic energy}} + \underbrace{m_0c^2}_{\text{fixed qty.}} = \underbrace{mc^2}_{\text{Called total energy}} = E$$

$$\boxed{E = mc^2}$$

कुल ऊर्जा = $E = \dfrac{m_0c^2}{\sqrt{1-v^2/c^2}}$ (Total Energy)

रैखीय संवेग = $p = \dfrac{m_0v}{\sqrt{1-v^2/c^2}}$ (linear momentum)

Now, $E^2 - p^2c^2$

$\Rightarrow m_0^2c^4$

(If you sub and simplify)

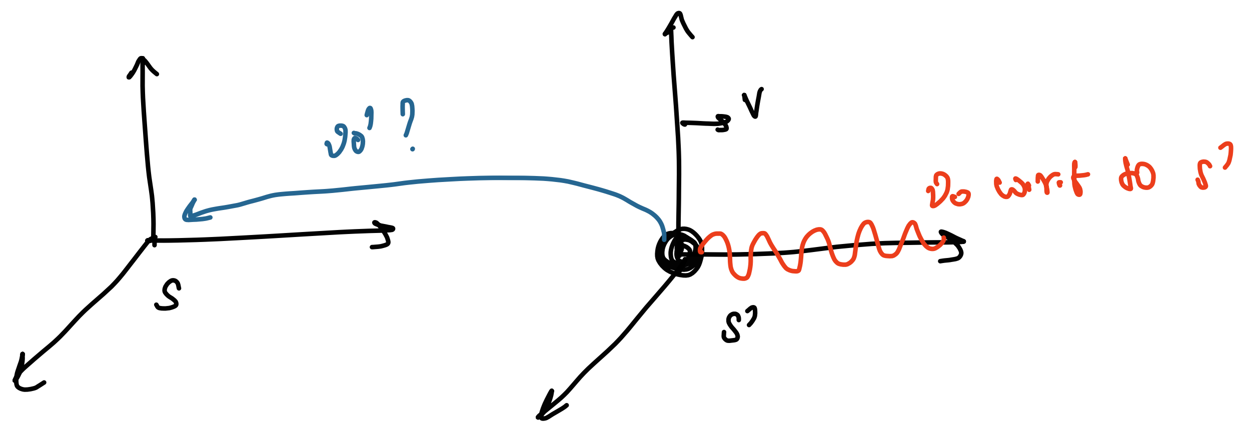
Only dependant on rest mass energy!

i.e inertial frame independant

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

कणों के समूह की कुल ऊर्जा E का रैखीय संवेग p हो तो : that will also be frame independant

Doppler Effect



S' frame:

E_1

E_2

at t_1': time @ which first crest comes

at $t_2' : t_1' + \Delta t'$ time @ which second crest comes

time diff b/w E_2 and E_1 ⇒ $\Delta t'$

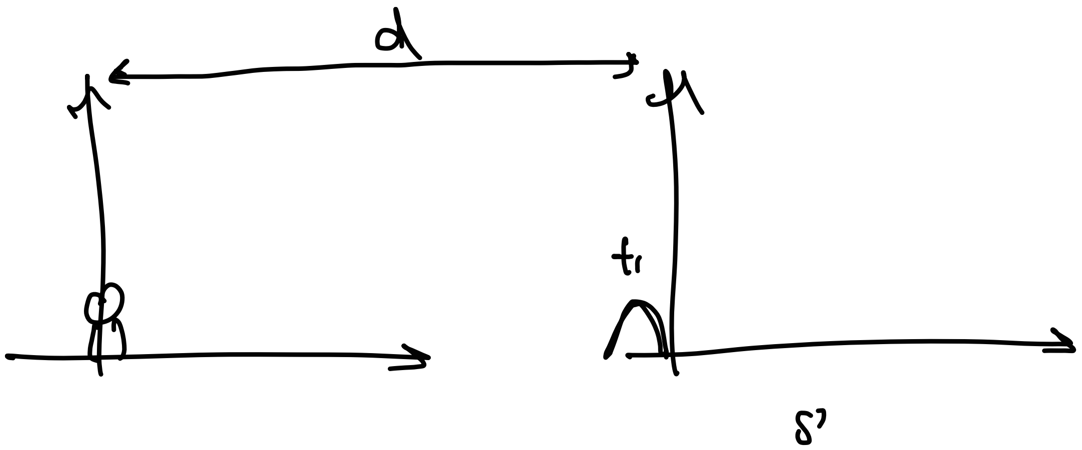
$$\nu_0 = \frac{1}{\Delta t'}$$

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{V^2}{c^2}}}$$

∵ E_1, E_2 happen at same x; the $\Delta t'$ thus measured is called proper time int → the smallest time intv b/w those 2 events in any frame.

Ie in Earth frame,

every Δt sec, a crest happens. But when does it reach, With respect to Earth

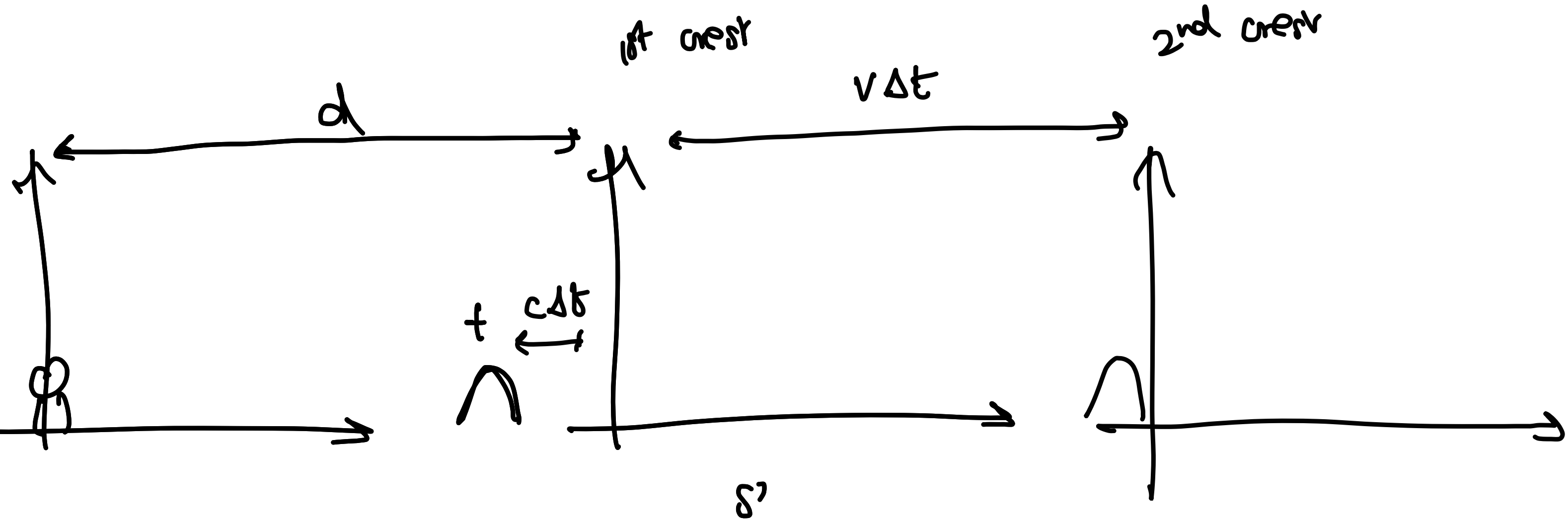


पहले crest के निकलने का समय : t_1

पहले crest के पहुँचने का समय : $t_1 + \frac{d}{c}$ → speed of wave

(here, luckily its frame independent, so even if the observer moved it would be v)

दूसरे crest के निकलने का समय : $t_1 + \Delta t$



दूसरे crest के निकलने का समय : $t_1 + \Delta t$

दूसरे crest के पहुँचने का समय :

∴ time diff b/w 2 crests:

$$\cancel{t_1} + \Delta t + \frac{\cancel{d} + v\Delta t}{\cancel{c}} - \cancel{t_1} - \frac{\cancel{d}}{\cancel{c}}$$

$$\Rightarrow \Delta t + \frac{v\Delta t}{c} \Rightarrow \Delta t\left(1+\frac{v}{c}\right) = \frac{\Delta t'\,(1+v)}{\sqrt{1-v^2/c^2}}$$

$$\frac{1}{\nu_0^*} \Rightarrow \frac{\left(1+\frac{v}{c}\right)}{(\nu_0)\sqrt{1-v^2/c^2}}$$

$\sqrt{a^2-b^2}$

$$\Rightarrow \nu_0^* = \frac{\nu_0\sqrt{1-v^2/c^2}}{1+v/c} = \nu_0\sqrt{\frac{1-v/c}{1+v/c}}$$

$$\boxed{\nu^* = \nu_0 \sqrt{\frac{c-v}{c+v}}}$$

RELATIVISTIC DOPPLER

EFFECT