1. Big omega Notation: prove that $g(n)=n^3+2n^2+4n$ is $\Omega(n^3)$

$g(n) \geq c \cdot n^3$

$g(n) = n^3+2n^2+4n$

for finding constant c and n_0

$n^3+2n^2+4n \geq c \cdot n^3$

Divide both sides with n^3

$\frac{1+2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1+\frac{2}{n}+\frac{4}{n} \geq c$

Here $\frac{2}{n}$ and $\frac{4}{n^2} \geq c$

$1+\frac{2}{n}+\frac{4}{n^2} \geq 1$

example $c = 1/2$

$1+2/n+4/n^2 \geq \frac{1}{2}$

$1+2/n+4/n^2 \geq 1$

$1+2/n+4/n^2 \geq 1/2$

Thus $g(n) = n^3+2n^2+4n$ is needed $\Omega(n^3)$

2. Big theta notation: determine whether $h(n) = 4n^2+3n$ is $\Theta(n^2)$ or not

$c_1 n^2 \leq h(n) \leq c \cdot n^2$

It upper bond $h(n)$ is $O(n^2)$

In lower bond $h(n)$ is $\Omega(n^2)$

upper bond $(O(n^2))$

$h(n) = 4n^2+3n$

$h(n) \leq c_2 n^2$

$4n^2+3n \leq c_2 n^2 \Rightarrow 4n^2+3n \leq cn^2$

Let $c_2 = 5$

divide both sides by n^2

$4+3/n \leq 5$

$h(n) = 4n^2+3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

$h(n) = 4n^2+3n$ is $\Theta(n^2)$

$$1 + \frac{n}{n\log n} \leq 2$$

$$1 + \frac{1}{\log n} \leq c_2$$

$$1 + \frac{1}{\log n} \leq 2$$

then $h(n)$ is $O(n \log n)$

lower bond

$$h(n) \geq c_1(n \log n)$$

$$h(n) = n\log n + n$$

$$n\log n + n \geq c_1 n \log n$$

divide both sides by $n \log n$

$$1 + \frac{n}{n\log n} \geq c_1$$

$$1 + \frac{1}{\log n} \geq c_1 \quad (\text{simpicity } c_1 = 1)$$

$$\frac{1}{\log n} \geq 0 \text{ for all } n > 1$$

$h(n)$ is $\Omega(\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n \log n)$

3. Solve the following recurrence relation & find the order of growth of solution $T(n) = 4T(n/2) + n^2$, $T(1) = 1$

Let $f(n) = n^3 - 2n^2 + n$ and $g(n) - n^2$ show whether $f(n) = \Omega$ $g(n)$ is true or false

$$f(n) \geq c_1 g(n)$$

Substituting $f(n)$ & $g(n)$ in to this inequality we get

$$n^3 - 2n^2 + n \geq c(-n^2)$$

$\forall$ and c and n_0 holds $n \geq n_0$

$$n^3 - 2n^2 + n \geq -cn^2$$

$$n^3 - 2n^2 + n + cn^2 \geq 0 \quad (n^3 \geq 0)$$

$$n^3 + ((c-2))n^2 + n \geq 0$$

$$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

$\therefore$ The statement $f(n) = \Omega g(n)$ is true,

4. Determine whether $h(n) = n\log n + n$ is $\Theta(\log n)$ prove a rigrous proof for you conclusion

$c_1 n\log n \le n \le c_2 n\log n$

upper bound

$h(n) \le c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \le c_2 n\log n$

divide both side by $n\log n$

$1 + \frac{n}{n\log n} \le 2$

$1 + \frac{1}{\log n} \le c_2$

$1 + \frac{1}{\log n} \le 2$

Then $h(n)$ is $O(n\log n)$.

lower bond

$h(n) \ge c_1 n_1 \log n$

$h(n) = n\log n + n$

$n\log n + n \ge c_1 n\log n$

divide both sides by $n\log n$

$1 + \frac{n}{\log n} \ge c_1$

$1 + \frac{1}{\log n} \ge c_1$

$\frac{1}{\log n} \ge 0$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

5. Solve the following recurrance relations and find the order of growth of solutions.

$T(n) = 4T(n/2) + n^2 \; T(1) = 1$

$T(n) = 4T(n/2) + n^2, \; T(1) = 1$

$T(n) = 4T(n/2) + n^2, \; T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4, b = 2, f(n) = n^2$

Applying master theoram

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - c})$

$f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + E})$, then $T(n) = f(n)$

$T(n) = aT(n/2) + n^2, \; T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log n)$

$T(n) = \Omega(n^{\log_b a + E})$, then $T(n) = f(n)$

calculating $\log_b a$

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = O(n^2)$ (comparing $f(n)$ with $n^{\log_b a}$)

$f(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$

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