1. Solve the following recurrence relation

a) $x_n = x(n-1)+5$ for $n>1$ with $x(1)=0$

1) write down the first term to identify the pattern

$x(1)=0$

$x(2)=x(1)+5=5$

$x(3)=x(2)+8=10$

$x(4)=x(3)+5=15$

2) identify the pattern (or) the general term

→ The first term $x(1)=0$

The common difference $d=5$

The general formula nth term on AP is

$x(n)=x(1)+x(n-1)d$

Substituting the given values

$x(n)=0(n-1):5=5(n-1)$

The solution is

$x(n)=5(n-1)$

b) $x(n)=3x(n-1)$ for $n>1$ with $x(1)=4$

1) write down first term identify the pattern

$x(1)=4$

$x(2)=3x-1=3\cdot4=12$

$x(3)=3x(2)=26$

$x(4)=3x(9)=108$

2) identify the general terms

→ the first term $x(1)=4$

The common ratio $r=3$

The general formula nth term of gp is

$x(n)=x(1)=r^{n-1}$

Substituting given value $x(n)=4\cdot3^{n-1}$

The sol is the $x(n)=4\cdot3^{n-1}$

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c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$)

for $n = 2^k$ we can write reference in terms of k

1) Substituting $n = 2^k$ in the reccerence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2) write down can write few terms identify the pattern

$x(1) = 1$

$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$

$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$

$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$

3) identify the general term finding the pattern

we observe that:

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + --$$

since $x(1) = 1$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + --$$

the geometric series with term $n=2$ and the last term 2^k except for additional term

the sum of geometrical series with ratio $r=2$ given by

$$S = a\frac{r^n - 1}{r - 1}$$

were $a = 2$, $r = 2$ and $n = k$

$$S = \frac{2^k - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

Adding the +1 term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

solution is $2^{k+1} - 1$

d) $x(n) = x(n/3)+1$ for $n>1$ with $x(1)=1$ (solve for $n=3^k$)

for $n=3^k$ we can write the recurrence the terms of k

1) substituting $n=3^k$ in the recurrence

$$x(3^k) = x(3^{k-1})+1$$

2) write down few terms identify the pattern

$$x(1)=1$$

$$x(3) = x(3^1)+1 = 1+1 = 2$$

$$x(9) = x(3^2) = x(3)+1 = 2+1 = 3$$

$$x(27) = x(3^3) = x(9)+1 = 3+1 = 4$$

3) identify the general term:

we observe that

$$x(3^k) = x(3^{k-1})+1$$

summing up the series

$$x(3^k) = 1+1+1+ \dots +1$$

$$x(3^k) = k+1$$

The solution

$$x(3^k) = k+1$$

2. Evaluate the following recurrence complexity

(i) $T(n) = T(n/2)+1$ where $n=2^k$ for all $k \le 0$

the recurrence reaction can be solved using iteration method

1) substituting $n=2^k$ recurrence

2) iterate the recurrence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(8) \pm T(n)+1 = T((1)+1)+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = T((1)+2)+1 = T(1)+3$

3) generalize the pattern

$$T(2^k) = T(1)+k$$

Since $n=2^k$, $k=\log_2 n$

$T(n)=T(2^k)=T(1)+\log_2 n$

4) Assume $T(1)$ is constant c

$T(n)=c+\log_2 n$

The solution is

$T(n)=O(\log n)$

ii) $T(n)=T(n/3)+T(2n/3)+cn$ where c is constant and n input size

The recurrence can solved by using masters theoram for divide_and_conquer recurrence from

$T(n)=aT(n/b)+f(n)$

where $a=2$, $b=3$, and $f(n)=cn$

Lets determine the value $\log_b a$

$\log_b a = \log_3 2$

using properties logarithams

$\log_3 2 = \frac{\log 2}{\log 3}$

Now we compare $f(n)=n$ with $n^{\log_3 2}$

$f(n)=O(n)$

$n=n^1$

since $\log_3 2$ we are third case master's theoram

$f(n)=O(n^c)$ with $c>\log_b a$

The sol is $T(n)=O(f(n))=O(cn)=O(n)$

3) consider the following recurrence algoritham

min [A(0 -- n-2)"

if n=1 return A [0]

Else temp = min [1A(0 -- n-2]

if temp < = A(n-1) return temp

else

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return A[n-1]

a) what does this algoritham compute

The given algoritham min[A[0--n-1] compute the min value in array 'A' from indexa 0 for n-1 it does by recursively finding the min value sub array A[0---n-2] and then comparing with lost element A[n-1] to determine the overall max value

~~To~~

To determine recurrence relation for algoritham basic operation count, let's analyze steps involved in the algoritham the basic operation comparision function calls

Recurrence relation setup

1) Base case when $n=1$, the algoritham performs single operation to return A[0].

2) recursive case when $n>1$ the algorithan

make recursive call to min [A(0--n-2]):

perform comparsion b/w temp and A[n-1] let T(n) represent the algoritham. perform array of size n

1) Base case:

$T(1)=1$

2) recursive case

$T(n) = T(n-1)+1$

Here T(n-1) ~~+~~ account for operation performed by the recu~~here T~~rsive call to min [A(0--n-2)] and the +1 account comparision b/w temp and A[n--1]

To solve the relation we can use iterate method

$T(n) = T(n-1)+1$

$= T((n-2)+1)+1$

$= T((n-3)+1)+1)+1$

$- \; --$

$= 1+(n-1)$

$= n$

~~the~~ solution is

$T(n) = n$

This means algoritham perform basic operation for an input array of size n

4) Analyze the order of growth

1) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $\Omega(g(n))$ rotation

to analyze the order of growth use the Ω rotation we need to compare the given function $F(n)$ & $g(n)$

Given function

$F(n) = 2n^2+5$

$g(n) = 7n$

order of growth using $\Omega(g(n))$ Notation

The notation $\Omega(g(n)$ describle lower bond on the growth rate sufficent large $f(n)$ grows at least $g(n)$

$F(n) = g(n)$

Less analyze $F(n) = 2n^2+5$ with respect to $g(n) = 7n$

1) identify Dominant terms:

→ The dominant term in $F(n)$ is $2n^2$. since it grows paster than constant term as n increase

→ the dominent term in $g(n)$ is $7n$

2) Establish the inequality

→ we want to find constant c and n_0 such that

$$2n^2+5 \geq c \cdot 7n \text{ for all } n \geq n_0$$

3) simplify the inequality

→ ignore the lower order than 5 for larger

$$2n^2 \geq 7cn$$

→ Divide both sides by n

$$2n \geq 7c$$

→ solve from n

$$n \geq 7c/2$$

4) choose constants

Let $c=1$

$$n \geq \frac{7.1}{2} = 3.5$$

for $n \geq n$ the inequality holds

$$2n^2+5 \geq 7n \text{ for all } n \geq n$$

we use shown the there exist constants $c \geq 1$ and $n_0 = n$

such that for all $n \geq n_0$

$$2n^2+5 \geq 7n$$

Thus, we can concude that

$$P(n) = 2n^2+5 = \Omega(7n)$$

in Ω notation the domian term $2n^2$ f(n) clearly grows

faster than n. Hence

$$f(n) = \Omega(n^2)$$

However for the specific comparision asked $f(n) = \Omega(7n)$

is also correct

showing that f(n) grows at least as past as 7n

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