5.1.1 : Design context free grammars for the following languages.

(a) The set $\{0^n 1^n \mid n \geq 1\}$, that is set of all strings of one or more 0's followed by an equal number of 1's

$L = \{01, 0011, 000111, \ldots\}$

$S \to 0S1 \mid 01$

$G = (V, T, P, S)$

$V = \{S\}$

$T = \{0, 1\}$

$P = S \to 0S1 \mid 01$

$S \to S$

(b) The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a diff. number of b's and c's, or both.

$S \to AB \mid CD$

$A \to aA \mid \varepsilon$

$B \to bBc \mid E \mid cD$

$C \to aCb \mid E \mid aA$

$D \to cD \mid \varepsilon$

$E \to bE \mid b$

The grammar works as.

1) A generates zero or more a's

2) D generates zero or more c's.

3) E generates one or more b's

4) B generates an equal number of b's and c's, the produces either one or more b's (via E) or one or more c's (via CD)

That is, B generates strings in $b^* c^*$ with an unequal number of b's & c's

5) C generates unequal number of a's then b's

6) AB generates strings in $a^* b^* c^*$ with an unequal number of b's & c's while CD generates strings in $a^* b^* c^*$ with an unequal number of a's & b's

Context free grammar.

$G = (V, T, P, S)$

$V = \{S, A, B, C, D, E\}$

$T = \{a, b, c\}$

P = Productions as listed above

$S = \{S\}$

(c) The set of all strings of a's and b's that are not of the ww; that is not equal to any string repeated.

$G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b\}$

$P = S \rightarrow AB \mid BA \mid A \mid B$

$A \rightarrow aAa \mid aAb \mid bAa \mid bAb \mid a$

$B \rightarrow aBa \mid aBb \mid bBa \mid bBb \mid b$

$S \rightarrow \{S\}$

(d) The set of all strings with twice as many 0's as 1's

$L = \{ \varepsilon, \cancel{00} \; 011, 001111, 000111111, \dots \}$

$S \rightarrow SS \mid 00S1 \mid 1S00 \mid 0S1S0 \mid \varepsilon$

CFG $\Rightarrow$ $G = (V, T, P, S)$

$V = \{S\}$

$T = \{0, 1\}$

$P = S \rightarrow SS \mid 00S1 \mid 1S00 \mid 0S1S0 \mid \varepsilon$

$\{S \rightarrow S1S0S0S \mid S0S1S0S \mid S0S0S1S\}$

$S = \{S\}$

S.1.2

The following grammar generates the language of regular expression $0^* 1 (0+1)^*$

$S \rightarrow A1B \rightarrow 1$

$A \rightarrow 0A \mid \varepsilon \rightarrow 3$ (2)

$B \rightarrow 0B \mid 1B \mid \varepsilon \rightarrow 6$ (4, 5)

Give leftmost and rightmost derivation of the following strings

① 00101

Leftmost derivation :-

$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 001B \Rightarrow 0010B$

$\Rightarrow 00101B \Rightarrow 00101$

Rightmost :-

$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A101B \Rightarrow A101 \Rightarrow 0A101$

$\Rightarrow 00A101 \Rightarrow 00101$

② 1001

Leftmost :-

$S \Rightarrow A1B \Rightarrow 1B \Rightarrow 10B \Rightarrow 100B \Rightarrow 1001B \Rightarrow 1001$

Rightmost :-

$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A100B \Rightarrow A1001B \Rightarrow A1001 \Rightarrow 1001$

③ 00011

Leftmost :-

$$S \overset{1}{\Rightarrow} A1B \overset{2}{\Rightarrow} 0A1B \overset{2}{\Rightarrow} 00A1B \overset{2}{\Rightarrow} 000A1B \overset{3}{\Rightarrow} 0001B \overset{5}{\Rightarrow} 00011B$$

$$\overset{6}{\Rightarrow} 00011$$

Rightmost :-

$$S \overset{1}{\Rightarrow} A1B \overset{5}{\Rightarrow} A11B \overset{6}{\Rightarrow} A11 \overset{2}{\Rightarrow} 0A11 \overset{2}{\Rightarrow} 00A11 \overset{2}{\Rightarrow} 000A11$$

$$\overset{3}{\Rightarrow} 00011$$

S.4.5

This question concerns the grammar from ex. 5.1.2 which we reproduce here:

$S \to A1B$

$A \to 0A \mid \varepsilon$

$B \to 0B \mid 1B \mid \varepsilon$

ⓐ S.T this grammar is unambiguous.

The gramma is said to be unambiguous when the given grammar can be derived from both right and leftmost derivation.

Ex:- 00101

Lm :- $S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 0010B \Rightarrow 00101B \Rightarrow 00101$

Rm :- $S \Rightarrow A1B \Rightarrow A10B \Rightarrow A101B \Rightarrow 0A101 \Rightarrow 00A101 \Rightarrow 00101$

(a) So the grammar is unambiguous.

(b) The grammar that is ambiguous for the language is 00101

$S \to A1B$

$A \to 0A|\varepsilon$

$B \to 1B|\varepsilon$

neglecting 0B

Left derivation

$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 0011$

Right derivation

$S \Rightarrow A1B \Rightarrow A11B \Rightarrow 0111$

Hence, 00101 is unambiguous as we cannot derive from both the derivation from the language.

<u>5.4.7</u>

Consider the following grammar:

$E \to +EE|*EE|-EE|x|y.$

(1) Find LMD and RMD and derivation tree for the string $+*-xyxy$.

$E \to +EE$ — (1)

$E \to *EE$ — (2)

$E \to -EE$ — (3)

$E \to x$ — (4)

$E \to y$ — (5)

Left most derivation :-

$$E \xRightarrow{1} +EE \xRightarrow{2} +*EE \xRightarrow{3} +*-EEEE \xRightarrow{4} +*-xEEE$$

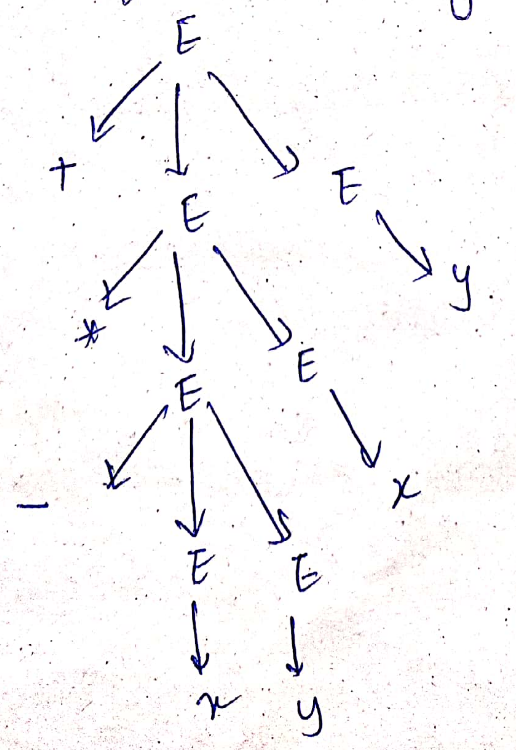
$$\xRightarrow{5} +*-xyEE \xRightarrow{4} +*-xyxE \xRightarrow{5} +*-xyxy$$

Right most derivation :-

$$E \xRightarrow{1} +EE \xRightarrow{5} +Ey \xRightarrow{2} +*EEy \xRightarrow{4} +*Exy$$

$$\xRightarrow{3} +*-EExy \xRightarrow{5} +*-Eyxy \xRightarrow{4} +*-xyxy$$

Parse tree for derivation of ~~+*~~ +*-xy xy



(8) As shown from leftmost & rightmost derivation of $+ * - xy.xy$. The grammar $+ * - xyxy$ can be derived from both the derivations

∴ The grammar given is unambiguous proved by ex.