From wormhole to time machine:
Remarks on Hawking's chronology protection conjecture

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The recent interest in "time machines" has been largely fueled by the *apparent* ease with which such systems may be formed in general relativity, given relatively benign initial conditions such as the existence of traversable wormholes or of infinite cosmic strings. This rather disturbing state of affairs has led Hawking to formulate his *chronology protection conjecture*, whereby the formation of "time machines" is forbidden. This paper will use several simple examples to argue that the universe appears to exhibit a "defense in depth" strategy in this regard. For appropriate parameter regimes, Casimir effects, wormhole disruption effects, and gravitational back reaction effects all contribute to the fight against time travel. Particular attention is paid to the role of the quantum gravity cutoff. For the class of model problems considered it is shown that the gravitational back reaction becomes large before the Planck scale quantum gravity cutoff is reached, thus supporting Hawking's conjecture.

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I. INTRODUCTION

This paper addresses Hawking's chronology protection conjecture [1, 2], and some of the recent controversy surrounding this conjecture [3].

The recent explosion of interest in "time machines" is predicated on the fact that it *appears* to be relatively easy to construct such objects in general relativity. Time machines have been based on the traversable wormholes of Morris and Thorne [4, 5], and on Gott's cosmic string construction [6].

These results have led to a flurry of papers aimed at either making one's peace with the notion of time travel [7–9], or of arguing that, despite *prima facie* indications, the construction of time machines is in fact impossible. Papers specifically addressing Gott's cosmic string construction include [10, 11]. In a more general vein Hawking's chronology protection conjecture [1, 2] is applicable to both wormhole constructions and cosmic string constructions.

This paper will focus on Hawking's chronology protection conjecture specifically as applied to traversable wormholes. The discussion will be formulated in terms of a simple model for a traversable wormhole which enables one to carefully explain the *seemingly* trivial manipulations required to turn a traversable wormhole into a time machine. To help illustrate the manner in which nature might enforce the chronology protection conjecture the utility of two particular approximation techniques will be argued. Firstly, it is extremely useful to replace the original dynamical system with a quasistationary approximation where one considers adiabatic variation of parameters describing a stationary spacetime. It shall be argued that this approximation captures essential elements of the physics of time machine construction while greatly simplifying technical computations, pedagogical

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issues, and conceptual issues. Secondly, for a particular class of wormholes, once one is sufficiently close to forming a time machine, it shall be shown that it is a good approximation to replace the wormhole mouths with a pair of planes tangent to the surface of the mouths. This approximation now reduces all computations to variations on the theme of the Casimir effect and permits both simple and explicit calculations in a well-controlled parameter regime.

The reader's attention is particularly drawn to the existence and behavior of a one-parameter family of closed "almost" geodesics which thread the wormhole throat. The wormhole is deemed to initially be well behaved so that the closed geodesics belonging to this one-parameter family are initially spacelike. The putative construction of a time machine will be seen to involve the invariant length of the members of this family of geodesics shrinking to zero—so that the family of geodesics, originally spacelike, becomes null, and then ultimately becomes timelike. Not too surprisingly, it is the behavior of the geometry as the length of members of this family of closed geodesics shrinks to invariant length zero that is critical to the analysis.

It should be emphasised that the Universe appears to exhibit a "defense in depth" strategy in this regard. For appropriate ranges of parameters describing the wormhole (such as masses, relative velocities, distances, and time shifts) Casimir effects (geometry induced vacuum polarization effects), wormhole disruption effects, and gravitational back reaction effects all contribute to the fight against time travel.

The overall strategy is as follows: start with a classical background geometry on which some quantum fields propagate — this is just the semiclassical quantum gravity approximation. In the vicinity of the aforementioned family of closed spacelike geodesics the vacuum expectation value of the renormalized stress-energy tensor may be calculated approximately — this calculation is a minor variant of the standard Casimir effect calculation. The

associated Casimir energy will diverge as one gets close to forming a time machine. Furthermore, as the length of the closed spacelike geodesics tends to zero the vacuum expectation value of the renormalized stress-energy tensor itself diverges, until ultimately vacuum polarization effects become larger than the exotic stress-energy required to keep the wormhole throat open, hopefully disrupting the wormhole before a time machine has a chance to form. Finally, should the wormhole somehow survive these disruption effects, by considering linearized gravitational fluctuations around the original classical background the gravitational back reaction generated by the quantum matter fields can be estimated. For the particular class of wormholes considered in this paper this back reaction will be shown to become large before one enters the Planck regime. It is this back reaction that will be relied upon as the last line of defence against the formation of a time machine. In this matter I am in agreement with Hawking [1, 2].

Units. Adopt units where $c \equiv 1$, but all other quantities retain their usual dimensionalities, so that in particular $G = \hbar/m_P^2 = \ell_P^2/\hbar$.

II. FROM WORMHOLE TO TIME MACHINE

The seminal work of Morris and Thorne on traversable wormholes [4] immediately led to the realization that, given a traversable wormhole, it *appeared* to be very easy to build a time machine [4, 5]. Indeed so easy is the construction that it *seemed* that the creation of a time machine might be the generic fate of a traversable wormhole [12].

In this regard it is useful to perform a gedanken experiment that clearly and cleanly separates the various steps involved in constructing a time machine: step 0—acquire a traversable wormhole; step 1—induce a "time shift" between the two mouths of the wormhole; step 2—bring the wormhole mouths close together. It is only the induction of the "time shift" in step 1 that requires intrinsically relativistic effects. It may perhaps be surprising to realize that the apparent creation of the time machine in step 2 can take place arbitrarily slowly in a nonrelativistic manner. (This observation is of course the ultimate underpinning for one's adoption of quasistationary adiabatic techniques.)

A. Step 0—Acquire a traversable wormhole

It is well known that topological constraints prevent the classical construction of wormholes *ex nihilo* [13–15]. The situation once one allows quantum gravitational processes is less clear-cut, but it is still quite possible that topological selection rules might constrain [16–19] or even forbid [20] the quantum construction of wormholes *ex nihilo*. If such proves to be the case then even an arbitrarily advanced civilization would be reduced to the mining of primordial traversable wormholes — assuming such primordial wormholes to have been built into the multiverse by the "first cause." Nevertheless, to get the discussion started, assume (following Morris and Thorne)

that an arbitrarily advanced civilization has by whatever means necessary acquired and continues to maintain a traversable wormhole.

In the spirit of Refs. [20–22] it is sufficient, for the purposes of this section, to take an extremely simple model for such a traversable wormhole — indeed to consider the wormhole as being embedded in flat Minkowski space and to take the radius of the wormhole throat to be zero. Thus our wormhole may be mathematically modeled by Minkowski space with two timelike world lines identified. For simplicity one may further assume that initially the two mouths of the wormhole are at rest with respect to each other, and further that the wormhole mouths connect "equal times" as viewed from the mouth's rest frame. Mathematically this means that one is considering 3+1 Minkowski space with the following two lines identified:

$$\ell_1^\mu(\tau) = \bar{\ell}_0^\mu + \frac{1}{2}\delta^\mu + V^\mu\tau, \quad \ell_2^\mu(\tau) = \bar{\ell}_0^\mu - \frac{1}{2}\delta^\mu + V^\mu\tau,$$

$$(2.1)$$

where V^μ is an arbitrary timelike vector, δ^μ is perpendicular to V^μ and so is spacelike, and $\bar{\ell}_0^\mu$ is completely arbitrary. The center of mass of the pair of wormhole mouths follows the line

$$\bar{\ell}^\mu(\tau) = \bar{\ell}_0^\mu + V^\mu\tau, \quad\quad (2.2)$$

and the separation of the wormhole mouths is described by the vector δ^μ.

The present construction of course does not describe a time machine, but one shall soon see that very simple manipulations *appear* to be able to take this "safest of all possible wormholes" and turn it into a time machine.

B. Step 1—Induction of a time shift

As the penultimate step in one's construction of a time machine it is necessary to induce a (possibly small) "time shift" between the two mouths of the wormhole. For clarity, work in the rest frame of the wormhole mouths. After a suitable translation and Lorentz transformation the simple model wormhole of step 0 can, without loss of generality, be described by the identification $(t, 0, 0, 0) \equiv (t, 0, 0, \delta)$. One wishes to change this state of affairs to obtain a wormhole of type $(t, 0, 0, 0) \equiv (t+T, 0, 0, \ell)$, where T is the time shift and ℓ is the distance between the wormhole mouths. Mathematically one wishes to force the vectors V^μ and δ^μ to no longer be perpendicular to each other so that it is possible to define the time shift to be $T = V^\mu\delta_\mu$, while the distance between the mouths is $\ell = \|(\delta^\mu{}_\nu + V^\mu V_\nu)\delta^\nu\|$. Physically this may be accomplished in a number of different ways.

1. The relativity of simultaneity

Apply identical forces to the two wormhole mouths so that they suffer identical accelerations (as seen by the initial rest frame). It is clear that if the mouths initially connect equal times (as seen by the initial rest frame), and

if their accelerations are equal (as seen by the initial rest frame), then the paths followed by the wormhole mouths will be identical up to a fixed translation by the vector δ^μ. Consequently the wormhole mouths will always connect equal times, as seen by the initial rest frame. Mathematically this describes a situation where δ^μ is a constant of the motion while V^μ is changing. Of course, after the applied external forces are switched off the two mouths of the wormhole have identical four-velocities V_f^μ not equal to their initial four-velocities V_i^μ. And by the relativity of simultaneity equal times as viewed by the initial rest frame is not the same as equal times as viewed by the final rest frame. If one takes the center of motion of the wormhole mouths to have been accelerated from four-velocity $(1, 0, 0, 0)$ to four-velocity $(\gamma, 0, 0, \gamma\beta)$ then the relativity of simultaneity induces a time shift

$$T = \gamma\beta\delta, \qquad (2.3)$$

while (as seen in the final rest frame) the distance between the wormhole mouths becomes $\ell = \gamma\delta$.

2. Special relativistic time dilation

Another way of inducing a time shift in the simple model wormhole of step 0 is to simply move one of the wormhole mouths around and to rely on special relativistic time dilation. This can either be done by using rectilinear motion [5] or by moving one of the mouths around in a large circle [23]. Assuming for simplicity that one mouth is unaccelerated, that the other mouth is finally returned to its initial four-velocity, and that the final distance between wormhole mouths is the same as the initial distance, then the induced time shift is simply given by

$$T = \int_i^f (\gamma - 1) dt. \qquad (2.4)$$

Mathematically, V^μ and ℓ are unaltered, while T and hence δ^μ are changed by this mechanism.

3. General relativistic time dilation

As an alternative to relying on special relativistic time dilation effects, one may instead rely on the general relativistic time dilation engendered by the gravitational redshift [12]. One merely places one of the wormhole mouths in a gravitational potential for a suitable amount of time

to induce a time shift:

$$T = \int_i^f (\sqrt{g_{00}} - 1) dt.$$

4. Comment

Of course, this procedure has not yet built a time machine. The discussion so far has merely established that given a traversable wormhole it is trivial to arrange creation of a traversable wormhole that exhibits a time shift upon passing through the throat — this time shift can be created by a number of different mechanisms so that the ability to produce such a time shift is a very robust result.

C. Step 2—Bring the wormhole mouths close together

Having constructed a traversable wormhole with time shift the final stage of time machine construction is deceptively simple: merely push the two wormhole mouths towards one another (this may be done as slowly as is desired). A time machine forms once the physical distance between the wormhole mouths ℓ is less than the time shift T. Once this occurs, it is clear that closed timelike geodesics have formed — merely consider the closed geodesic connecting the two wormhole mouths and threading the wormhole throat. As an alternative to moving the mouths of the wormhole closer together one could of course arrange for the time shift to continue growing while keeping the distance between the mouths fixed — such an approach would obscure the distinctions between steps 1 and 2.

The advantage of clearly separating steps 1 and 2 in one's mind is that it is now clear that the wormhole mouths may be brought together arbitrarily slowly — in fact adiabatically — and still *appear* to form a time machine. If one makes the approach adiabatic (so that one can safely take the time shift to be a constant of the motion) then the wormhole may be mathematically modeled by Minkowski space with the lines $(t, 0, 0, z(t))$ and $(t + T, 0, 0, \ell_0 - z(t))$ identified. The physical distance between the wormhole mouths is $\ell(t) = \ell_0 - 2z(t)$. The family of closed "almost" geodesics alluded to in the Introduction is just the set of geodesics (straight lines), parametrized by t, connecting $\ell_1^\mu(t)$ with $\ell_2^\mu(t)$. Specifically, with $\sigma \in [0, 1]$,

$$\begin{aligned} X^\mu(t, \sigma) &= \sigma\ell_1{}^\mu(t) + \{1 - \sigma\}\ell_2{}^\mu(t) \\ &= \Big(t + \{1 - \sigma\}T, 0, 0, \{1 - \sigma\}\ell_0 - \{1 - 2\sigma\}z(t)\Big). \end{aligned} \qquad (2.5)$$

Here σ is a parameter along the closed geodesics, while t parametrizes the particular closed geodesic under consideration. These curves are true geodesics everywhere except at $(\sigma = 0) \equiv (\sigma = 1)$, the location of the worm-

hole mouths, where there is a "kink" in the tangent vector induced by the relative motion of the wormhole mouths. The invariant length of the members of this family of closed geodesics is easily seen to be

$$\delta(t) = \|\ell_1{}^\mu(t) - \ell_2{}^\mu(t)\| = \|(T, 0, 0, \ell(t))\| = \sqrt{\ell(t)^2 - T^2}.$$

$$(2.6)$$

Once $\ell(t) < T$ a time machine has formed. Prior to time machine formation the vector $(T, 0, 0, \ell(t))$ is, by hypothesis, spacelike. Therefore it is possible to find a Lorentz transformation to bring it into the form $(0, 0, 0, \delta)$ with $\delta = \sqrt{\ell^2 - T^2}$. A brief calculation shows that this will be accomplished by a Lorentz transformation of velocity $\beta = T/\ell(t) = T/\sqrt{T^2 + \delta^2(t)}$, so that $\gamma = \ell/\delta$ and $\gamma\beta = T/\delta$. In this new Lorentz frame the wormhole connects "equal times," and this frame can be referred to as the frame of simultaneity (synchronous frame). Note, in particular, that as one gets close to building a time machine $\delta(t) \to 0$, so that $\beta \to 1$, and the velocity of the frame of simultaneity approaches the speed of light.

D. Discussion

The construction process described above clearly separates the different effects at work in the putative construction of time machines. Given a traversable wormhole, arbitrarily small special and/or general relativistic effects can be used to generate a time shift. Given an arbitrarily small time shift through a traversable wormhole, arbitrarily slow adiabatic motion of the wormhole mouths towards each other *appears* to be sufficient to construct a time machine. This immediately leads to all of the standard paradoxes associated with time travel and is very disturbing for the state of physics as a whole.

As a matter of logical necessity precisely *one* of the following alternatives must hold: (1) the boring physics conjecture, (2) the Hawking chronology protection conjecture, (3) the Novikov consistency conspiracy, (4) the radical rewrite conjecture.

The boring physics conjecture may roughly be formulated as "suffer not traversable wormholes to exist." Merely by asserting the nonexistence of traversable wormholes all time travel problems of the particular type described in this paper go quietly away. However, to completely forbid time travel requires additional assumptions, such as the nonexistence of (infinite length) cosmic strings and limitations on the tipping over of light cones. For instance, requiring the triviality of the fundamental group $\pi_1(\mathcal{M})$ (the first homotopy group) implies that the manifold possesses no closed noncontractible loops. Such a restriction would preclude both (1) traversable wormholes (more precisely: that class of traversable wormholes that connect a universe to itself), and (2) cosmic strings (both finite and infinite). But even this is not enough — time travel can seemingly occur in universes of trivial topology [1, 2, 24], so even stronger constraints should be imposed. In this regard Penrose's version of the strong cosmic censorship conjecture [25] implies the causality protection conjecture via the equivalence (strong cosmic censorship) $\Leftrightarrow$ (global hyperbolicity) $\Rightarrow$ (strong causality) $\Leftrightarrow$ ($\exists$ global time function). The "boring physics conjecture" might thus best be formulated as strong cosmic censorship together with triviality of the fundamen-

tal group $\pi_1(\mathcal{M})$. There is certainly no experimental evidence against the boring physics conjecture, but this is a relatively uninteresting possibility. In particular, considering the relatively benign conditions on the stress-energy tensor required to support a traversable wormhole it seems to be overkill to dispose of the possibility of wormholes merely to avoid problems with time travel.

Less restrictively the Hawking chronology protection conjecture allows the existence of wormholes but forbids the existence of time travel [1, 2]: "Suffer not closed nonspacelike curves to exist." If the chronology protection conjecture is to hold then there must be something wrong with the *apparently* simple manipulations described above that *appeared* to lead to the formation of a time machine. In fact, that is the entire thrust of this paper — to get some handle on precisely what might go wrong. It proves to be the case that the problem does not lie with the induction of a time shift — as previously mentioned this is a robust result that can be obtained through a number of different physical mechanisms. Rather, it is the apparently very simple notion of pushing the wormhole mouths together that is at fault. A number of physical effects seem to conspire to prevent one from actually bringing the wormhole mouths close together. This will be discussed in detail in subsequent sections.

More radically, the Novikov consistency conspiracy is willing to countenance the existence of both traversable wormholes and time travel, but asserts that the multiverse *must* be consistent no matter what — this point of view is explored in [7–9]: "Suffer not an inconsistency to exist."

More disturbingly, once one has opened Pandora's box by permitting closed timelike curves (time travel) I would personally be rather surprised if something as relatively mild as the Novikov consistency conspiracy would be enough to patch things up. Certainly the sometimes expressed viewpoint that the Novikov consistency conspiracy is the unique answer to the causality paradoxes is rather naive at best. The Novikov consistency conspiracy is after all still firmly wedded to the notion of spacetime as a four-dimensional Hausdorff differentiable manifold.

For a rather more violently radical point of view permit me to propound the radical rewrite conjecture wherein one posits a radical rewriting of all of known physics from the ground up. Suppose, for instance, that one models spacetime by a non-Hausdorff differentiable manifold. What does this mean physically? A non-Hausdorff manifold has the bizarre property that the dimensionality of the manifold is *not* necessarily equal to the dimensionality of the coordinate patches [26]. From a physicist's perspective, this idea has been explored somewhat by Penrose [25]. Crudely put: while coordinate patches remain four-dimensional in such a spacetime, the dimensionality of the underlying manifold is arbitrarily large, and possibly infinite. Local physics remains tied to nicely behaved four-dimensional coordinate patches. Thus one can, for instance, impose the Einstein field equations in the usual manner. Every now and then, however, a passing wave front (generated by a "branching event") passes

by and suddenly duplicates the whole Universe. It is even conceivable that a branching non-Hausdorff spacetime of this type might be connected with or connectable to the Everett "many worlds" interpretation of quantum mechanics [27, 28].

If one wishes an even more bizarre model of reality, one could question the naive notion that the "present" has a unique fixed "past history." After all, merely by adding a time reversed "branching event" to our non-Hausdorff spacetime one obtains a "merging event" where two universes merge into one. Not only is predicibility more than somewhat dubious in such a universe, but one appears to have lost retrodictability as well. Even more so than time travel, such a cognitive framework would render the Universe unsafe for historians, as it would undermine the very notion of the existence of a unique "history" for the historians to describe.

Such radical speculations might further be bolstered by the observation that if one takes Feynman's "sum over paths" notion of quantum mechanics seriously then all possible past histories of the Universe should contribute to the present "state" of the Universe.

I raise these issues not because I particularly believe that that is the way the Universe works, but rather, because once one has opened Pandora's box by permitting time travel, I see no particular reason to believe that the only damage done to our notions of reality would be something as facile as the Novikov consistency conspiracy.

III. QUANTUM EFFECTS

A. Vacuum polarization effects

Attention is now drawn to the effect that the wormhole geometry has on the propagation of quantum fields. Generically, one knows that a nontrivial geometry modifies the vacuum expectation value of the renormalized stress-energy tensor. One way of proceeding is to recall that the wormhole is modeled by the identification

$$(t, 0, 0, 0) \equiv (t + T, 0, 0, \ell). \tag{3.1}$$

This formulation implies that one is working in the rest frame of the wormhole mouths. For definiteness, suppose one is considering a quantized scalar field propagating in this spacetime. One could then, in principle, investigate solutions of d'Alembert's equation in Minkowski space subject to these identification constraints, find the eigenfunctions and eigenvalues, quantize the field and normal order, and in this manner eventually calculate $\langle 0|T^{\mu\nu}|0\rangle$. Alternatively, one could adopt point-splitting techniques. Such calculations are decidedly nontrivial. For the sake of tractability it is very useful to adopt a particular approximation that has the effect of reducing the problem to a generalized Casimir effect calculation.

Perform a Lorentz transformation of velocity $\beta = T/\ell$, so that in this new Lorentz frame the wormhole connects "equal times." In this frame of simultaneity (synchronous frame) the wormhole is described by the identification

$$(\gamma t, 0, 0, \gamma \beta t) \equiv (\gamma t, 0, 0, \gamma \beta t + \delta). \tag{3.2}$$

The discussion up to this point has assumed "pointlike" mouths for the wormhole throat. To proceed further one will need to take the wormhole mouths to have some finite size, and will need to specify the precise manner in which points on the wormhole mouths are to be identified. Two particularly simple types of pointwise identification are of immediate interest.

(1) Synchronous identification. $\forall x_1 \in \partial\Omega_1, x_2 \in \partial\Omega_2, x_1^\mu \equiv x_2^\mu = x_1^\mu + s\hat{\delta}^\mu$, where $\hat{\delta}^\mu$ is a fixed (spacelike) unit vector and s is a parameter to be determined. A particular virtue of synchronous identification is that once one goes to the synchronous frame, all points on the wormhole mouths are identified at "equal times," that is, $x_1 = (t, \mathbf{x}_1) \equiv (t, \mathbf{x}_2) = (t, \mathbf{x}_1 + s\hat{z}) = x_2$.

(2) Time-shift identification. $\forall x_1 \in \partial\Omega_1, x_2 \in \partial\Omega_2, x_1 = (t, \mathbf{x}_1) \equiv (t + T, \mathbf{x}_2) = x_2$, with $\mathbf{x}_1, \mathbf{x}_2$ ranging over the mouths of the wormhole in a suitable manner. This "time-shift" identification is the procedure adopted by Kim and Thorne and is responsible for many of the technical differences between that paper and this. A particularly unpleasant side effect of "time-shift" identification is that there is no one unique synchronous frame for the entire wormhole mouth.

Though these identification schemes apply to arbitrarily shaped wormhole mouths, one may for simplicity take the wormhole mouths to be spherically symmetric of radius R as seen in their rest frames. The resulting spacetime is known as a *capon* spacetime. Adopting "synchronous identification," in the frame of simultaneity the wormhole mouths are oblate spheroids of semimajor axis R, and semiminor axis $R/\gamma = R\delta/\ell$. Let us work in the parameter regime $\delta \ll \ell$ and $\delta \ll R$, then one may safely approximate the wormhole mouths by flat planes. One is then seen to be working with a minor generalization of the ordinary Casimir effect geometry — a Casimir effect with a time shift. (For a nice survey article on the Casimir effect see [29]. See also the textbooks [30, 31].)

Alternatively, one could work with the cubical wormholes of Ref. [21]. When two of the flat faces of such a wormhole are directly facing each other, the technical differences between synchronous and time-shift identifications vanish for those faces. With this understanding the following comments can also be applied to cubical wormholes with either identification scheme.

The model for the wormhole spacetime is now Minkowski space with two planes identified:

$$(\gamma t, x, y, \gamma \beta t) \equiv (\gamma t, x, y, \gamma \beta t + \delta), \tag{3.3}$$

or, going back to the rest frame,

$$(t, x, y, 0) \equiv (t + T, x, y, \ell). \tag{3.4}$$

The net effect of (1) allowing for a finite size for the wormhole throat, of (2) either adopting synchronous identification or of restricting attention to flat faced wormholes, combined with (3) the approximation $\delta \ll R$, has thus been to replace the identification of world lines with the identification of planes. The same effect could have been obtained by staying in the rest frame, giving the

wormhole mouths finite radius R, displacing the wormhole mouths to $z = -R$ and $z = \ell + R$, respectively, and letting $R \to \infty$. The advantage of the argument as presented in the frame of simultaneity is that it makes clear that for any synchronously identified traversable wormhole close enough to forming a time machine this approximation becomes arbitrarily good.

Continuing to work in the frame of simultaneity, the manifest invariance of the boundary conditions under rotation and/or reflection in the xy plane restricts the stress-energy tensor to be of the form

$$\langle 0|T_{\mu\nu}|0\rangle = \begin{bmatrix} T_{tt} & 0 & 0 & T_{tz} \\ 0 & T_{xx} & 0 & 0 \\ 0 & 0 & T_{xx} & 0 \\ T_{tz} & 0 & 0 & T_{zz} \end{bmatrix}. \tag{3.5}$$

By considering the effect of the boundary conditions one may write eigenmodes of the d'Alembertian in the separated form

$$\phi(t, x, y, z) = e^{-i\omega t} e^{ik_x x} e^{ik_y y} e^{ik_z z}, \tag{3.6}$$

subject to the constraint $\phi(t, x, y, z) = \phi(t, x, y, z + \delta)$. Therefore the boundary condition on k_z implies the classical quantization $k_z = \pm n2\pi/\delta$, while k_x and k_y are unquantized (i.e., arbitrary and continuous). The equation of motion constrains ω to be

$$\omega = \sqrt{(n2\pi/\delta)^2 + k_\perp^2}. \tag{3.7}$$

Now, recalling that $\langle 0|T_{tz}|0\rangle \propto \langle 0|\partial_t\phi\partial_z\phi|0\rangle$, and performing a mode sum over n, it is clear that the positive values of k_z exactly cancel the negative values so that $\langle 0|T_{tz}|0\rangle = 0$. Furthermore, one may note that the quantization conditions on ω and k_z depend only on δ, not on β or γ. Thus, without loss of generality one may immediately apply the result for $\beta = 0$ to the case $\beta \neq 0$ and obtain

$$\langle 0|T_{\mu\nu}|0\rangle = -\frac{\hbar k}{\delta^4} \left(\eta_{\mu\nu} - 4n_\mu n_\nu\right). \tag{3.8}$$

Here $n_\mu = \delta_\mu/\delta$ is the tangent to the closed spacelike geodesic threading the wormhole throat. In the frame of simultaneity $n_\mu = (0, 0, 0, 1)$, while in the rest frame $n_\mu = (-T/\delta, 0, 0, \ell/\delta)$. The constant k is a dimensionless numerical factor that it is not worth the bother to completely specify. In a more general context the argument given above applies not only to scalar fields but also to fields of arbitrary spin that are (approximately) conformally coupled. In general k will depend on the nature of the applied boundary conditions (twisted or untwisted), the spin of the quantum field theory under consideration, etc. Furthermore, as δ decreases, massive particles may be considered as being approximately massless once $\hbar/\delta \gg m$. Thus k should be thought of as changing in stepwise fashion as $\delta \to 0$. This behavior is entirely analogous to the behavior of the R parameter in $e^+ e^-$ annihilation. A complete specification of k would thus require complete knowledge of the asymptotic behavior of the spectrum of elementary particles, and it is for this reason that one declines further precision in the specification of k.

The form of the stress-energy tensor can also be determined by noting that the identifications (3.3) and (3.4) describing the wormhole can be easily solved by considering an otherwise free field in Minkowski space subject to the constraint

$$\phi(x^\mu) \equiv \phi(x^\mu \pm n\delta^\mu), \quad n = 0, 1, 2, \ldots . \tag{3.9}$$

Since these constraints do not depend on V, the four-velocity of the wormhole mouth, one may directly apply the results obtained for the ordinary Casimir effect [29–31].

At this stage it is also important to realize that the sign of k is largely immaterial. If k is positive then one observes a positive Casimir energy which tends to repel the wormhole mouths, and so tends prevent the formation of a time machine. On the other hand if k is negative one finds a large positive stress threading the wormhole throat. This stress tends to act to collapse the wormhole throat, and so also tends to prevent the formation of a time machine.

Note the similarities — and differences — between this result and those of Hawking [1, 2] and Kim and Thorne [3]. The calculation presented here is in some ways perhaps more general in that it is clear from the preceding discussion that this calculation is capable of applying to the generic class of "synchronously identified" wormholes once $\delta \ll R$. Furthermore, instead of the value of the stress-energy tensor being related to the temporal distance to the "would be Cauchy horizon," it is clear in this formalism that it is the existence and length of closed spacelike geodesics that controls the magnitude of the stress-energy tensor.

The limitations inherent in the type of wormhole model currently under consideration should also be made clear: the model problem is optimally designed to make it easy to thwart the formation of a time machine (i.e., closed timelike curves). It is optimal for thwarting in two senses: (1) The choice of synchronous identification prevents defocusing of light rays when they pass through the wormhole (i.e., it makes $f = 0$ in the notation of Hawking [1, 2]). If an advanced civilization were to try to create a time machine, that civilization presumably would optimize their task by choosing f arbitrarily large, and not optimize nature's ability to thwart by choosing $f = 0$. (2) The adiabatic approximation requires the relative motion of the wormhole mouths to be arbitrarily slow (i.e., h arbitrarily close to zero, in the notation of Hawking [1, 2]). This is an idealized limit; and again, it is the case that this idealization makes it easier for nature to thwart the time machine formation, because it gives the growing vacuum polarization an arbitrarily long time to act back on the spacetime and distort it.

B. k positive — Repulsive Casimir energy

For the synchronously identified model wormholes we have been discussing one may calculate the four-momentum associated with the stress-energy tensor by

$$P^\mu = \oint \langle 0|T^{\mu\nu}|0\rangle d\Sigma_\nu = \langle 0|T^{\mu\nu}|0\rangle \pi R^2 \delta n_\perp^\mu. \tag{3.10}$$

Here $n_\perp^\mu = (1,0,0,0)$ in the frame of simultaneity, so that $n_\perp$ is perpendicular to n. Thus

$$P^\mu = \frac{\hbar k}{\delta^4} \, \pi \, R^2 \, \delta \, n_\perp^\mu. \tag{3.11}$$

By going back to the rest frame one obtains

$$P^\mu = \frac{\hbar k}{\delta^4} \pi R^2 \delta \left(\frac{\ell}{\delta}, 0, 0, \frac{T}{\delta} \right), \tag{3.12}$$

so that one may identify the Casimir energy as

$$E_{\text{Casimir}} = P^\mu V_\mu = \frac{\hbar k \pi R^2 \ell}{\delta^4}. \tag{3.13}$$

Take $k > 0$ for the sake of discussion, then this Casimir energy would by itself give an infinitely repulsive hard core to the interaction between the wormhole mouths. However one should exercise some care. The calculation is certainly expected to break down for $\delta < \ell_P$, so that one may safely conclude only that there is a finite but large potential barrier to entering the full quantum gravity regime. Fortunately this barrier is in fact very high. For $T \gg T_P$ one can estimate

$$E_{\text{barrier}} \approx E_P \left(\frac{R}{\ell_P} \right)^2 \left(\frac{T}{T_P} \right), \tag{3.14}$$

while for $T \ll T_P$ (i.e., $T \approx 0$) one may estimate

$$E_{\text{barrier}} \approx E_P \left(\frac{R}{\ell_P} \right)^2. \tag{3.15}$$

For macroscopic wormholes these barriers are truly enormous. For microscopic wormholes ($R \ll \ell_P$) one really does not care what happens, firstly because it is not too clear whether wormholes with $R \ll \ell_P$ can even exist and secondly because, even if such wormholes do exist, they would in no sense be traversable [20], and so would be irrelevant to the construction of usable time machines. The nontraversability of such microscopic wormholes is easily seen from the fact that they could only be probed by particles of Compton wavelength much less than their radius, implying that

$$E_{\text{probe}} > \hbar/R \gg E_P. \tag{3.16}$$

On the other hand, for macroscopic wormholes one may safely assert that

$$R/\ell_p > 2m/m_P, \tag{3.17}$$

since otherwise each wormhole mouth would be enclosed by its own event horizon and the wormhole would no longer be traversable. Combining these results, it is certainly safe to assert that the barrier to full quantum gravity satisfies

$$E_{\text{barrier}} \gg E_P \left(\frac{2m}{m_P} \right)^2 \approx \frac{m^2}{m_P}. \tag{3.18}$$

To get a better handle on the parameter regime in question, consider the combined effects of gravity and the Casimir repulsion. In linearized gravity the combined potential energy is

$$V = - \left(\frac{m}{m_P} \right)^2 \frac{\hbar}{\ell} + \frac{\hbar k \pi R^2 \ell}{(\ell^2 - T^2)^2}. \tag{3.19}$$

The Casimir energy dominates over the gravitational potential energy once

$$\delta^4 \approx \frac{k \pi R^2 \ell^2}{(m/m_P)^2} > k \pi \ell_P^2 \ell^2, \quad \text{i.e., } \delta \geq \sqrt{\ell_P \ell}. \tag{3.20}$$

One may safely conclude that (1) for k positive there is an enormous potential barrier to time machine formation, and (2) for "reasonable" wormhole parameters the repulsive Casimir force dominates long before one enters the Planck regime.

C. k negative — Wormhole disruption effects

At first glance, should the constant k happen to be negative, one would appear to have a disaster on one's hands. In this case the Casimir force seems to act to help rather than hinder time machine formation. Fortunately yet another physical effect comes into play. Recall, following Morris and Thorne [4] that a traversable wormhole must be threaded by some exotic stress energy to prevent the throat from collapsing. In particular, at the throat itself (working in Schwarzschild coordinates) the stress-energy tensor takes the form

$$T_{\mu\nu} = \frac{\hbar}{\ell_P^2 R^2} \begin{bmatrix} \xi & 0 & 0 & 0 \\ 0 & \chi & 0 & 0 \\ 0 & 0 & \chi & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{3.21}$$

On general grounds $\xi < 1$, while χ is unconstrained. On the other hand, the vacuum polarization effects just considered contribute to the stress-energy tensor in the region between the wormhole mouths an amount

$$\langle 0|T_{\mu\nu}|0\rangle = \frac{\hbar k}{\delta^4} \begin{bmatrix} 4(T/\delta)^2 + 1 & 0 & 0 & 4T\ell/\delta^2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 4T\ell/\delta^2 & 0 & 0 & 4(\ell/\delta)^2 - 1 \end{bmatrix}. \tag{3.22}$$

In particular, the tension in the wormhole throat, required to prevent its collapse is $\tau = \hbar/(\ell_P^2 R^2)$, while the tension contributed by vacuum polarization effects is

$$\tau = -\frac{\hbar k}{\delta^4} \left[4(\ell/\delta)^2 - 1 \right] \approx \frac{-4\hbar k \ell^2}{\delta^6}. \tag{3.23}$$

Vacuum polarization effects dominate over the wormhole's internal structure once

$$\delta^3 < \ell_P \ell R / \sqrt{|k|}, \tag{3.24}$$

and it is clear that this occurs well before reaching the Planck regime.

By looking at the *sign* of the tension it appears that for negative values of k these vacuum polarization effects will tend to decrease R, that is, will tend to collapse the wormhole throat. Conversely, for positive values of k these vacuum polarization effects would appear to tend

to make the wormhole grow.

Concentrate on negative values of k. Well before δ shrinks down to the Planck length, the vacuum polarization will have severely disrupted the internal structure of the wormhole — presumably leading to wormhole collapse. It should be pointed out that there is no particular need for the collapse to proceed all the way down to $R = 0$ and subsequent topology change. It is quite sufficient for the present discussion if the collapse were to halt at $R \approx \ell_P$. In fact, there is evidence, based on minisuperspace calculations [32–34], that this is indeed what happens. If indeed collapse is halted at $R \approx \ell_P$ by quantum gravity effects then the Universe is still safe for historians since there is no reasonable way to get a physical probe through a Planck scale wormhole [20].

D. Summary

Vacuum polarization effects become large as $\delta \to 0$. Depending on an overall undetermined sign either (1) there is an arbitrarily large force pushing the wormhole mouths apart, or (2) there are wormhole disruption effects at play which presumably collapse the wormhole throat down to the size of a Planck length. Either way, usable time machines are avoided.

Unfortunately, because the class of wormholes currently under consideration is optimally designed to make it easy to thwart the formation of a time machine, it is unclear to what extent one may draw generic conclusions from these arguments.

IV. GRAVITATIONAL BACK REACTION

There is yet another layer to the Universe's "defense in depth" of global causality. Consider linearized fluctuations around the locally flat background metric describing the synchronously identified model wormhole:

$$g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} - \frac{1}{2}\bar{h}\eta_{\mu\nu}. \tag{4.1}$$

By going to the transverse gauge $\bar{h}^{\mu\nu},_\nu = 0$, one may write the linearized Einstein field equations as

$$\Delta\bar{h}_{\mu\nu} = \frac{-16\pi\ell_P^2}{\hbar}\langle 0|T_{\mu\nu}|0\rangle. \tag{4.2}$$

Now, working in the rest frame of the wormhole mouths, the time translation invariance of the geometry implies $\partial_t\bar{h}_{\mu\nu} = 0$. Similarly, within the confines of the Casimir geometry approximation, the translation symmetry in the x and y directions implies $\partial_x\bar{h}_{\mu\nu} = 0 = \partial_y\bar{h}_{\mu\nu}$. Thus the linearized Einstein equations reduce to

$$\partial_z^2\bar{h}_{\mu\nu} = \frac{16\pi k\ell_P^2}{\delta^4}\left(\eta_{\mu\nu} - 4n_\mu n_\nu\right). \tag{4.3}$$

Note that the tracelessness of $T_{\mu\nu}$ implies the tracelessness of $\bar{h}_{\mu\nu}$ so that $\bar{h}_{\mu\nu} = h_{\mu\nu}$. Using the boundary condition that $h_{\mu\nu}(z = 0, t = 0) = h_{\mu\nu}(z = \ell, t = T)$ the linearized Einstein field equations integrate to

$$h_{\mu\nu}(z) = \frac{16\pi}{2}(z - \ell/2)^2\frac{k\ell_P^2}{\delta^4}\left(\eta_{\mu\nu} - 4n_\mu n_\nu\right). \tag{4.4}$$

To estimate the maximum perturbation of the metric calculate

$$\delta g_{\mu\nu} = h_{\mu\nu}(z = 0)$$
$$= h_{\mu\nu}(z = \ell) = \frac{16\pi k\ell_P^2\ell^2}{8\delta^4}\left(\eta_{\mu\nu} - 4n_\mu n_\nu\right). \tag{4.5}$$

In particular,

$$\delta g_{tt} = \frac{-16\pi k\ell_P^2\ell^2}{8\delta^4}\left(1 + 4\frac{T^2}{\delta^2}\right). \tag{4.6}$$

Now if $T \gg T_P$, which is the case of interest for discussing the putative formation of time machines, this back reaction certainly becomes large well before one enters the Planck regime, thus indicating that the gravitational back reaction becomes large and important long before one needs to consider full quantum gravity effects.

Even if $T = 0$ one has $\delta g_{tt} = -(16\pi/8)k(\ell_P^2/\ell^2)$. While the back reaction is in this case somewhat smaller, it still becomes large near the Planck scale.

It might be objected that the frame-dependent quantity δg_{tt} is not a good measure of the back reaction. *Au contraire*, one need merely consider the manifestly invariant object

$$\varphi = h_{\mu\nu}V^\mu V^\nu. \tag{4.7}$$

Here V^μ is the four-velocity of the wormhole throat, and φ is the physical gravitational potential governing the motion of the wormhole mouths. For instance, the geodesic acceleration of the wormhole mouths is easily calculated to be

$$a^\mu = V^\mu,_\nu V^\nu = \Gamma^\mu_{\alpha\beta}V^\alpha V^\beta = -\frac{1}{2}\nabla^\mu\varphi. \tag{4.8}$$

Because of the delicacy of these particular issues, it may be worthwhile to belabor the point. Suppose one redoes the entire computation in the frame of simultaneity. One still has

$$\Delta\bar{h}_{\mu\nu} = \frac{-16\pi\ell_P^2}{\hbar}\langle 0|T_{\mu\nu}|0\rangle, \tag{4.9}$$

where now the boundary conditions are $h_{\mu\nu}(z = \beta t, t) = h_{\mu\nu}(z = \beta t + \delta, t)$. This immediately implies that $h_{\mu\nu}$ is a function of $(z - \beta t)$. Indeed the linearized Einstein equations integrate to

$$h_{\mu\nu}(z, t) = \frac{16\pi}{2}\frac{(z - \beta t - \delta/2)^2}{(1 - \beta^2)}\frac{k\ell_P^2}{\delta^4}\left(\eta_{\mu\nu} - 4n_\mu n_\nu\right). \tag{4.10}$$

So that at the mouths of the wormhole, $(z = \beta t) \equiv (z = \beta t + \delta)$, factors of δ and $\gamma = 1/\sqrt{1 - \beta^2}$ combine to yield

$$\delta g_{\mu\nu} = h_{\mu\nu}(z = \beta t, t)$$
$$= h_{\mu\nu}(z = \beta t + \ell, t) = \frac{16\pi k\ell_P^2\ell^2}{8\delta^4}\left(\eta_{\mu\nu} - 4n_\mu n_\nu\right), \tag{4.11}$$

which is the same tensor as the result calculated in the rest frame (as of course it must be).

The only real subtlety in this case is to realize that, as previously argued, δg_{tt} as measured in the frame of simultaneity is *not* a useful measure of the gravitational back reaction.

Yet another way of seeing this is to recall that in linearized Einstein gravity the gravitational potential of a pair of point particles is determined by their masses, relative separation, and their four-velocities by [35, 36]

$$V = \left\{\frac{Gm_1m_2}{r}\right\}\left\{\frac{2(V_1 \cdot V_2)^2 - 1}{\gamma_1\gamma_2}\right\}. \qquad (4.12)$$

Applied to the case presently under discussion, one immediately infers that the difference between the synchronous frame and the rest frame induces an enhancement factor of γ^2.

While the four-velocity of the wormhole throat does not enter into the computation of the stress-energy tensor it is important to realize that the four-velocity of the throat does, via the naturally imposed boundary conditions, have an important influence on the gravitational back reaction.

In any event, the gravitational back reaction will radically alter the spacetime geometry long before a time machine has the chance to form. Note that for k negative, so that the Casimir energy is negative and attractive, the *sign* of δg_{tt} obtained from this linearized analysis indicates a repulsive back reaction and furthermore hints at the formation of an event horizon should δ become sufficiently small. Unfortunately a full nonlinear analysis would be necessary to establish this with any certainty.

V. GENERALITIES

To put the analysis of this paper more properly in perspective it is useful to abstract the essential ingredients of the calculation. Consider a stationary, not necessarily static, but otherwise arbitrary Lorentzian spacetime of nontrivial topology. Specifically, assume that $\pi_1(\mathcal{M}) \neq e$, that is, that the first homotopy group (the fundamental group) is nontrivial. By definition the nontriviality of $\pi_1(\mathcal{M})$ implies the existence of closed paths not homotopic to the identity. This is the *sine qua non* for the existence of a wormhole. By smoothness arguments there also exist smooth closed paths not homotopic to the identity. Take one of these smooth closed paths and extremize its length in the Lorentzian metric. One infers the existence of at least one smooth closed geodesic in any spacetime with nontrivial fundamental group. If any of these closed geodesics is timelike or null then the spacetime is diseased and should be dropped from consideration. Furthermore, since the metric can be expressed in a t independent manner one may immediately infer the existence of an infinite one-parameter family of closed geodesics parameterized by t. A slow adiabatic (i.e., quasistationary) variation of the metric will preserve this one-parameter family, though the members of this family may now prove to be only approximately geodesics. The formation of a time machine is then signaled by this one-parameter family of closed "almost" geodesics switching over from spacelike character to null

and then timelike character.

If one is desirous of proceeding beyond the adiabatic approximation this can be done at the cost of additional technical machinery. Consider now a completely arbitrary Lorentzian spacetime of nontrivial topology. Pick an arbitrary base point x. Since $\pi_1(\mathcal{M})$ is nontrivial there certainly exist closed paths not homotopic to the identity that begin and end at x. By smoothness arguments there also exist smooth closed paths not homotopic to the identity — though now there is no guarantee that the tangent vector is continuous as the path passes through the point x where it is pinned down. Take one of these smooth closed paths and extremize its length in the Lorentzian metric. One infers the existence of many smooth (except at the point x) closed geodesics passing through every point x in any spacetime with nontrivial fundamental group. Again, if any of these closed geodesics is timelike or null then the spacetime is diseased and should be dropped from consideration.

To get a suitable one-parameter family of closed geodesics that captures the essential elements of the geometry, suppose merely that one can find a well-defined throat for one's Lorentzian wormhole. Place a clock in the middle of the throat. At each time t as measured by the wormhole's clock there exists a closed "pinned" geodesic threading the wormhole and closing back on itself in "normal space." This geodesic will be smooth everywhere except possibly at the place that it is "pinned" down by the clock. This construction thus provides one with a one-parameter family of closed geodesics suitable application of the preceding analysis.

Pick one of these closed geodesics. Let $\sigma \in [0,1]$ be a parametrization of the geodesic. Extend this to a coordinate patch $\{t,x,y,\sigma\}$ covering a tubelike neighborhood surrounding the geodesic. By adopting the spacelike generalization of comoving coordinates one may without loss of generality write the metric in the form

$$ds^2 = g_{ij}(t,x,y,\sigma)dx^idx^j + \delta^2(t,x,y)d\sigma^2. \qquad (5.1)$$

Here $dx^i \in \{dt,dx,dy\}$, while the fact that the curve $x = y = t = 0$ is a geodesic is expressed by the statement $\partial_i\delta(0,0,0) = 0$. By considering the three-dimensional Riemann tensor on slices of constant σ one may define a quantity

$$R^{-2} = \max_{\sigma \in [0,1]} \sqrt{R_{ijkl}R^{ijkl}}. \qquad (5.2)$$

This quantity R estimates the minimum "radius of curvature" of the constant σ hypersurfaces and so may usefully be interpreted as a measure of the minimum radius of the wormhole throat.

For a traversable wormhole, precursor to a traversable time machine, one wishes R to be macroscopic and to remain so. Now consider what happens as δ shrinks. Eventually $\delta \ll R$, at which stage the length of the closed geodesic is much less than the length scale of the transverse dimensions of the wormhole throat. By going to Riemann normal coordinates one may now approximate the metric by

$$ds^2 = \eta_{ij}dx^idx^j + \delta^2(t,x,y)d\sigma^2 \quad (x,y \ll R), \qquad (5.3)$$

which is just a generalization of the Casimir geometry expressed in the synchronous frame. Nasty "fringe" effects occur for $x, y \geq R$, but these may be quietly neglected as is usual and reasonable within the confines of the Casimir approximation. Of greater significance is the behavior of $\delta(t, x, y)$ as a function of t, x, and y. If the variation of $\delta(t, x, y)$ is not particularly rapid one may safely make the further approximation of modeling the situation by the ordinary Casimir effect. In that case the conclusions of the previous section then follow in this more general context.

This happy circumstance is automatically satisfied if the wormhole mouths are synchronously identified, since then $\delta(t, x, y) \leq \delta(t, 0, 0) + O(R/\gamma) = \delta(t, 0, 0)[1 + O(R/\ell)]$. On the other hand, the time shift identification adopted by Kim and Thorne leads to a very rapid growth of $\delta(t, x, y)$. This rapid variation of $\delta(t, x, y)$ with position makes analysis considerably more difficult. Fortunately, it is still relatively easy to convince oneself that the vacuum expectation value of the renormalized stress-energy tensor along the central geodesic must be proportional to δ^{-4}. Therefore, at least along the central geodesic, the exotic matter supporting the wormhole throat is overwhelmed by the renormalized stress energy. The extent to which this disruption along the central geodesic might lead to disruption of the traversability of the wormhole is unfortunately not calculable by the present techniques.

The whole point of this aspect of the discussion is, of course, to see what exactly is special about the simple model wormholes considered earlier — the analysis presented in this paper is seen to be limited to adiabatic (spatial and temporal) changes in $\delta(t, x, y)$.

VI. COMPARISONS WITH PREVIOUS WORK

There are several manners in which one might attempt to parametrize the strength of the divergences in the stress-energy tensor and the gravitational back reaction. The estimates of Kim and Thorne [3] make extensive use of the "proper time to the Cauchy horizon" as measured by an observer who is stationary with respect to one of the wormhole mouths. Hawking [1, 2] advocates the use of an "invariant distance to the Cauchy horizon," which is equal to the proper time to the Cauchy horizon as measured by an observer who is stationary with respect to the frame of simultaneity. On the other hand, this paper has focused extensively on the invariant length of closed geodesics.

The calculations of this paper suggest that the use of the "proper time to the Cauchy horizon," whether measured by an observer in the rest frame or in the synchronous frame is not a useful way of parametrizing the singularities in the stress-energy tensor. To see this, transcribe the "order of magnitude estimates" of Kim and Thorne [3] into the notation of this paper (Δt is the proper distance to the Cauchy horizon as measured in the rest frame). Consider a pair of slowly moving wormhole mouths with relative velocity $\beta_{\rm rel}$ (not to be confused with the totally different $\beta \equiv T/\ell$ associated with the transformation from the rest frame to the synchronous frame). For a small relative velocity, the wormhole is

modeled by the identification of world lines

$$(t, 0, 0, 0) \equiv (t + T, 0, 0, \ell - \beta_{\rm rel}\, t), \tag{6.1}$$

so that $\delta^2(t) = (\ell - \beta_{\rm rel}\, t)^2 - T^2$. (There is nothing particularly sacred about taking a small relative velocity — it just simplifies life in that one only has a simple linear equation to solve instead of a quadratic.) The Cauchy horizon forms when $\delta^2(\Delta t) = 0$, that is, when

$$\Delta t = \frac{(\ell - T)}{\beta_{\rm rel}}. \tag{6.2}$$

Resubstituting into $\delta^2 \equiv \delta^2(t = 0)$, noting that $\ell + T \approx 2\ell$, and being careful to retain all factors of $\beta_{\rm rel}$, one obtains, for the Kim-Thorne estimates,

$$\delta^2 \approx 2\ell(\ell - T) = 2\ell\, \beta_{\rm rel}\, \Delta t, \tag{6.3}$$

$$\langle 0|T^{\mu\nu}|0\rangle \approx (R/\ell)^\zeta \, \frac{1}{\ell(\beta_{\rm rel}\Delta t)^3} \tag{6.4}$$

$$= (R/\ell)^\zeta \, \frac{1}{\ell(\ell - T)^3}, \tag{6.5}$$

$$\delta g_{\mu\nu} \approx (R/\ell)^\zeta \, \frac{\ell_P^2}{\ell\beta_{\rm rel}\Delta t} \tag{6.6}$$

$$= (R/\ell)^\zeta \, \frac{\ell_P^2}{\ell(\ell - T)}. \tag{6.7}$$

Here ζ is an integer exponent that depends on the homotopy class of the closed geodesic, and on whether or not one is close to either mouth of the wormhole. (The factors of $\beta_{\rm rel}$ were unfortunately omitted in the discussion portion of Ref. [3].)

To see the inadequacy of the use of Δt at a conceptual level, observe that the use of Δt intrinsically "begs the question" of the creation of a time machine by explicitly asserting the existence of a Cauchy horizon and then proceeding to measure distances from that presumed horizon. In particular, one may consider a wormhole in which one keeps the distance between the mouths fixed (i.e., set the relative velocity of the mouths to zero). By definition, this implies that the Cauchy horizon never forms and that $\Delta t = +\infty$. (If one really wishes to be technical, consider a collection of wormholes of the type considered by Kim and Thorne and take the limit as the relative velocity goes to zero.)

To get a better handle on the actual state of affairs, go to the Casimir limit by adopting synchronous identification and taking $\delta \ll R$. In this case one may safely neglect the geometrical factors $(R/\ell)^\zeta$. By adopting a point-splitting regularization the (renormalized) propagator may easily be seen to be approximated by

$$\langle 0|\phi(x)\phi(x)|0\rangle \approx \delta^{-2}. \tag{6.8}$$

The stress-energy tensor is then computable by taking two derivatives of the propagator, yielding

$$\langle 0|T^{\mu\nu}|0\rangle \approx \hbar\delta^{-4}t^{\mu\nu}. \tag{6.9}$$

Here $t^{\mu\nu}$ is a dimensionless tensor built up out of the metric and tangent vectors to the closed spacelike geodesic. While the *components* of $t^{\mu\nu}$ may be large in some Lorentz frames, the only sensible invariant measure of the size, $t^{\mu\nu}t_{\mu\nu}$, is of order 1. In particular, looking at

the tt component, as viewed from the rest frame of one of the wormhole mouths, yields

$$t^{tt} \approx \ell^2 \delta^{-2} \quad \Rightarrow \quad \langle 0|T^{tt}|0\rangle \approx \hbar \ell^2 \delta^{-6} \approx \hbar \ell^{-1}(\ell - T)^{-3}.$$
(6.10)

This is the analogue of the estimate of Kim and Thorne (including insertion of the appropriate factor of $\beta_{\rm rel}$). However, one sees that this estimate is somewhat misleading in that it is a componentwise estimate that is highly frame dependent.

Returning to the tensorial formulation, a double spatial integration gives an estimate for linearized metrical fluctuations:

$$\delta g_{\mu\nu} \approx \ell_P^2 \ell^2 \delta^{-4} t^{\mu\nu}.$$
(6.11)

This estimate (which is backed up by the earlier explicit calculation) is radically different from that of Kim and Thorne. The difference can be traced back to the choice of synchronous identification versus time-shift identification, and the effect that these different identification procedures have on the Van Vleck determinant.

Turning to other matters, to understand Hawking's "invariant" distance to the Cauchy horizon, go to the synchronous frame. One requires $\beta_{\rm rel}^{\rm synch} \ll \beta \equiv T/\ell$, so that the wormhole may be described by the identification

$$(t, 0, 0, \beta t) \equiv (t, 0, 0, (\beta - \beta_{\rm rel}^{\rm synch})t + \delta),$$
(6.12)

then $\delta(t)^2 = (\delta - \beta_{\rm rel}^{\rm synch} t)^2$. (Warning, $\beta_{\rm rel}^{\rm synch}$ is now the relative velocity as measured in the synchronous frame.) The Cauchy horizon forms at

$$t = \delta/\beta_{\rm rel}^{\rm synch}.$$
(6.13)

Hawking now takes this object $t = \delta/\beta_{\rm rel}^{\rm synch}$, which is in fact an invariant measure of the distance to the Cauchy horizon, as his parameter governing the strength of the singularities encountered when trying to build a time machine. Of course, this parameter suffers from deficiencies analogous to the Kim-Thorne parameter in that it is intrinsically incapable of properly reflecting the divergence structure that is known to occur in the simple stationary (or indeed quasistationary) models considered in this paper.

Turning to the question of the quantum gravity cutoff, Kim and Thorne assert that this cutoff occurs at

$$\Delta t = \frac{(\ell - T)}{\beta_{\rm rel}} \approx \ell_P \Rightarrow \ell - T \approx \beta_{\rm rel}\, \ell_P.$$
(6.14)

Hawking claims that the cutoff occurs at

$$t = \delta/\beta_{\rm rel}^{\rm synch} \approx \ell_P \Rightarrow \delta \approx \beta_{\rm rel}^{\rm synch}\, \ell_P.$$
(6.15)

I beg to differ. Both of these proposed cutoffs exhibit unacceptable dependence on the relative motion of the wormhole mouths.

As an improved alternative cutoff, consider the following: Pick a point x in spacetime. Since, by hypothesis, the spacetime has nontrivial topology there will be at least one closed geodesic of nontrivial homotopy that runs from x to itself. If the length of this geodesic is less than a Planck length, then the region surrounding the point x should no longer be treated semiclassically. Presumably, one should also supplement this requirement by a bound on the curvature: If $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} > \ell_P^{-4}$ then the region surrounding the point x should no longer be treated semiclassically.

The advantage of the cutoff expressed in this manner is that it appears to be eminently physically reasonable. This cutoff gives meaningful answers in the case of a relatively stationary pair of wormhole mouths, does not beg the question by requiring the existence of a Cauchy horizon to formulate the cutoff, and furthermore can be concisely and clearly stated in complete generality for arbitrary spacetimes.

VII. DISCUSSION

This paper has examined at some length a series of physical effects that lend support to Hawking's chronology protection conjecture. Explicit calculations have been exhibited for simple models in suitable parameter regimes. In particular much has been made of the use of adiabatic techniques combined with minor variations on the theme of the Casimir effect.

These calculations suggest that the Universe exhibits a defense in depth strategy with respect to global causality violations. Effects contributing to the hoped for inability to manufacture a time machine include vacuum polarization effects, wormhole disruption effects, and the gravitational back reaction induced by the vacuum expectation value of the renormalized stress-energy tensor.

Though the calculations in this paper have been phrased in terms of the traversable wormhole paradigm, the general result to be abstracted from this analysis is that (1) any spacetime of nontrivial topology contains closed (spacelike) geodesics and (2) it appears that the Universe reacts badly to closed spacelike geodesics attempting to shrink to invariant length zero.

I would conclude by saying that the available evidence now seems to favor Hawking's chronology protection conjecture. Both the experimental evidence (the nonappearance of hordes of tourists from the future) and the theoretical computations now support this conjecture. Paraphrasing Hawking, it seems that the Universe is indeed safe for historians.

Perhaps most importantly, the single most serious objection to the existence of traversable wormholes has always been the *apparent* ease with which they might be converted into time machines. Thus adopting Hawking's chronology protection conjecture immediately disposes of the single most serious objection against the existence of traversable wormholes. This observation should be interpreted as making the existence of traversable wormholes a much more reasonable hypothesis. Investigations of questions such as the averaged weak energy condition, and all other aspects of traversable wormhole physics, thus take on a new urgency [37–41].

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