2|1|26

EE 201 - Signals & Systems.

Signal

processing of cause to produce Effect.

An information containing entity to ~~produce~~ /useful for user

- Noise is an Signal. → Also has info, but unwanted.
- Signal is not defined by usefulness

Books:

1. Alam V Open heim
2. BP Lathe
3. Simon Hayking

→ Schaum Series.

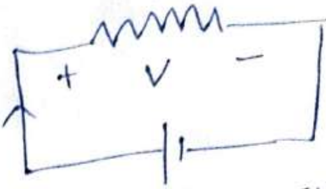
~~Audio Signal~~

Speech signals → 300 Hz - 3.5 KHZ
(TALK)
Normal 1.5 KHZ
Singers 2.5 KHZ
Opera sig - 3.5 KHZ

Audio Signal → 20HZ-20,000 HZ
(HEAR)

VEDIO Signal - 0 to 4.5 MHZ
└ Analog TV baseband video.

Ohms law



Emf   Emf → i → V

$V \propto i$ → cause
└ effect

Cause → $\boxed{\text{System}}$ → Effect

$V \propto i$ - function

$i \rightarrow \boxed{\text{RCKt}} \rightarrow V$

- linear
- Memoryless
- Time variant

$V(i) = Ri$
└ independent

$V(t) = Ri(t)$

AI
└ Intilligent
└ I/P
└ processing.
└ O/P.

How does machiens amplify signals.?

1. Antenna size
2. Long dist Transmission
3. Avoid interfernce

Ex: Mic.

Speech signal → MIC — low freq — Modulator — High freq → Transmits

800 - 3.5KHZ

MIC | Transoluces

Modulator: ~~Amplifies~~ shifts the low freq to High freq.

Antenna.

Converts into Electrical d/c signal
(converts energy from a One form to another)

- Assignment: Maxwells law 4 Equations.

High freq ↓ Amplifier → Transmits

Amplifier: To strengthen signal (signal strength)

Transmits: Electrical → EM waves

Increse Amplitude medium

Amplifiers - wireless.
Repeaters - wired.
regenarate digital signals.

Reciving End (speaker)

Ampr — High freq — DE-Modulator — low freq → Transducer

Transducer: electrical → sound.

↓

O/P AUDIO signal

20-20KHZ.

electrical Input → Electrical System → electrical output

- This course is for ~~foon~~ to generate mathametical Expression for input and Modelling for electrical systems.

Signal.

- Any entity having some associated information/ data within itself.

FM signals
— 92.5 Big channel

Range 87 MHz – 108 MHz

Eg:

$x(t)$ – continous Signal

$x[n]$ – Discrete signals

• MAXWELL

How electrical & Magnetic fields

1. Guass law of Electricity $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_E}{\varepsilon_0}$

2. Guass law of Magnetism $\oint_S \vec{B} \cdot d\vec{A} = 0$

3. Faraday law of EMI $\oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

4. Amphere-Maxwell law $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

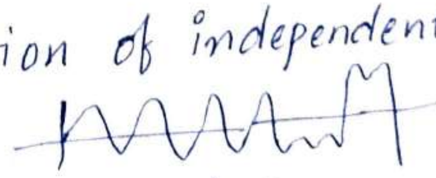
5/01/26

Image → $S(x,y,z)$
Spatial representation.

$v(t), i(t)$
Time variant

Signal: Mathametical Representation of independent variable

2 types – Continous signal



– Discrete signal.

Continous:- $S(t)$ – continous domain

Obtained from continuum value.

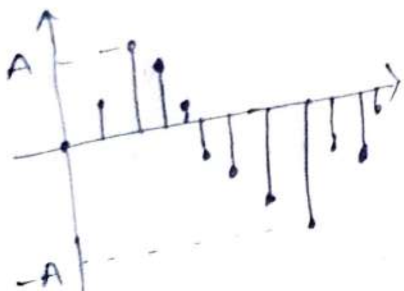
Ex: voltage, current, audio, vedio, light, sinosondial, Exponentials
(raw) e^{-ax}, e^{ax}

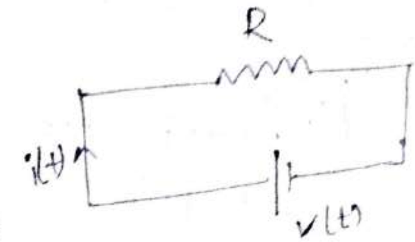
Discrete :

- Discrete domain
- Discrete or integer value.
- Independent values are discrete / Integer value

Ex: count of students.

$x[n] =$





- v(t) and i(t) from t_1 to t_2.

$v(t)$ — continous.

Instantenous power — Discrete At $t = t_1$

Power — continous At (t_1, t_2)

Instantenous power $= v(t)\, i(t)$

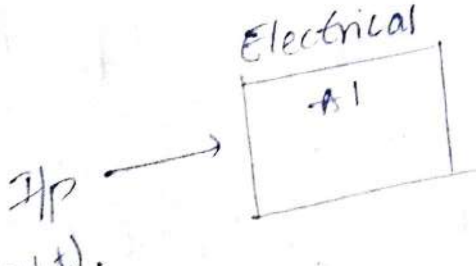
$$p(t) = \frac{1}{R} v^2(t)$$

(W)

- Energy $E = \int_{t_1}^{t_2} p(t)\, dt = \int_{t_1}^{t_2} \frac{v^2(t)}{R} dt = \int_{t_1}^{t_2} i^2(t) \cdot R\, dt$.

└ discrete ⇒ Σ

Generallizing,



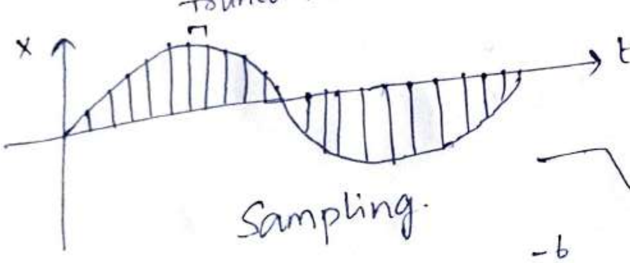
$$\text{Energy} = \int_{t_1=-\infty}^{t=\infty} |x(t)|^2 dt. \quad \text{Continous.}$$

$$\text{Energy (J)} = \sum_{n=n_1}^{n_2} |x[n]| \quad \text{discrete.}$$

→ while discretizing a continous signal; there would be loss of signal.

Assignment
Why energy & power? Energy - Total signal content
Power - How strong is signal.
- If you know Energy, you can change circutry.

Truncation Error.



Algo → digital
So it has always.
Truncation Error

→ while digitalized it always has Truncation Error even if you go to billions.

- Truncation Error $< 10^{-6}$
- Sampling Error.

- digital system [Accuracy ✗ / feasibility ✓]

1.101432143
1.1014321432632 4567

Cont $E = \int_{t=-\infty}^{t=\infty} |x(t)|^2 dt \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)^2| dt = E_\infty$

Dis $E = \sum_{N=n_1}^{n_2} |x[N]|^2 \triangleq \lim_{\substack{N\to\infty \\ n_2 = N}} \sum^{N} |x[n]|^2 = E_\infty$

Continous
AVG POWER $= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt$

$= \frac{1}{T-(-T)} \int_{-T}^{T} |x(t)|^2 dt = \frac{E_\infty}{2T}$

$= 0.$

$\frac{E_\infty}{2T}$ = AVG POWER (W)

AVG POWER OF SIGNAL
$= \frac{E_\infty}{2T}$

Discrete
AVG Power $= \frac{1}{n_2 - n_1 + 1} \sum_{N=n_1}^{n_2} |x[n]|^2 = \frac{E_\infty}{2N+1}$

6/1/25
$P_\infty = \lim_{T\to\infty} \frac{1}{2T}$
2T : tota
E_∞ [F / I
Ex:
Not
1. A
2. T

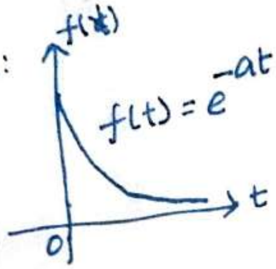
6/11/25

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{E_\infty}{2T}$$

2T : total time period, replace with t/x

$$E_\infty \begin{cases} \text{Finite Energy } (E_\infty < \infty) \rightarrow \boxed{P_\infty = 0} \\ \text{Infinite Energy.} \end{cases}$$

Ex:



$$E = \lim_{T\to\infty} \int_0^T e^{-2at} dt$$

$$= \lim_{T\to\infty} \frac{-1}{2a}\left[e^{-2at}\right]_0^T$$

$$= \lim_{T\to\infty} \cancel{\frac{1}{2a}} \cdot \frac{1}{2a}\left[e^{-2aT} - 1\right]$$

$$= \frac{1}{2a} \lim_{T\to\infty} \left[1 - \frac{1}{e^{2aT}}\right]$$

$$= \frac{1}{2a}.$$

Finite Energy.

Power = 0.

Note:

1. AVG Power of Finite Energy is zero Always.
2. These signals whose power is zero and Finite Energy are called Energy Signals.

Case 2: Infinite Energy, Finite Power.

$$x(t) = A\sin(w_0 t)$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} A^2 \sin^2(w_0 t) dt$$

$$E = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

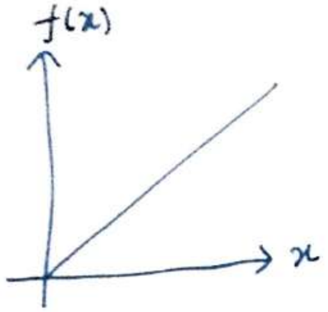
$$= \frac{1}{T_0}\int_0^{T_0} A^2 \sin^2 w_0 t \, dt$$

$$= \frac{A^2}{T_0}\left[\frac{t}{2}\right]_0^{T_0} = \frac{A^2}{\cancel{T_0}}\left(\frac{\cancel{T_0}}{2}\right) = \frac{A^2}{2}.$$

$E_\infty = \infty$ & P_∞ is Finite

→ These signals are called power signals.

Case 3: $E_\infty = \infty$ & $P_\infty = \infty$



$$r(t) = t \cdot u(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$E = \int_{-\infty}^{\infty} |r(t)|^2 dt = \int_0^{\infty} t^2 dt = \left[\frac{t^3}{3}\right]_0^{\infty} = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T t^2 dt$$

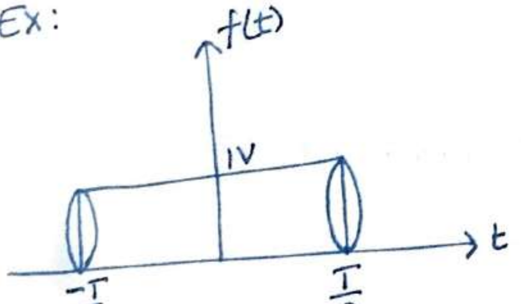
$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{T^2}{6}$$

$$\boxed{P = \infty}$$

– It is Neither Energy nor Power signals.

Ex:



$$f(t) = \begin{cases} 1 & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & \text{otherwise.} \end{cases}$$

$$E_\infty = \int f^2(x) dt$$

= Area under curve

$$= \left[\frac{T}{2} - \left(-\frac{T}{2}\right)\right] \cdot 1$$

$$\boxed{E_\infty = T}$$

Rectangular Function

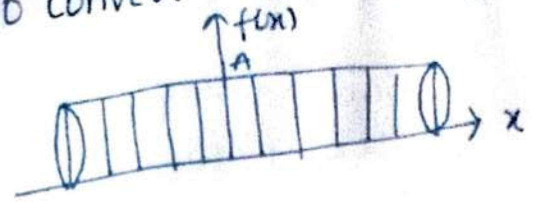
Gate Function

Gt(x) | Rect(x)

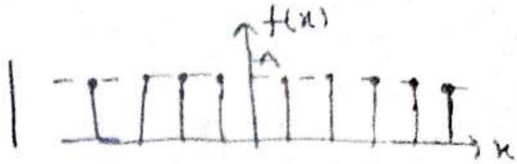
AVG Power, $P_\infty = 0$.

– Energy signal.

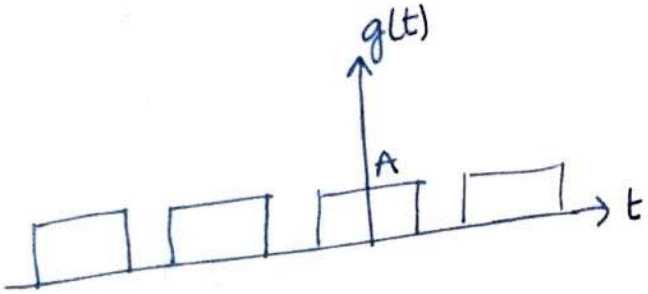
To convert this into digital



$\Rightarrow$

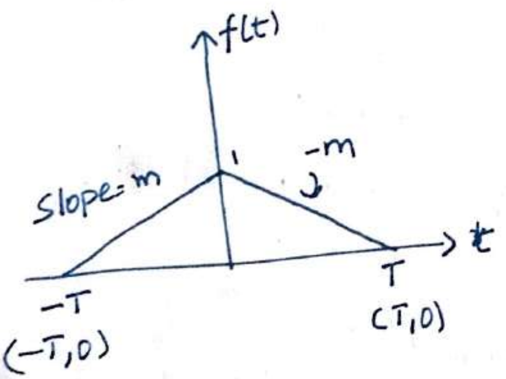


$$x[n] = \begin{cases} 1 & \text{for } n \in N^+ \\ 0 & \text{otherwise.} \end{cases}$$



$$P_\infty = A^2$$

Triangular Function



$y = mx + c$

$c = 1,\ (-T, 0)$, solve for m

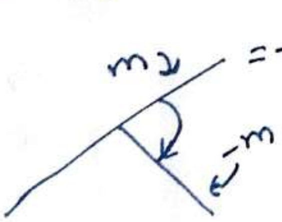
$0 = m(-T) + 1$

$m \cdot T = 1$

$m = \frac{1}{T}$

$$f(t) = \begin{cases} \frac{1}{T}t + 1 & -T \le t \le 0 \\ \frac{-1}{T}t + 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

change in slope $= \frac{-2}{T}$



$= -[m + m]$

$= -2m$

$$E = 2\int_0^T \left(1 - \frac{t}{T}\right)^2 dt \qquad u = 1 - \frac{t}{T}$$

$$= 2\int_1^0 u^2 \cdot (-T\,du)$$

$$= 2T \int_0^1 u^2\,du$$

$$= 2T\left[\frac{u^3}{3}\right]_0^1$$

$$E = \frac{2T}{3}$$

$$P = \lim_{L\to\infty} \frac{1}{2L}\int_{-L}^{L} |f(t)|^2 dt$$

$$P = \lim_{L\to\infty} \frac{E}{2L}$$

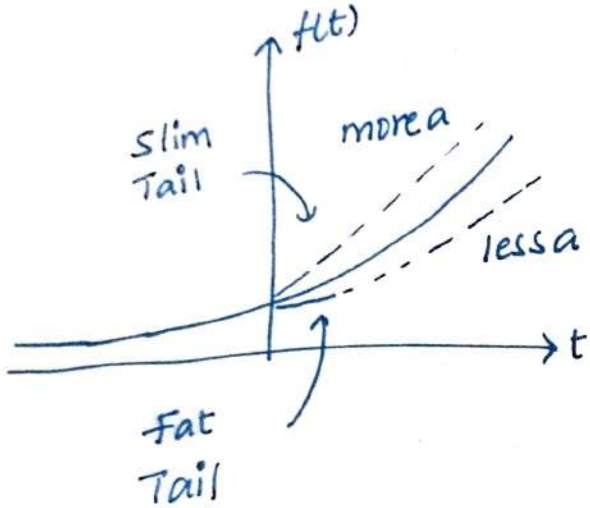
$P = 0.$

Comb Structure

Infinite Energy
Finite Power.

→ power signal.

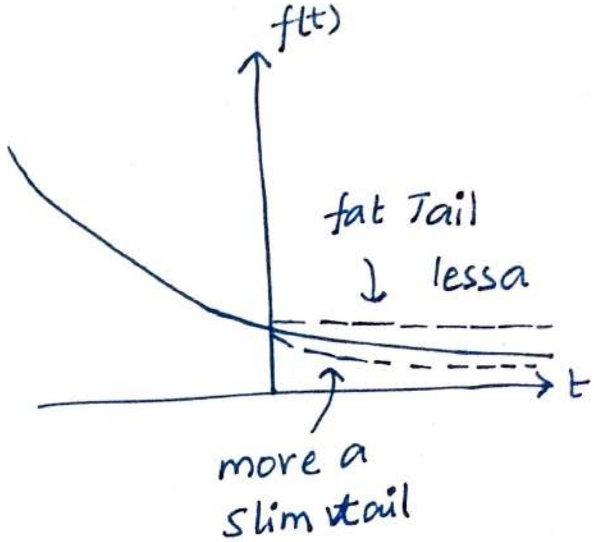
Exponential Signals :-



Exponential Increasing Function

$f(t) = e^{at}$

Energy & power → Infinite



Exponential Decreasing Function

$f(t) = e^{-at}$

→ Finite Energy

- Zero Crossover Signals ie., change in the Amplitude of the signals +ve to −ve side instant to the change occurs in what we denote ZC

Ex: Sinosodual, zero cross overs: $\pi, 2\pi, 3\pi$ .....
Cosinusodial, zero cross overs: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ ....

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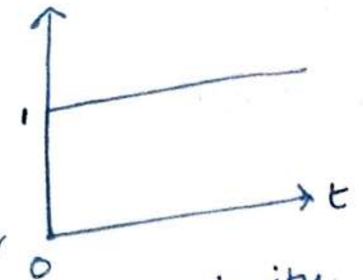
Unit step Signal / Function

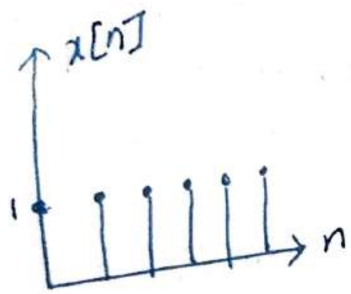


If $A \neq 1$ its step Function.

$$f(x) = u(t) = \begin{cases} 1/A & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Amplitude remains const until End.

↳ disscontinuity.



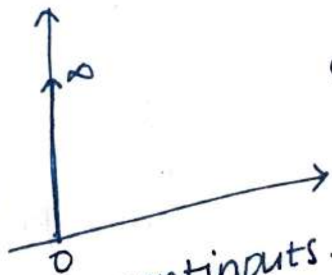
- Discrete unit step function
- discontinous at every integer

$$u(t) = \begin{cases} 1 & t \geq 0 \text{ \& } t \in \{N^+, 0\} \\ 0 & \text{elsewhere} \end{cases}$$

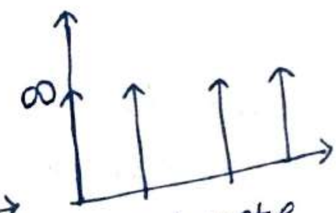
Impulse Signal

$\delta(t)$

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{else} \end{cases}$$



continuouts.



Discrete

$$\delta[t] = \begin{cases} \infty & t \in \{N, 0\} \\ 0 & \text{else} \end{cases}$$

$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 =$ Area under curve. → Even signal

- Impulse signal is a rectangle, length $\propto \frac{1}{\text{width}}$

width $\rightarrow 0 \Rightarrow$ length $\rightarrow \infty$

Area $= \infty \cdot 0 = 1$.

↳ Absolutely Integrable Function

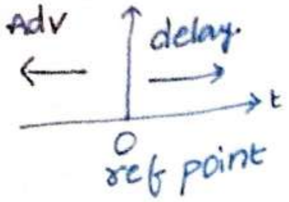
Transformation of Independent Variables.

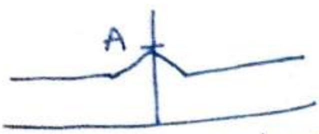
↳ shifting in time domain
- Delay
- Advancing

My class at 2:10 PM
↳ Goes at 2:00 PM - Advancement
↳ Goes at 2:15 PM - Delay , Delay time = 5 min.

$x(t) = x(t - t_0)$ delaying time

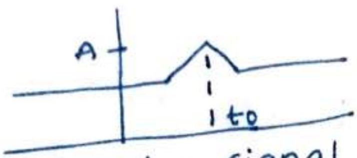
$x(t) = x(t - (-t_0))$ Advancing of time.





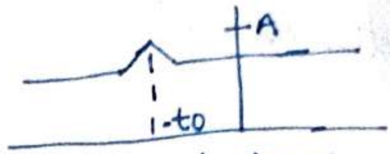
Normal signal

$x(t)$



Delay signal

$x(t-t_0)$



Advanced signal

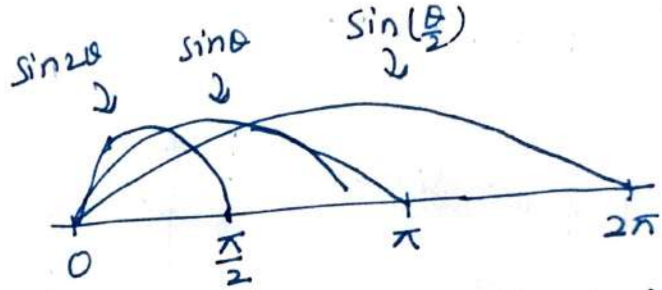
$x(t+t_0)$

Ex: Doppler Effect.

2. Scaling.

$x(at)$

$\begin{cases} 0<a<1 & \text{Expanding.} \\ a>1 & \text{Compression.} \end{cases}$



Ex: Playback speed in vedio streaming platforms (Yt, Netflix etc..)

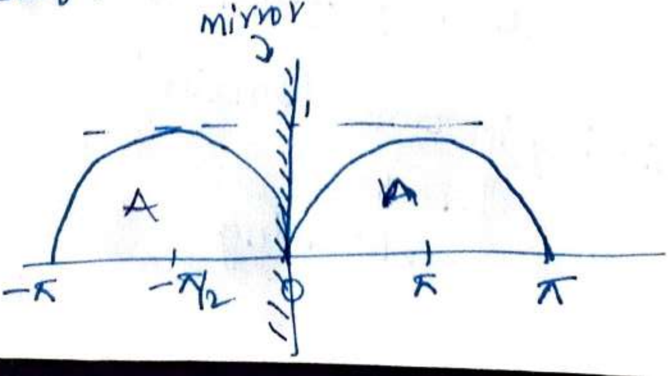
0.25x, 0.5x } Expanding
1x - Normal speed
1.5x, 2.5x } Compression.

. Reversal.

Extension of Scaling with $a = -1$

180° phase shift along ~~x~~ (not x) independent var axis

Mirror image of itself, mirror on y-axis.



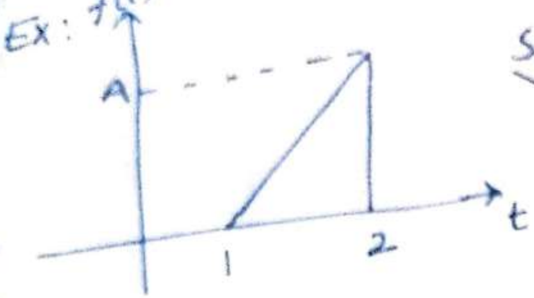
- $x(ax+b)$

Do 1^{st} - shifting

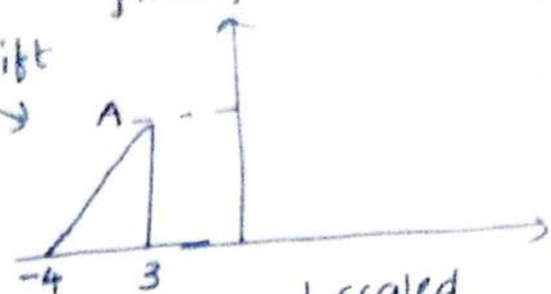
2^{nd} - Scaling | Reversal

find $f(2t+5)$

Ex: $f(t)$

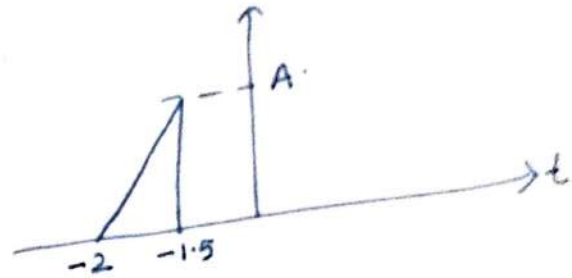


Shift



↓ scaled

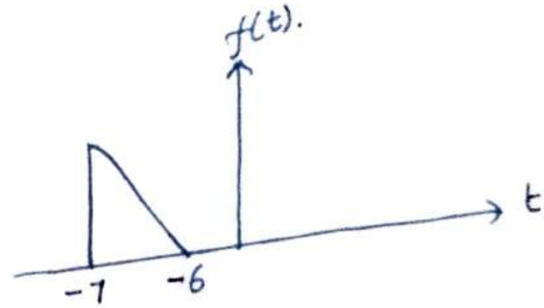
Don't do in reverse order



t	1	2
$2t$	$\frac{1}{2}$	1
$2t+5$	$-\frac{9}{2}$	-4

If you want to do like this

$2(t+5/2)$	$-\frac{3}{2}$	$-2.$



H.W

Q) $f(-t-5)$

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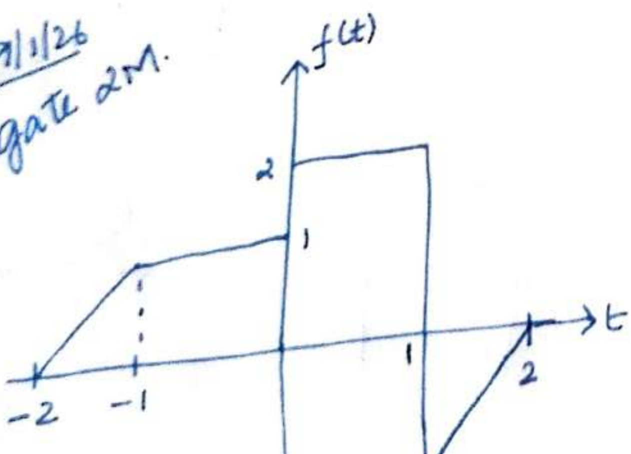
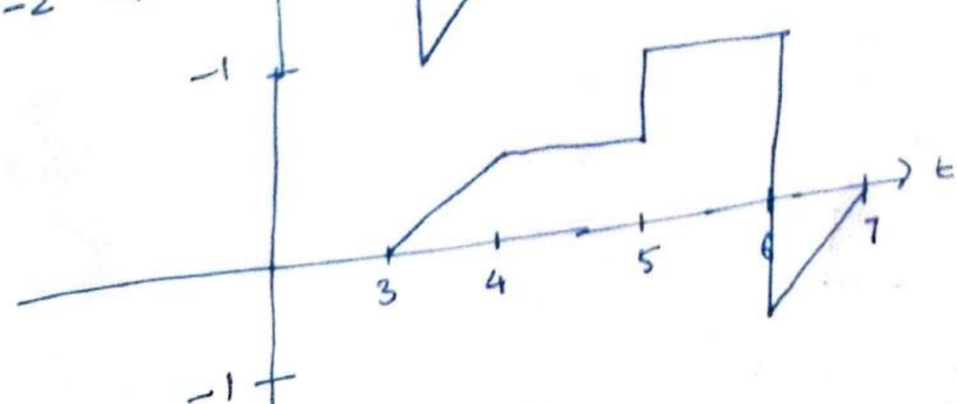
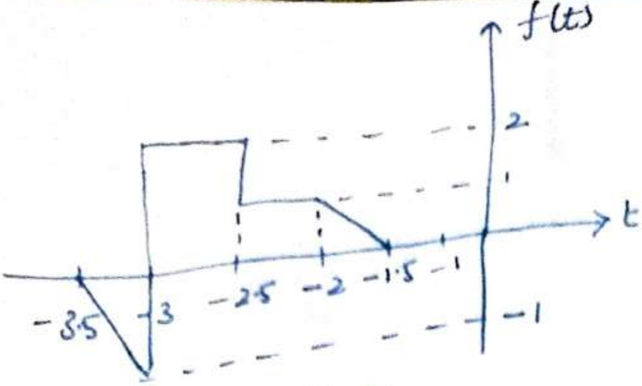
gate 2M.

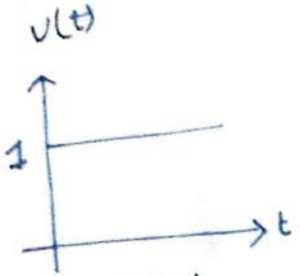


$\Rightarrow f(-2t-5)$

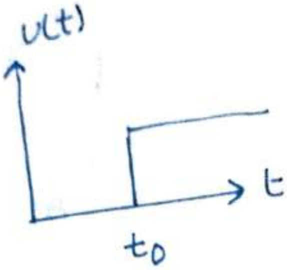




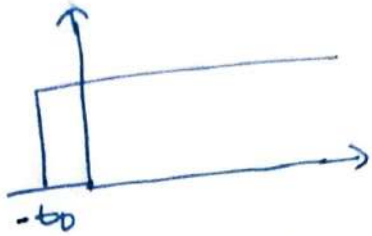
→ Unit step function:



u(t)

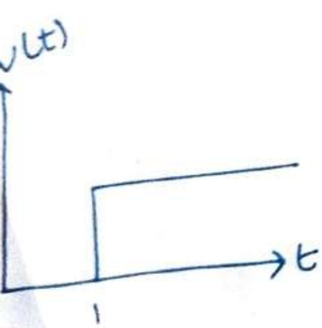


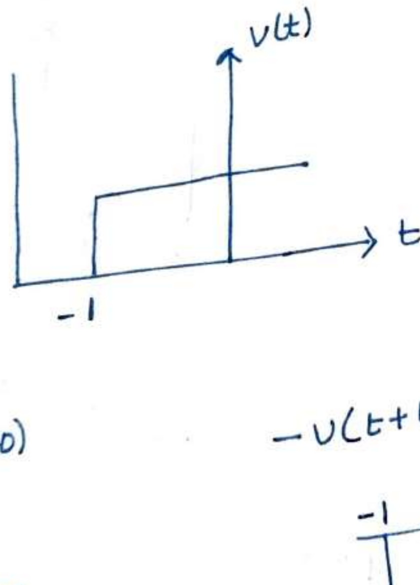
$u(t-t_0)$

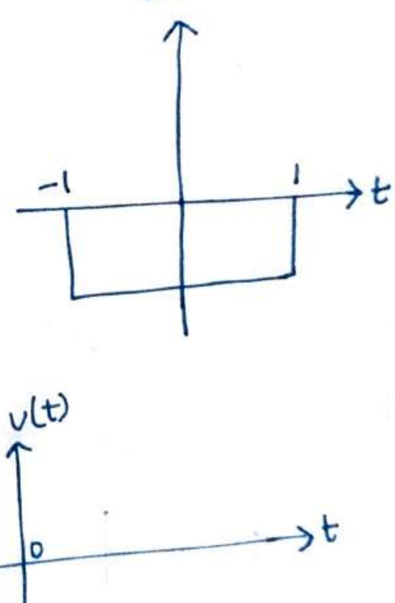


$u(t+t_0)$

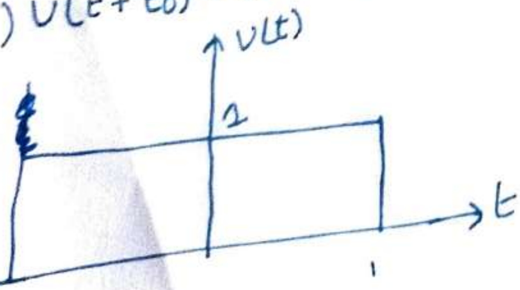
Q) $u(t-t_0) - u(t+t_0)$ where $t_0 = 1$



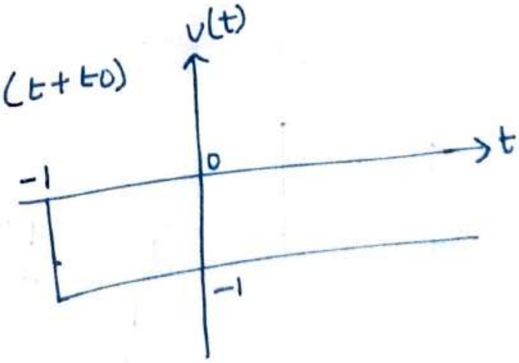




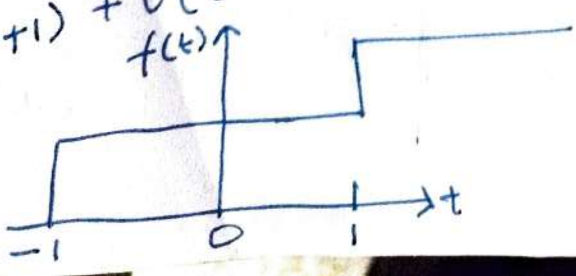
) $u(t+t_0) - u(t-t_0)$



$-u(t+t_0)$



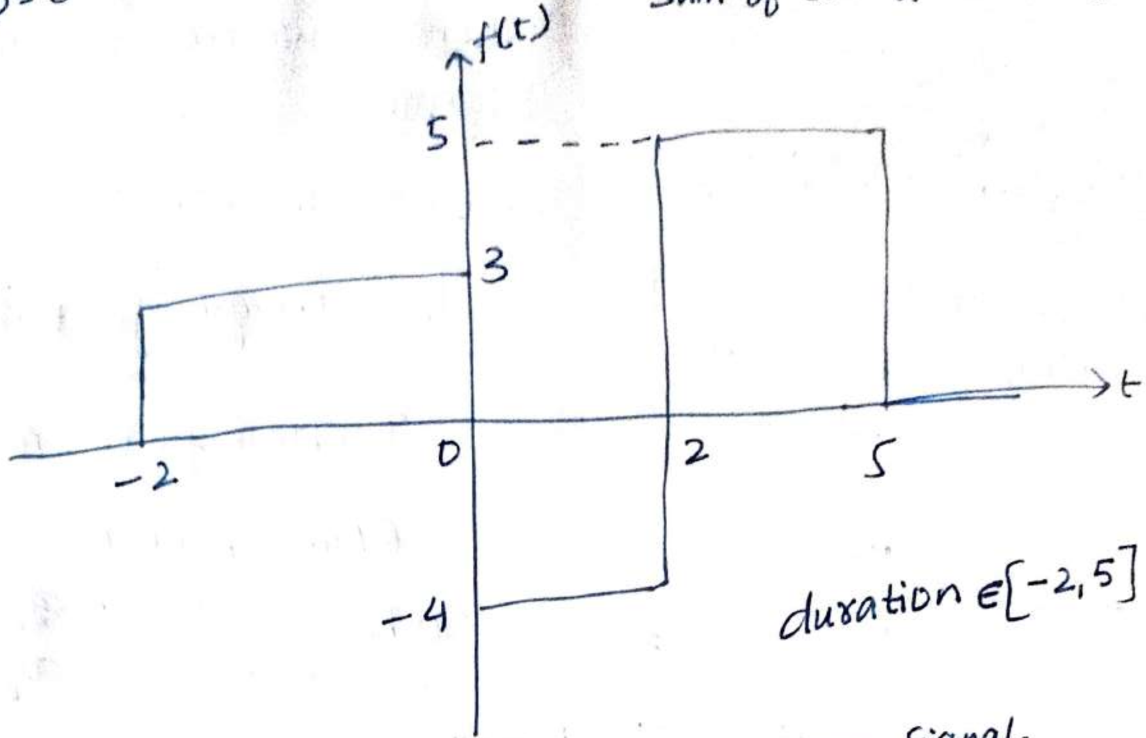
$t+1) + u(t-1)$



Q) H.W

$f(t) = 3u(t+2) - 7u(t) + 9u(t-2) - 5u(t-5)$

Sum of co-efficient = 0.



duration $\in [-2, 5]$

Sum of coefficients is zero for fixed duration signal.

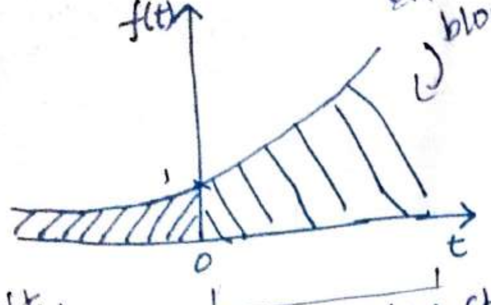
- ~~Any signal having step change alone in it, then it can be always be defined as sum of no of shifted signals with suitable coefficients where shift u(t), represents the time instant where a~~

- Any signal having step change alone in it then it can always be defined as a sum of number of shifted signals with suitable co-efficients where shift given to u(t) represents time instant where a change in step is required and co-efficients indicates the amount of step change required at shift (which is given to your u(t) signal).

12/01/26

$f(t) = e^{at}$, $a \in \mathbb{N}$

energy blows up



left sided. Truncation using step

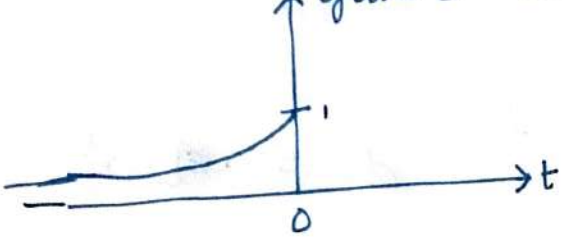
Where do we get Energy signal?

at $t = \infty$, $f(t) \to 0$ to get E.S.

$\Big| e^{at}$ Energy $= \int_{-\infty}^{\infty} |x(t)|^2 dt$

So, multiply $f(t) \times u(-t) = g(t)$

$g(t) = e^{at} \cdot u(-t)$



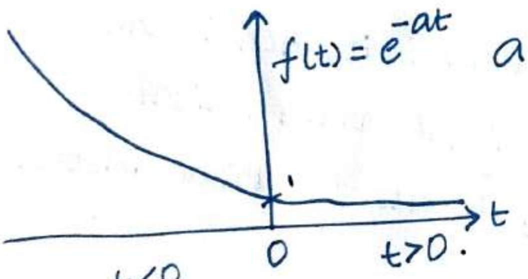
To obtain finite energy time should be $-\infty$ to ∞.

→ To have finite energy signal must decay.

$E|_{t=\infty} = 0 \Rightarrow P|_{t=\infty}$

→

$f(t) = e^{-at}$ $a \in \mathbb{N}$



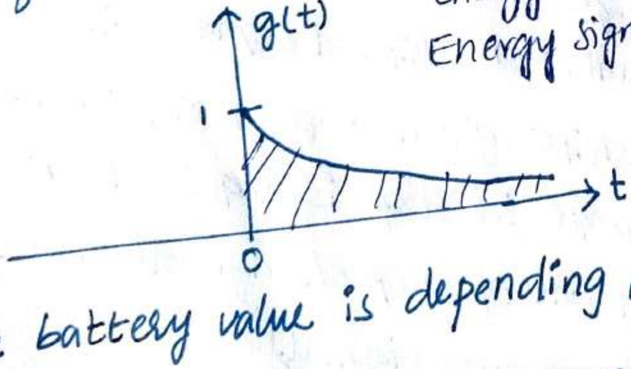
~~fix~~ t → independent var

To aviod confusion

$f(t) = e^{-at} = e^{-|a|t}$.

a	t	total sign
+ve	+ve	−ve
−ve	+ve	+ve
+ve	−ve	+ve
−ve	−ve	−ve

Right sided $g(t) = e^{-at} \cdot u(t)$.



Energy → finite
Energy signal.

→ The battery value is depending on this graph → 63% value.

At $t = \frac{1}{a}$ in e^{-at} the 63% energy is dispated

phones battery is calulculated using this concept.

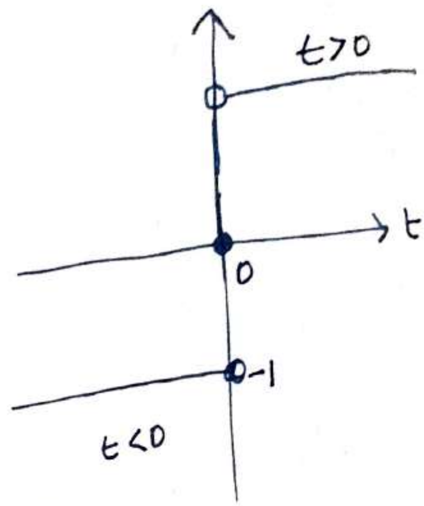
$u(t) = 1 \quad t>0$

$\frac{1}{2} \quad t=0$ ← Based on Even & odd signals

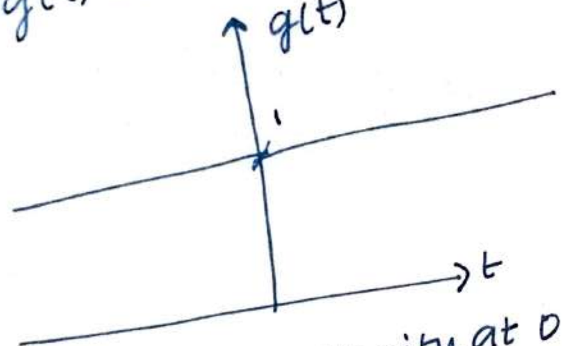
$0 \quad t<0$ $\qquad u(t) \Rightarrow$ odd signals

for continous cases, $u(t)\big|_{t=0} = 1$ is considered. (Right derivative)

$u(t) - u(-t)$



$g(t) = u(t) + u(-t)$



No discontinuity at 0.

$u(t)\big|_{t=0} = \dfrac{u(t^-) + u(t^+)}{2}$

$= \dfrac{1+1}{2}$

$= 1$

It is
→ ~~Not~~ a signum function.

Signum function $f(x)\big|_{x=0} \rightarrow$ ~~undefined~~ 0

but here $f(x)|_{x=0} = 0$.

left
$\lim_{t\to 0^-} u(t) + u(-t) = 1$

Right
$\lim_{t\to 0^+} u(t) + u(-t) = 1$

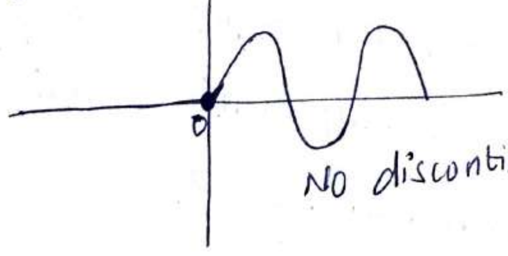
H.W

$P_1(t) = \cos t \cdot u(t)$

$S_1(t) = \sin(t) \cdot u(-t)$

$S_2(t) = \sin(t) \cdot u(t)$.

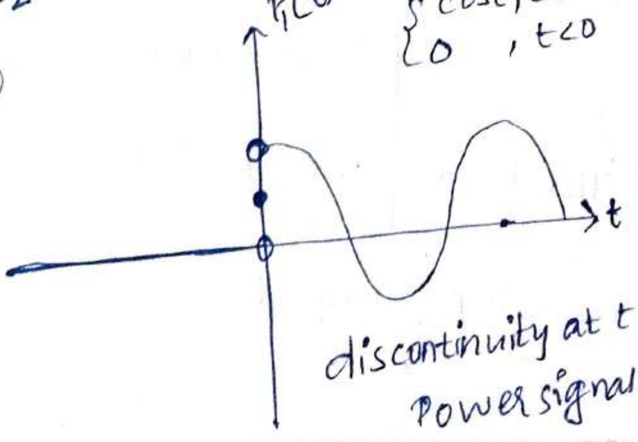
$S_2(t)$:



No discontinuity

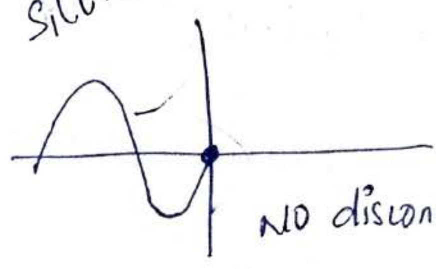
1.

$P_1(t) = \begin{cases} \cos t, & t \geq 0 \\ 0, & t<0 \end{cases}$



discontinuity at $t=0$
Power signal

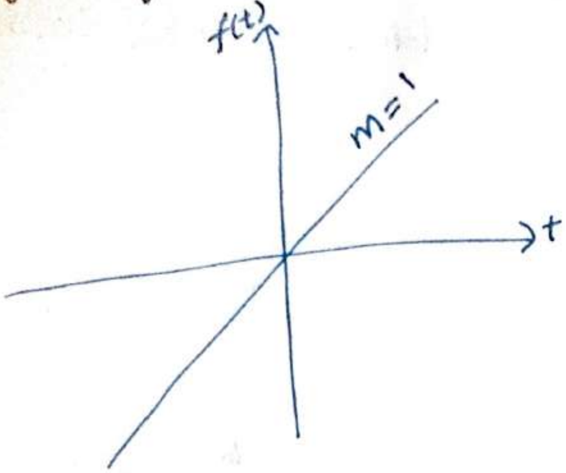
$S_1(t)$:



No discontinuity

→ Multiplication of slope with step unit size step function gives you ramp.

$y = mt + c$ ↳ Bias, DC component

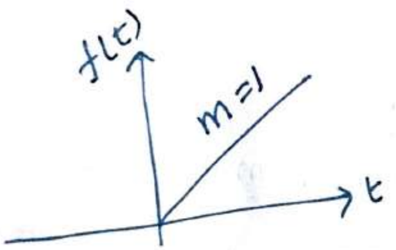


$f(t) = t \begin{cases} t & \geq 0 \\ -t & \leq 0. \end{cases}$

$c = 0 \Rightarrow y(t) = mt\, u(t) \Rightarrow$ Ramp

$m = 0 \Rightarrow y(t) = c \cdot u(t)$

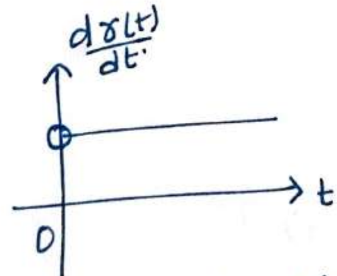
$r(t) | g(t) = f(t) \cdot u(t)$



Ramp

$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

$\frac{d\,r(t)}{dt} = \begin{cases} 1 & t \geq 0. \\ 0 & \text{otherwise} \end{cases}$



If you differentiate slope you get step (stepsize = m)

If you differentiate unit/step function we get discontinous delta function ($\delta(t)$)

differentiation of ramp function → unit step function

$\frac{d}{dt} r(t) = u(t)$

$\frac{d}{dt} [mt\, u(t)] = m u(t) + \underline{mt\, \delta(t)}$ Impulse term

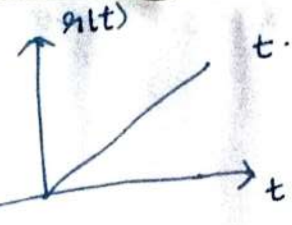
$\frac{d}{dt} u(t) = \delta(t)$

Ramp → step

Step → impulse

Line → step → Impulse.

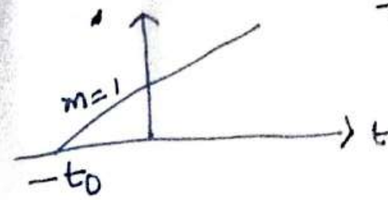
$r(t) =$



$t \cdot u(t)$

What $r(t+t_0)$?

$t_0 \Rightarrow +ve$

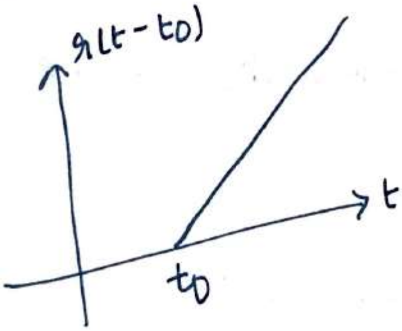


$$r(t+t_0) = \begin{cases} t+t_0 \; u(t+t_0) & t \geq t_0 \\ 0 & \text{Otherwise} \end{cases}$$

Advancment.

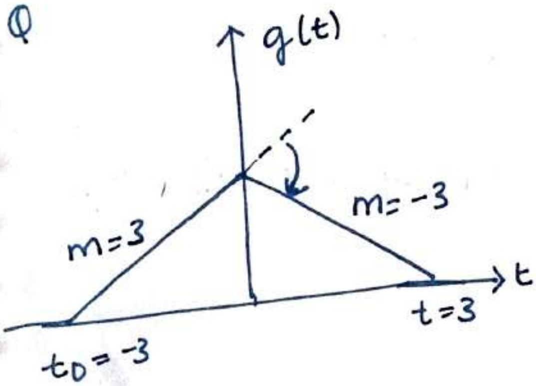
$r(t-t_0)$

$= (t-t_0)\, u(t-t_0) \quad t \geq t_0.$



$\Leftarrow$ delayed version.

Q

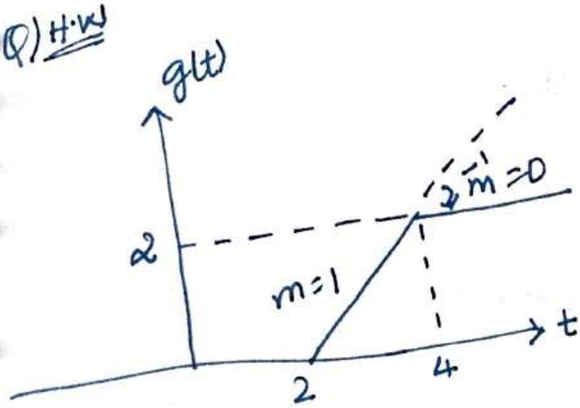


Find function of $g(t)$ in terms of $r(t)$.

Sol Amount of change in slope

$$g(t) = 3r(t+3) - 6r(t) + 3r(t-3)$$

Q) H.W



$g(t)$ in combination of $r(t)$ & $u(t)$?

Step 1: Slope $=0$ $\quad t<2$

Step 2: $r(t-2)$

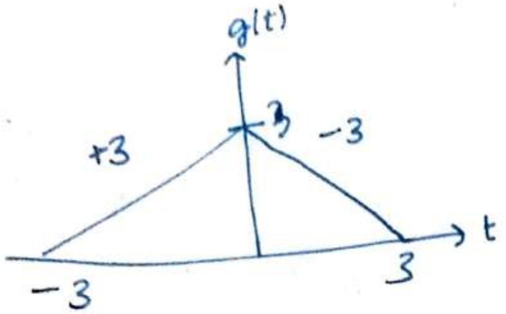
Step 3: Cancel $r(t-2)$ to inc and ~~u(t)~~ its const

change $= -1$, so

$$g(t) = r(t-2) - r(t-4)$$

Q) plot $f(t) = 2r(t+5) - 2r(t+2) - 2r(t-2) + 2r(t-5)$

H.W.



Ramps control slope

Differences of ramps create corners

- We add & Subtract ramps to change slopes at the correct points.

Step 1: slope = 0

Step 2: At $t = -3$, slope must incrose by +3

$+3r(t+3)$

Step 3: At $t=0$, slope must change from +3 to −3

change in slope = −6

So subtract a ramp starting at 0 with slope 6

$-6r(t)$

$3r(t+3) - 6r(t)$

step 4: At $t=3$, slope must change from −3 to 0

So add $+3r(t-3)$

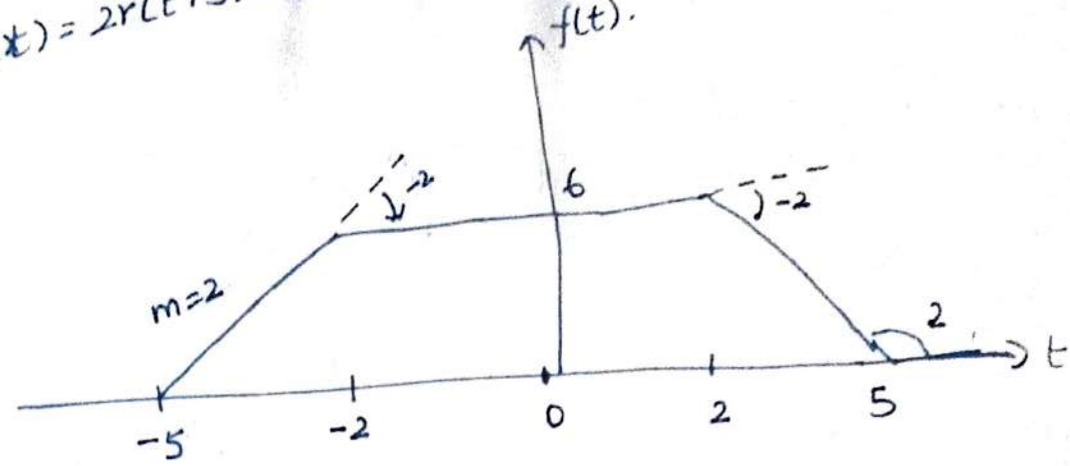
$g(t) = 3r(t+3) - 6r(t) + 3r(t-3)$

Universal formula:

$$\text{Signal} = \sum (\text{slope change}) \times r(t - t_0)$$

Q) plot

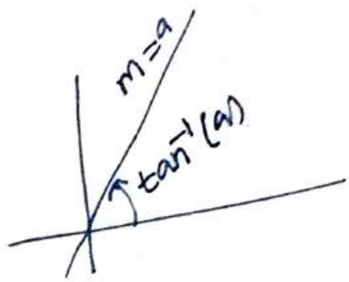
$f(t) = 2r(t+5) - 2r(t+2) - 2r(t-2) + 2r(t-5)$



14 jan 26

$r(t) =$ unit ramp signal. $(m=1)$

$r(at) =$ slope $= a$.

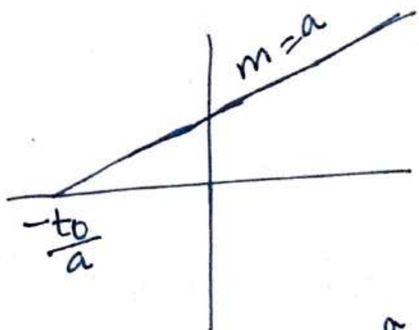


$r(at) = \begin{cases} at & t \geq 0. \\ 0 & \text{otherwise.} \end{cases}$

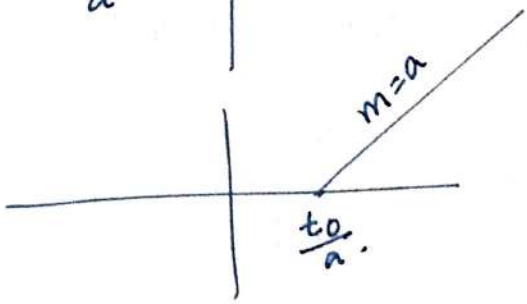
$r(at) = a \cdot r(t)$

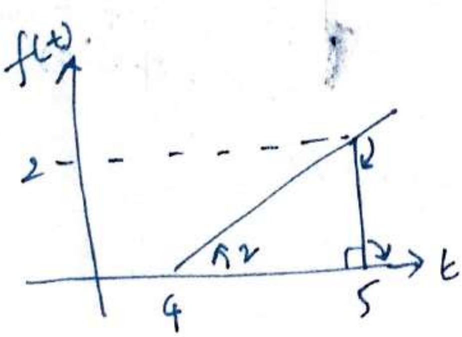
Time Scaling = Magnitude Scaling of unit ramp] does it work for $a = -ve$ numbers?

$r(at+t_0) = \begin{cases} at & t \geq t_0 \\ 0 & \text{otherwise.} \end{cases}$



$r(at-t_0) = \begin{cases} at & t > t_0 \\ 0 & \text{otherwise} \end{cases}$





$g(t) = f(t) + f(-t+9)$

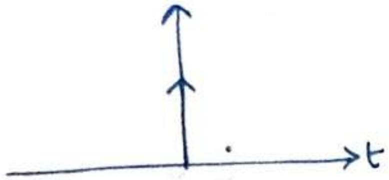
$g(t)$ in terms of $r(t)$ & $u(t)$?

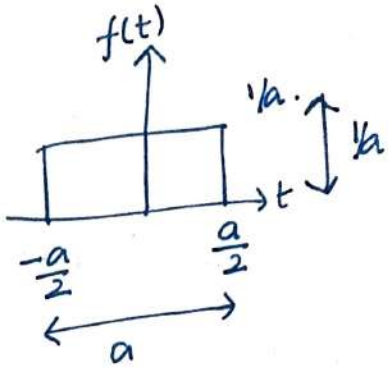
Q) $x(4t+7)$ & $x(-2t-5)$
H.W.

Dirac Delta Function:

$$\boxed{\delta(t) = \frac{d}{dt}u(t)} \quad \delta(t) = \lim_{\Delta t \to 0} f(t)$$

duration $\Delta t \to 0$





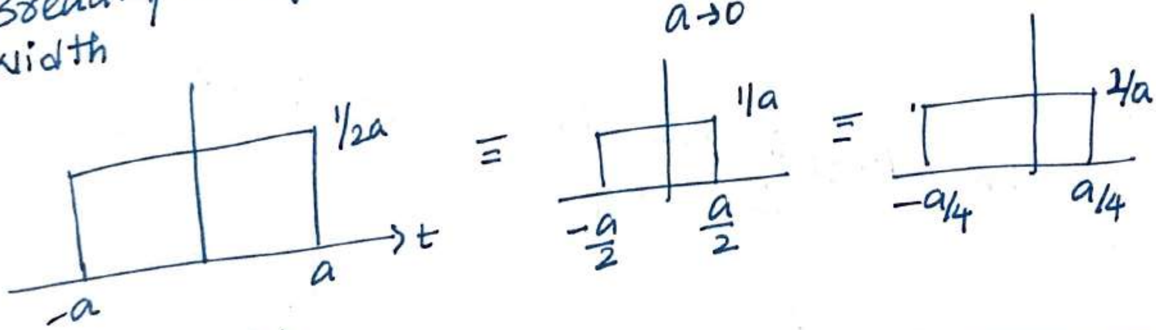
$$\delta(t) = \begin{cases} \infty & \text{at } t=0 \\ 0 & \text{Otherwise.} \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Area = 1

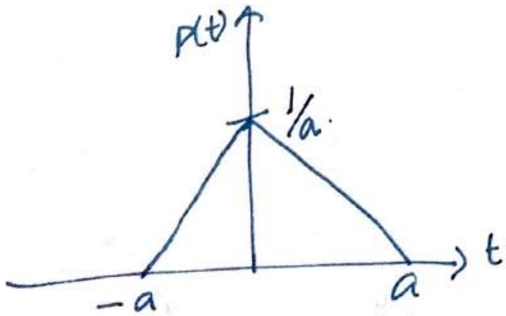
$$\lim_{a \to 0} f(t) = \delta(t)$$

Breadth/width $\propto \frac{1}{\text{length}}$



(Not to scale)

duration $\propto \frac{1}{\text{Amplitude}}$

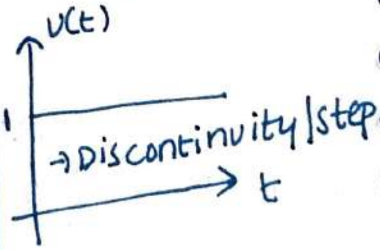


$$\delta(t) = \lim_{a \to 0} P(t)$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

What's the use of this $\delta(t)$?

$$\delta(t) = \frac{d}{dt}u(t)$$

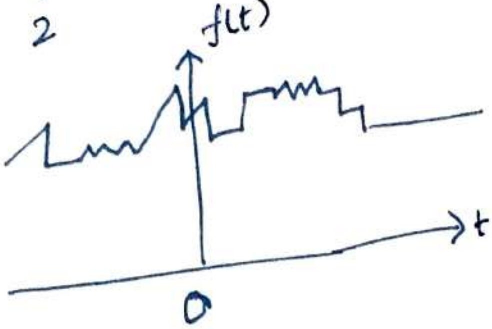


diff of step = Area of $\delta(t)$

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\,dt.$$

The $\delta(t)$ is an impulse function

$$\int_2^5 \delta(t)\,dt = 0$$ → whats role of this?



$f(t) \rightarrow$ Speech signal.

Note.

1. Dirac delta function is absolutely integrable (Area under curve is finite)

$f(t) * \delta(t) =$

Sampling of a function

$$\boxed{\int_{-\infty}^{\infty} f(t) * \delta(t)\,dt = f(0).}$$ } Time sifting.

$$\int_2^3 f(t) * \delta(t)\,dt = 0.$$

- Filtering, sorting, or selecting relevant information over time

Ex: Analyze historical data to predict patterns.

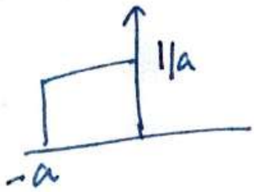
Cyclones, moonsoons etc

- data analysis, ML.

$\delta(t)$ ✓ what about $\delta(bt) = \lim_{a \to 0} p(bt)$, b can be +ve/-ve.

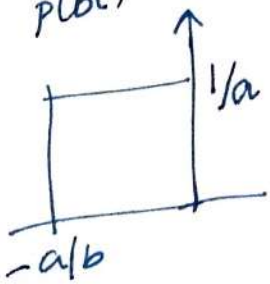
$$\boxed{\delta(bt) = \frac{1}{|b|}\delta(t)}$$

$p(t) =$



⇒

$p(bt)$



$$\int_{-\infty}^{\infty} \cos t * \delta(t)\, dt$$

$$\int_{-\infty}^{\infty} \sin\left(t - \frac{\pi}{2}\right) * \delta(t-\pi)\, dt$$

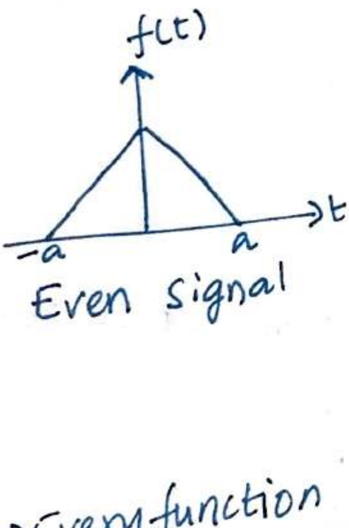
$$\int_{-\infty}^{\infty} \sin\left(t - \frac{\pi}{2}\right) * \delta(3t-\pi)\, dt$$

Even & Odd Signals.

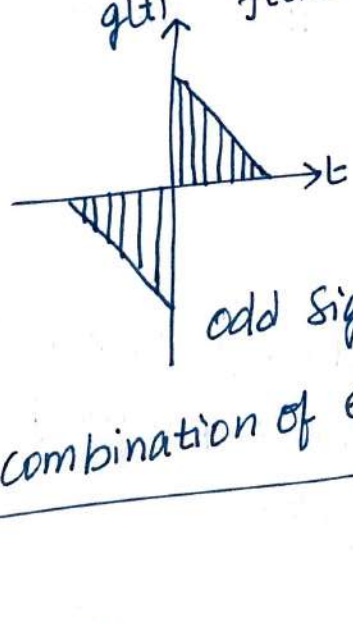
Even: Symmetric signals $f(t) = f(-t)$ Ex: cos t

Odd: Asymmetric Signals $f(t) = -f(-t)$ Ex: sin t.

$f(t) = -f(-t)$



Even signal



odd signal

→ Every function is combination of even & odd functions.

H.W

$r(t) = t \cdot u(t)$

$\frac{d(r(t))}{dt}$ = discontinuity = step size

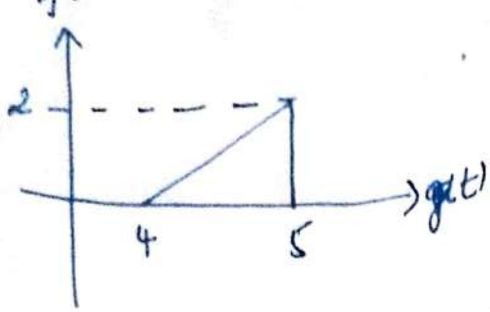
ramp $\underset{\int}{\overset{\frac{d}{dx}}{\rightleftarrows}}$ unit $\underset{\int}{\overset{\frac{d}{dx}}{\rightleftarrows}}$ dirac

$\frac{d^2(r(t))}{dt^2} = \delta(t)$.

d(slope) = step size.

H.W

1)



$g(t) = f(t) + f(-t+9)$

$g(t)$ in terms of $r(t)$ & $u(t)$?

Ans:

$$f(t) = \begin{cases} 0 & t<4 \\ \text{linear} & 4<t<5 \\ 0 & t>5 \end{cases}$$

$\text{slope} = \frac{2-0}{5-4} = 2$

$f(t)$ in ramp $= 2r(t-4) - 2r(t-5)$

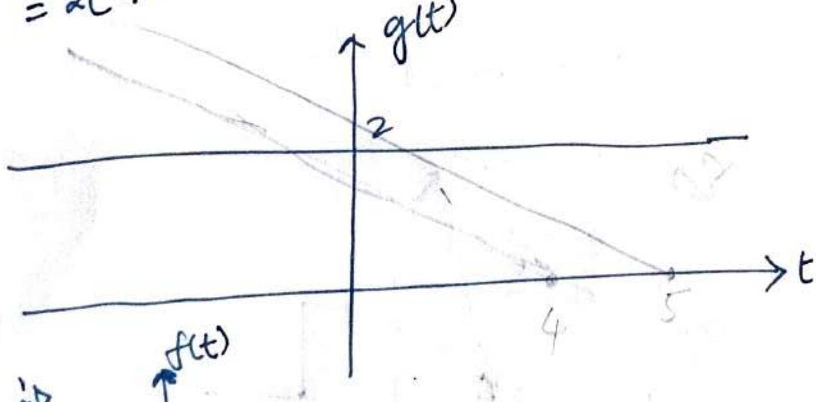
$f(t) = 2[(t-4)u(t-4) - (t-5)u(t-5)]$

$f(-t+9) = 2[(-t+9-4)u(-t+9-4) - (-t+9-5)u(-t+9-5)]$

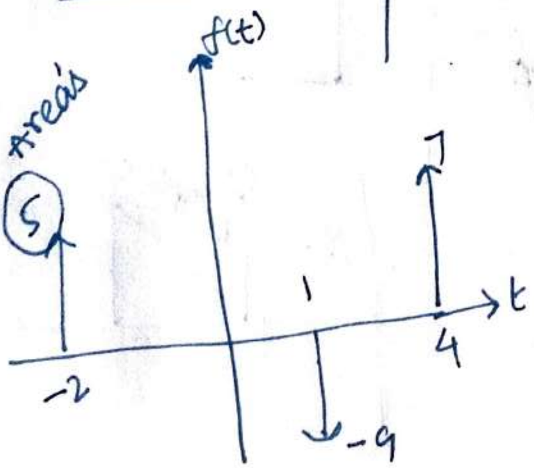
$= 2[r(-t+5) - r(-t+4)]$

$g(t) = [f(t) + f(-t+9)]$

$= 2[r(t-4) - r(t-5) + r(5-t) - r(4-t)]$



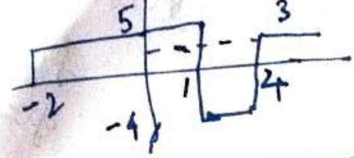
$$r(4-t) = \begin{cases} 4-t & t<4 \\ 0 & t>4 \end{cases}$$



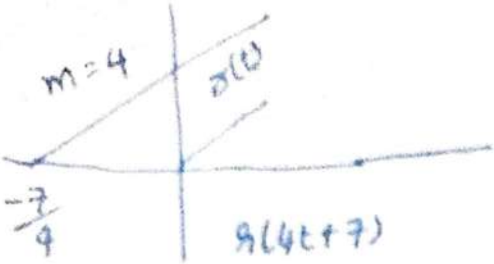
$f(t) = 5\delta(t+2) - 9\delta(t-1) + 7\delta(t-4)$

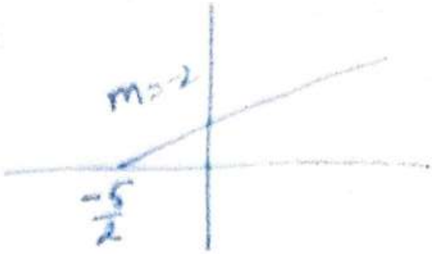
Integrator (high pass filter)

$g(t) = \int f(t)\,dt$



$x(4t+7)$ & $x(-2t-5)$





$$\int_{-\infty}^{\infty} \cos t \cdot \delta(t)\,dt$$

$= \cos 0$

$= 1$

sifting property

$$\int_{\infty}^{\infty} f(t)\,\delta(t-a)\,dt = fa$$

Scaling property

$$\delta(at-b) = \frac{1}{|a|}\,\delta\left(t-\frac{b}{a}\right)$$

$$\int_{-\infty}^{\infty} \sin\left(t-\frac{\pi}{2}\right)\delta(t-\pi)\,dt$$

$= \sin\left(\pi-\frac{\pi}{2}\right)$

$= \sin\frac{\pi}{2}$

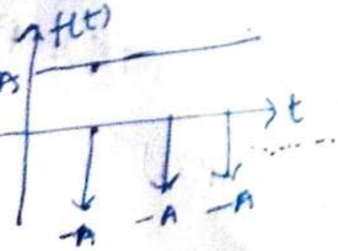
$= 1$

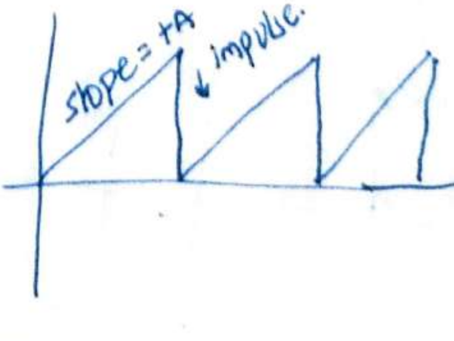
$$\int_{-\infty}^{\infty} \sin\left(t-\frac{\pi}{2}\right)\delta(3t-\pi)\,dt = \frac{1}{3}\int_{-\infty}^{\infty} \sin\left(t-\frac{\pi}{2}\right)\delta\left(t-\frac{\pi}{3}\right)dt$$

$= \frac{1}{3}\sin\left(\frac{\pi}{3}-\frac{\pi}{2}\right)$

$= -\frac{1}{3}\times\frac{1}{2}$

$= \frac{-1}{6}$





$g(t) = \int f(t)\,dt$

$f(t) = A - A\sum_K \delta(t-t_K)$

$g(t) = At - A\sum_K u(t-t_K)$

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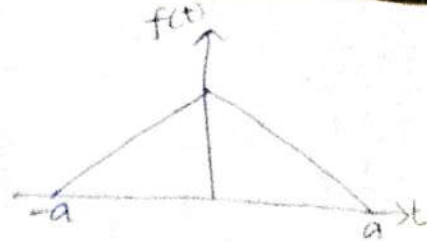
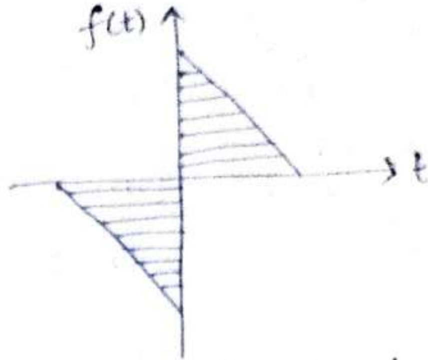
Even & Odd Signals



Even: Symmetric Signals $f(t) = f(-t)$

Ex: $\cos t$, t^2, $|t|$



Odd: AntiSymmetric Signals

$f(t) = -f(-t)$

$\Rightarrow f(-t) = -f(t)$

Ex: $\sin t$, t^3, $\text{sgn}(t)$

→ Every function is combination of even & odd signals.

$$x(t) = x_e(t) + x_o(t)$$

$x(t) = x_e(t) + x_o(t)$ —— 1

Sub $t = -t$

$x(-t) = x_e(-t) + x_o(-t)$

$x(-t) = x_e(t) - x_o(t)$ —— 2

Sub 1 & 2

$x(t) - x(-t) = 2x_o(t)$

$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

Add 1 & 2

$x(t) + x(-t) = 2x_e(t)$

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

Here $x(t)$ is real function.

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Real & Complex valued function

Real function: $f(t) = 5t$

Complex function: $f(t) = e^{jt} = \cos t + j\sin t$

C_n: complex domain coefficient

$g(t) = e^{jt} \cdot e^{2jt} = (\cos t + j\sin t)(\cos 2t + j\sin 2t)$

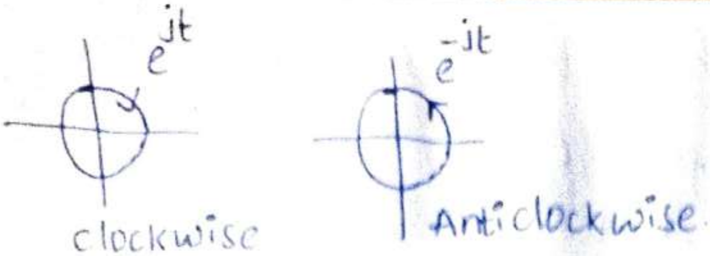
$= e^{3jt}$ (easy to compute in Exponential way)

$h(t) = \frac{e^{2jt}}{e^{jt}} = \text{Exp}(jt)$.

To map into freq domains we need to do 3 steps

1. Constant
2. Cosinusoidal
3. Sinusoidal

It's called fourier transform.



Periodic & Aperiodic Signals

A periodic continous time signal $x(t)$ has the property that there is a positive value of T for which

$$x(t) = x(t+T), \text{ for all values of } t$$

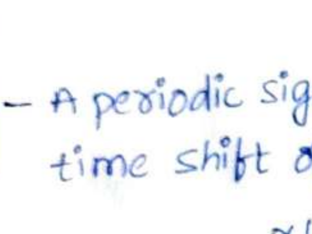
- A periodic signal has the property that it is unchanged by a time shift of T

$$x(t) = x(t+T) = x(t+mT) \quad m = 1,2,3,4 \ldots$$

$x(t)$ is also periodic with period $T, 2T, 3T, 4T, \ldots$

- Fundamental Time period (T_0): Smallest value of T so that $x(t) = x(t+T)$
- Fundamental frequency $= \frac{2\pi}{T_0}$.

Ex:



← unit step train

← perodic train

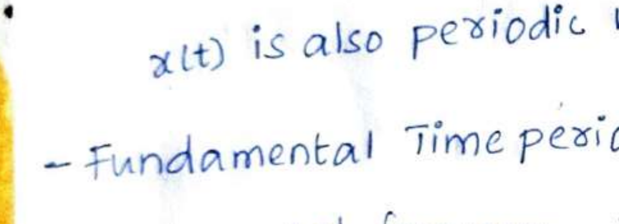
$g(t) = \int f(t)\,dt$

$f(t) = A - A\Sigma\,\delta(t-t_K)$

$g(t) = \int f(t) = \int A - A\Sigma\,\delta(t-t_K)\,dt$

$= \int A\,dt - \int A\Sigma\,\delta(t-t_K)\,dt$

$g(t) = At - A\Sigma\,u(t-t_K)$

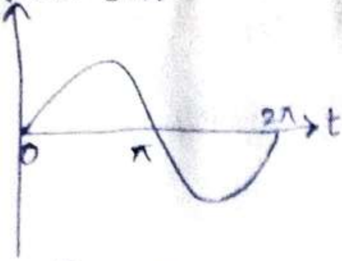


periodic signal

fundamental time period = 1 sec.

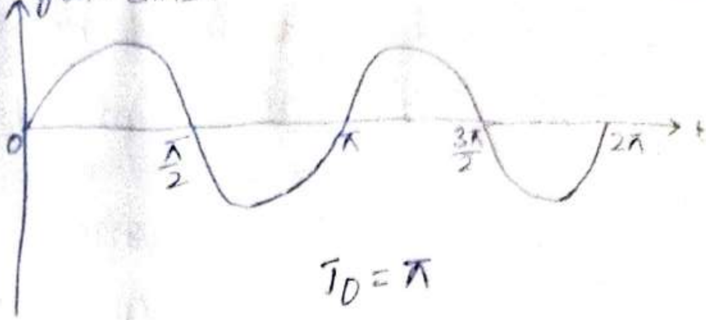
fundamental frequency = 2π Hz.

$f(t) = \sin t$



$T_0 = 2\pi$

$g(t) = \sin 2t$



$T_0 = \pi$

$h(t) = f(t) + g(t) = \sin t + \sin 2t$

fundamental Time period $= \text{lcm}(2\pi, \pi)$
$= 2\pi$.

- Fundamental Time period of combined signals
$= \text{lcm}$(individual ~~per~~ fundamental Time periods)

- For a continous time signal min time period of combined signals will be least common multiple (lcm) of all the Time signals.

H.W.

Q) $h(t) = \cos(\sqrt{3}\pi t) + \cos(2\sqrt{3}\pi t + 6)$

$T_{0_1} = \frac{2\pi}{\sqrt{3}\pi} = \frac{2}{\sqrt{3}}$ $\quad T_{0_2} = \frac{2\pi}{2\sqrt{3}\pi} = \frac{1}{\sqrt{3}}$

$\frac{T_{0_1}}{T_{0_2}} = 2$ (p/q) so $h(t)$ is periodic.

$T_0 = \text{lcm}\left(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$ sec., $f_0 = \frac{\sqrt{3}}{2}$ Hz.

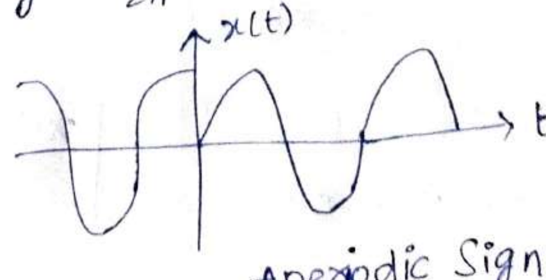
Q) $\sin(t), \sin(2t), \sin(3t), \sin(4t)$

fundamental Time period $= \left(2\pi, \pi, \frac{2\pi}{3}, \frac{\pi}{2}\right) = 2\pi$.

fundamental frequency $= \frac{2\pi}{2\pi} = 1$.

H.W.

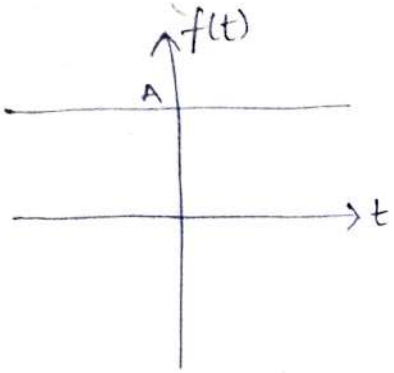
Q) $x(t) = \begin{cases} \cos t & t < 0 \\ \sin t & t \geq 0 \end{cases}$



Aperiodic Signal.
discontinuity at $t = 0$.

→ For a Aperiodic signal, fundamental time period is infinite

Aperiodic signal is also periodic, but $T_0 = \infty$ & $w_0 = 0$.



Constant function

Aperiodic Signal

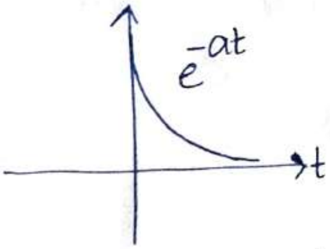
T_0 = undefined

$w_0 = 0$

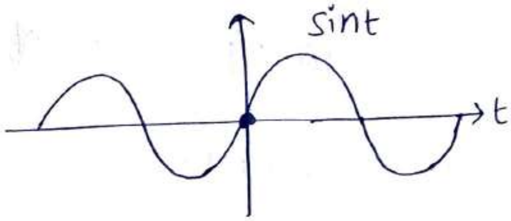
$f_0 = 0$.

Note:

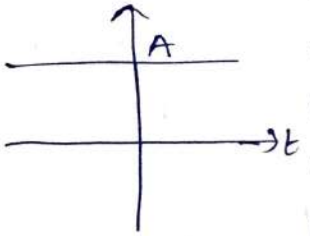
1. Energy signals are Aperiodic signals
2. Power signals may be periodic or Aperiodic in nature.



Energy Signal
Aperiodic



Power Signal
Periodic



Power Signal
Aperiodic.

-Energy signals always Aperiodic in nature because at $t=\infty$ value is zero. So it doesnt follow $x(t) = x(t+T)$

ssignment

$$(t) = \begin{cases} \cos t & t<0 \\ \sin t & t \geq 0 \end{cases}$$

can this function turn into periodic in nature?

- Make both sides compatible

$$x(t) = \begin{cases} \cos t & t<0 \\ \sin(t+\frac{\pi}{2}) = \cos t & t \geq 0 \end{cases} \quad \Bigg| \quad x(t) = \begin{cases} \cos(t-\frac{\pi}{2}) = \sin t & t<0 \\ \sin t & t \geq 0. \end{cases}$$

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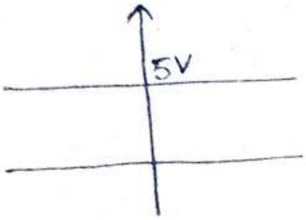
Fundamental Time period (T_0)

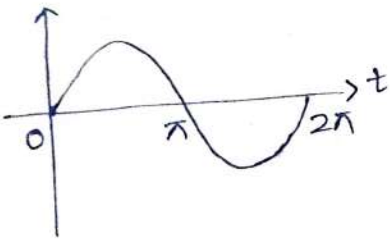
fundamental frequency (f_0)
fundamental angular frequency (w_0)



T_0 = undefined

$w_0 = \frac{2\pi}{T_0} = 0$

$f_0 = 0$

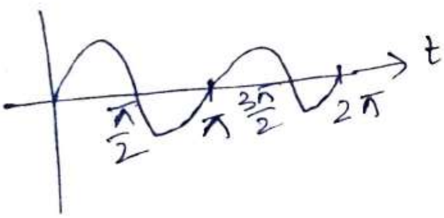


$f(t) = \sin t$

$\int_0^{2\pi} \sin t = 0$

$T_0 = 2\pi$

$f_0 = \frac{1}{2\pi}$

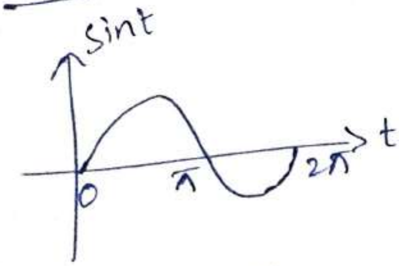


$f(t) = \sin 2t$

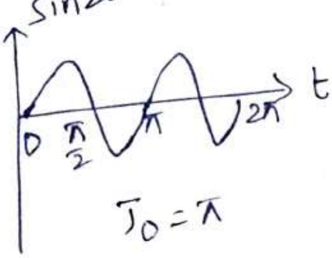
$T_0 = \pi$

$f_0 = \frac{1}{\pi}$

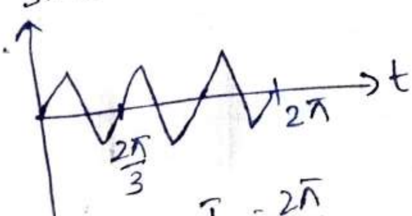
Common Time period (CTP)



$T_0 = 2\pi$



$T_0 = \pi$



$T_0 = \frac{2\pi}{3}$

$\sin kt \Rightarrow T_0 = \frac{2\pi}{k}$

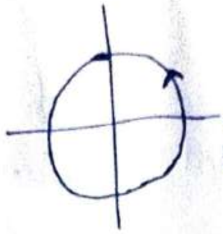
Common time period for these signals = 2π.

→ If fundamanental Time period for $f(t)$ is T_0, then fundamental Time period (FTP) is $\frac{T_0}{a}$.

$\sin w_0 t$

$w_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{w_0}$



$\frac{2\pi}{w_0}$

$\int_0^{2\pi} x \Rightarrow \int_0^{2\pi/w_0} \sin w_0 t \, dt = 0.$

$\sin(K w_0 t)$

$FTP = \frac{2\pi}{K w_0} = T_0$

$CTP = \frac{2\pi}{w_0}$

$\sin(w_0 t + \phi)$

ϕ = phase, which is 0 in above case.

phasers.

e^{j2t}	e^{-2jt}
e^{jt}	e^{-jt}
e^{3jt}	e^{-3jt}
e^{Kjt}	e^{-Kjt}

e^{Kjt} or e^{-Kjt}

$T_0 = \frac{2\pi}{K}$

"$-$ve" indicates direction($\uparrow$ or $\downarrow$)

$e^{K w_0 jt}$ or $e^{-K w_0 jt}$ $T_0 = \frac{2\pi}{|K w_0|}$

Take Away Questions

$\int_0^{2\pi/w_0} \sin w_0 t \sin 2 w_0 t \, dt = 0$

$\int_0^{T_0} \cos w_0 t \, dt = 0$

$\int_0^{2\pi/w_0} e^{j w_0 t} (e^{j w_0 t})^* dt = \frac{2\pi}{w_0}$

$e^j \times (e^j)^* = 1$

$\rightarrow$ sin and cosines are Orthogonal Signals.

$f(t) = Ae^{j(w_0 t + \phi)}$

$= A\cos(w_0 t + \phi) + jA\sin(w_0 t + \phi)$

$A\sin(w_0 t + \phi)$ = Amp of Imaginary part.

$f(t) \rightarrow$ Complex valued Signal.

Its

Even Conjugate/
Conjugate Symmetric : $f(t) = f^*(-t)$

Odd conjugate/
Conjugate Anti Symmetric : $f(t) = -f^*(-t)$

$-f(t) = e^{jt}$

$f^*(t) = e^{-jt}$

$f^*(-t) = e^{-j(-t)}$

$= e^{jt}$

Conjugate Symmetric

$f(t) = te^{jt}$

$f^*(+t) = te^{-jt}$

$f^*(-t) = -te^{+jt}$

Conjugate Anti Symmetric.

→ For conjugate Symmetric

Real part is ~~always~~ always is Even

Img part is always is Odd.

→ For Conjugate Anti Symmetric

Real part is Odd.

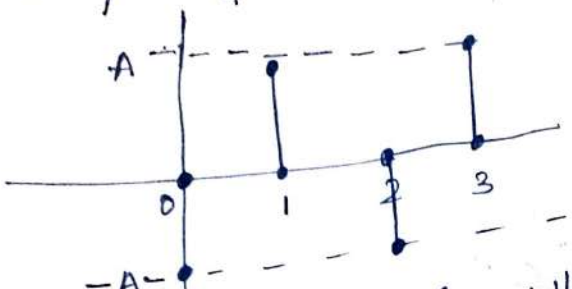
Img part is Even.

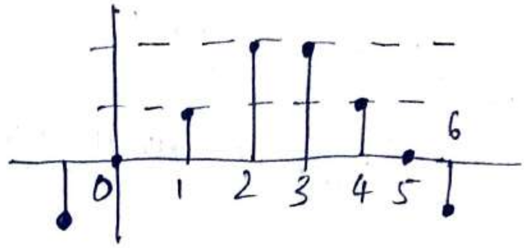
$\rightarrow e^{jt} = \underset{\text{Even}}{\cos t} + \underset{\text{odd}}{j\sin t}$

$je^{jt} = \underset{\text{Odd}}{j\cos t} + \underset{\text{even}}{jt\sin t}$

- Discrete Signals

L sample Size are always discrete



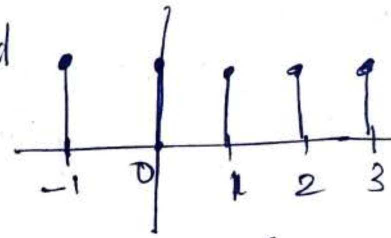


→ Loss of information while discretizing a continous Signal

Ex:

Continous

discretized ⟹



loss of information.

→ Discretization is done by uniform sampling.

Traling Errors.

→ If sample is more then Errors is

more sample
less Truncation Error.

→ Digital v/s Discrete

Based on Amp	Based on Time (finite set of Integers
$\delta(n)$ - Kronecker delta	$\delta(t)$ → delta function doesn't Exists in discrete.

Exists in discrete

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Discerte Time Signals.

$f(t)$: continous time signal
$f(n)$: Discerte time signal.

- Discerte time signals are defined only at discrete times, and independent variable takes on only discerte set of values. $f(n)$: Discerte domain, $n \in \mathbb{Z}$

- There is the inaccuracy from approximating an continous signal into a finite discerte signal leads to errors is Truncation error

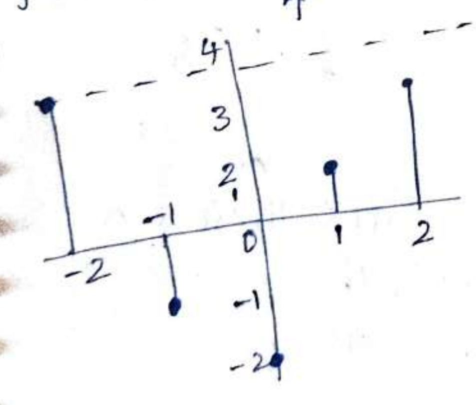
$$\text{Truncation Errors} \propto \frac{1}{\text{sample size}}$$

Ex: $f(n) = \{4, -1, \underset{\uparrow}{-2}, 1, 3\}$ ↑ This indicates for origin i.e., $f(0) = -2$

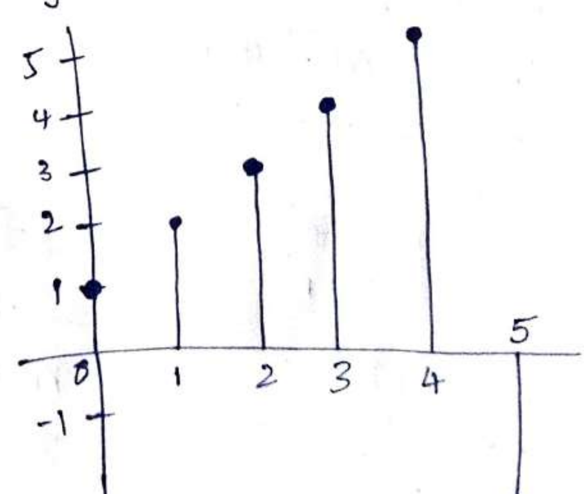
n	-2	-1	0	1	2
$f(n)$	4	-1	-2	1	3

Note: If no arrow is there, take first value as origin, if changed needs to specify.

$f(n) = \{4, -1, \underset{\uparrow}{-2}, 1, 3\}$



$f(n) = \{1, 2, 3, 4, 5, -2\}$



$$f(n) = \begin{cases} n+6, & n = -2 \\ n & n = -1, 1 \\ n-2 & n = 0 \\ n+1 & n = 2 \end{cases}$$

- We cannot write like this for Continous Signal.

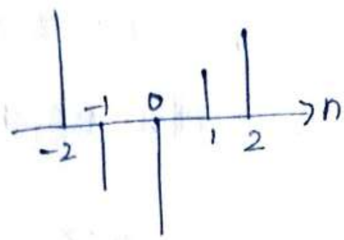
Periodicity.

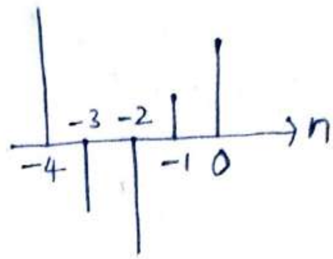
- periodic signals are defined analogously in discrete Time, if signal periodic with period N then

$$x[N] = x[n+N] \text{ or } f(n) = f(n \pm N)$$

Shifting.

$f(n) = \{4, -1, \underset{\uparrow}{-2}, 1, 3\}$

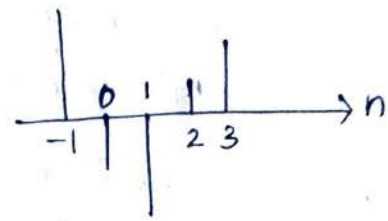




$f(n-2) = \{4, -1, -2, 1, \underset{\uparrow}{3}\}$

f(n)

$f(n+1) = \{4, \underset{\uparrow}{-1}, -2, 1, 3\}$



Scaling.

Two types
1. Decimation
2. Interpolation
   a. zero Interpolation
   b. Unit Interpolation
   c. Avg Interpolation

$f(an) : a > 1$
- Decimation
- ax time speed/faster.

$f(n) = \{4, -1, -2, 1, 3\}$

$f(2n) = \{4, -2, 3\}$

$\quad\quad\quad -1 \quad 0 \quad 1$

- lost some data.
- Compression
- destruction.

$f(an) : a < 1$
- Interpolation
- ax slower

$f(n) = \{4, -1, \underset{\uparrow}{-2}, 1, 3\}$

$f(\frac{n}{2}) = \{4, -, -1, -, \underset{\uparrow}{-2}, -, 1, -, 3\}$

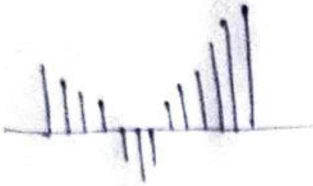
'–' shows new spaces, these are newly created, The method of filling/padded are made diff Interpolations

a. zero: $f(\frac{n}{2}) = \{4, 0, -1, 0, \underset{\uparrow}{-2}, 0, 1, 0, 3\}$

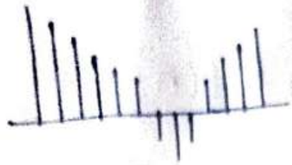
b. Unit: $f(\frac{n}{2}) = \{4, 1, -1, 1, -2, 1, 1, 1, 3\}$

c. AVG: $f(\frac{n}{2}) = \{4, \frac{3}{2}, -1, \frac{-1}{2}, -2, \frac{-1}{2}, 1, 2, 3\}$

Rerasal



$f(n)$

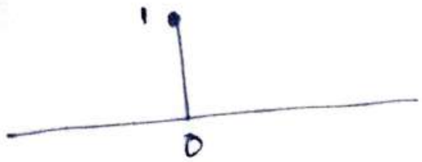


$f(-n)$

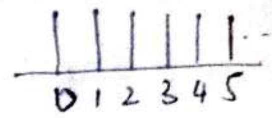
It's called reversal. reflection about $n=0$.

Kronecker delta function

$$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{otherwise} \end{cases}$$



$$\delta(n) = u(n) - u(n-1)$$



$u(n)$

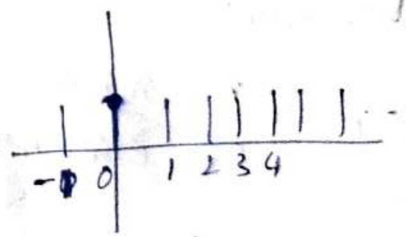
$$f(n) = \{4, -1, \underset{\uparrow}{-2}, -1, 3\}$$

$$f(n) = 4\,\delta(n+2) - \delta(n+1) - 2\delta(n) + \delta(n-1) + 3\delta(n-2)$$

(Analogus to Time sifting) $f(n) = \sum_{K \in D} f(K)\,\delta(n-K)$

$u(n-1)$:

$u(n+1)$



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$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

$$\boxed{u(n) = \sum_{K=-\infty}^{\infty} u(K)\,\delta(n-K)}$$

Analogus to $u(t) = \int_{t \in [-\infty, \infty)} \delta(t)\,dt$.

$$u(n) = \sum_{K=-\infty}^{\infty} \delta(n-K) \cdot u(K)$$

Proof:

$$\sum_{K=-\infty}^{\infty} u(K) \cdot \delta(n-K) = \delta(n) + \delta(n-1) + \delta(n-2) + \dots$$

for $n<0$ $u(n) = 0$. so, $u(K)\delta(n-K) \; \forall K<0 = 0$.

for $n \geq 0$ $u(n) = 1$ so, $u(K) \cdot \delta(n-K) = 1 \cdot \delta(n-K) \; \forall K \geq 0$.

→ conversely, the discrete time unit step is the running sum of unit sample

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

running sum in eq is 0 for $n<0$

$$u[n-n_0] = \sum_{K=0}^{\infty} \delta[n-K_0]$$

$$u[n] = \sum_{K=-\infty}^{n} \delta[K]$$

case 1: $n<0$.

$u[n] = \delta[-\infty] + \dots + \delta[n]$

$0 = \delta[-\infty] + \dots + \delta[n]$

case 2: $n=0$.

$$\underbrace{u[0]}_{1} = \underbrace{\delta[-\infty] + \dots}_{0} \underbrace{\delta[0]}_{1}$$

$u[0] = \delta[0] = 1$

case 3: $n>0$.

$u[n] = \delta[0] + \delta[1] + \delta[2] + \dots$

$= 1$

$$u[n-2] = \sum_{K=-\infty}^{n} \delta[k-2]$$

$\delta[k-2]$ at 2 is 1

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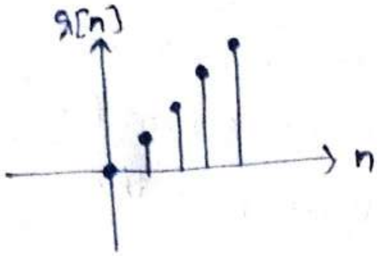
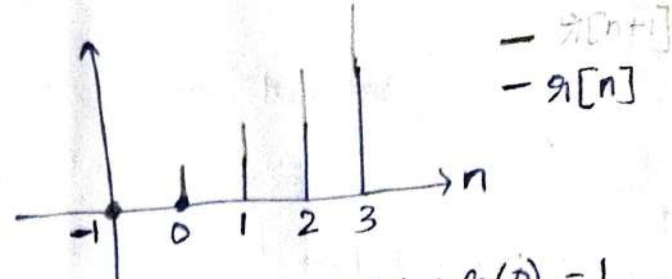
ramp function.

$$r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$u[n] = r[n+1] - r[n]$



at $n=0$, $u(0) = r(1) - r(0) = 1$

at $n=-1$ $u(-1) = r(0) - r(-1) = 0$

at $n=1$ $u(1) = r(2) - r(1) = 1$

relation:

$r[n] = n\,u[n]$

$u[n] = r[n+1] - r[n]$

$\rightarrow$ Analogus to $u(t)\big|_{t=0} = \dfrac{dr(t)}{dt}$

Note.

1. $\delta[n] = u[n] - u[n-1]$
2. $u[n] = \sum_{k=-\infty}^{n} \delta[k]$
3. $r[n+1] - r[n] = u[n]$
4. $r[n] = \sum_{k=-\infty}^{n} u[k]$.

$$\text{ramp} \underset{\text{cum}}{\overset{\text{diff}}{\rightleftarrows}} \text{step} \underset{\text{cum}}{\overset{\text{diff}}{\rightleftarrows}} \text{Kronecker}$$

Assignment.

$\sin(3\omega_0 n) = \dfrac{2\pi}{3\omega_0}$.

$\cos(2\omega_0 n) = \dfrac{2\pi}{2\omega_0} = \dfrac{\pi}{\omega_0}$.

periodicity.

periodic.

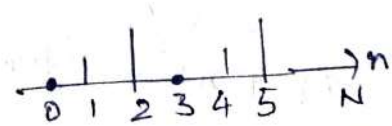
$N_0 = 2$.

periodic

$N_0 = 3$.

Power signal

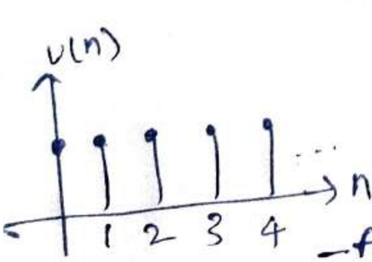
$\text{Power} = \dfrac{1}{2N+1} \sum_{-N}^{N} |f(n)|^2$



periodic $(n \geq 0)$

$N_0 = 3$

$\text{Power} = \dfrac{1}{N} \sum_{n=0}^{N} |f(n)|^2$



No value for $n>0$ is 0 but $n<0$ is.

– Aperiodic

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Discrete Time Signal.

$\cos 2w_0 n$ rational Integer ✓

$N_0 = \frac{2\pi}{w_0} \times r$

Integer representation.

N_0 - rational.

$\left.\begin{array}{l} N_0 = \pi \\ N_0 = \sqrt{3} \\ N_0 = j \end{array}\right]$ Not accepted.

DTS is Aperiodic in those cases.

→ $e^{j3\pi n}$ $N_0 = \frac{2\pi}{3\pi} r$

$= \frac{2}{3} \times r$

$r = 3, N_0 = 2.$

periodic.

Ex: $\cos n = \cos 1 \cdot n$

$w_0 = 1$ $N_0 = 2\pi$

Aperiodic Signal.

Ex: $\cos \sqrt{2} n = \cos \underset{w_0}{\sqrt{2}} \cdot n$

$N_0 = \sqrt{2}\pi$

Aperiodic Signal.

$$\boxed{\frac{w_0}{2\pi} = \frac{r}{N}}$$

$w_0 N$ must be multiple of 2π

→ The DTS signal is periodic if $w_0/2\pi$ is a rational number and is not periodic otherwise.

→ This difference is beacuse in CTS it takes every value in real line, in other hand it accepts the value from the Domain.

→ fundamental period

$N = r\left(\frac{2\pi}{w_0}\right)$

fundamental frequency

$\frac{2\pi}{N}.$

Ex: $f[n] = e^{j\sqrt{2}\pi n}.$

$w_0 = \sqrt{2}\pi$ $N_0 = \frac{2\pi}{w_0} \times r$

$= \sqrt{2}.$

→ Discrete Aperiodic Signal.

$e^{jw_0 n}$ $1 \angle w_0 n$

$e^{j\frac{\pi}{4} n}$ $1 \angle \frac{\pi}{4} n$

$e^{j\pi n}$ $1 \angle \pi n$

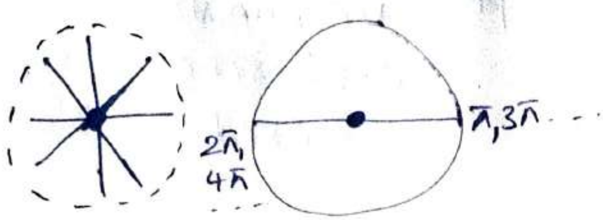
$e^{j2\pi n}$ $1 \angle 2\pi n$

$e^{j3\pi n}$ $1 \angle 3\pi n$

$e^{j\sqrt{2}\pi n}$ $1 \angle \sqrt{2}\pi n.$ ✗

- Phasors

$e^{j\omega_0 n}$ $\qquad\qquad$ $e^{j\pi n}$

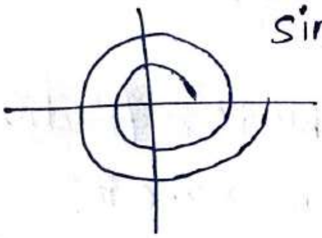


→ periodic only phasor Exists.

$e^{j\sqrt{2}n}$ → No phasor
Aperiodic
Non - recuring.

→ $\sin\sqrt{t}$ - Aperiodic CTS
$\sin\sqrt{2}t$ - periodic CTS

$\sin\sqrt{t}$



- It doesn't accept -ve values
- repeats after
$0, \pi^2, 4\pi^2, 9\pi^2, 16\pi^2 \ldots\ldots$
No common interval
So, Aperiodic.

$$f(t) = 20 + 10\sin\left(\frac{\pi}{7}t + \frac{\pi}{4}\right) + 5\cos\left(\frac{6\pi}{5}t + \frac{\pi}{6}\right) + 7\sin\left(\frac{5\pi}{6}t + \frac{\pi}{3}\right)$$

If we ignore the DC compnent / DC Shift, what would be fundamental time period?

$$\underbrace{\frac{\omega_0}{\omega_1} = \frac{5}{42} \quad \frac{\omega_1}{\omega_2} = \frac{36}{25} \quad \frac{\omega_3}{\omega_1} = \frac{35}{6}}$$

All are in p/q form.

$10\sin\left(\frac{\pi}{7}t + \frac{\pi}{4}\right)$

$\omega_0 = \frac{\pi}{7}$

$5\cos\left(\frac{6\pi}{5}t + \frac{\pi}{6}\right)$

$\omega_0 = \frac{6\pi}{5}$

$7\sin\left(\frac{5\pi}{6}t + \frac{\pi}{3}\right)$

$\omega_0 = \cancel{\frac{12}{5}}\ \frac{5\pi}{6}$

$$\text{Hcf}(\omega_{0_1}, \omega_{0_2}, \omega_{0_3}) = \frac{\text{Hcf}(1,6,5)\pi}{\text{lcm}(7,5,6)} = \frac{\pi}{210}$$

$$N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi/210} = 420.$$

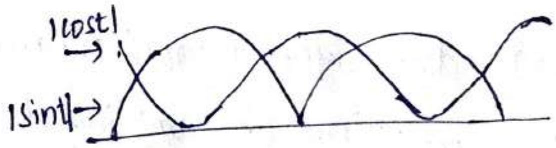
$$\boxed{N_0 = 420}$$

<u>Assignment</u>

Ex: $f(t) = |\sin t| + |\cos t|$

$|\sin t| \longrightarrow \pi$

$|\cos t| \longrightarrow \pi$



$$f\left(t + \frac{\pi}{2}\right) = \left|\sin\left(t + \frac{\pi}{2}\right)\right| + \left|\cos\left(t + \frac{\pi}{2}\right)\right|$$
$$= |\cos t| + |-\sin t|$$
$$= |\cos t| + |\sin t|$$

$f\left(t + \frac{\pi}{2}\right) = f(t)$

→ period $= \frac{\pi}{2}$.

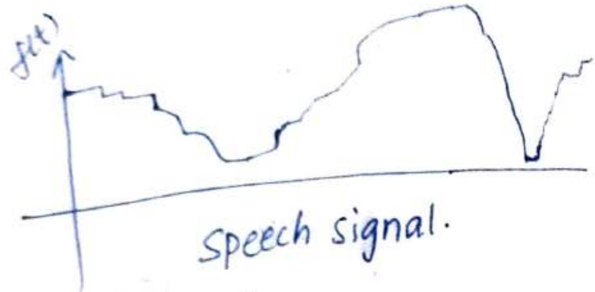
Random & Determenstic

$f(t) = \sin t$

$t=0, f(t)=0$

$t=\frac{\pi}{2}, f(t)=1$

Determenstic signal



Speech signal.

Random signal.

- No fixed Mathametical Expression
- probablitic measure of random variable.
- Random variable

Ex: Tossing a coin.

Head 0V

Toss -10V

→ Random signal is nothing much but mapping of sampling data.

- Mapping of Experiment on the Number line.
  - Random signal.

Random variable

3 Types

1. Continous
2. Discrete
3. Mixed.

Ex: Roll a die.

- Odd Number
  - Toss a coin & count head.
- Even Number
  - Continue.

$S.S = \begin{cases} \{1,H\} & \{1,T\} \\ \{3,H\} & \{3,T\} \\ \{5,H\} & \{5,T\} \end{cases}$

Random variable is a function

Q) How can we measure it?

- probability Density function.

  P.D.F.

- Cummilative Density function

  C.D.F

Analog & Digital Signals

- Its not continous & discrete (indepen.. var) (Time Based)
- This is value based / Amplitude.
- 3 Types.

1. Continous Time Analog signal Ex: Speech
2. Discrete Time Analog signal - Temp.
3. Discrete Time Digital signal - MP3

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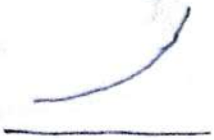
Analog signals:

Continous | Discrete

$f(t) = \sin t$ | $f(n) = \sin[n]$

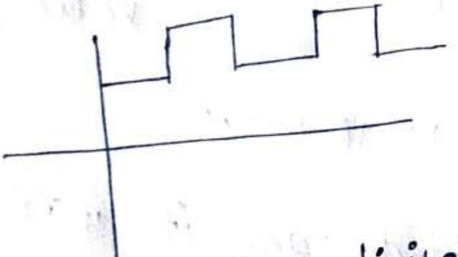


$f(t) = e^{at}$ | $f(n) = e^{an}$.

Digital signals:

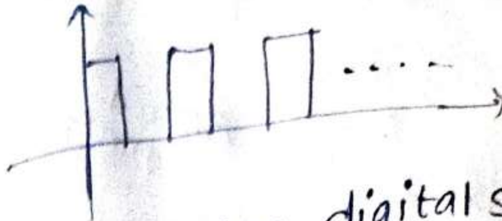
Value/Amp of function is fixed.

Ex:



Continous time digital Signal

Continous Time digital signal



discrete time digital signal

Electricity - Analog signal
Continous signal.

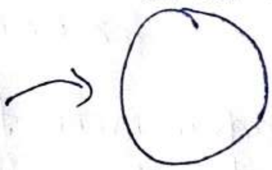
Laptop Screens - Digital signal
Discrete signal

Dish TV (DTH) - Digital Signal
Discrete signal

OTT platform - Digital signal.
Continous Signal

Image
$a = 1 \times 30$
$V = 1 \times 100$ } →

domain O.S.



$a' = 1 \times 100$
$30 + P$
70.

vedio ↓

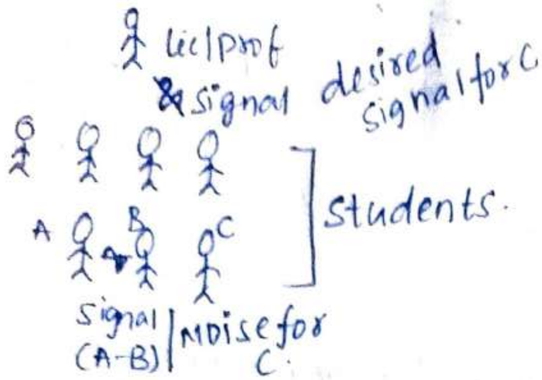
vector spaces

vedio + Image are Interpolated.

$f(an)$ $a<1$ interpolation is done

Note: why digital?
- Digital domain is useful for noice reduction.
- reconstruction of digital Signal Easy.
- Better signal Quality.

Noise:



The signal from A to B is Noise to C, the main signal of C is from prof.

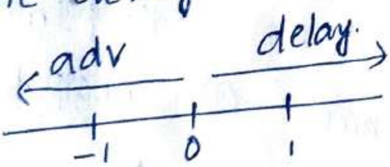
undesiarable signal at that particular ~~signal~~ time instant is noise.

Causal & Non-Causal Signals.

causal signal.

$f(t) = 0$ for $t<0$.

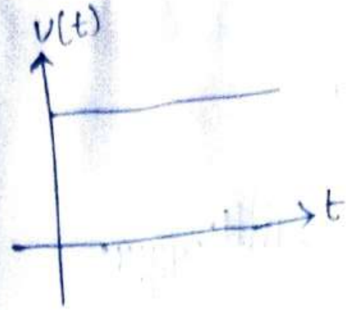
1. Time advancment $f(t+1)$
2. Time delay. $f(t-1)$



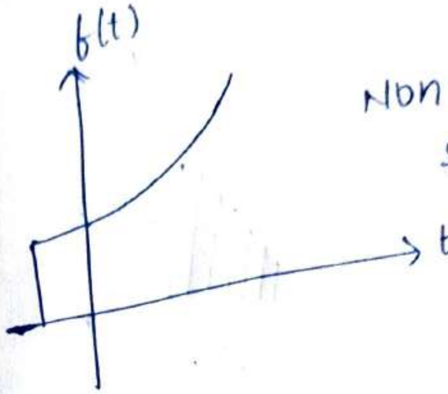
Non-Causal Signal

$f(t) \neq 0$ for $t<0$.

Signal may be in
delay for causal signals
Adv, delay for Non-causal signal



Causal Signal.



Non causal signal.

Bounded signals:

i) $|f(t)| \leq N$ N - some finite value.

ii) $f(t) \to 0$ at $t \to \infty$

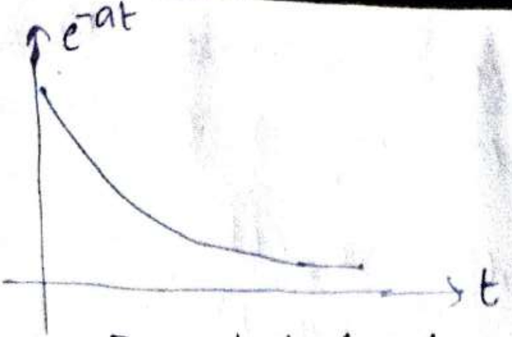
- Bounded signal. if i/ii True

Ex:

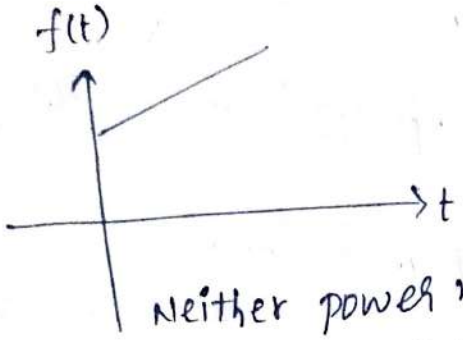
Power signal.

periodic signal.

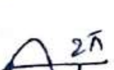
Bounded signal.



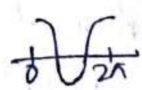
Bounded signal
Energy signal

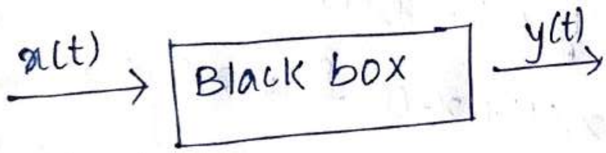


Neither power nor Energy
Non-Bounded Signal.



System





Any charesteric change in signal is caused by an Entity, then entity is called System.

System:
1. Representation of o/p wrt i/p
11. Physical Composition
111. Differntial Equation/difference
IV. Unit Impulse responses.

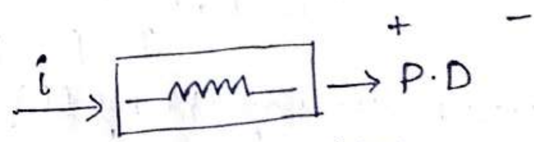
System:

def: A device, process or mathematical model that takes Input and transform into Output through some operation, establishing a relationship.
Input: cause/ Excitation
Output: Effect / responce

Linear Systems:

A System is said to linear if it follows
i. Homogenity.
11. Super position.

$y(t) \propto x(t)$



resistive System.

$PD = f(i)$

if $i \rightarrow ai$ $i' = ai$

$f(i') = PD'$
$= PD \cdot a$

Homogenity satisfies.

$f_1(i) = PD_1$ $f_1 + f_2(i) = PD_1 + PD_2$
$f_2(i) = PD_2$
$f_3(i) = PD_3$

Super Position Satisfies.
So, resistance is linear device.

3/02

To be a linear System:

1. Homogenity
2. Superposition.

Super-Position states that weighted sum would get a output of scaled cummilative sum desired.

$$y(t) = x(t)$$

$$x(t) \xrightarrow{S} y(t)$$

$$a x(t) \longrightarrow a y(t)$$

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

$$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$$

Super-Position (Additive Property)

$$\boxed{a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)}$$

a, b ∈ complex constants.

$\rightarrow \quad y(t) = t\, x(t)$

$$x(t) \longrightarrow \boxed{\text{Black box}} \longrightarrow y(t)$$

$$x_1(t) \xrightarrow{S} y_1(t) = t\, x_1(t)$$

$$a x_1(t) \rightarrow a t\, x_1(t) = a y_1(t)$$

Homogenity satisfies.

$\rightarrow$ | $\xrightarrow{S}$: passing through System

$$x_1(t) \xrightarrow{S} y_1(t) = t \cdot x_1(t)$$

$$x_2(t) \xrightarrow{S} y_2(t) = t \cdot x_2(t).$$

$$a x_1(t) \xrightarrow{S} a y_1(t)$$

$$b x_2(t) \xrightarrow{S} b y_2(t)$$

$$a x_1(t) + b x_2(t) \xrightarrow{S} a y_1(t) + b y_2(t)$$

$$a x_1(t) + b x_2(t) \xrightarrow{S} a t x_1(t) + b t x_2(t)$$

Super position Satisfies

$\rightarrow$ System is linear.

Q) $y(t) = x(t) + 3.$

Non-Linear System

$$y_1(t) = x_1(t) + 3$$

$$y_2(t) = x_2(t) + 3.$$

$$x_1(t) + x_2(t) \xrightarrow{S} x_1(t) + x_2(t) + 3$$

but $y_1(t) + y_2(t) = x_1(t) + x_2(t) + 6.$

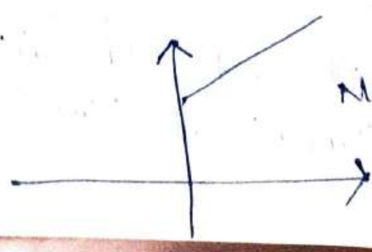
It violates Super Position.

$$x_1(t) = 5 x(t)$$

$$y_1(t) = 5 \cdot x(t) + 3$$

but $5 \cdot y_1(t) = 5 \cdot x(t) + 15$

So, It also violates homogenity also.



Not linear

Due to DC shift it is not an linear system anymore

Q) $y(t) = e^{x(t)}$

Non linear.

Homogenity

$y(t) = e^{a \cdot x(t)}$

but $a \cdot y(t) = a \cdot e^{x(t)}$

Super Position.

$y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y_1(t) + y_2(t) = e^{x_1(t)} + e^{x_2(t)}$

but $y_1(t) + y_2(t) = e^{x_1(t) + x_2(t)}$

$\vec{Q}$. $y(t) = Re\{x(t)\}$

where $x(t) = a(t) + jb(t)$

<u>sol</u> $y(t) = a(t)$

$5 \cdot x(t) = 5 \cdot a(t) + 5 \cdot j b(t)$

$5 \cdot y(t) = 5 \cdot a(t)$

$5 \cdot y(t) = 5 \cdot a(t)$

Homogenity satisfies. for real

$y_1(t) = a_1(t)$

$y_2(t) = a_2(t)$

$x_1 + x_2(t) \xrightarrow{S} a_1(t) + a_2(t)$

$= y_1 + y_2(t)$

Linear System.

$j x(t) = j a(t) - b(t)$

$j(y(t)) = j a(t)$

but

$j(x(t)) \xrightarrow{S} -b(t)$

So, Homogenity doesn't satisfy.

→ Non linear System.

<u>Assignment:</u>

Q) $y(t) = Re\{|x(t)|\}$

→ Non linear System.

Q) $y_1(t) = Re\{x^*(t)\}$

Q) $y(t) = Re\{x(t) \cdot x^*(t)\}$

Time Invariant System

A System is Time Invariant (TI) if shifting the input signal in time causes exactly the same shift in the output.

If input: $x(t) \xrightarrow{S} y(t)$

Then shifted input: $x(t-t_0)$

Should produce output: $y(t-t_0)$

If this holds → Time Invariant

if it does not hold → Time varying.

Test:

For a System: $y(t) = S[x(t)]$

step 1: Shift the Input first

Replace $x(t)$ with $x(t-t_0)$

$$y_1(t) = S[x(t-t_0)]$$

Step 2: Shift the output

Shift original output:

$$y_2(t) = y(t-t_0)$$

Step 3: Compare

If $y_1(t) = y_2(t)$ → Time Invariant

If not equal $y_1(t) \neq y_2(t)$ → Time varying.

Ex: $y(t) = x^2(t)$

S1: $y_1(t) = [x(t-t_0)]^2$

S2: $y(t) = x^2(t)$

$y_2(t) = x^2(t-t_0)$

– Time Invariant

Non Linear.

Ex: $y(t) = x(t^2)$

S1: $y_1(t) = x((t-t_0)^2)$

$= x(t^2+t_0^2-2tt_0)$

S2: $y_2(t) = x(t^2-t_0)$

So, Time varing

Linear

Ex: $y[n] = x^2[n]$

S1: $y_1[n] = (x[n-n_0])^2$

S2: $y_2[n] = x^2[n-n_0]$

Time Invariant.

Non-Linear

Ex: $y(t) = \sin(x(t))$

S1: $y_1(t) = \sin(x(t-t_0))$

S2: $y_2(t) = \sin(x(t-t_0))$

Time Invariant

Non-Linear

Ex: $y(t) = t\,x(t)$

S1: $y_1(t) = t\,x(t-t_0)$

S2: $y_2(t) = (t-t_0)\,x(t-t_0)$

Time varying.

Linear

Ex. $y[n] = x[n^2]$

S1: $y_1[n] = x[n^2-n_0]$

$y_2[n] = x[(n-n_0)^2]$

$y_1[n] \neq y_2[n]$

Time varying

Linear

Ex: $y(t) = 5x(t)$

S1: $y_1(t) = 5x(t-t_0)$

S2: $y_2(t) = 5x(t-t_0)$

Time Invariant

Linear.

Ex: $y(t) = \frac{d}{dx} x(t)$

S1: $y_1(t) = \frac{d}{dx} x(t-t_0)$

S2: $y_2(t) = \frac{d}{dx} x(t-t_0)$

Time invariant

Linear

Ex: $y[n] = n\,x[n]$

S1: $y_1[n] = n\,x[n-n_0]$

S2: $y_2[n] = (n-n_0)\,x[n-n_0]$

Time varing.

Music Player Volume Control - Time Invariant

Car Suspension - Time Invariant

Cooking Gas Stove - Time Invariant

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~~Q $y(t) = \int_{-\infty}^{t} x(t-t_0)dt$~~

~~St~~ Q. $y(t) = \int_{-\infty}^{t} x(t)\,dt$

S1: Input change

$$y_1(t) = \int_{-\infty}^{t} x(t-t_0)\,dt$$

let $t - t_0 = u$

differentiate on both sides

$dt = du$

$$y_1(t) = \int_{-\infty}^{t-t_0} x(u)\,du.$$

limits:

$t \to -\infty \quad u \to -\infty$

$t \to t \quad u \to t-t_0$

S2: Output change

$$y_2(t-t_0) = \int_{-\infty}^{t-t_0} x(t)\,dt$$

$\int_{\beta}^{\alpha} x(t)dt$ — dummy variable

real variables.

$y_1 = y_2$

Time variant System.

Assignment:

) $y(t) \xrightarrow{S} x(-t)$

. $y_1(t) = x(-t+t_0)$

$y_2(t) = x(-t-t_0)$

ime variant

inear system.

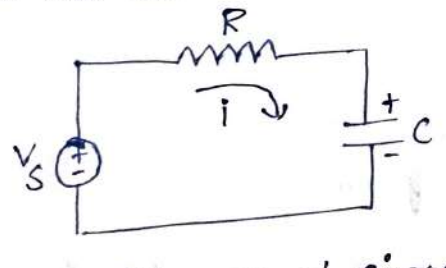
Casual & Non-Casual Systems:

A System is casual if the output at any time depends on values of the input at only the present and past times.

- The systems which output at any time depends on values of the input at only the present, past and future times are Non-Casual times.

- The Systems which output at any time depends on values of the input at only the Future times are Anti-Casual times.

Note:
If two inputs to a casual system are identical up to some point in time t_0 or n_0, the corresponding outputs must also be equal up to this same time.



The RC circuit is casual, since the capacitor voltage responds only to the present and past values of the Source voltage.

Note:
1. All casual systems are memory emmbedded Systems.
2. All memoryless Systems are casual

historical stock market analysis - casual signals.

$$y[n] = \frac{1}{2M+1} \sum_{k=-M}^{+M} x[n-k]$$ - Non-Casual Signals.

Ex: $y(t) = x(-t)$

at $t=0$, $y(0) = x(-0)$ present

at $t=2$, $y(2) = x(-2)$ past

at $t=-2$ $y(-2) = x(2)$ Future.

Non Casual Systems.

Assignment

Ex: $y[n] = x[-n]$

$y[-n] = x[n]$ future

$y[+n] = x[-n]$ past

$y[0] = x[0]$ present

Non casual systems.

Ex: $y(t) = x(t^2)$

at $t=0$, $y(0) = x(0)$ present

at $t=-1$ $y(-1) = x(1)$ Future.

Non Casual Systems.

Ex: $y[\sqrt{n}] = x[\sqrt{n}]$

for all $n \geq 1$: $\sqrt{n} \leq n$ past/present

Casual system.

Ex: $y[n] = x[\frac{-n}{3}]$

$n=3$ $\frac{-3}{3} = -1$ past input

$n=-3$ $-\frac{-3}{3} = 1$ Future input

- Laptop Screens

Screen shows frames as they arrive.

It does NOT wait for future frame.

Sometimes previous frame (buffering / smoothing)

Casual Systems.

- Netfilx / Youtube - Non Casual

Live streaming - Casual

Editing Software - Non-Casual

AI Frame Generation - Non Casual.

Static Memory (present I/P)

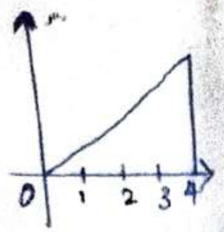
- Casual Systems.

Dynamic Memory

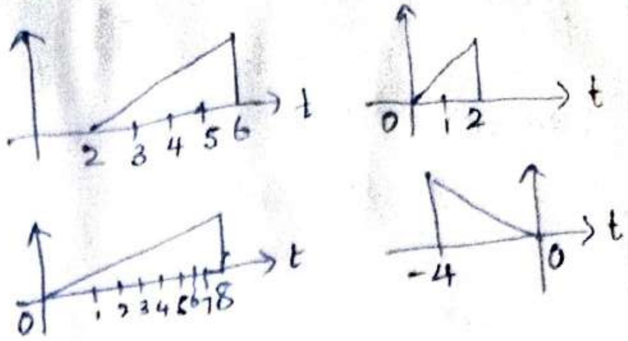
(present + past I/p)

- casual Systems.

1) Sketch



1) $x(t-2)$
2) $x(2t)$
3) $x(t/2)$
4) $x(-t)$



2) Find the period of signal:

a. $x(t) = \cos(t + \frac{\pi}{4})$

b. $x(t) = \sin \frac{2\pi}{3} t$

c. $x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$

d. $x(t) = \cos t + \sin \sqrt{2} t$

a. 2π

b. 3

c. $\frac{\frac{\pi}{3}}{\frac{\pi}{4}} = \frac{4}{3} \quad \frac{p}{q}$ 24

d. we cannot find period. aperiodic - ∞

3) Find Energy signal|power|Neither

i) $x(t) = e^{-at} \cdot u(t)$

ii) $x(t) = A\cos(\omega_0 t + \theta)$

iii) $x(t) = t \cdot u(t)$

i) Energy $E = \frac{1}{2a}$ $P = 0$

ii) power $E = \infty$ $P = \frac{A^2}{2}$

iii) Neither $E = \infty$ $P = \infty$

4) Show that

$\delta(t) = \begin{cases} \infty & \text{at } t = 0 \\ 0 & \text{elsewhere} \end{cases}$

a) $t\,\delta(t) = 0$.

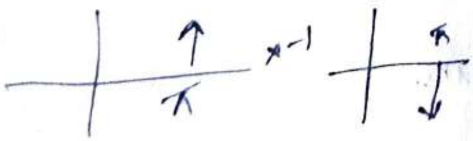
b) $\sin t\, \delta(t) = 0$.

c) $\cos t \; \delta(t-\pi) = -\delta(t-\pi)$

at 0 $t \cdot \delta(t) = 0$.

$t \cdot \sin t = 0$.

$\delta(t-\pi) = \begin{cases} \infty & \text{at } \pi \\ 0 & \text{elsewhere} \end{cases}$



at π $\cos \pi = -1$

Q) $x_1(t) = je^{j10t}$

$= -\sin 10t + j\cos 10t$

$\frac{2\pi}{10} = \frac{\pi}{5}$ $\frac{2\pi}{10}, \frac{2\pi}{10}$

Q) $\int_{-1}^{1} \delta(2t-1)\,dt = 1/2$

→ periodic signals are power signal.

Q) $\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt = 1$

Area under curve

Area = 1

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Static + Dynamic.

Static - Systems that depends upon present value.
Dynamic - Systems that depends upon present + past values

Static Systems - Its neither causal nor Anticausal nor Non casual.

$y(t) = A \cdot x(t)$. A = Amplification value.

Ex: Amplifier.

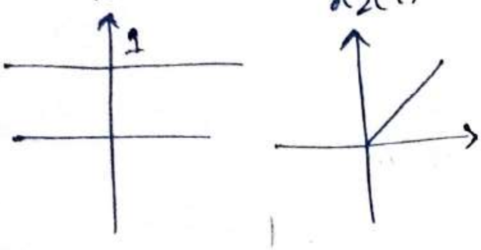
- A memoryless System.

Feedback is always dynamic in nature.

Stable & unstable Systems.

- A System which produces bounded signal as output irrespective of input (bounded) ~~unbounded input)~~ is called as stable system.

$x_1(t)$ $x_2(t)$



Stable System

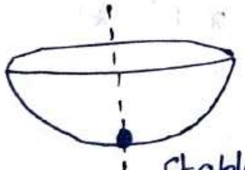
$x_1(t) \longrightarrow \boxed{\text{System}} \longrightarrow y_1(t)$

$y_1(t) \leq |M|$

~~Fig: Stable System~~

def: A stable system is a system that produces a bounded output for given bounded input maintining control, predictability and returning to equilibrivm after distrabnance.

Ex:



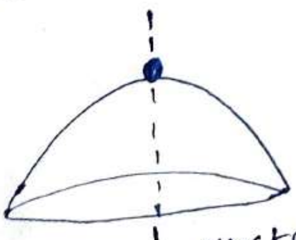
Stable System

$x_1(t) \longrightarrow \boxed{\text{System}} \longrightarrow y_1(t) (\leq |M|)$

Stable System

def: A unstable system is a system that produces a unbounded output for given bounded input after distrubance.

Ex:



unstable System

$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) (\nleq |M|)$

unstable System.

- For BIBO stability, The input should be absolutely integrable.

$x(t) = -2t$

$S = \frac{d}{dt}(x(t))$

- stable but not BIBO stable

$y(t) = x(t^2)$, $x(t) = M$

BIBO stable

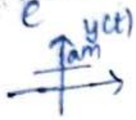
$x(t) = M$

$y(t) = x(t - t_0)$

BIBO stable.

$x(t) = M$

$y(t) = e^{a x(t)}$



BIBO Stable.

Assignment

$y(t) = e^{x(t)}$, $x(t) = u(t)$

Comment nature of System

1. Linear or non Linear — Linear
2. Time variance or invariance — Time varying
3. Causal ~~Or~~ Non Causal. — casual
4. stable ~~Or~~ unstable — unstable

Q. $y(t) = \int_{-\infty}^{t} x(t)\,dt$

$x(t) = u(t)$ - ~~Bo~~ unstable.

Q. $y(t) = \int_{-\infty}^{\infty} x(t)\,dt$, $x(t) = \delta(t)$

BIBO stable.

10/02/2026

1 Neccessary

2 Sufficient condition

BIBO 1 Absolute integrable for input.

2 Stability

Investible & Non-Investible System.

def: A investible System is one that produces a unique output for every input, allowing the original input signal to be uniquely recovered from the output Signal

$$\xrightarrow{x(t)} \boxed{BB} \xrightarrow{y(t)} \boxed{\frac{1}{BB}} \xrightarrow{x(t)}$$

System (Investible System) — Inverse System

- Inverse of System exist for every Investible System.
- One-to-one mapping: There must be a one to one mapping between the input & output Signals.

$$y(t) = T[x(t)]$$

$$\Rightarrow T^{-1}[y(t)] = x(t).$$

Ex:

$y(t) = 2x(t)$ ✓

$y(t) = x(t-3)$ ✓

$y[n] = x[-n]$ ✓

Non-Investiable has: many to one mapping

$y(t) = x^2(t)$ ✗ +x & -x

$y(t) = \cos(x(t))$ ✗

$y(t) = \sin(x(t))$ ✗

$y(t) = 0$ ✗

Non-Invertible System.

S = Compressor

$y(t) = x(4t)$

S^{-1} = Expandaer

$x(t) = y\left(\frac{t}{4}\right)$

Invertible.

Q

$$x(t) \longrightarrow \boxed{d/dt} \xrightarrow{y(t)} \boxed{\int dt} \xrightarrow{z(t)}$$

Invertible.

Ex: $y(n) = \sum_{K=-\infty}^{n} x(K)$

$= \sum_{K=-\infty}^{n-1} x(K) + x(n)$

$= y(n-1) + x(n)$

$x(n) = y(n) - y(n-1)$

Invertible

Assignment

1) $y(t) = e^{a\,Re\{x(t)\}}$

2) $y(n) = \begin{cases} x(n) & \text{for } n<0. \\ 0 & \text{for } n=0. \\ x(n+1) & \text{for } n\geq 0 \end{cases}$

Comment about

	1st	2nd
i) Linearity	Nonlinear	Linear
ii) Time variance	Time variant	Time variant
iii) Static/ Dynamic	Static	Dynamic
iv) Stability	Stable	Stable
v) Invertability.	Not Invertiable	Not Invertiuble
vi) Casuality.	casual.	Non-Casual.

LTI

Linear time Invariant System.

$$x(n) \longrightarrow \overset{h(n)}{\boxed{BB}} \longrightarrow y(n)$$

$y(n) = S[x(n) \,\&\, h(n)]$

$y(n)$ is a function of $x(n)$ & $h(n)$

Let say $h(n)$ is unit impulse

let us say $\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{else.} \end{cases}$

$f(n) = x(n) \times \delta(n)$

$x = \{\underset{\uparrow}{2}, 4\}$

$f(n) = 2\delta(n) + 4\delta(n-1)$

Mathametical representation

$x(n) = \{2, -4\}$

$f(n) = 2\delta(n) - 4\delta(n-1)$

In general.

$$f(n) = \sum_{K=-\infty}^{\infty} f(K)\,\delta(n-K)$$

$y(n) = \{1, 3\}$

$x(n) = \delta(n)$

$\delta(n) \xrightarrow[S]{} \{1, 3\}$

$x(n) \xrightarrow[S]{} y(n)$

$h(n)$ is impulse response

$f(n) = \{2, 4\}$

$= 2\delta(n) + 4\delta(n-1)$

$x(n) = \delta(n)$

$y(n) = \{1, 3\}$

$y(n) = \delta(n) + 3\delta(n-1) = h(n)$

$f(n) = \{2, 4\}$

$x(n) = \delta(n) \rightarrow \boxed{\text{LTI system}} \rightarrow y(n) = h(n)$

$2\delta(n) \rightarrow 2h(n)$

$4\delta(n-1) \rightarrow 4h(n-1)$

→ Neat | shorted version

$x(n) = \delta(n) \rightarrow \boxed{h(n)} \rightarrow y_1(n) = \{1, 3\}$

$y_1(n) = h(n) = \delta(n) + 3\delta(n-1)$

because $x(n) =$ unit impulse

$x(n) = f(n) \rightarrow \boxed{h(n)} \rightarrow y_2(n)$

where

$f(n) = \{2, 4\}$

$y_2(n) = \{2, 10, 12\}$

$y_2(n) = 2 \cdot h(n) + 4h(n-1)$

$= 2(\delta(n) + 3\delta(n-1))$

$+ 4(\delta(n) + 3\delta(n-1))$

$= 2\delta(n) + 10\delta(n-1) +$

$12\delta(n-2)$

$x(n) = f(n) \rightarrow \boxed{h(n)} \rightarrow$

$f(n) =$

$2h(n) + 4h(n-1) = f(n) \mid x(n)$

$2h(n) + 4h(n-1) + 12\delta(n-2) = y(n)$

$2\delta(n) + 6\delta(n-1) + 4\delta(n-1) + 12\delta(n-2)$

$y(n) = 2\delta(n) + 10\delta(n-1) + 12\delta(n-2)$

$\delta(t) \rightarrow \underset{S.}{\boxed{\dfrac{d}{dx}}} \rightarrow$ Doublet signal

unbounded signal

$\delta(t)$: unstable system.

Marginally stable.

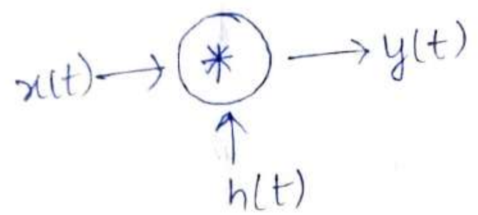
$u(t) / r(t)$: stable system

Convulution:

Opeoation that computes the output $y(t)$ of LTI system by blending an input signal $x(t)$ with the signals systems impulse response $h(t)$.

$$y(n) = \sum_{-\infty}^{\infty} x(t) * h(n-t)$$

$$y(n) = x(n) * h(n)$$



$$y(n) = \sum_{K=-\infty}^{\infty} x[k]\, h[n-k]$$

if system has unit impulse it rather $\delta(t) / \delta(n)$.

- convolution is commutative, assouiative, distoibutive.

- continous domains:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

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Convolution :-

Convolution operator $\circledast$

Maths: Multiplication of 2 variables.

Interaction of $h(n)$

Ex: $h(n) = \delta(n)$

$I(n) = \{1, 2, 3, 4\}$

$y(n) = x(n) \circledast h(n)$.

LTI System

$h(t) = \delta(t)$ $h(n) = \delta(n)$

$y(t)$ or $y(n)$ are unit Impulse response.

dis: $y(n) = x(n) \circledast h(n)$

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) \cdot \underline{h(n-K)}$$

reversal

→ shifting.

$y(t) = x(t) \circledast h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Ex: $h(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$.

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

$y(n) = ?$

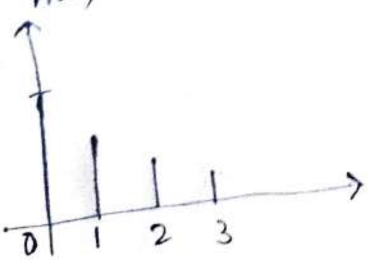
Sol: $y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K)$

$h(n) = 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$

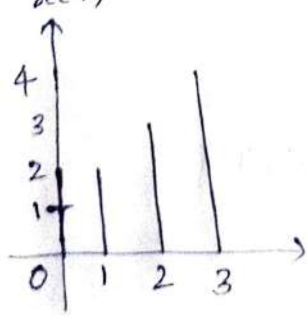
$h(n) = 4\delta(n) + 3\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$

$x(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2) + 4\delta(n-3)$

h(n)

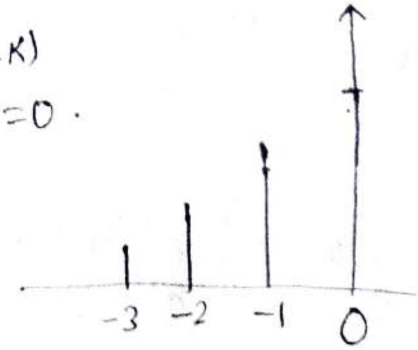


x(n)

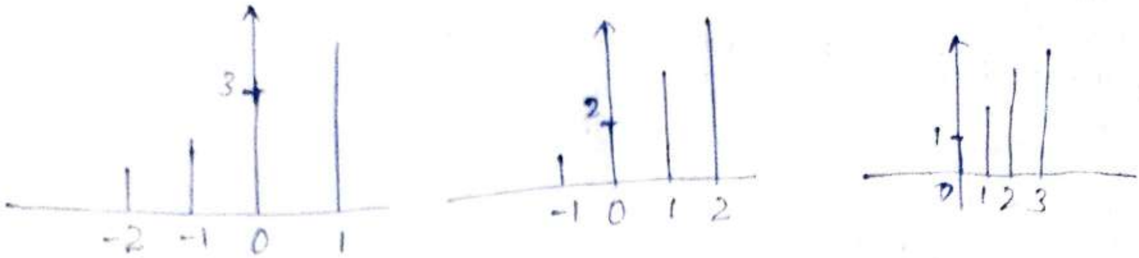


h(-K)

n = 0.



$n=1,\ h(1-k)$ $\qquad\qquad$ $n=2,\ h(2-k)$



$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\,h(-k)$$

$$= x(0)\cdot h(0) + \cancel{x(1)h(-1) + x(2)h(-2)} + x(3)h(-3)$$

$$= 1\cdot 4 + \cancel{2\cdot 3 + 3\cdot 2 + 4\cdot 1}$$

Multiply only overlapping ones

$$= 4$$

$$y(1) = x(0)h(1) + x(1)h(0)$$
$$= 1\cdot 3 + 2\cdot 4$$
$$= 11$$

$$y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$$
$$= 2 + 6 + 12$$
$$= 20$$

$$y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$$
$$= 1 + 4 + 9 + 16$$
$$= 30$$

$y(4) = 20$

$y(5) = 11$

$y(6) = 4$

$$y(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$$

Ex: $\delta(n-n_0)$

$f'(n) = f(n) \circledast \delta(n-n_0)$

$= f(n-n_0)$

Ex: $\delta(n-n_0)$

$f(n-n_1)$

$f'(n) = f(n-n_1) \circledast \delta(n-n_0)$

$= f(n-n_0-n_1)$

$$f(t) = \int_{\tau=-\infty}^{\infty} f(\tau)\,\delta(t-\tau)\,d\tau$$

~~$\delta(n-n_0)\, f(t)$~~

~~$f'(n) = f($~~

$\delta(t-t_0)\, f(t)$

$f'(t) = f(t-t_0)$

$\delta(t-t_0)\, f(t-t_1)$

$f'(t) = \delta(t-t_0) * f(t-t_1)$

$= f(t-t_0-t_1)$